Quantum Time crystals

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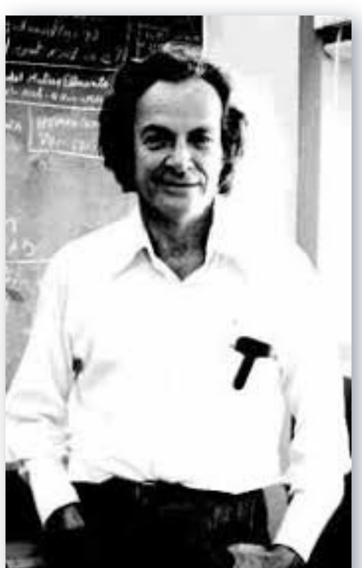
Quantum Simulators

“What & Why”

Many-body systems … “hard to understand”:

Frustrated quantum magnets
High-Tc superconductors
…

Quantum simulator: an experimental *controllable* system that reproduces the physics of a given model Hamiltonian



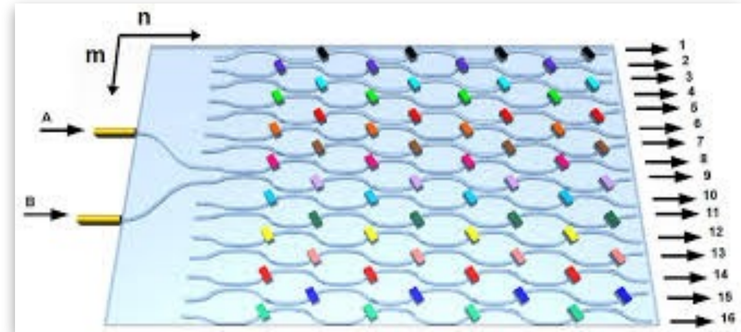
Questions not accessible via classical computation because the exp-large dimension of the Hilbert space (e.g. complex quantum ground states or dynamics in solid state systems)

Questions that are not directly “tractable/identified” in “nature” (e.g. thermalisation, defect formation, …)

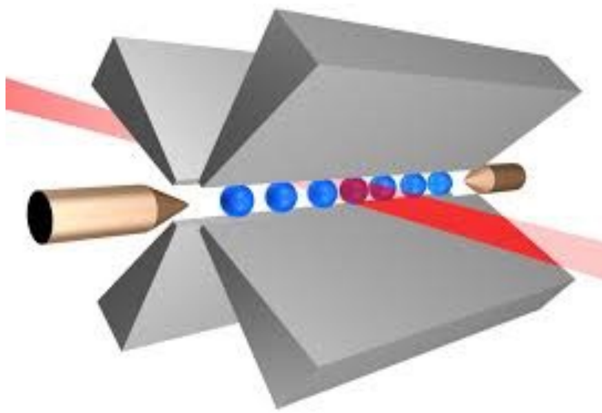
Refine our control on many-body systems

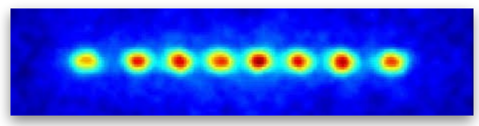
New states of matter (e.g. time-crystals,exotic qp,…)

Exp platforms

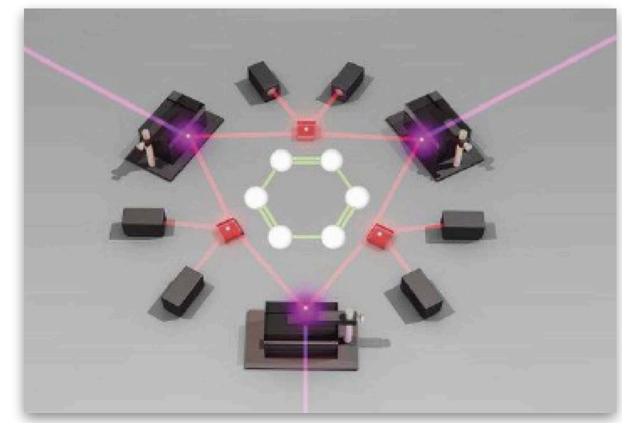


Photonic quantum simulators

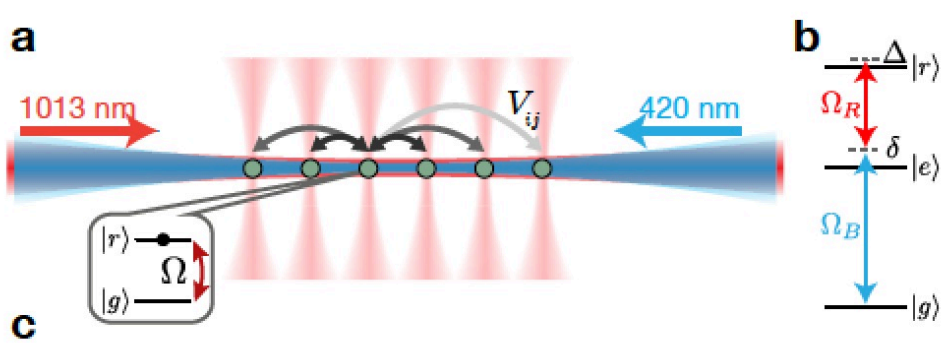


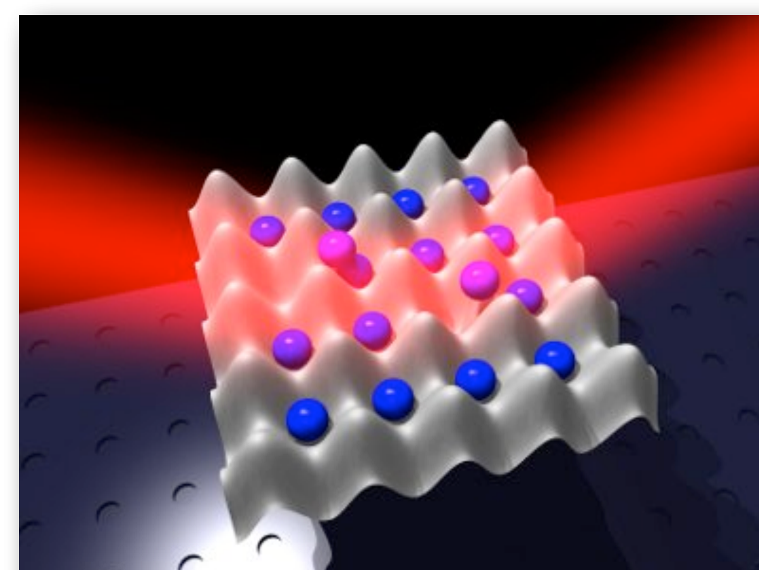


Trapped ions



Rydberg atoms



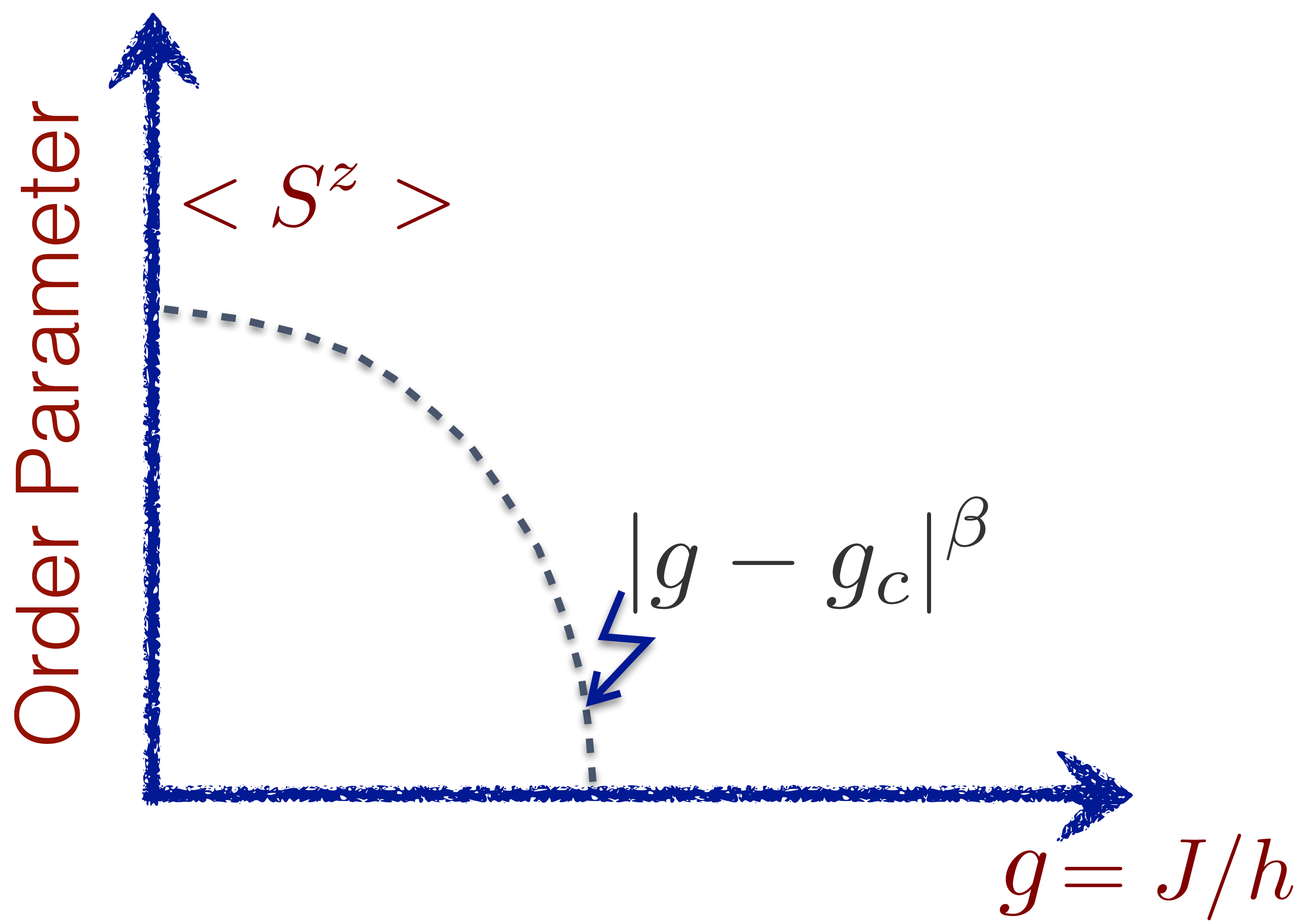


Optical lattices

}

Spontaneous symmetry breaking *[Example]*

$$H(h) = -\,J \sum_{<ij>}^{N} S_i^z\, S_j^z - 2\,h \sum_{i}^{N} S_i^x$$



Can time-translational invariance be spontaneously broken?

Time-crystal

F. Wilczek, Physical Review Letters **109**, 160401 (2012)

- Do laws of nature allow for the existence of a time-crystalline phase? ✓✓
- …if yes, how to define/characterise a time crystal? ✓
- …where to look for it? ✓
- How much do we know of its relations to other phenomena?
- Is it “useful”?

Outline

- **Introduction time crystals**
  - Problem & definition (Wilczek 2013)
  - No-go theorem (Watanabe & Oshikawa 2015)
  - Floquet time-crystals (Else *et al* & Khemani *et al* 2013)
  - First exps on FTC (Choi *et al* & Zhang *et al* 2016)
- **Floquet time crystals in clock models**
  - Conditions for the existence
  - Direct transitions between different crystals
- **Floquet time crystals in clean systems**
  - Infinite range systems
- **Continuous (Boundary) time crystals**
  - Connections to many-body open quantum systems
- **Time crystals & Synchronization**

In collaboration with

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A. Russomanno, F. Iemini, M. Dalmonte, R.F., Phys. Rev. B **95**, 214307 (2017)
F. Iemini, A. Russomanno, J. Keeling, M. Schirò, M. Dalmonte, and R. F., Phys. Rev. Lett. **121**, 035301 (2018)
O. Scarlatella, R. F., and M. Schirò, Phys. Rev. B **121**, 064511, (2019)
F. M. Surace, A. Russomanno, M. Dalmonte, A. Silva, R. F., and F. Iemini, Phys. Rev. B **99**, 104303 (2019)
R. Khasseh, R. Fazio, S. Ruffo, A. Russomanno, Phys. Rev. Lett. **123**, 184301 (2019)
A. Russomanno, S. Notarnicola,F.M. Surace, R.F., M. Dalmonte, and Markus Heyl Phys. Rev. Research. **2,** 012003(R) (2020)

Definition of a time crystal

Time-independent Hamiltonian $\mathcal{H}$

$$< O >= g(t)$$

Bad definition:

Even Rabi oscillations would fit into the category of time-crystals

Definition of a time crystal

Definition TTSB: $\phi(\vec{x}, t)$ local order parameter

$$\lim_{V \to \infty} \langle \phi(\vec{x}, t)\phi(\vec{x'}, t')\rangle \xrightarrow[|\vec{x} - \vec{x'}| \to \infty]{} f(t - t')$$

No-go theorem:(*) systems in the ground state or in thermal equilibrium cannot manifest any time-crystalline behaviour

Watanabe & Oshikawa 2015

(*) with sufficiently short-interactions

Floquet time crystals

(TTSB in periodically driven systems)

Theory
D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).
V. Khemani, A. Lazarides, R. Moessner, and S. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

Experiments
J. Zhang *et al*, Nature **543**, 217 (2017)
S. Choi *et al*, Nature **543**, 221 (2017).

$$\mathcal{H}(t+T) = \mathcal{H}(t)$$

Hamiltonian **Period T** → **Spontaneous breaking** → Observables **Period nT**

$$f(t) = \lim_{N\to\infty} \langle \psi | \hat{O}(t) | \psi \rangle$$

$$f(t+\tau_B) = f(t) \qquad \tau_B = nT$$

Floquet time crystals

T

Interactions + disorder	Flip pulse	Interactions + disorder	Flip pulse	Interactions + disorder
$H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z$	$\prod_i \sigma_i^x$	$H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z$	$\prod_i \sigma_i^x$	$H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z$
↑ ↑ ↓ ↑ ↓		↓ ↓ ↑ ↓ ↑		↑ ↑ ↓ ↑ ↓

$$\hat{U} = \exp\left[-i\pi \sum_i^N \hat{\sigma}_i^x\right] \exp\left[-i\hat{H}(\hat{\sigma}_i^z)\right]$$

$$m_z = \frac{1}{N} \sum_i \langle \sigma_i^z(t) \rangle$$

will oscillate with a double period

The periodic drive, under generic conditions will heat the system up to infinite temperatures

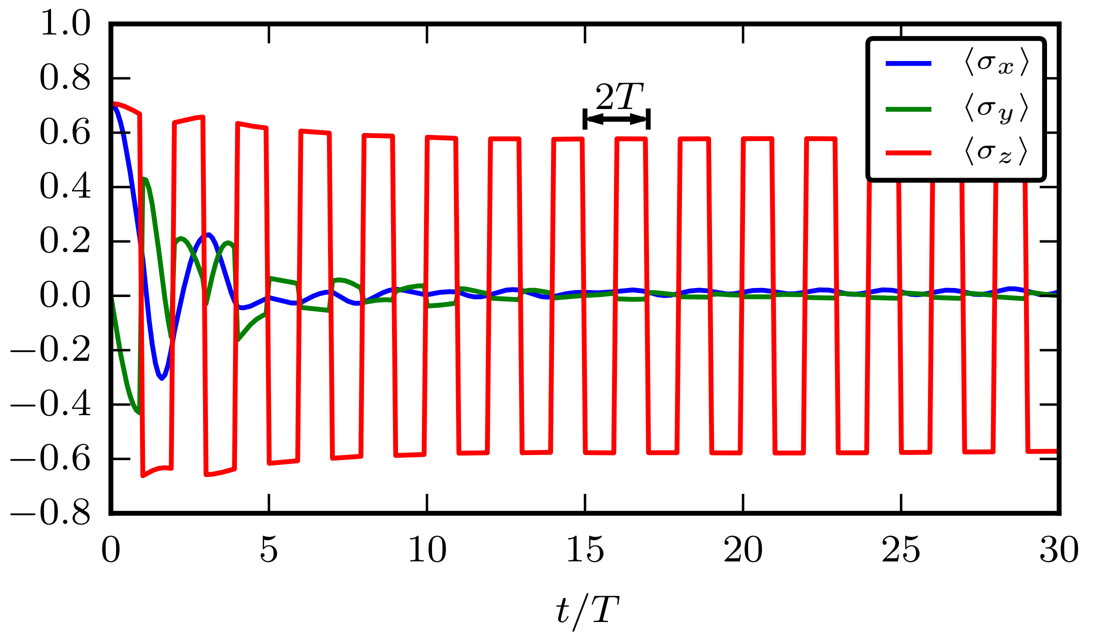
Rigidity: no fine-tuned Hamiltonian parameters.

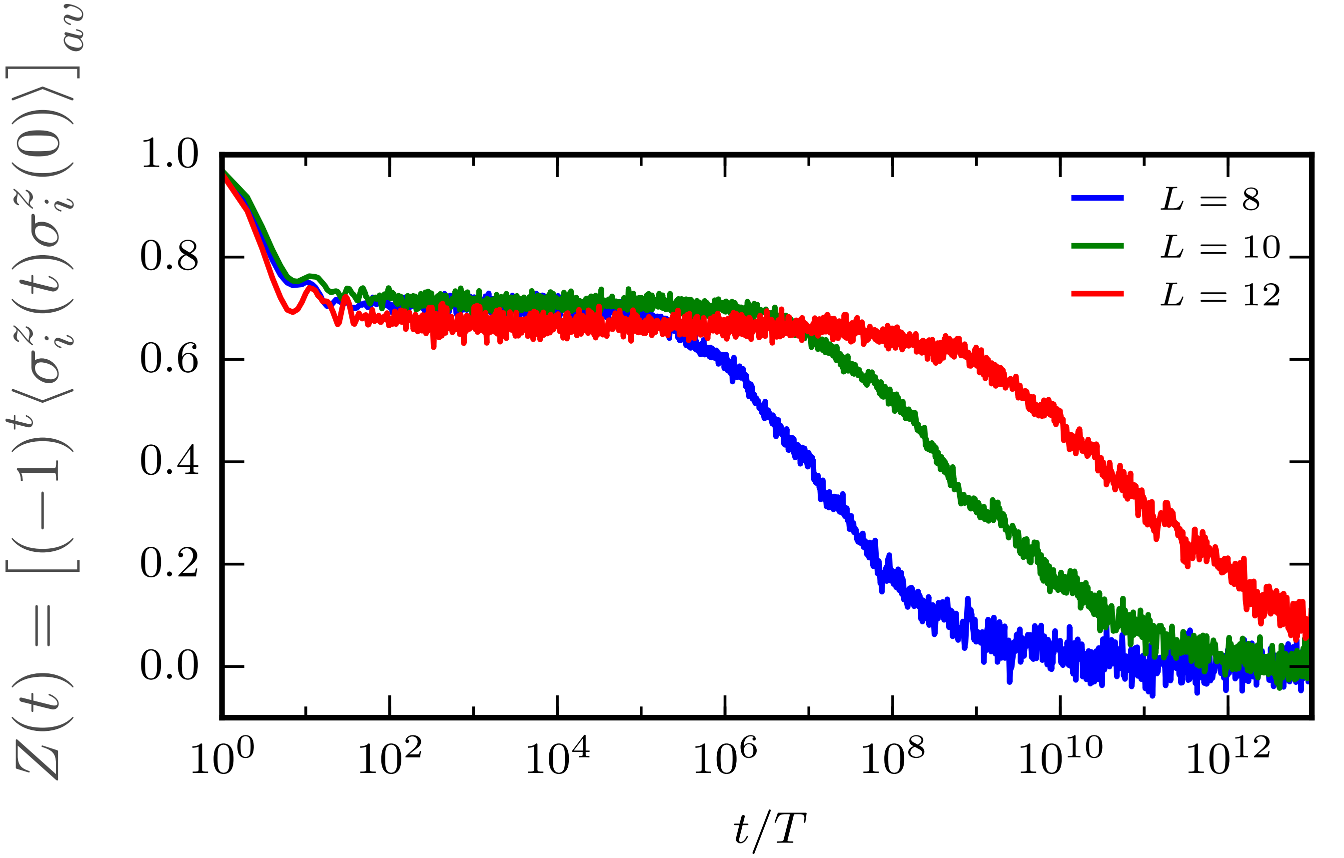
Persistence: the non-trivial oscillation must persist for infinitely long time when first taking the thermodynamic limit.

Floquet time crystals

Disordered Ising model in an external field

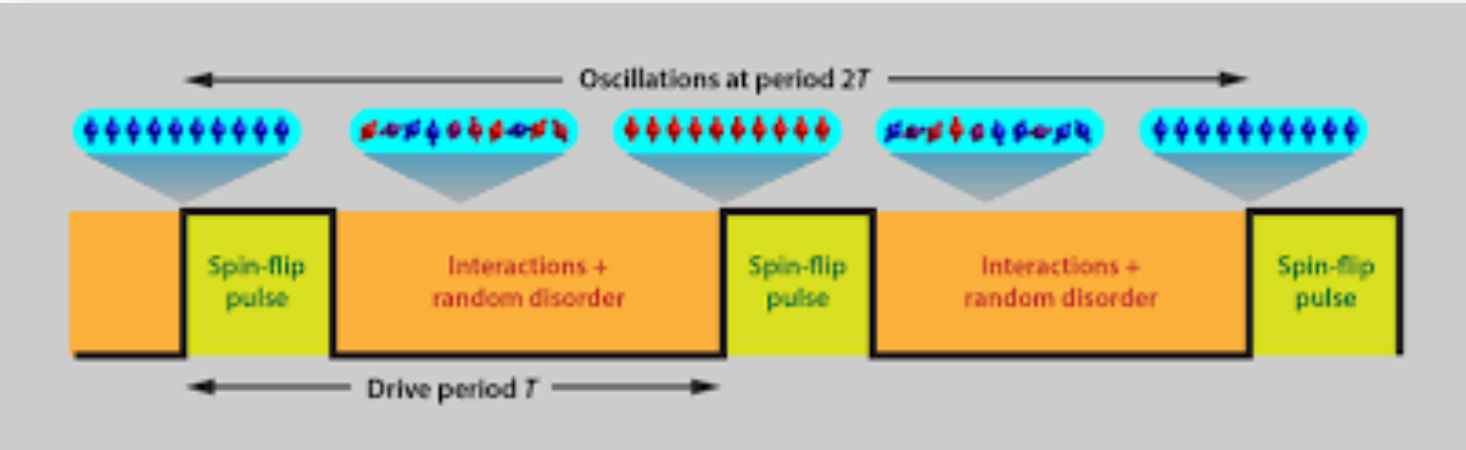
Else *et al* 2016

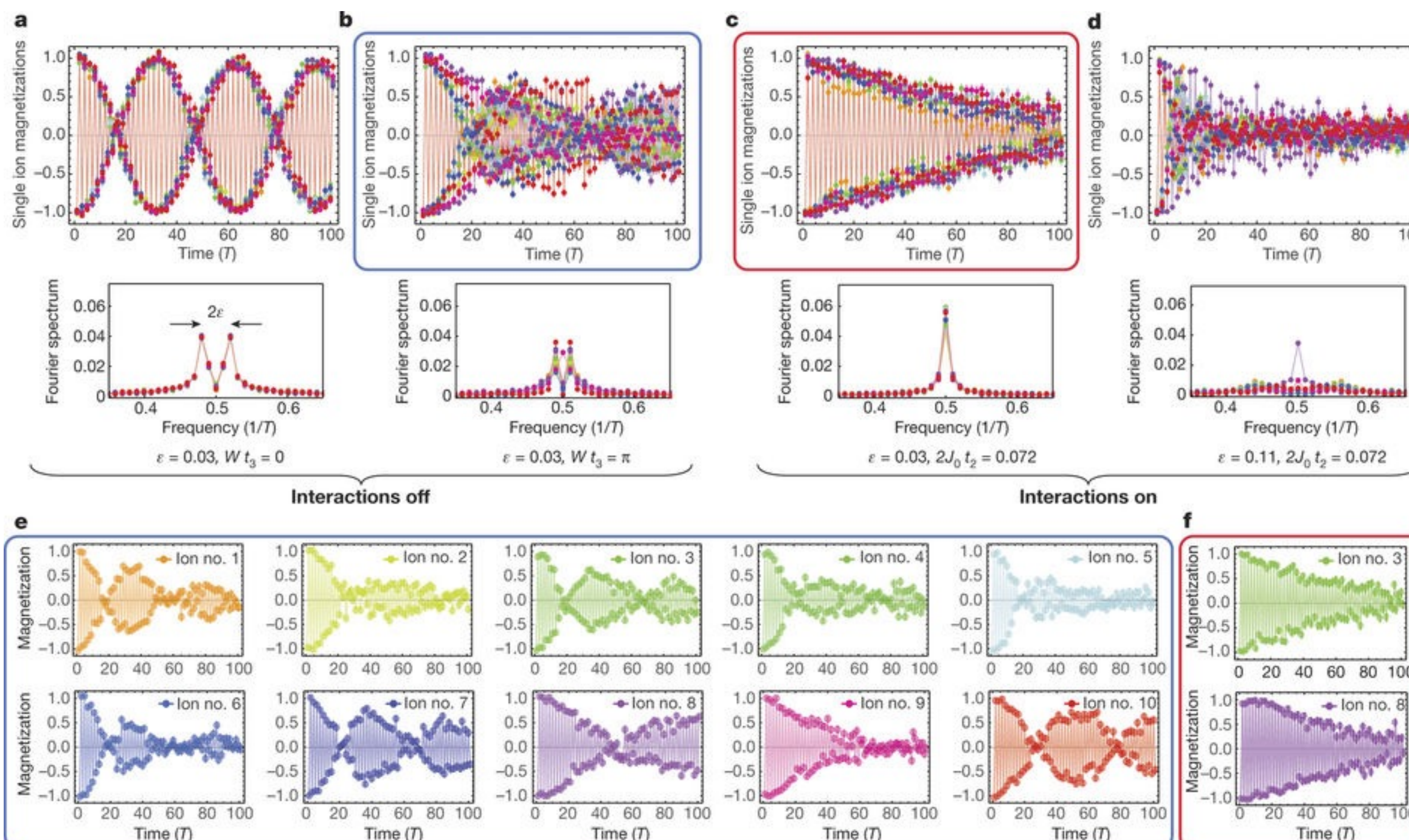




Time-crystals

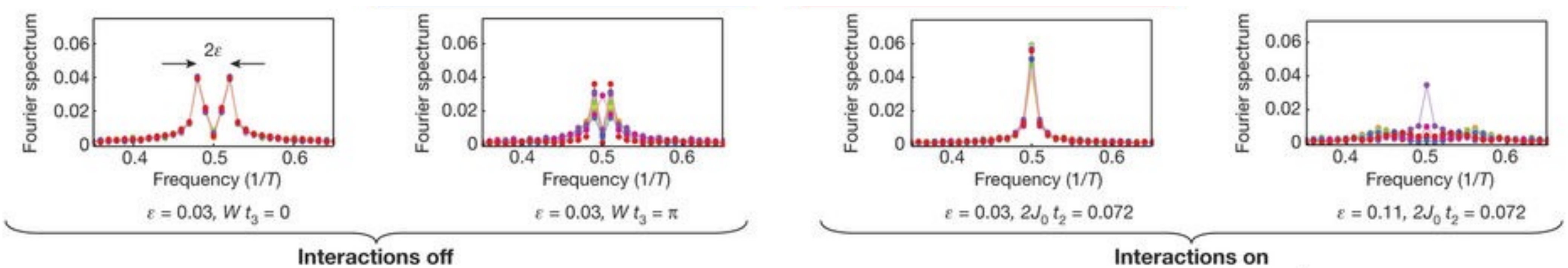
Experimental realisation of Floquet time-crystals
Choi *et al* 2016
Zhang *et al* 2016



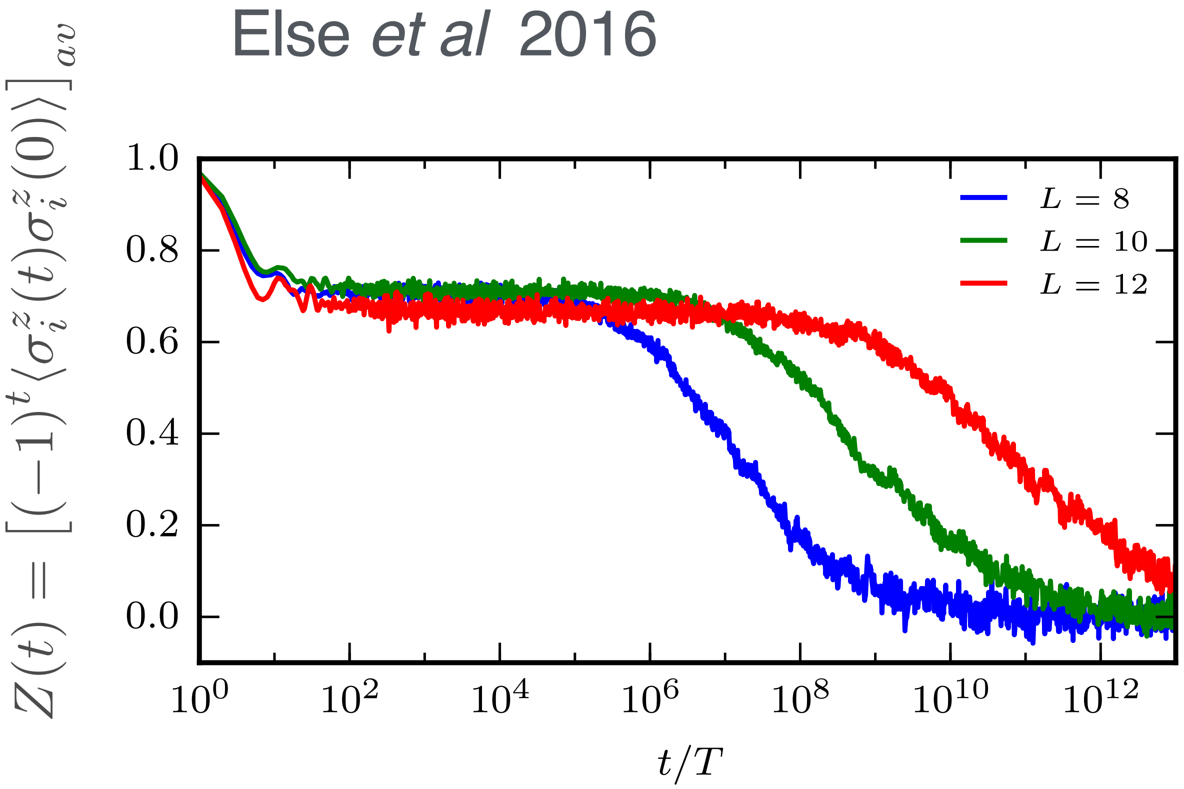


Floquet time crystals

Choi *et al* 2016
Zhang *et al* 2016



Else *et al* 2016



Interactions + disorder → **Stability**

More recent experiments

Rovny *et al* 2018
Pal *et al* 2016

- **Is it possible to have a Floquet time-crystal in the absence of disorder?**
- **How to observe direct transitions between different crystalline phases**

Floquet time-crystals in infinite-range models

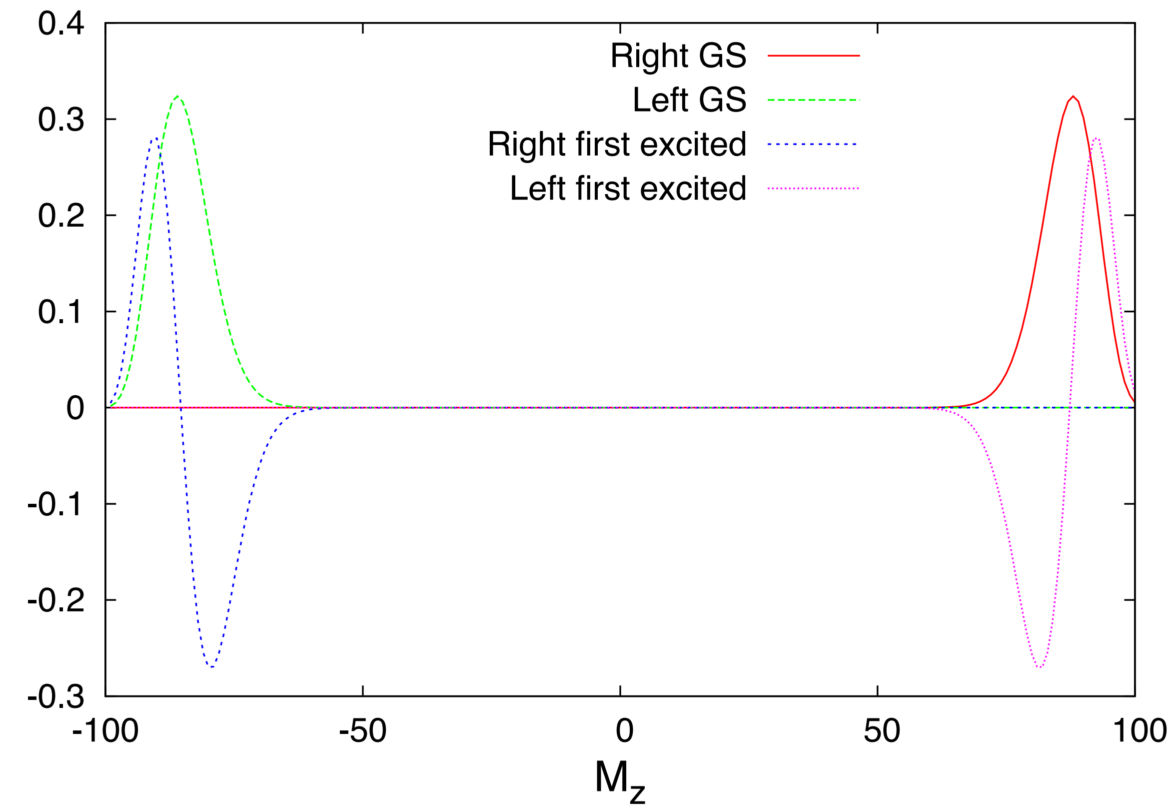
Floquet time crystals in the LMG models

$$\mathcal{H}(h_0) = -\frac{2J}{N}\sum_{i,j}^{N}\hat{\sigma}_i^z\hat{\sigma}_j^z - 2h_0\sum_i^N\hat{\sigma}_i^x$$

When $h_0 < J$ there is Z_2 symmetry breaking that involves a finite fraction of all the spectrum.

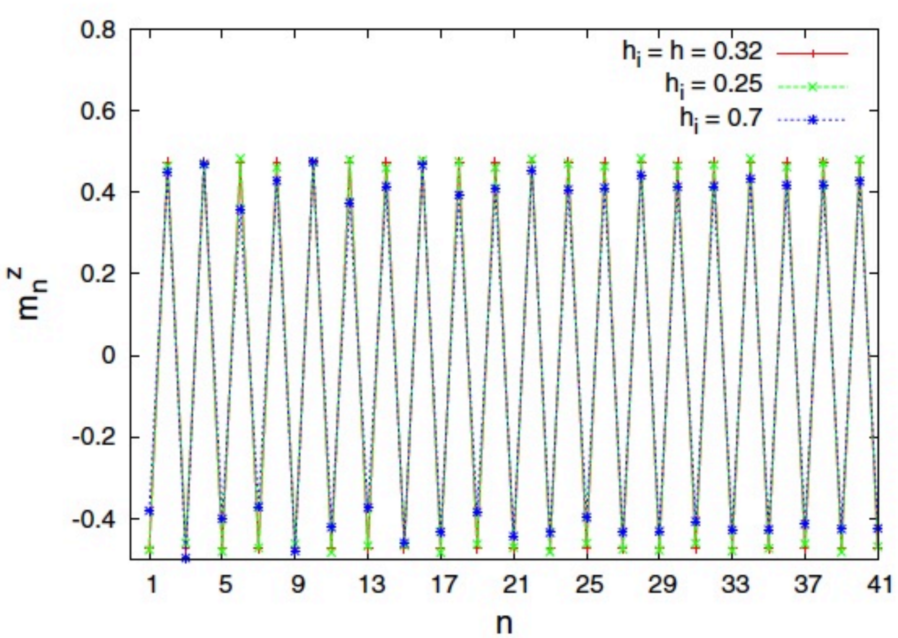
In the thermodynamic limit below the broken symmetry edge E* the corresponding energy eigenstates appear in degenerate doublets. Each member of the pair is localised in the basis of the eigenstates M_z.

For finite sizes, the eigenstates are the even and odd superpositions of each doublet (with a spitting exponentially small in N).



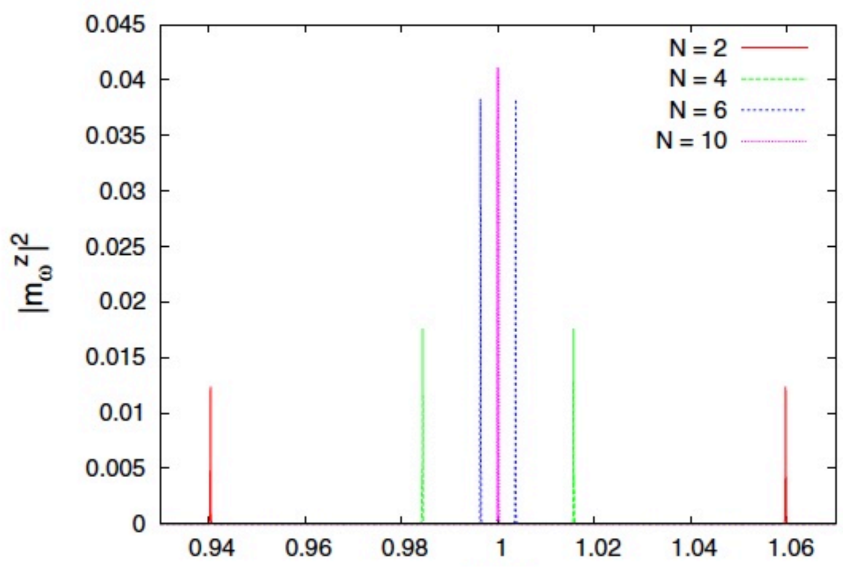
Floquet time-crystals in the LMG model

$$\widehat{U} = \widehat{U}_{\text{kick}} \exp\left[-i\widehat{H}(h)\tau\right] \text{ with } \widehat{U}_{\text{kick}} \equiv \exp\left[-i\phi\sum_i^N \widehat{s}_i^x\right],$$

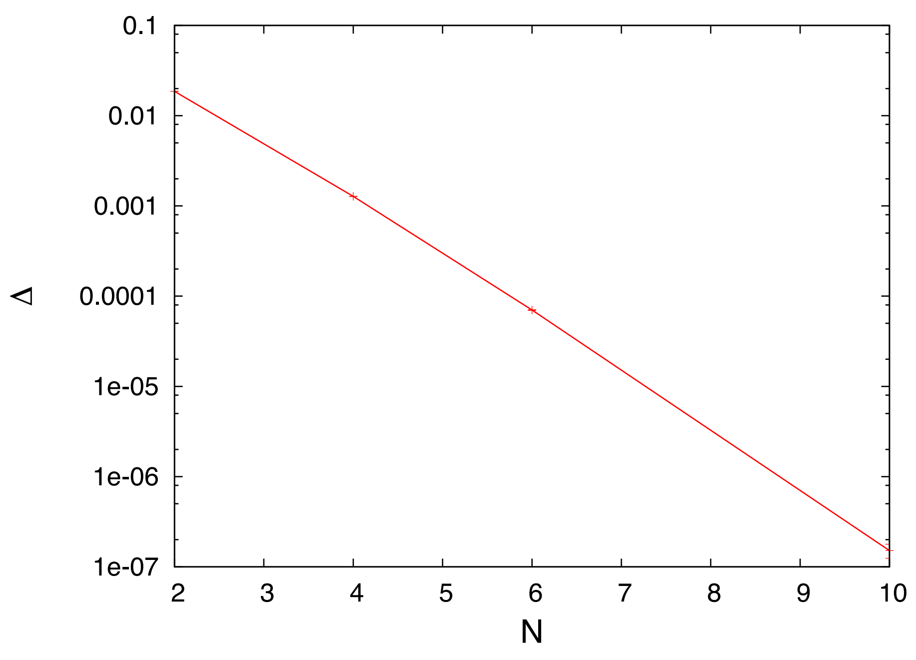


$$m_z = \frac{1}{N}\sum_i \langle \sigma_i^z(t)\rangle$$

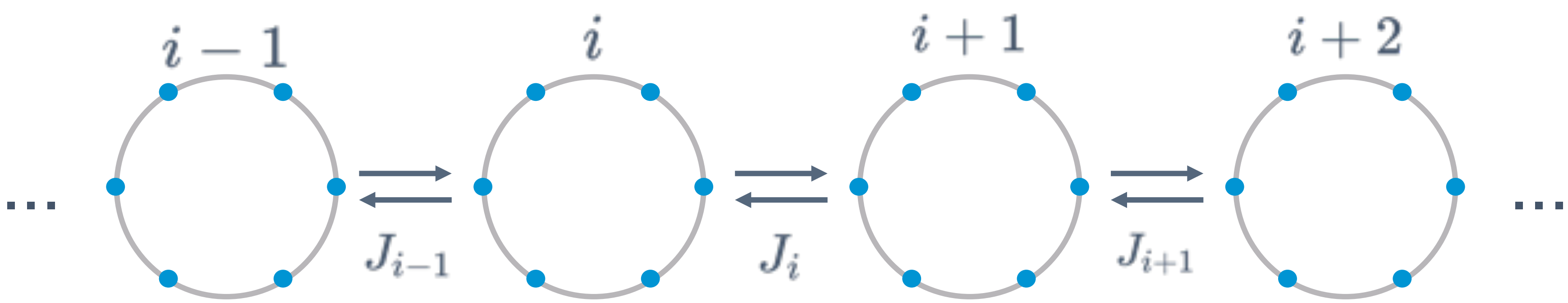
will oscillate with a double period

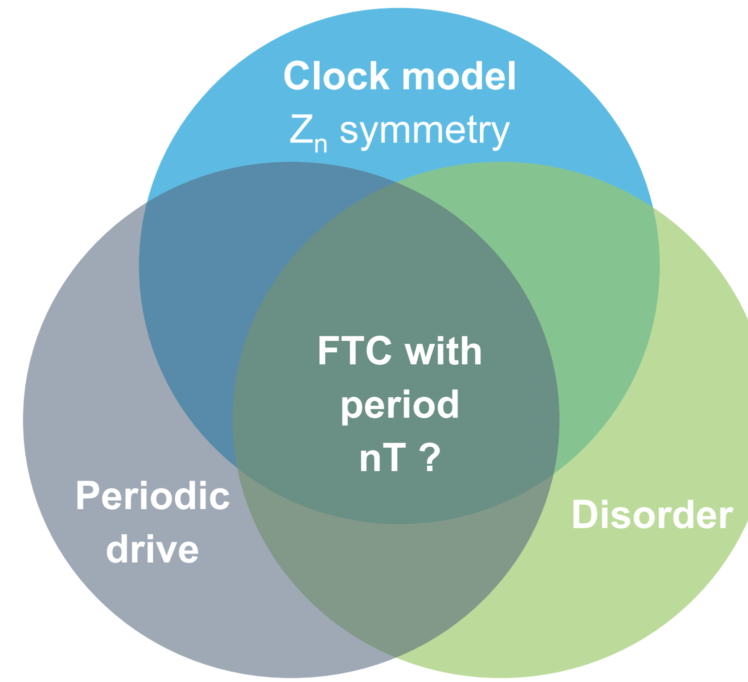


The gap closes in the thermodynamic limit



Floquet time crystals in clock models





$$\tau = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \sigma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \omega^{n-1} \end{pmatrix}$$

$$X = \prod_i \tau_i$$

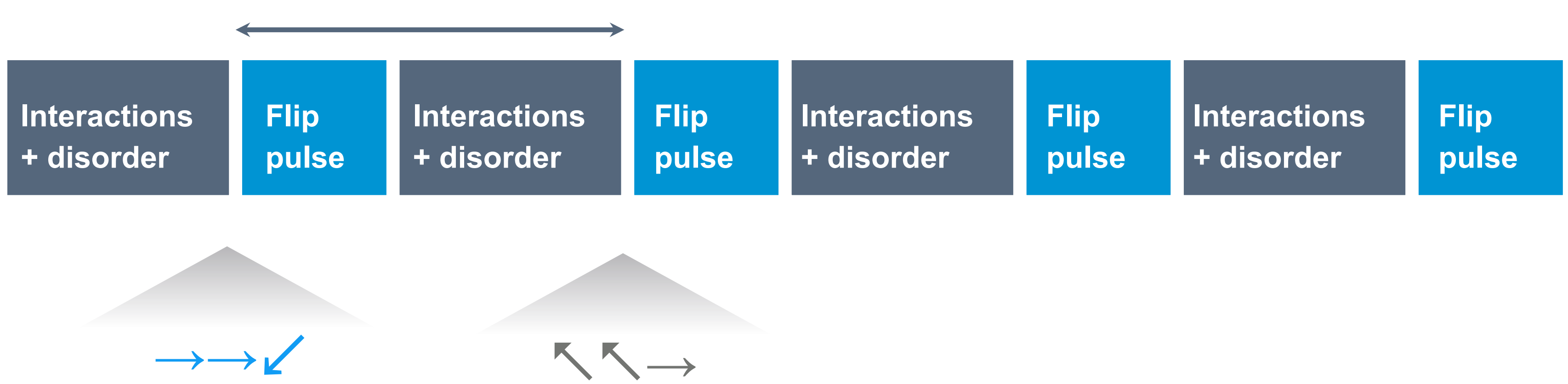
$$H = \sum_i J_i (e^{i\varphi} \sigma_i^\dagger \sigma_{i+1} + h.c.) + \sum_i h_i^z (e^{i\varphi_z} \sigma_i + h.c.) + \sum_i h_i^x (e^{i\varphi_x} \tau_i + h.c.)$$

$$\omega = e^{2\pi i/n}$$

$$\sigma\tau = \omega\tau\sigma$$

$$\sigma^n = 1$$

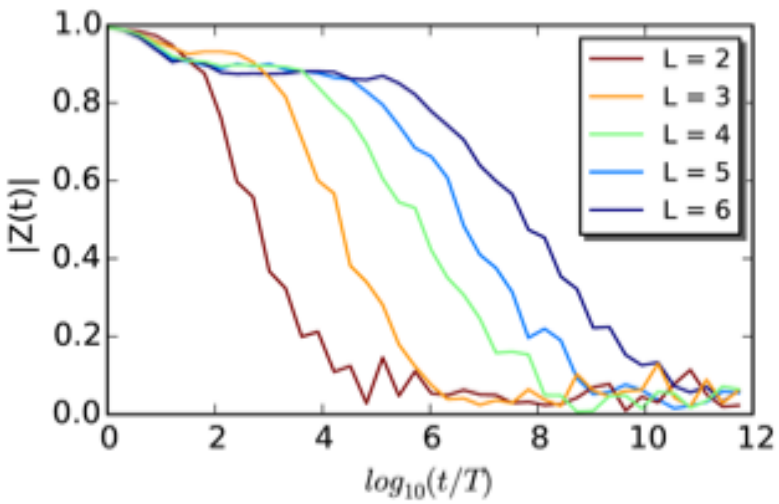
$$\tau^n = 1$$



Floquet time crystals in clock models

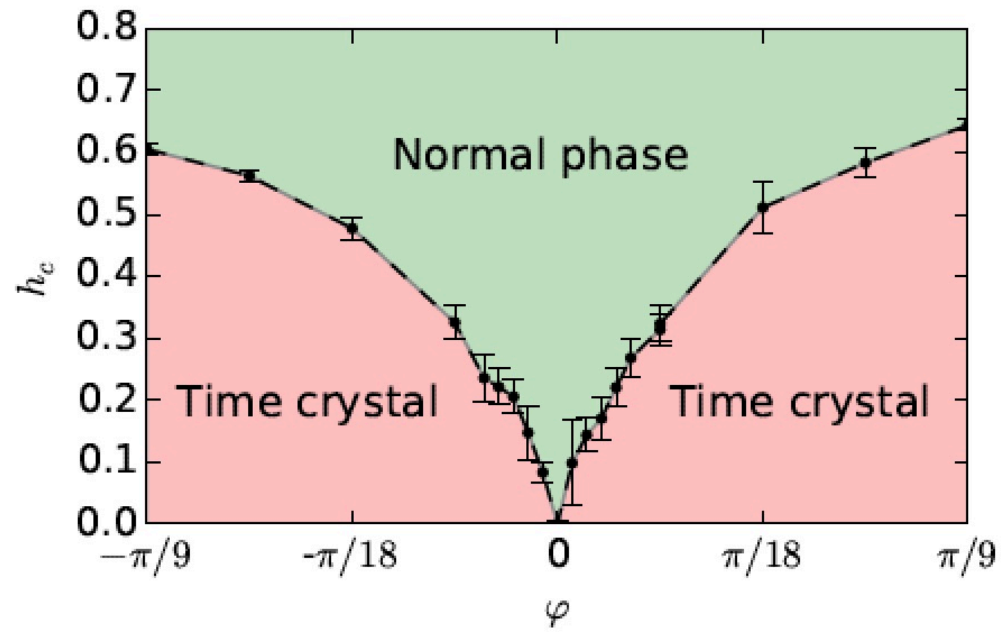
$$H = \sum_i J_i(e^{i\varphi}\sigma_i^\dagger\sigma_{i+1} + h.c.) + \sum_i h_i^z(e^{i\varphi_z}\sigma_i + h.c.) + \sum_i h_i^x(e^{i\varphi_x}\tau_i + h.c.)$$

CHIRAL MODEL



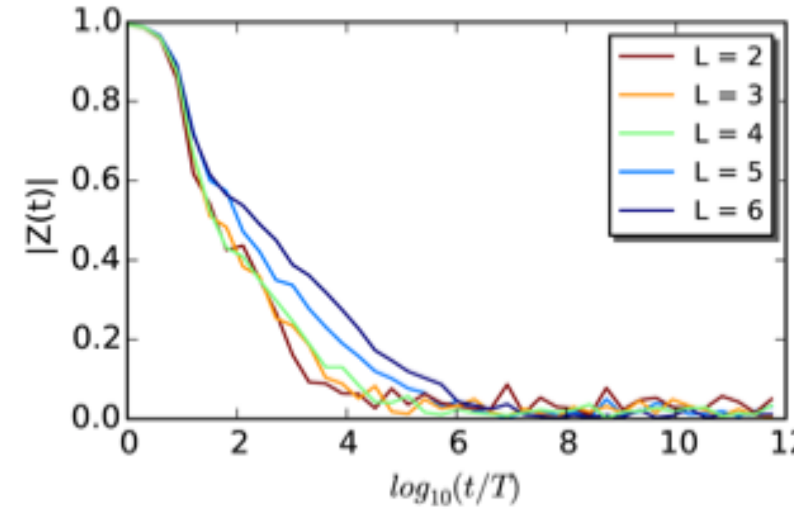
Decay time $t^* \sim O(e^{cL})$

➜ **Robust**



$h_z = 0.3$

NON-CHIRAL MODEL



For Z_n possibility to observe transitions between different crystalline phases

(Boundary) time crystals in dissipative systems

- **Is the coupling to a bath always detrimental?**

“Many-body” limit cycles as time-crystals in open systems …

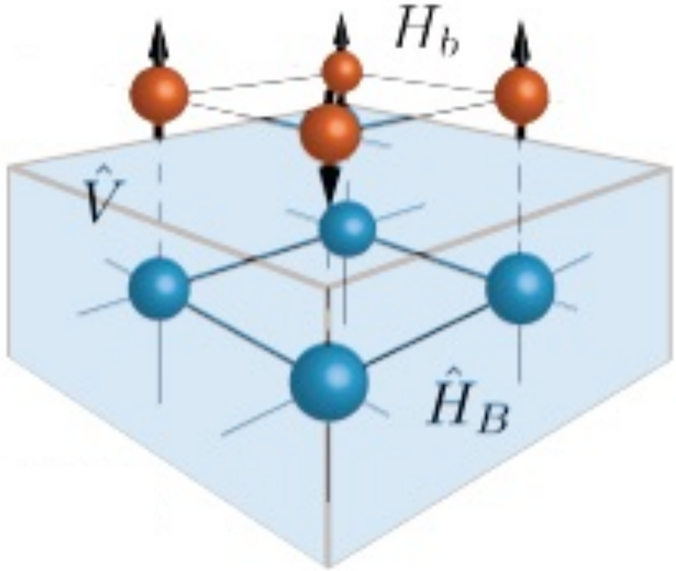
… they can be interpreted as “boundary” phases

The no-go theorem does not apply since the steady state is non-equilibrium …

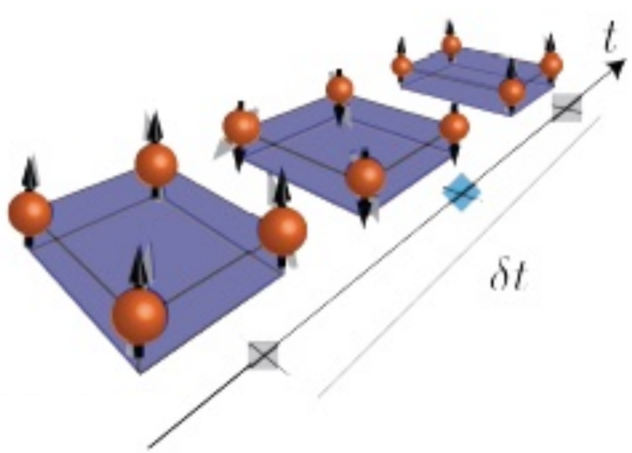
… an idealised model vs possible exp realisations

These limit cycles can be understood as a macroscopic synchronised dynamics characterised by a time-dependent order parameter

$$\hat{H} = \hat{H}_{\mathrm{B}} + \hat{H}_{\mathrm{b}} + \hat{V}$$



$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$$



$$\hat{\rho}_{\mathrm{b}} = \mathrm{Tr}_{\mathrm{B}}\left(|\psi(t)\rangle\langle\psi(t)|\right)$$

$$\frac{d}{dt}\hat{\rho}_{\mathrm{b}} = \hat{\mathcal{L}}\left[\hat{\rho}_{\mathrm{b}}\right]$$

A toy model

$$\hat{H}_{\mathrm{b}} = \omega_0 \sum_j \hat{\sigma}_j^x$$

The steady state diagram of the model has two distinct phases

$\omega_0/\kappa < 1$	$\omega_0/\kappa > 1$
$\langle \hat{S}^z \rangle \neq 0$	$\langle \hat{S}^z \rangle = 0$

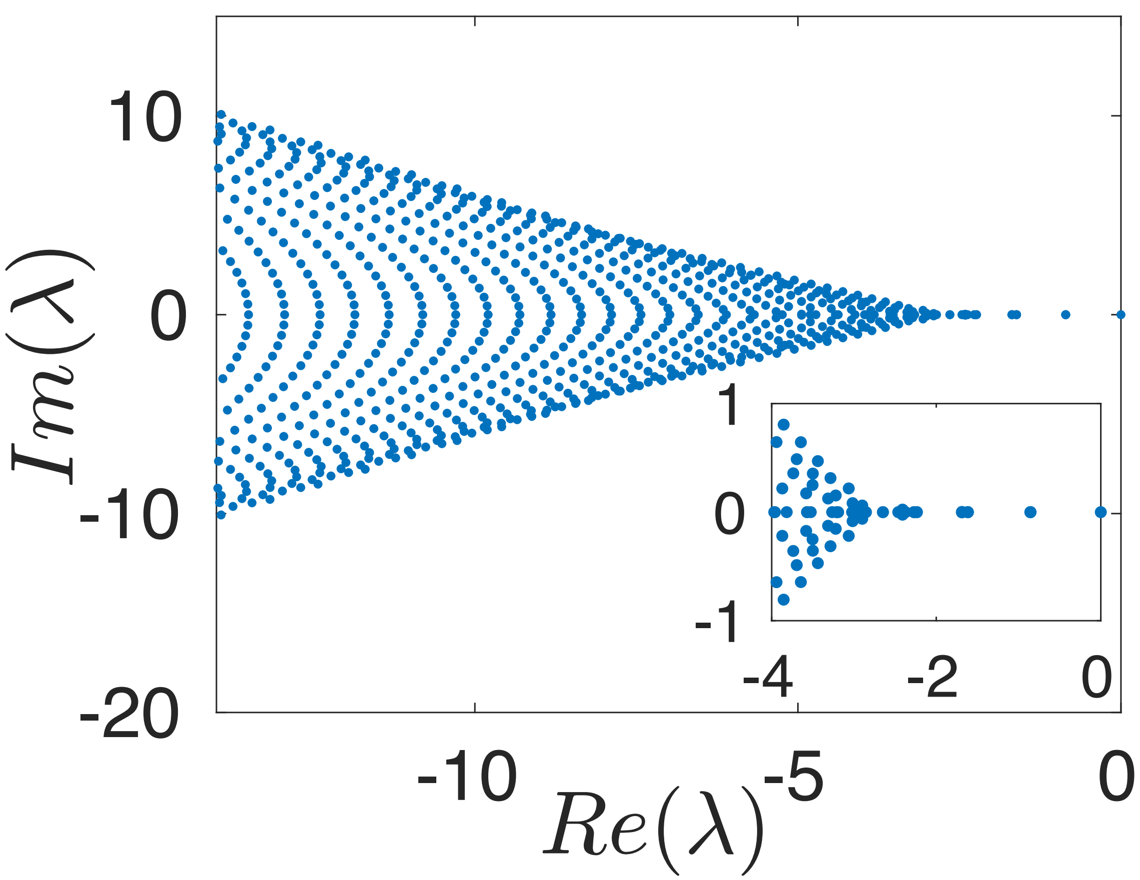
$$\hat{S}^\alpha = \sum_j \hat{\sigma}_j^\alpha$$

J. Hannukainen and J. Larson Phys. Rev. A **98**, 042113 (2018)

$$\frac{d}{dt}\hat{\rho}_{\mathrm{b}} = i\omega_0[\hat{\rho}_{\mathrm{b}}, \hat{S}^x] + \frac{\kappa}{S}\left(\hat{S}_-\hat{\rho}_{\mathrm{b}}\hat{S}_+ - \frac{1}{2}\{\hat{S}_+\hat{S}_-, \hat{\rho}_{\mathrm{b}}\}\right)$$

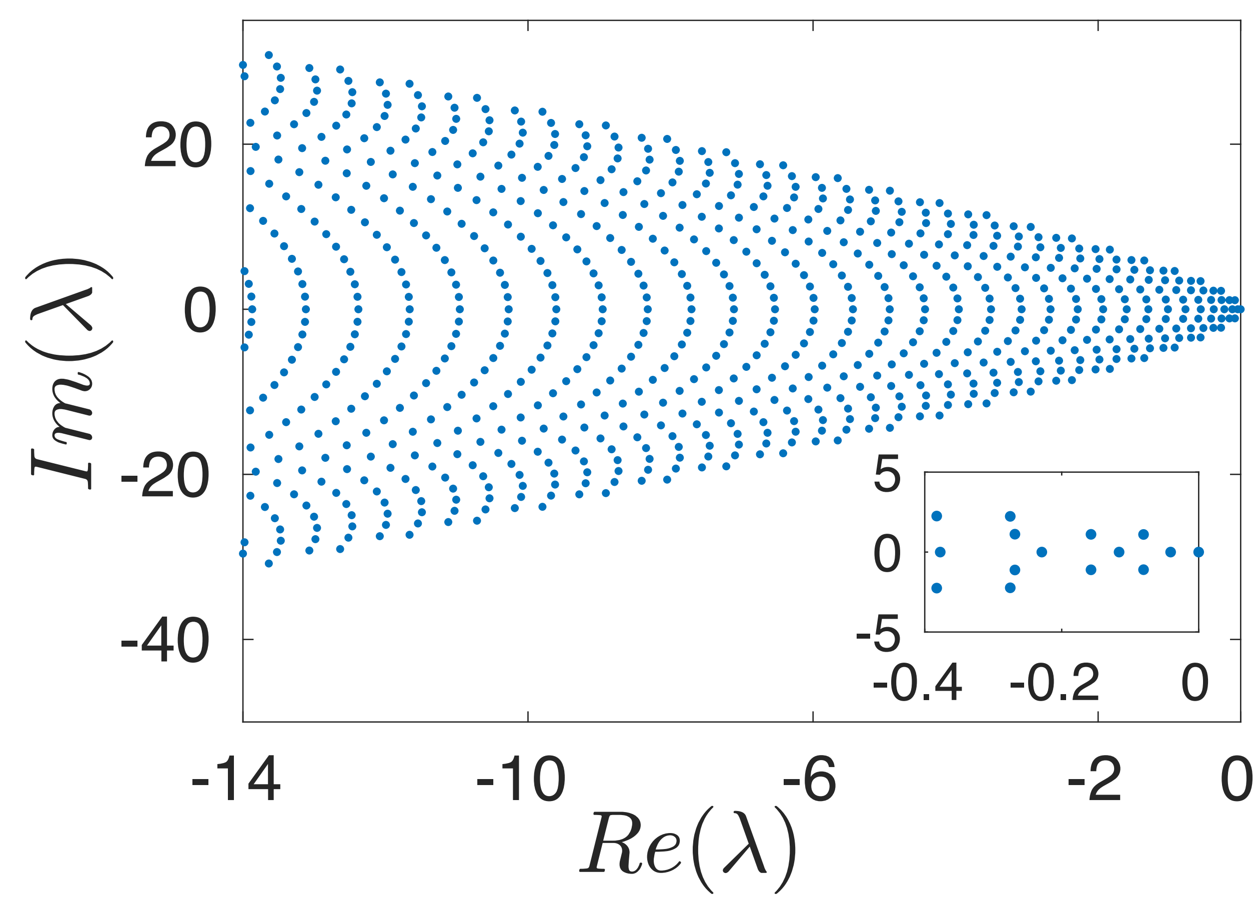
A toy model

$$\omega_0/\kappa > 1$$

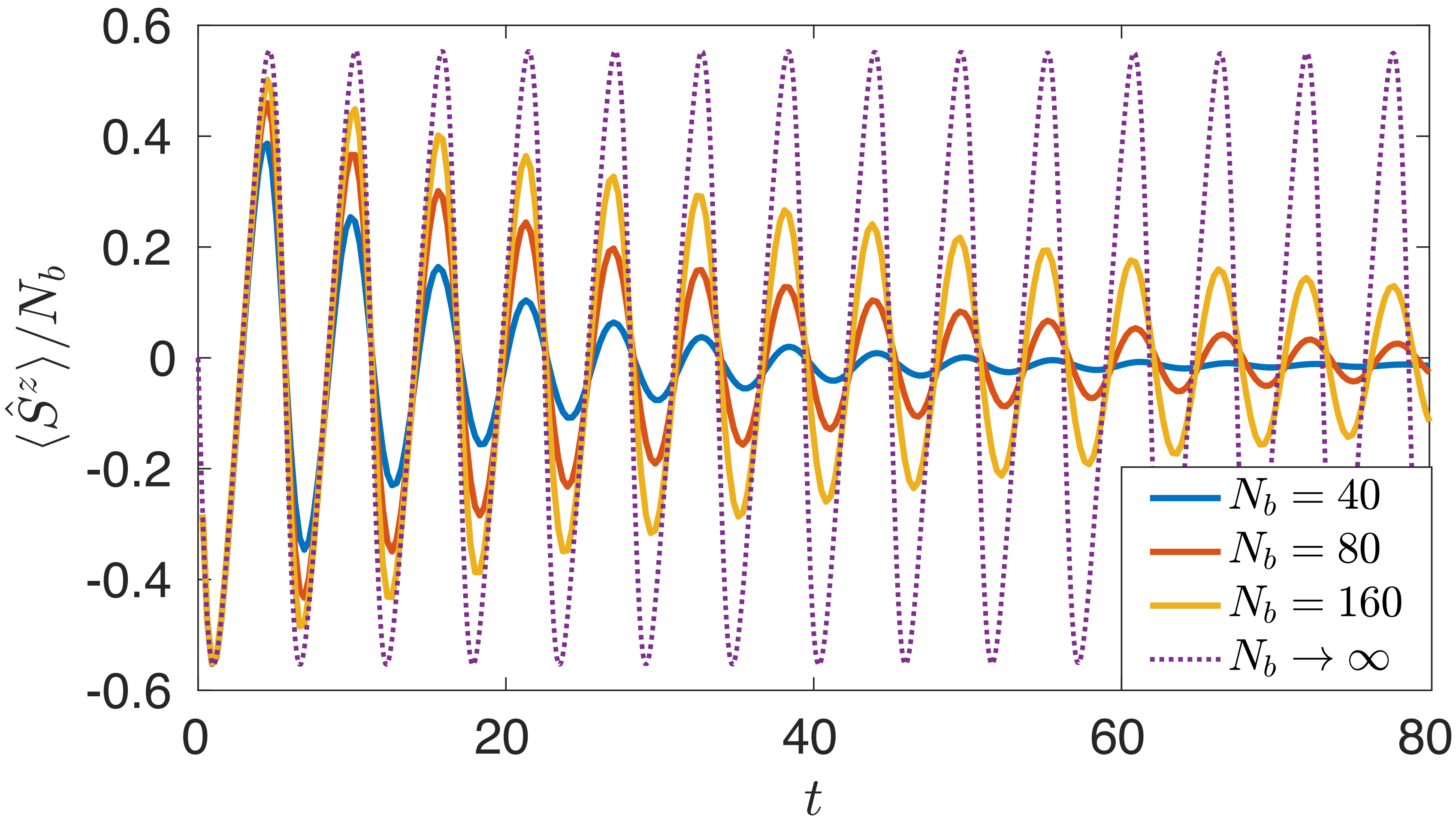


The spectrum is gapped and the low-lying eigenvalues of the Liouvillian have purely real values

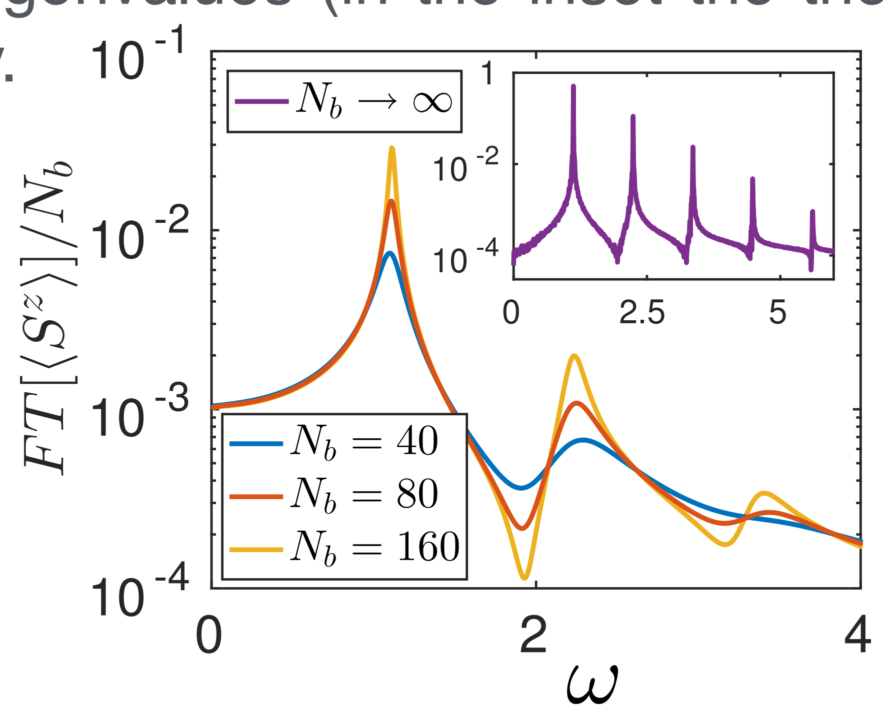
$$\omega_0/\kappa < 1$$

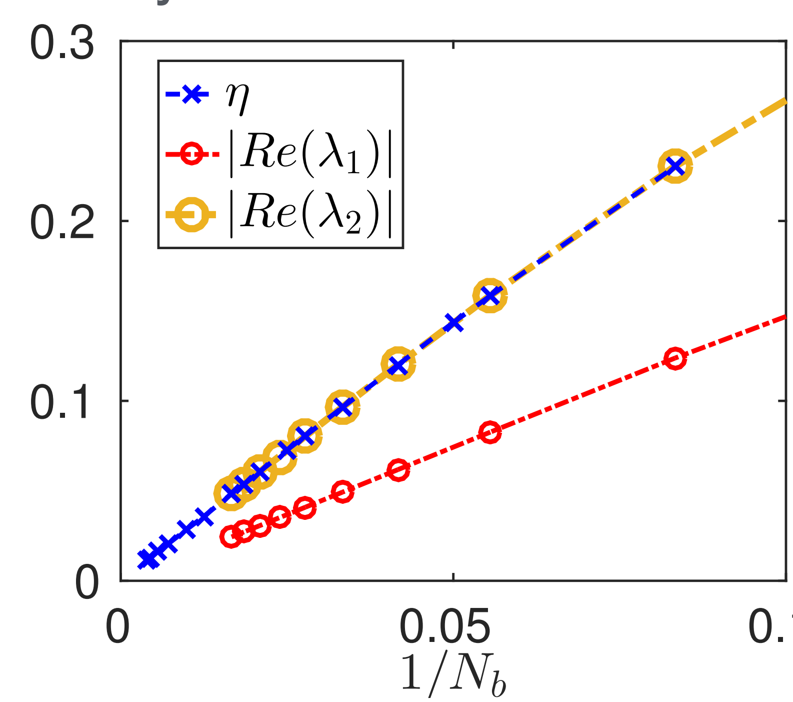


The spectrum becomes gapless and the low-lying excited eigenvalues have a non zero imaginary part



The peaks in the Fourier transform are associated to the band separations in the imaginary part of the Lindblad eigenvalues (in the inset the thermodynamic limit where the oscillations persist indefinitely.



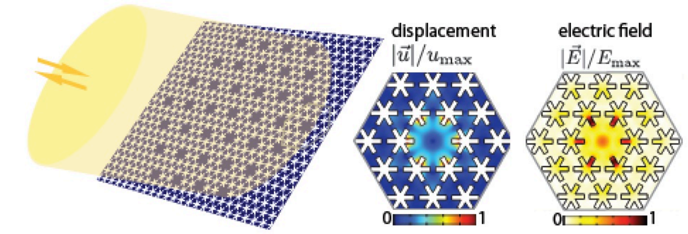


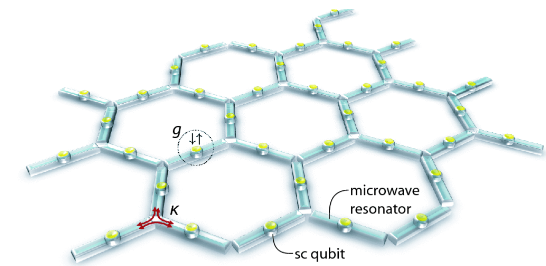
The decay rate of the oscillations scales as N_b^{-1}

Quantum synchronisation in many-body open systems

Identical features have been already seen in model systems of interacting Rydberg atoms, opto-mechanical arrays, coupled cavity arrays and interacting spin-systems.

These phases were all found however in a mean-field approximation, it is not clear to which extent they will survive when fluctuations are included.

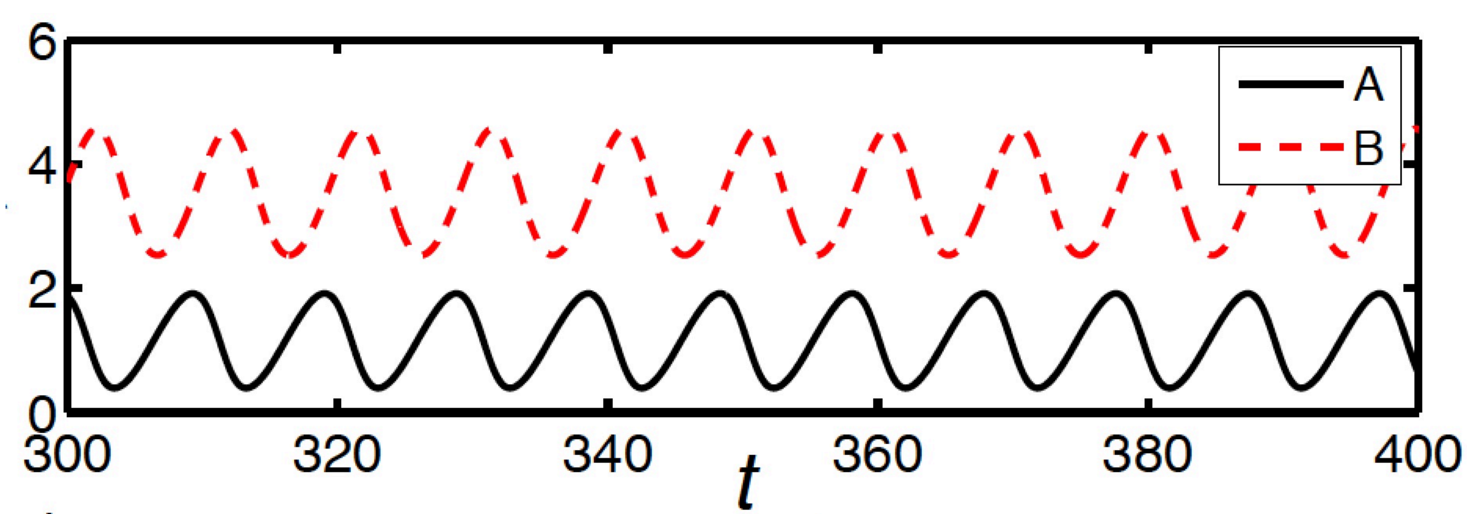




Lee, Haffner, and Cross (2011)
M. Ludwig and F. Marquardt,(2013)
Jin, *et al* (2013)
Schiro', *et al* (2016)
Chan, Lee, and Gopalakrishnan (2015)
….

$\hat{O}$ macroscopic order parameter

$$\langle \hat{O} \rangle_{ss} = \mathrm{Tr} \rho_{ss} \hat{O} = f(t)$$



Conclusions

- Floquet time crystals in clock models
- Infinite range models support time-crystalline phase
- Transitions between different symmetry broken phases
- Many-body synchronisation