1. What is active matter and why is it interesting?

2. Background 1: low Reynolds number hydrodynamics

3. Background 2: nematic liquid crystals

4. Active nematics and active turbulence

5. Self-propelled topological defects

6. Confining active turbulence

7. Bacteria: the hare and the tortoise

8. Eukaryotic cells as an active system
nematic symmetry
Topological defects in nematic liquid crystals

\begin{align*}
m &= + \frac{1}{2} \\
m &= - \frac{1}{2}
\end{align*}

\text{topological charge} \quad m = \frac{1}{2\pi} \int_{dS} d\theta
Active turbulence: bacteria

Dense suspension of microswimmers
Dogic group Brandeis
Active turbulence

Fluorescence Confocal Microscopy
Continuum equations of liquid crystal hydrodynamics

\[(\partial_t + u_k \partial_k)Q_{ij} - S_{ij} = \Gamma H_{ij}\]
Continuum equations of liquid crystal hydrodynamics

\[
(\partial_t + u_k \partial_k)Q_{ij} - S_{ij} = \Gamma H_{ij}
\]

\[
S_{ij} = (\lambda E_{ik} + \Omega_{ik})(Q_{kj} + \delta_{kj}/3) + (Q_{ik} + \delta_{ik}/3)(\lambda E_{kj} - \Omega_{kj}) - 2\lambda(Q_{ij} + \delta_{ij}/3)(Q_{kl} \partial_k u_l)
\]

\[
E_{ij} = (\partial_i u_j + \partial_j u_i)/2
\]

\[
\Omega_{ij} = (\partial_j u_i - \partial_i u_j)/2
\]
Continuum equations of liquid crystal hydrodynamics

\[
(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij}
\]

\[
S_{ij} = (\lambda E_{ik} + \Omega_{ik})(Q_{kj} + \delta_{kj}/3) +
\]

\[
(Q_{ik} + \delta_{ik}/3)(\lambda E_{kj} - \Omega_{kj}) - 2\lambda(Q_{ij} + \delta_{ij}/3)(Q_{kl} \partial_k u_l)
\]

\[
E_{ij} = (\partial_i u_j + \partial_j u_i)/2
\]

\[
\Omega_{ij} = (\partial_j u_i - \partial_i u_j)/2
\]

\[
H_{ij} = -\delta \mathcal{F} / \delta Q_{ij} + (\delta_{ij}/3) \text{Tr}(\delta \mathcal{F} / \delta Q_{kl})
\]

\[
\mathcal{F} = K(\partial_k Q_{ij})^2/2 + AQ_{ij} Q_{ji}/2 + BQ_{ij} Q_{jk} Q_{ki}/3 + C(Q_{ij} Q_{ji})^2/4
\]
Continuum equations of liquid crystal hydrodynamics

\[ \rho \left( \partial_t + u_k \partial_k \right) u_i = \partial_j \Pi_{ij} \]
Continuum equations of liquid crystal hydrodynamics

\[ \rho (\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij} \]

\[ \Pi_{ij}^{viscous} = 2\mu E_{ij} \]
Continuum equations of liquid crystal hydrodynamics

\[ \rho (\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij} \]

\[ \Pi_{ij}^{\text{viscous}} = 2\mu E_{ij} \]

\[ \Pi_{ij}^{\text{passive}} = -P \delta_{ij} + 2\lambda (Q_{ij} + \delta_{ij}/3)(Q_{kl} H_{lk}) - \lambda H_{ik}(Q_{kj} + \delta_{kj}/3) \]

\[ -\lambda(Q_{ik} + \delta_{ik}/3) H_{kj} - \partial_i Q_{kl} \frac{\delta F}{\delta \partial_j Q_{lk}} + Q_{ik} H_{kj} - H_{ik} Q_{kj} \]

Tumbling parameter
Continuum equations of liquid crystal hydrodynamics

\[
(\partial_t + u_k \partial_k)Q_{ij} - S_{ij} = \Gamma H_{ij}
\]

couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

\[
\rho(\partial_t + u_k \partial_k)u_i = \partial_j \Pi_{ij}
\]

viscous + passive stress
Active turbulence: bacteria

Dense suspension of microswimmers
Continuum equations of liquid crystal hydrodynamics

\[ (\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij} \]

couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

\[ \rho (\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij} \]

viscous + passive
Continuum equations of active liquid crystal hydrodynamics

\[(\partial_t + u_k \partial_k)Q_{ij} - S_{ij} = \Gamma H_{ij}\]

Couples nematic order and shear flows

Relaxation to minimum of Landau-de Gennes free energy

\[\rho(\partial_t + u_k \partial_k)u_i = \partial_j \Pi_{ij}\]

Viscous + passive + active stress

\[\Pi_{ij}^{active} = -\zeta Q_{ij}\]
Active stress => active turbulence

Active contribution to the stress

\[ -\zeta Q \]

Gradients in the magnitude or direction of the order parameter induce flow.

Hatwalne, Ramaswamy, Rao, Simha, PRL 2004
nematic ordering is unstable to bend instabilities
Active stress => active turbulence

Active contribution to the stress

\[-\zeta Q\]

Gradients in the magnitude or direction of the order parameter induce flow.

Linear stability analysis =>

nematic state is unstable to vortical flows

Hatwalne, Ramaswamy, Rao, Simha, PRL 2004
Active stress => active turbulence

Active contribution to the stress

\[- \zeta Q\]

Gradients in the magnitude or direction of the order parameter induce flow.

Linear stability analysis => nematic state is unstable to vortical flows

What happens instead is active turbulence

Hatwalne, Ramaswamy, Rao, Simha, PRL 2004
Active turbulence

Dense suspension of microswimmers

Vorticity field
Modelling active turbulence
Active turbulence: topological defects are created and destroyed
Active turbulence: topological defects are created and destroyed
Flow fields around defects

\[ m = \pm \frac{1}{2} \]

1. the viscous stress, \( \Pi_{\text{viscous}} \)

\[ \Pi_{\text{viscous}} = 2 \mu E_{ij} \]
Dogic group, Brandeis
Active turbulence

Fluorescence Confocal Microscopy
Sanchez, Chen, DeCamp, Heymann, Dogic, Nature 2012
L. Giomi, M.J. Bowick, Ma Xu, M.C. Marchetti, PRL 110, 228101
Martínez-Prat et al
FC Keber et al, Science, 2014
1. What is active matter and why is it interesting?
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Jordi Ignes-Mullol
Teresa Lopez-Leon
Francesc Sagues
Why study confined active systems?

To control active turbulence

Many active systems are finite in extent – tissues, organoids, tumours

The physics:

Two competing length scales

Motile topological defects
States of an Active Nematic in a Channel

\[ \text{Activity number } A = \sqrt{z h^2 / K} \]

Voituriez et al
EPL (2005)
States of an Active Nematic in a Channel
Ceilidh Dance
Vortex lattice and active topological microfluidics
No flow $\Rightarrow$ laminar flow $\Rightarrow$ the Ceilidh dance $\Rightarrow$ active turbulence

Increasing activity $\Rightarrow$

$\leq$ Increasing confinement
Microtubules and kinesin motors in channels

Hardouin et al, Communications Physics 2 (2019)

Widths 30 – 400 microns
The dancing state in confined microtubule – kinesin mixtures

Distribution of defects across the channel:

Blue  -1/2

Green  +1/2
Shear + periodic bursts of defects

Distance between defects is set by the channel width
Shear + bursts of defects

Defect bursts are periodic