Quantum Time crystals

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Quantum Simulators
Many-body systems … “hard to understand”:

Frustrated quantum magnets

High-Tc superconductors

…

Quantum simulator: an experimental controllable system that reproduces the physics of a given model Hamiltonian

Questions not accessible via classical computation because the exp-large dimension of the Hilbert space (e.g. complex quantum ground states or dynamics in solid state systems)

Questions that are not directly “tractable/identified” in “nature” (e.g. thermalisation, defect formation, …)

Refine our control on many-body systems

New states of matter (e.g. time-crystals, exotic qp, …)
Exp platforms

Trapped ions

Rydberg atoms

Optical lattices

Photonic quantum simulators
Spontaneous symmetry breaking

\[ H(h) = -J \sum_{i,j}^N S_i^z S_j^z - 2h \sum_i^N S_i^x \]

Order Parameter

\[ \langle S^z \rangle \]

\[ |g - g_c|^\beta \]

\[ g = J/h \]
Can time-translational invariance be spontaneously broken?

Time-crystal

Do laws of nature allow for the existence of a time-crystalline phase?

…if yes, how to define/characterise a time crystal?

…where to look for it?

How much do we know of its relations to other phenomena?

Is it “useful”?
Outline

- **Introduction time crystals**
  - Problem & definition (Wilczek 2013)
  - No-go theorem (Watanabe & Oshikawa 2015)
  - Floquet time-crystals (Else et al & Khemani et al 2013)
  - First exps on FTC (Choi et al & Zhang et al 2016)

- **Floquet time crystals in clock models**
  - Conditions for the existence
  - Direct transitions between different crystals

- **Floquet time crystals in clean systems**
  - Infinite range systems

- **Continuous (Boundary) time crystals**
  - Connections to many-body open quantum systems

- **Time crystals & Synchronization**
Definition of a time crystal

Time-independent Hamiltonian $\mathcal{H}$

$$\langle O \rangle = g(t)$$

**Bad definition:**

Even Rabi oscillations would fit into the category of time-crystals
Definition of a time crystal

**Definition TTSB:** \( \phi(\vec{x}, t) \) local order parameter

\[
\lim_{V \to \infty} \langle \phi(\vec{x}, t) \phi(\vec{x}', t') \rangle \to f(t - t') \quad |\vec{x} - \vec{x}'| \to \infty
\]

No-go theorem: (*) systems in the ground state or in thermal equilibrium cannot manifest any time-crystalline behaviour

Watanabe & Oshikawa 2015

(*) with sufficiently short-interactions
Spontaneous breaking Hamiltonian

Period $T$

Observables

Period $nT$

Theory

Experiments

Floquet time crystals (TTSB in periodically driven systems)

$$\mathcal{H}(t + T) = \mathcal{H}(t)$$

$$f(t) = \lim_{N \to \infty} \langle \psi | \hat{O}(t) | \psi \rangle$$

$$f(t + \tau_B) = f(t) \quad \tau_B = nT$$
Floquet time crystals

\[ H = \sum_i J_i \sigma_i^x \sigma_{i+1}^z \]

\[ \prod_i \sigma_i^x \]

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\[ H = \sum_i J_i \sigma_i^x \sigma_{i+1}^z \]

\[ \hat{U} = \exp \left[ -i \pi \sum_i^N \hat{\sigma}_i^x \right] \exp \left[ -i \hat{H} (\hat{\sigma}_i^z) \right] \]

\[ m_z = \frac{1}{N} \sum_i \langle \sigma_i^z(t) \rangle \]

The periodic drive, under generic conditions will heat the system up to infinite temperatures

will oscillate with a double period
**Rigidity:** no fine-tuned Hamiltonian parameters.

**Persistence:** the non-trivial oscillation must persist for infinitely long time when first taking the thermodynamic limit.
Floquet time crystals

Disordered Ising model in an external field

\[ Z(t) = \left[ (-1)^t \langle \sigma_i^z(t) \rangle \sigma_i^z(0) \right]_{av} \]
Time-crystals

Experimental realisation of Floquet time-crystals
Choi et al 2016
Zhang et al 2016
Floquet time crystals

Choi et al 2016
Zhang et al 2016

Else et al 2016

Interactions + disorder
Stability

More recent experiments
Rovny et al 2018
Pal et al 2016
Is it possible to have a Floquet time-crystal in the absence of disorder?

How to observe direct transitions between different crystalline phases

Floquet time-crystals in infinite-range models
When $h_0 < J$ there is $Z_2$ symmetry breaking that involves a finite fraction of all the spectrum.

In the thermodynamic limit below the broken symmetry edge $E^*$ the corresponding energy eigenstates appear in degenerate doublets. Each member of the pair is localised in the basis of the eigenstates $M_z$.

For finite sizes, the eigenstates are the even and odd superpositions of each doublet (with a splitting exponentially small in $N$).
Floquet time-crystals in the LMG model

\[ \hat{U} = \hat{U}_{\text{kick}} \exp \left[ -i \hat{H}(h) \tau \right] \quad \text{with} \quad \hat{U}_{\text{kick}} \equiv \exp \left[ -i \phi \sum_{i} \hat{s}_i^x \right], \]

\[ m_z = \frac{1}{N} \sum_i \langle \sigma_i^z(t) \rangle \]

will oscillate with a double period}

The gap closes in the thermodynamic limit
Floquet time crystals in clock models

\[ \tau = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \sigma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \omega^{n-1} \end{pmatrix} \]

\[ X = \prod_i \tau_i \]

\[ H = \sum_i J_i (e^{i\varphi} \sigma_i^+ \sigma_{i+1} + \text{h. c.}) + \sum_i h_i^x (e^{i\varphi} \sigma_i + \text{h. c.}) + \sum_i h_i^x (e^{i\varphi} \tau_i + \text{h. c.}) \]

\[ \omega = e^{2\pi i/n} \]

\[ \sigma \tau = \omega \tau \sigma \]

\[ \sigma^n = 1 \]

\[ \tau^n = 1 \]
Floquet time crystals in clock models

\[ H = \sum_i J_i (e^{i\varphi} \sigma_i^\dagger \sigma_{i+1} + h.c.) + \sum_i h_i^z (e^{i\varphi} \sigma_i + h.c.) + \sum_i h_i^x (e^{i\varphi} \tau_i + h.c.) \]

**CHIRAL MODEL**

Decay time \( t^* \sim O(e^{cL}) \)

\( \Rightarrow \) Robust

**NON-CHIRAL MODEL**

For \( Z_n \) possibility to observe transitions between different crystalline phases
(Boundary) time crystals in dissipative systems

Is the coupling to a bath always detrimental?

“Many-body” limit cycles as time-crystals in open systems ...

... they can be interpreted as “boundary” phases

The no-go theorem does not apply since the steady state is non-equilibrium ...

... an idealised model vs possible exp realisations

These limit cycles can be understood as a macroscopic synchronised dynamics characterised by a time-dependent order parameter
\[ \hat{H} = \hat{H}_B + \hat{H}_b + \hat{V} \]

\[ |\psi(t)\rangle = e^{-i\hat{H}t} |\psi(0)\rangle \]

\[ \hat{\rho}_b = \text{Tr}_B (|\psi(t)\rangle \langle \psi(t)|) \]

\[ \frac{d}{dt} \hat{\rho}_b = \hat{\mathcal{L}} [\hat{\rho}_b] \]
A toy model

\[ \hat{H}_b = \omega_0 \sum_j \hat{\sigma}^x_j \]

\[ \hat{S}^\alpha = \sum_j \hat{\sigma}^\alpha_j \]

The steady state diagram of the model has two distinct phases

<table>
<thead>
<tr>
<th>Condition</th>
<th>Phase 1</th>
<th>Phase 2</th>
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<tbody>
<tr>
<td>( \omega_0/\kappa &lt; 1 )</td>
<td>( \langle \hat{S}^z \rangle \neq 0 )</td>
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\[ \frac{d}{dt} \hat{\rho}_b = i \omega_0 [\hat{\rho}_b, \hat{S}^x] + \frac{\kappa}{\mathcal{S}} \left( \hat{S}^- \hat{\rho}_b \hat{S}^+ - \frac{1}{2} \{ \hat{S}^+, \hat{S}^-, \hat{\rho}_b \} \right) \]
A toy model

The spectrum is gapped and the low-lying eigenvalues of the Liouvillian have purely real values.

\[ \omega_0/\kappa > 1 \]

The spectrum becomes gapless and the low-lying excited eigenvalues have a non-zero imaginary part.

\[ \omega_0/\kappa < 1 \]
The peaks in the Fourier transform are associated to the band separations in the imaginary part of the Lindblad eigenvalues (in the inset the thermodynamic limit where the oscillations persist indefinitely.

The decay rate of the oscillations scales as $N_b^{-1}$.
Quantum synchronisation in many-body open systems

Identical features have been already seen in model systems of interacting Rydberg atoms, opto-mechanical arrays, coupled cavity arrays and interacting spin-systems.

These phases were all found however in a mean-field approximation, it is not clear to which extent they will survive when fluctuations are included.

\[
\langle \hat{O} \rangle_{ss} = \text{Tr} \rho_{ss} \hat{O} = f(t)
\]

\( \hat{O} \) macroscopic order parameter

Lee, Haffner, and Cross (2011)
M. Ludwig and F. Marquardt, (2013)
Chan, Lee, and Gopalakrishnan (2015)
Conclusions

- Floquet time crystals in clock models
- Infinite range models support time-crystalline phase
- Transitions between different symmetry broken phases
- Many-body synchronisation