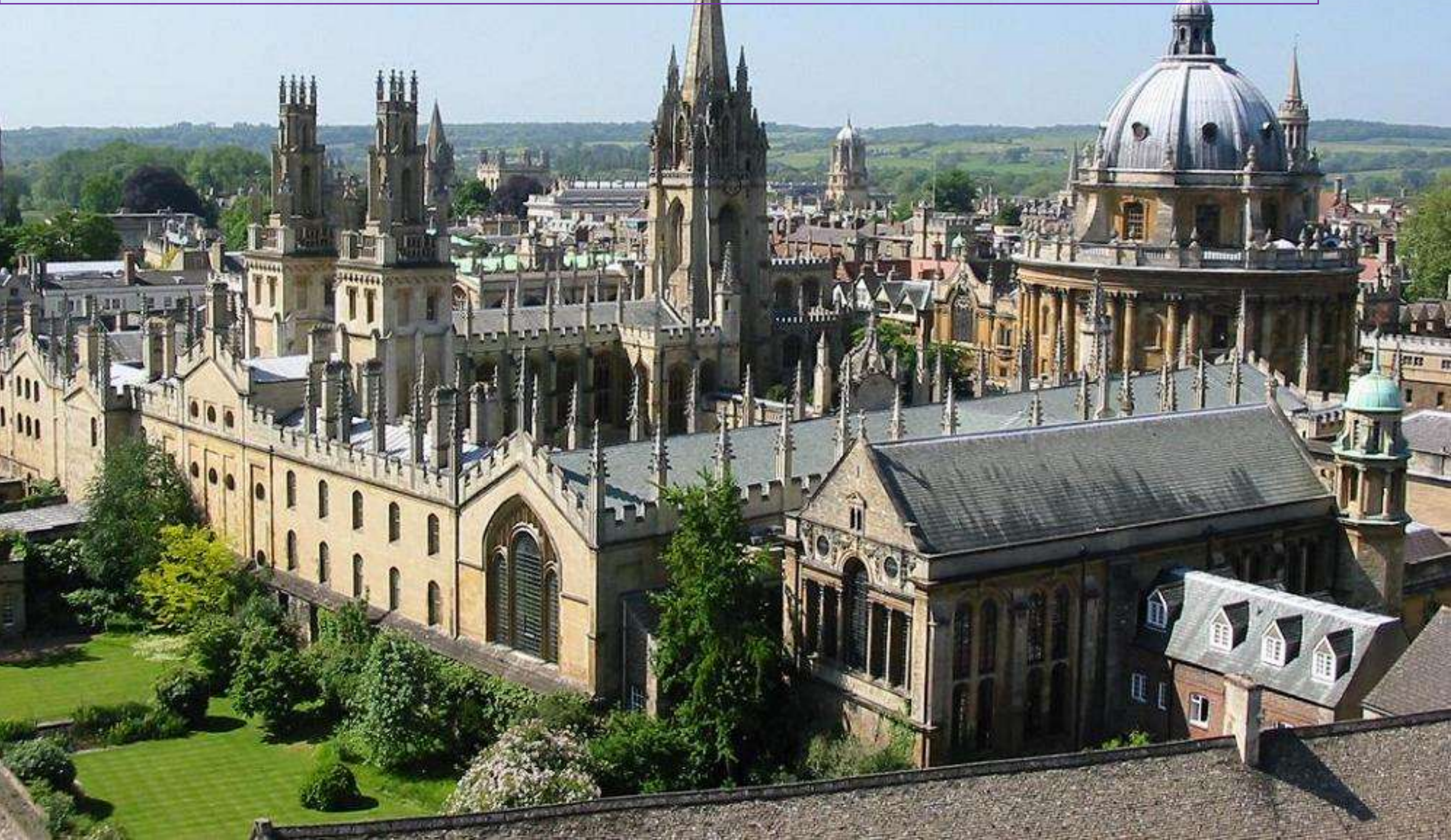
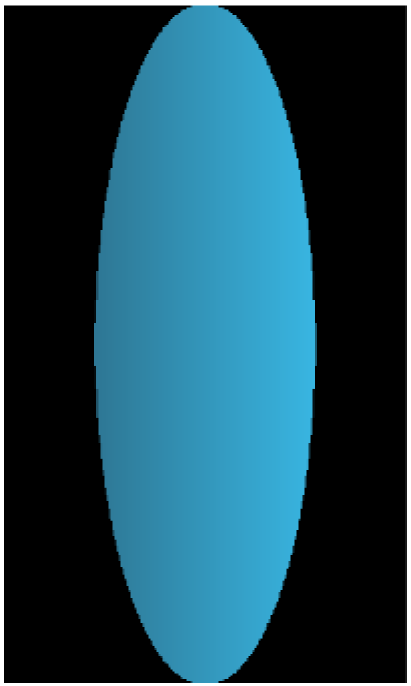
Active Matter 2

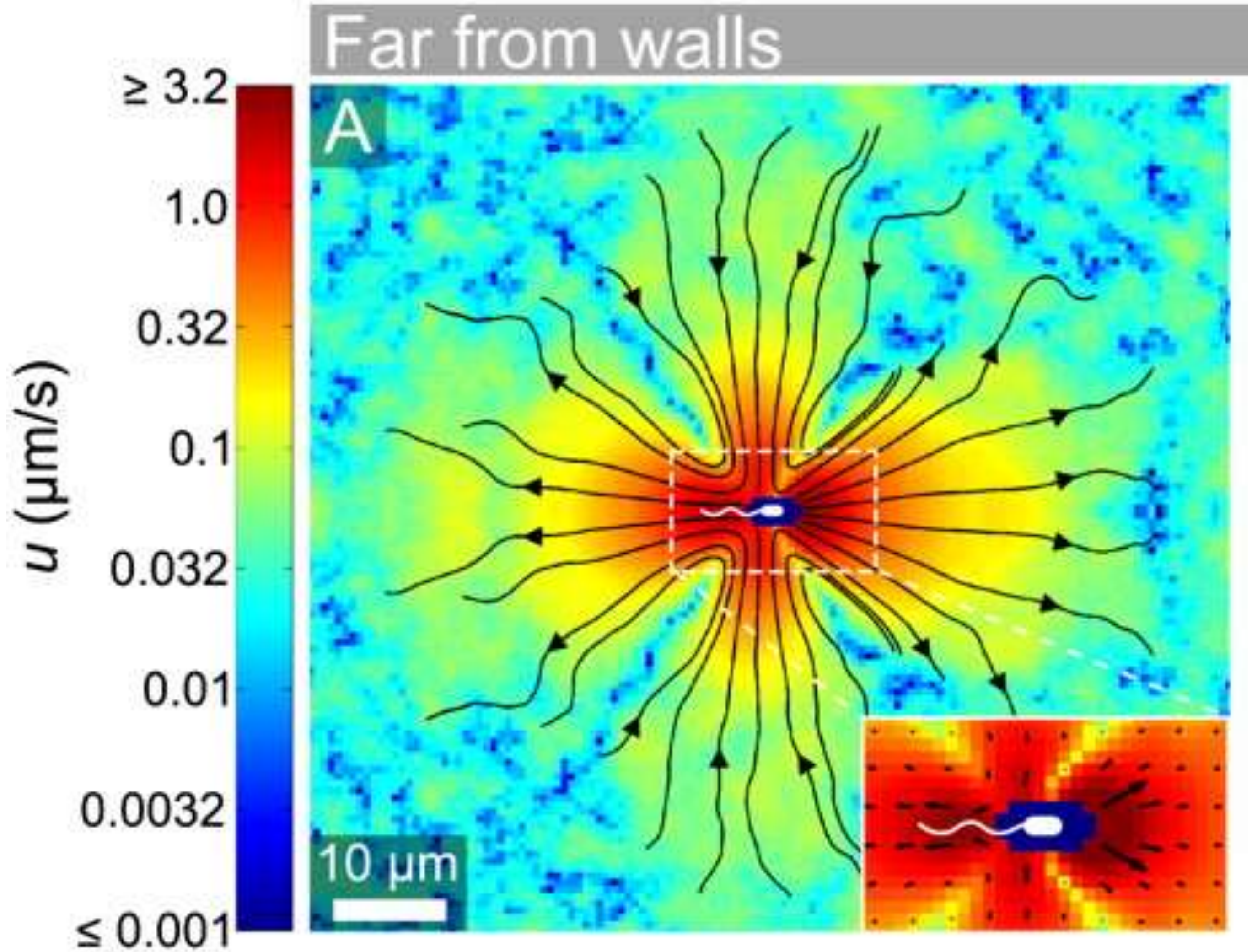
Julia Yeomans
University of Oxford



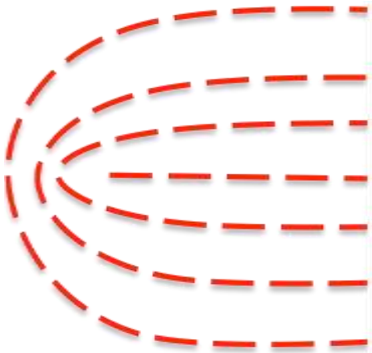
1. What is active matter and why is it interesting?
2. Background 1: low Reynolds number hydrodynamics
3. Background 2: nematic liquid crystals
4. Active nematics and active turbulence
5. Self-propelled topological defects
6. Confining active turbulence
7. Bacteria: the hare and the tortoise
8. Eukaryotic cells as an active system

nematic symmetry

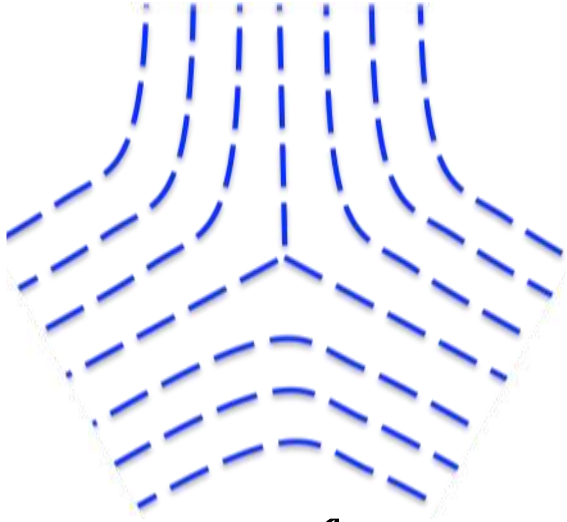




Topological defects in nematic liquid crystals



$$m = +\frac{1}{2}$$

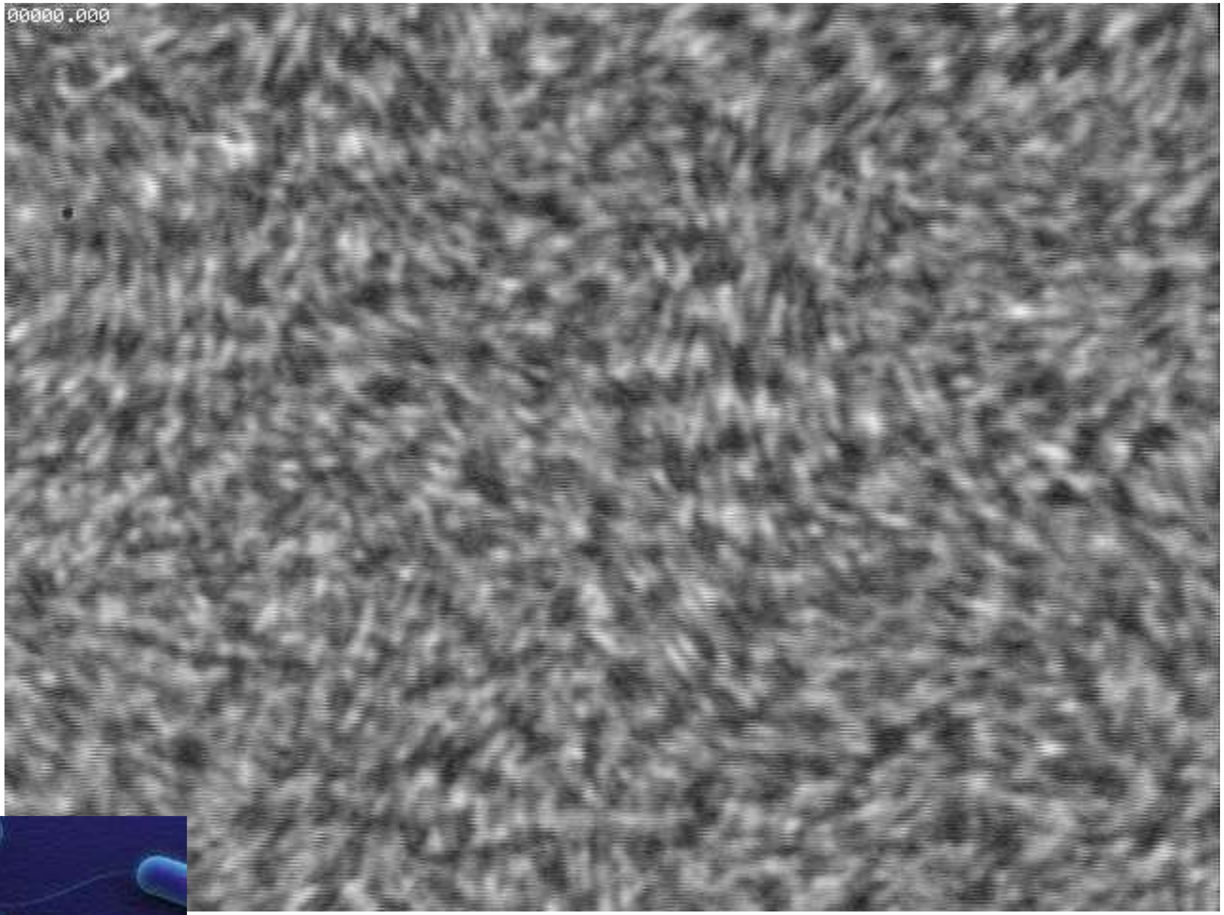


$$m = -\frac{1}{2}$$

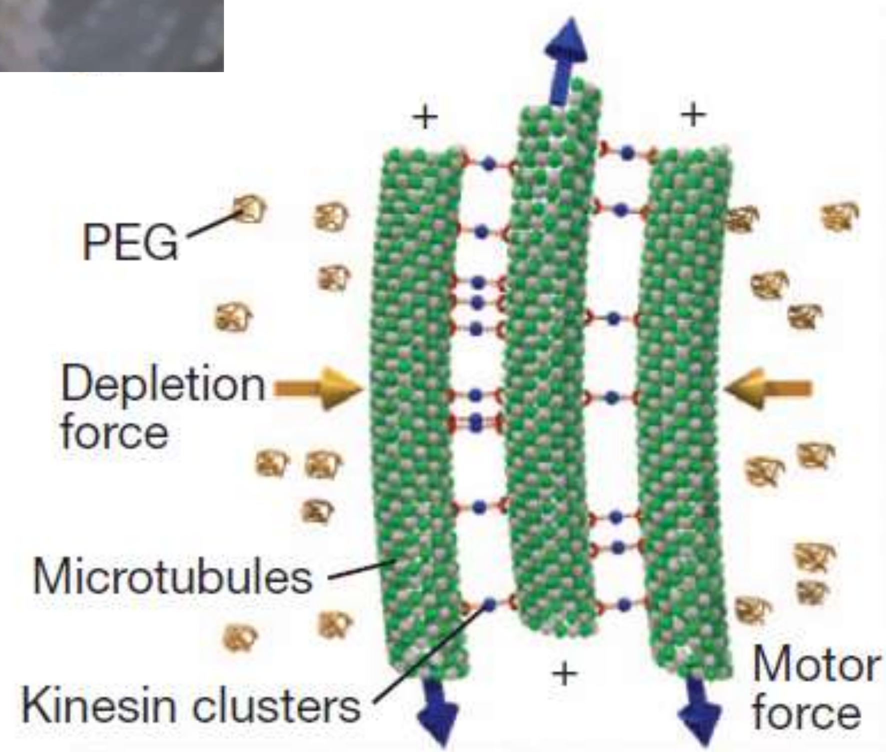
topological charge

$$m = \frac{1}{2\pi}\int_{dS} d\theta$$

Active turbulence: bacteria



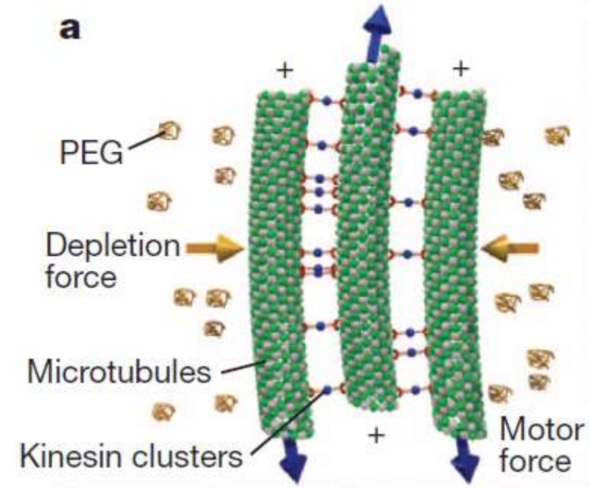
Dense suspension of microswimmers

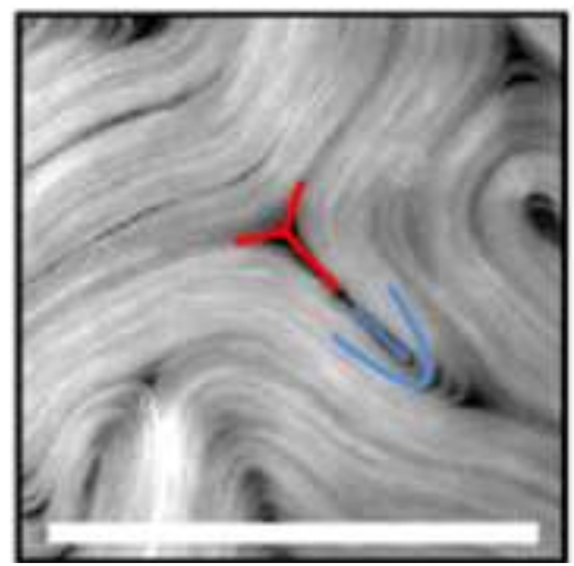


Dogic group Brandeis

Active turbulence

Fluorescence Confocal Microscopy





Continuum equations of liquid crystal hydrodynamics

$$\boxed{(\partial_t + u_k\partial_k)Q_{ij} - S_{ij} = \Gamma H_{ij}}$$

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$$S_{ij} = (\lambda E_{ik} + \Omega_{ik})(Q_{kj} + \delta_{kj}/3) +$$

$$(Q_{ik} + \delta_{ik}/3)(\lambda E_{kj} - \Omega_{kj}) - 2\lambda(Q_{ij} + \delta_{ij}/3)(Q_{kl}\partial_k u_l)$$

$$E_{ij} = (\partial_i u_j + \partial_j u_i)/2$$

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$$H_{ij} = -\delta\mathcal{F}/\delta Q_{ij} + (\delta_{ij}/3)\mathrm{Tr}(\delta\mathcal{F}/\delta Q_{kl})$$

$$\mathcal{F} = K(\partial_k Q_{ij})^2/2 + AQ_{ij}Q_{ji}/2 + BQ_{ij}Q_{jk}Q_{ki}/3 + C(Q_{ij}Q_{ji})^2/4$$

Continuum equations of liquid crystal hydrodynamics

$$\boxed{\rho(\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij}}$$

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$$\Pi_{ij}^{viscous} = 2\mu E_{ij}$$

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$$\begin{aligned}\Pi_{ij}^{passive} = {} & -P\delta_{ij} + 2\lambda(Q_{ij} + \delta_{ij}/3)(Q_{kl}H_{lk}) - \lambda H_{ik}(Q_{kj} + \delta_{kj}/3) \\ & -\lambda(Q_{ik} + \delta_{ik}/3)H_{kj} - \partial_i Q_{kl}\frac{\delta\mathcal{F}}{\delta\partial_j Q_{lk}} + Q_{ik}H_{kj} - H_{ik}Q_{kj}\end{aligned}$$

Tumbling parameter

Continuum equations of liquid crystal hydrodynamics

$$\boxed{(\partial_t + u_k\partial_k)Q_{ij} - S_{ij} = \Gamma H_{ij}}$$

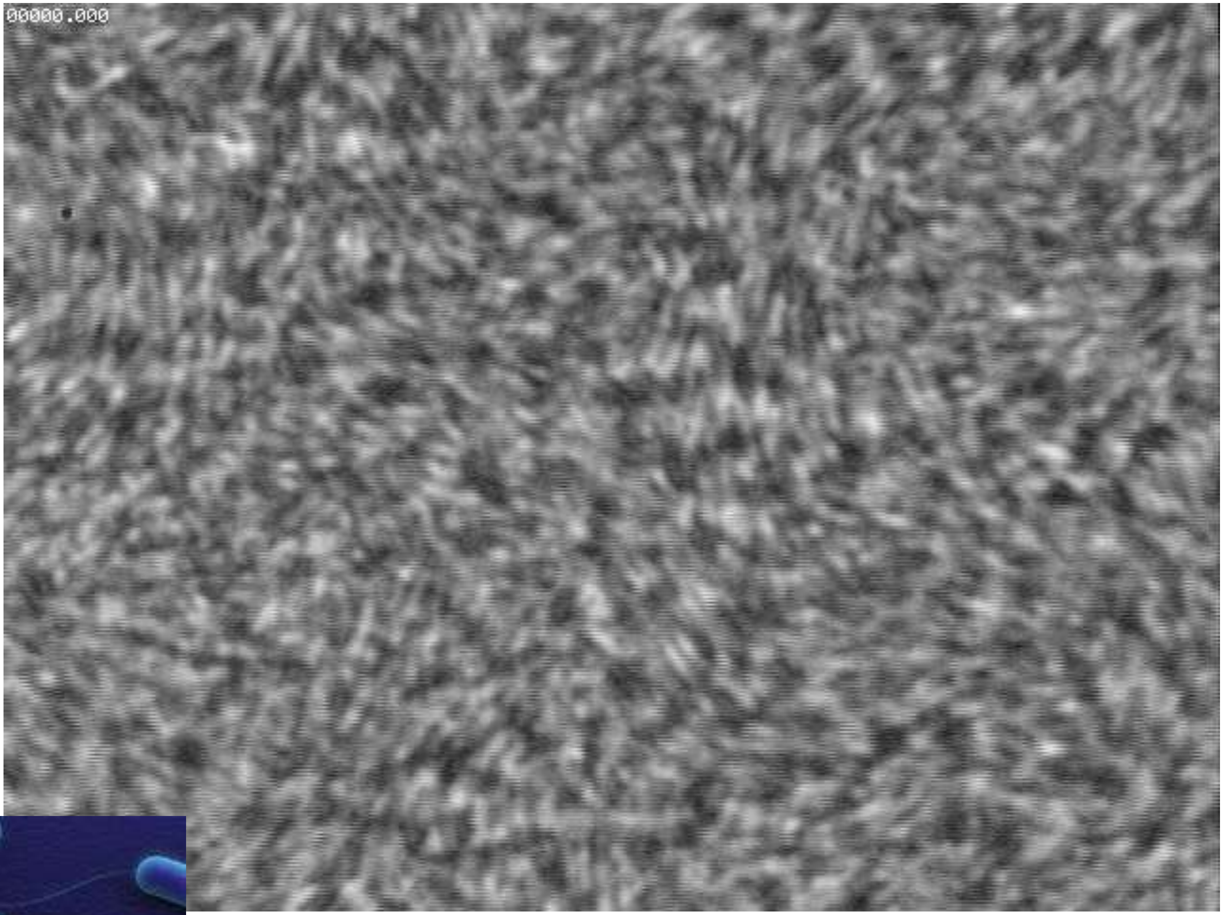
couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

$$\boxed{\rho(\partial_t + u_k\partial_k)u_i = \partial_j \Pi_{ij}}$$

viscous + passive stress

Active turbulence: bacteria



Dense suspension of microswimmers

Continuum equations of liquid crystal hydrodynamics

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viscous + passive

Continuum equations of active liquid crystal hydrodynamics

$$\boxed{(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij}}$$

couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

$$\boxed{\rho(\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij}}$$

viscous + passive + active stress

$$\boxed{\Pi_{ij}^{active} = -\zeta Q_{ij}}$$

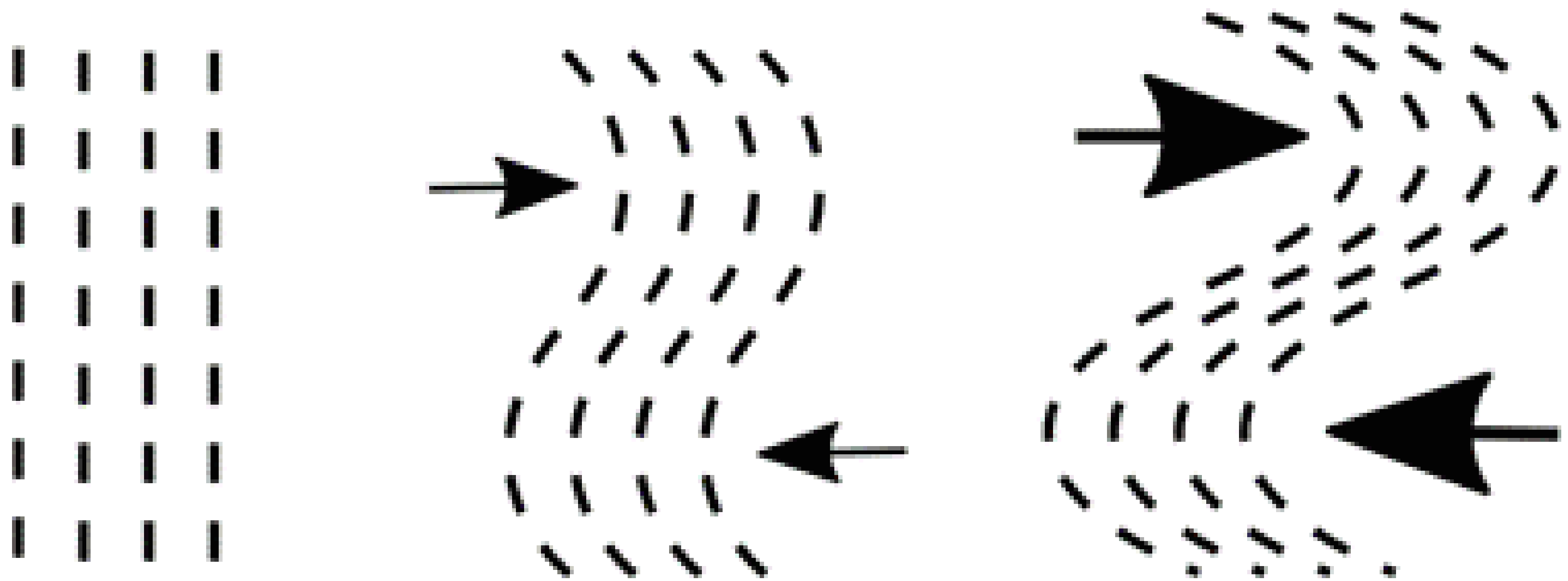
Active stress => active turbulence

Active contribution to the stress $-\zeta Q$

Gradients in the magnitude or direction of the order parameter induce flow.

Hatwalne, Ramaswamy, Rao, Simha, PRL 2004

nematic ordering is unstable to bend instabilities



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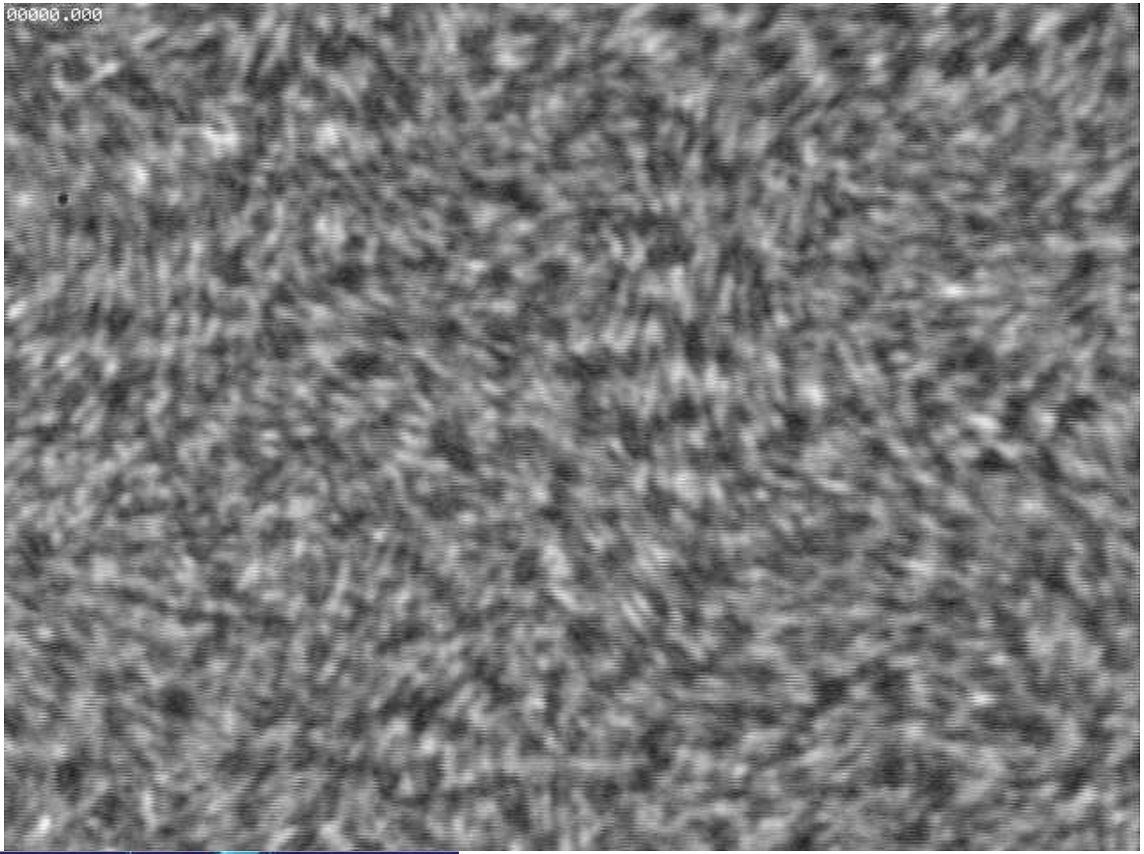
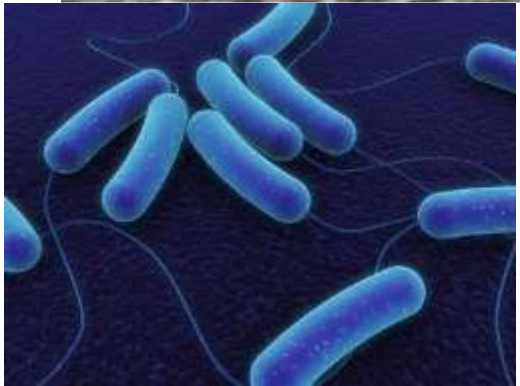
What happens instead is active turbulence

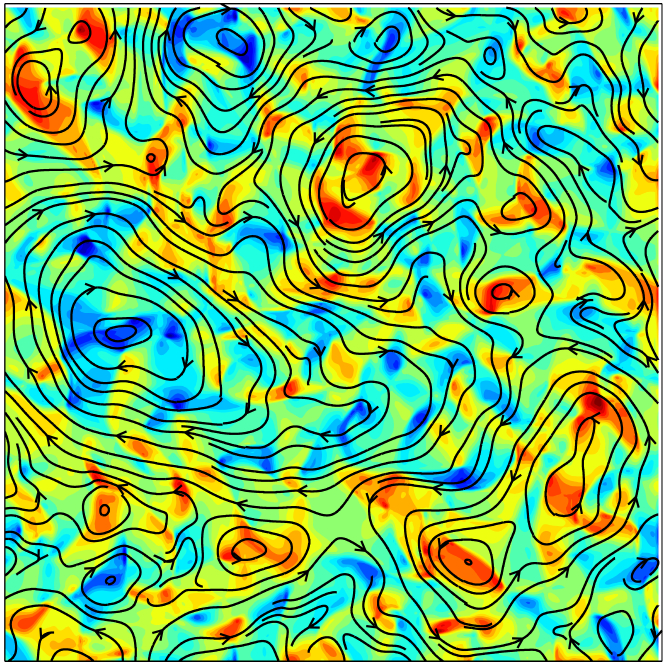
Hatwalne, Ramaswamy, Rao, Simha, PRL 2004

Active turbulence



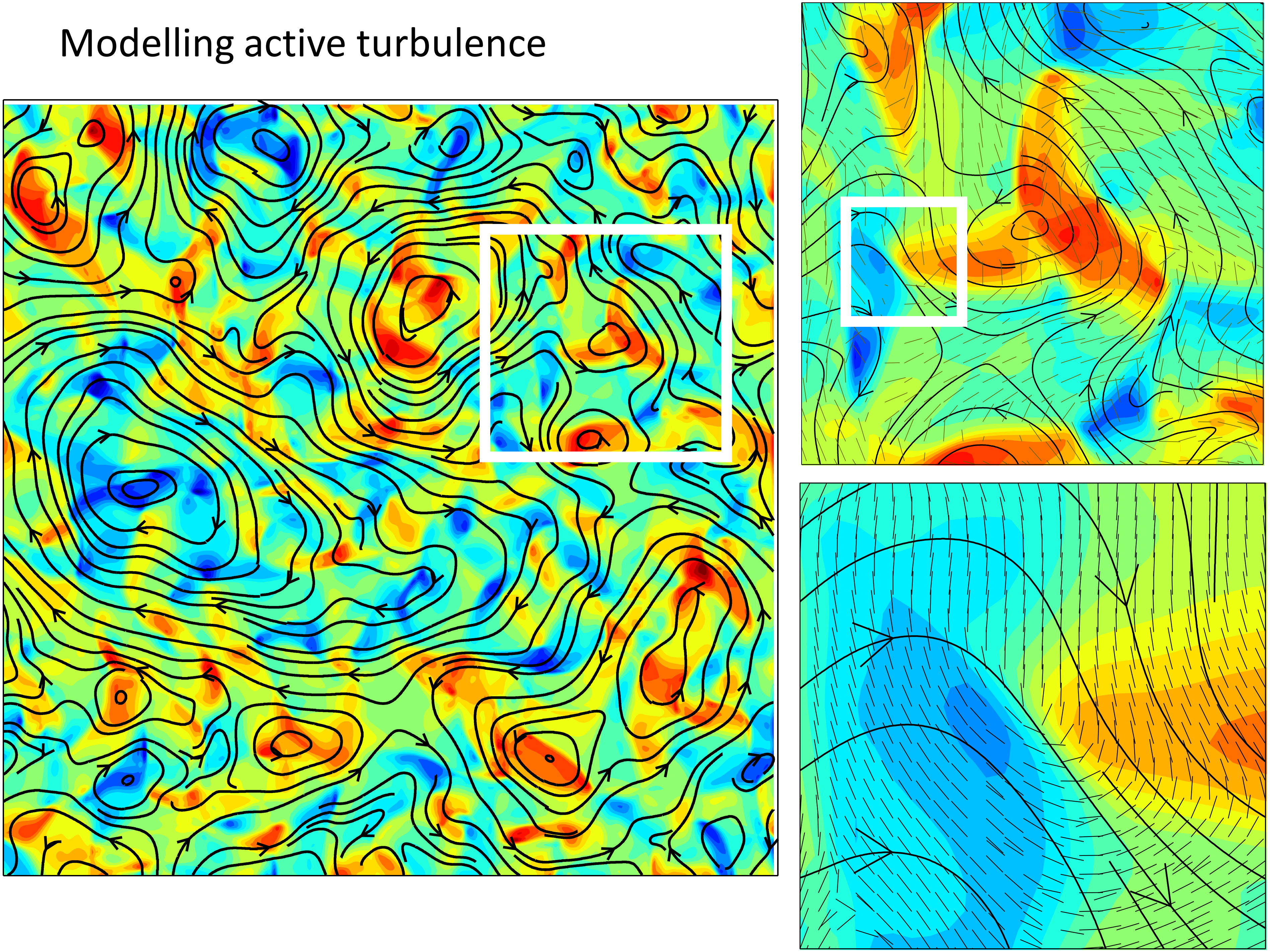


Dense suspension of microswimmers

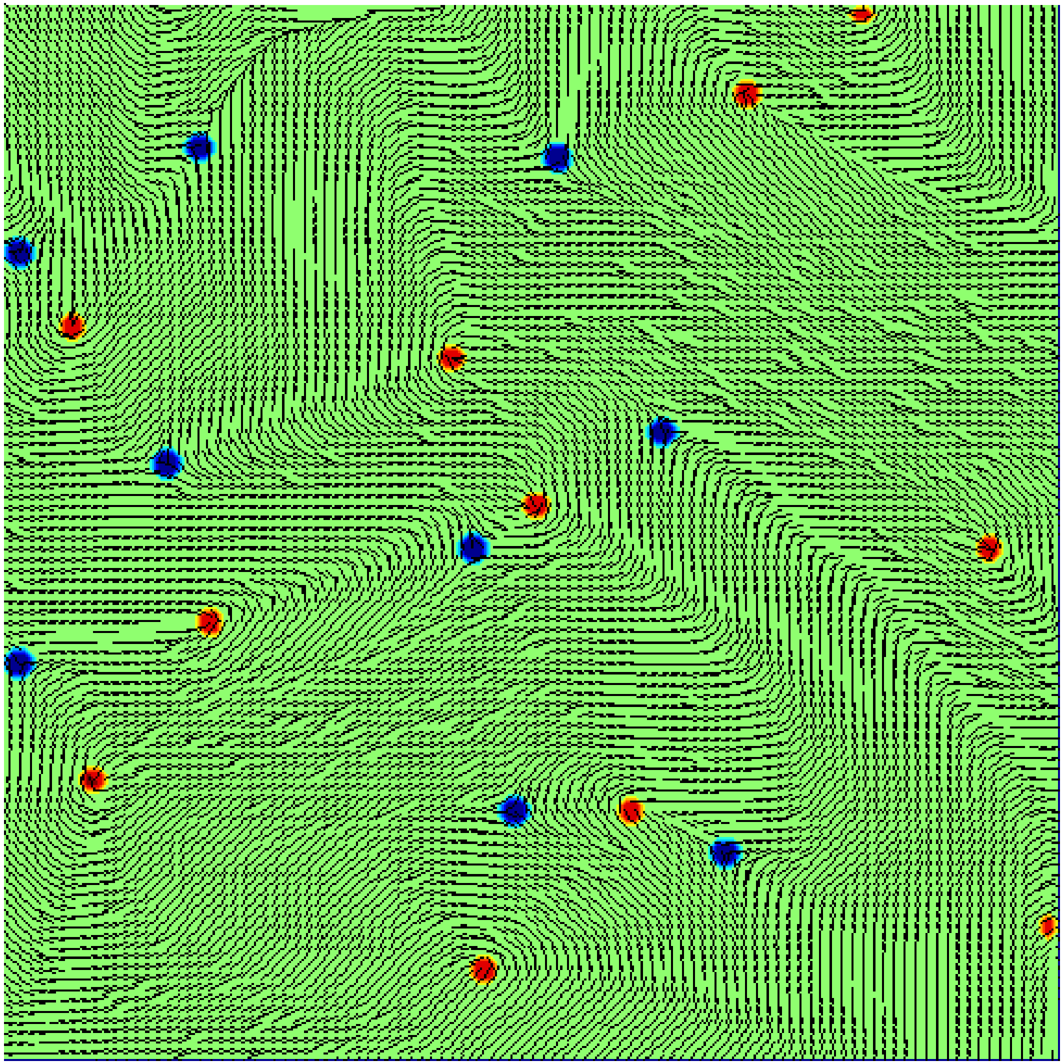


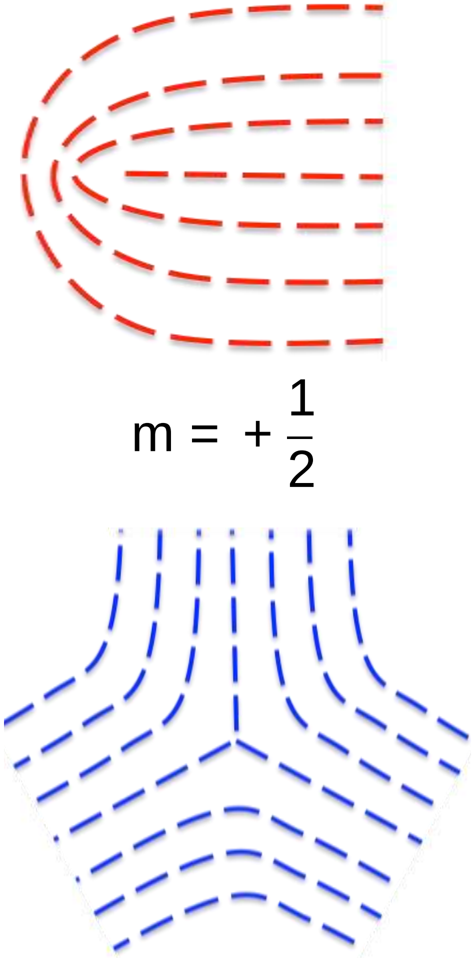
Vorticity field

Modelling active turbulence



Active turbulence: topological defects are created and destroyed

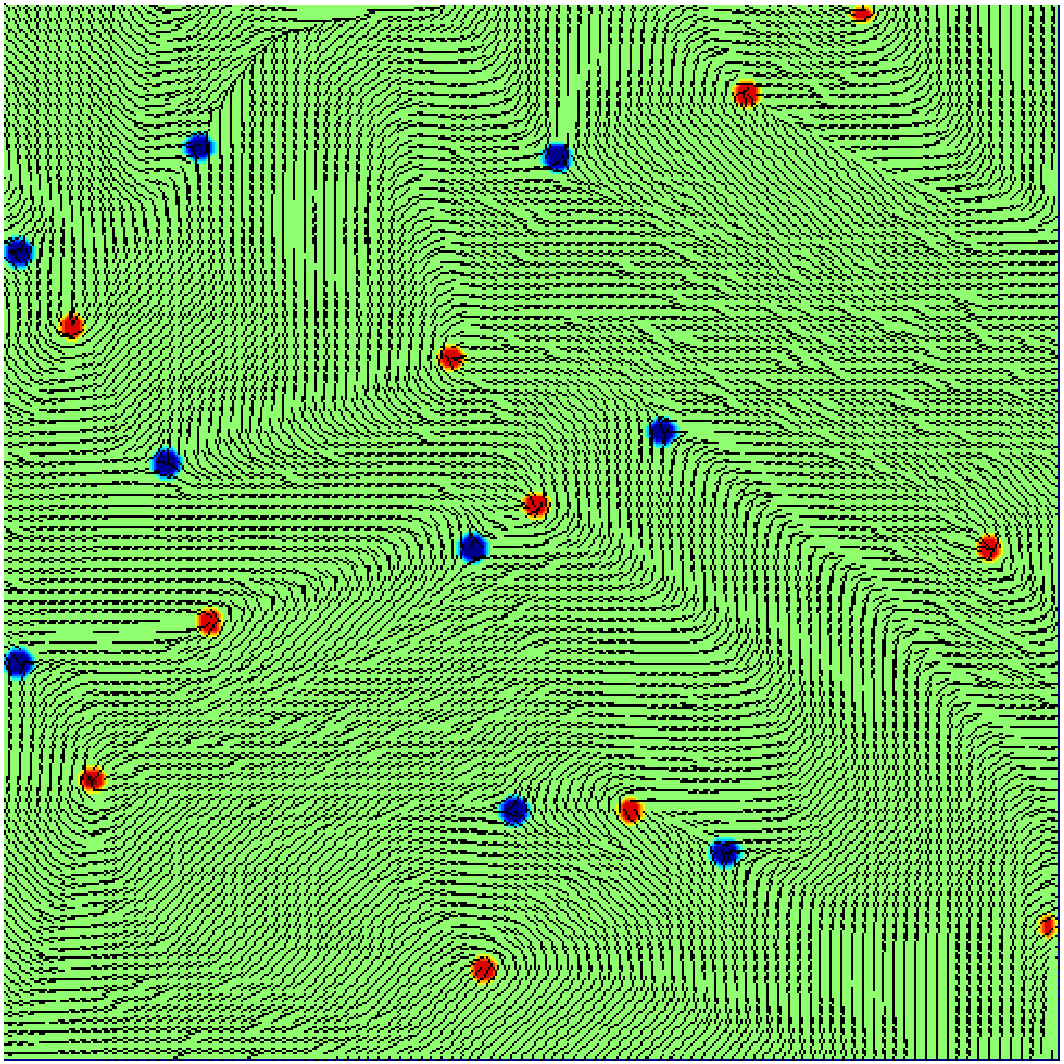




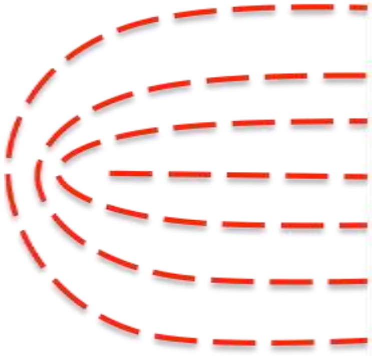
$$m = +\frac{1}{2}$$

$$m = -\frac{1}{2}$$

Active turbulence: topological defects are created and destroyed



Topological defects are self motile



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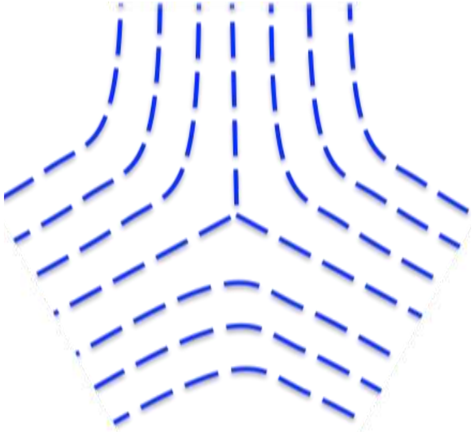
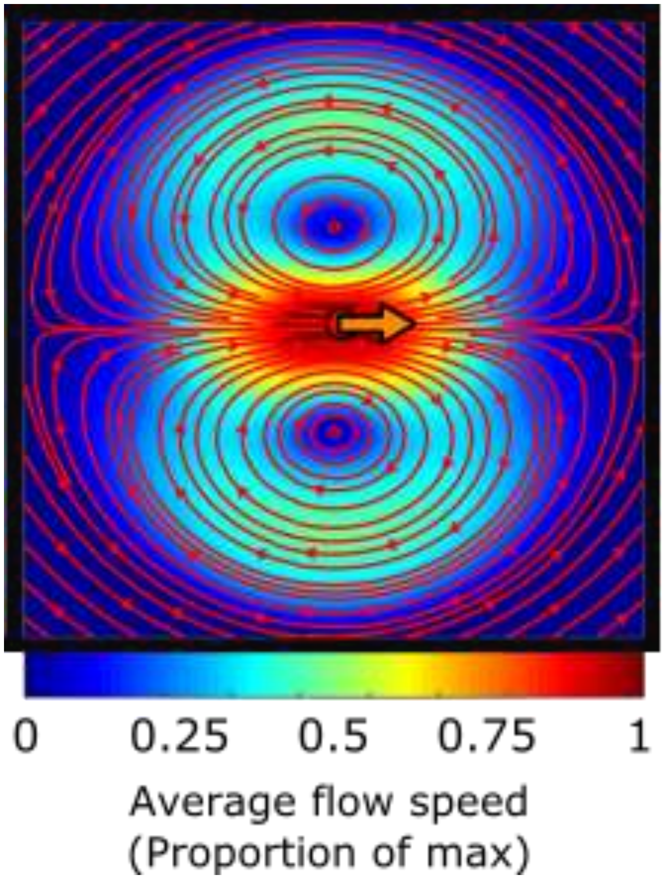


$$m = -\frac{1}{2}$$

Flow fields around defects

$$m = +\frac{1}{2}$$



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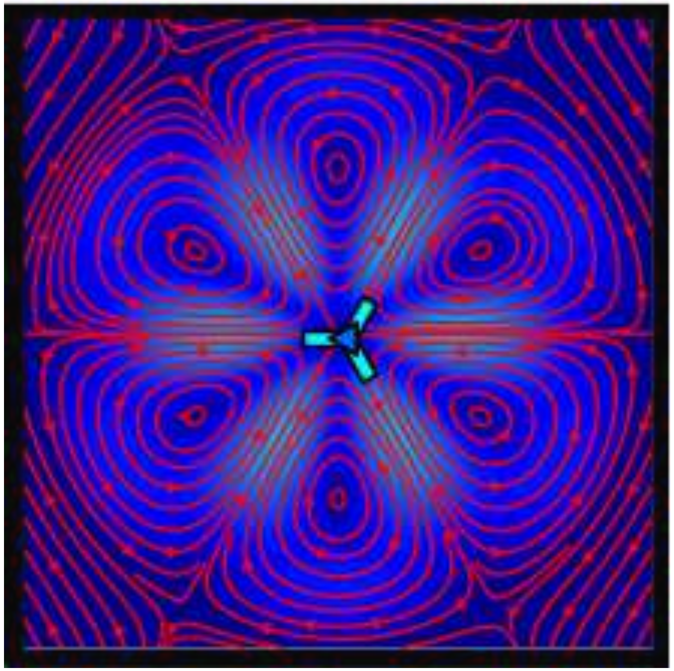
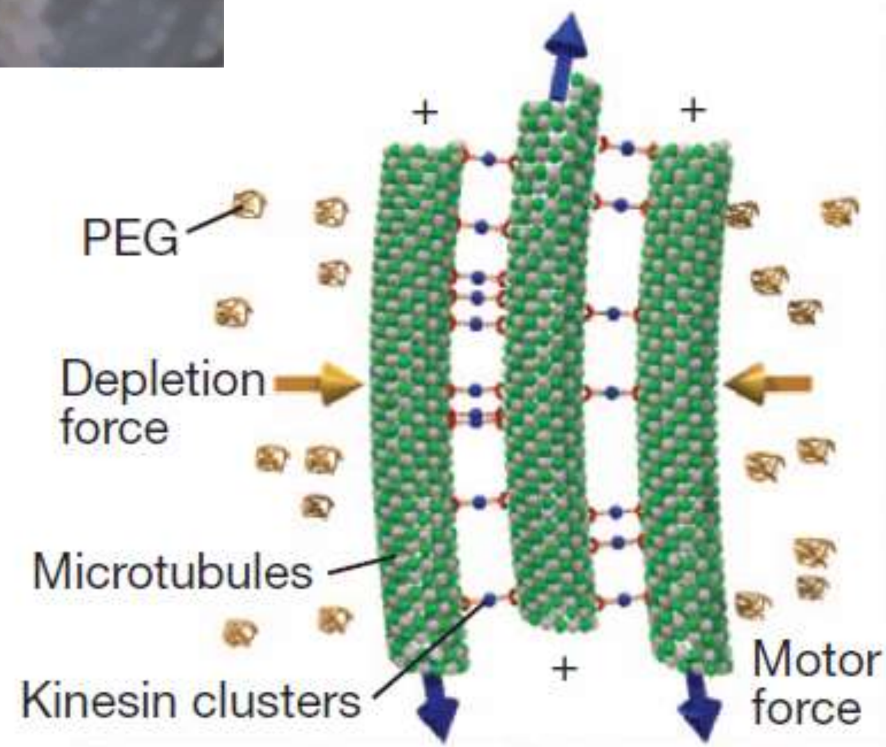
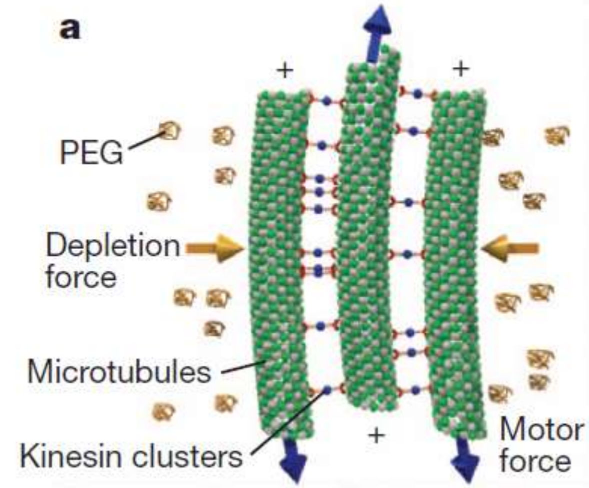


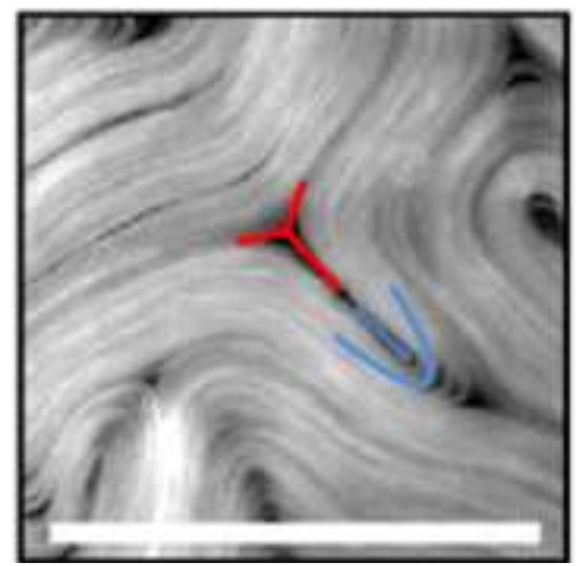


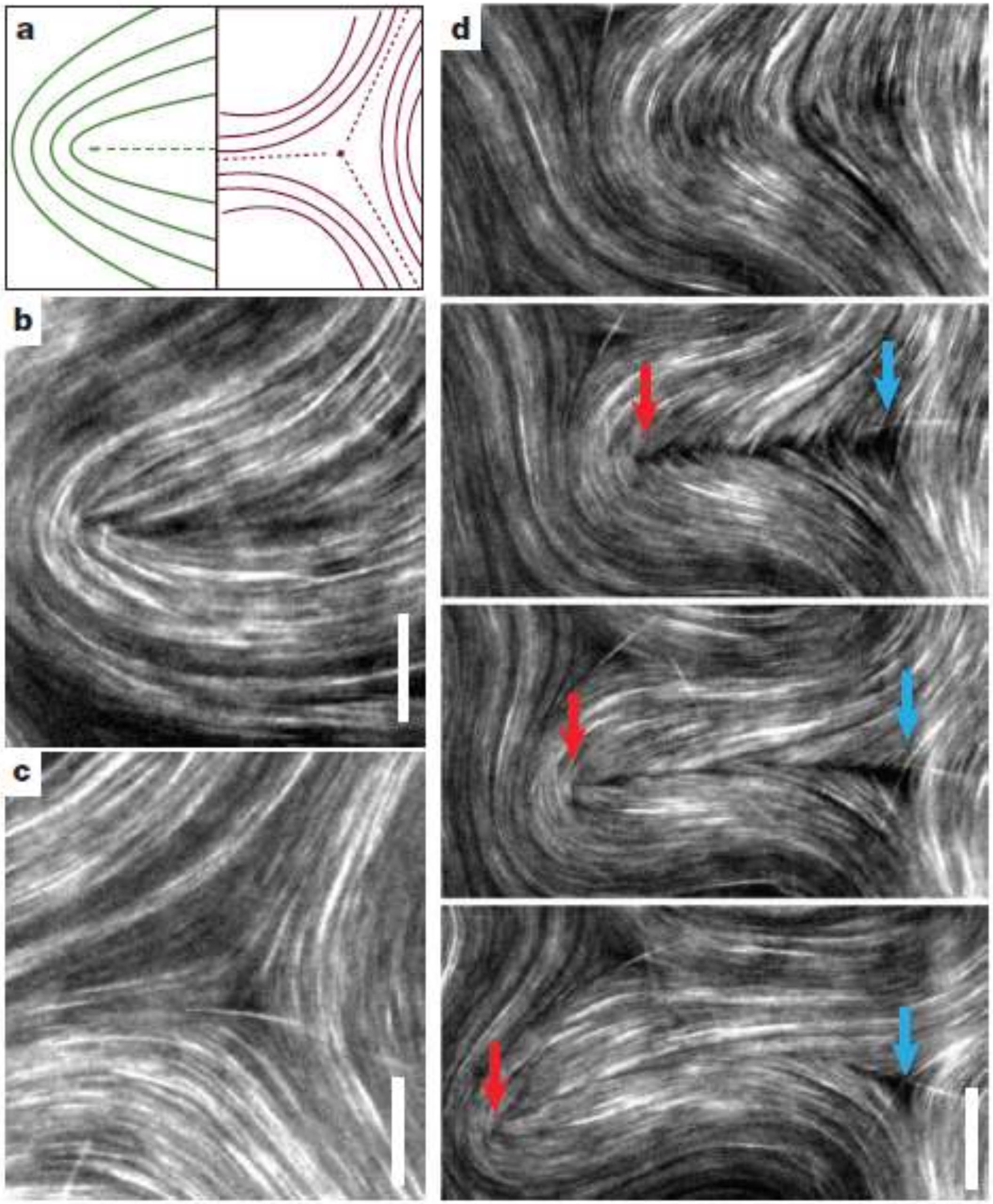
Dogic group, Brandeis

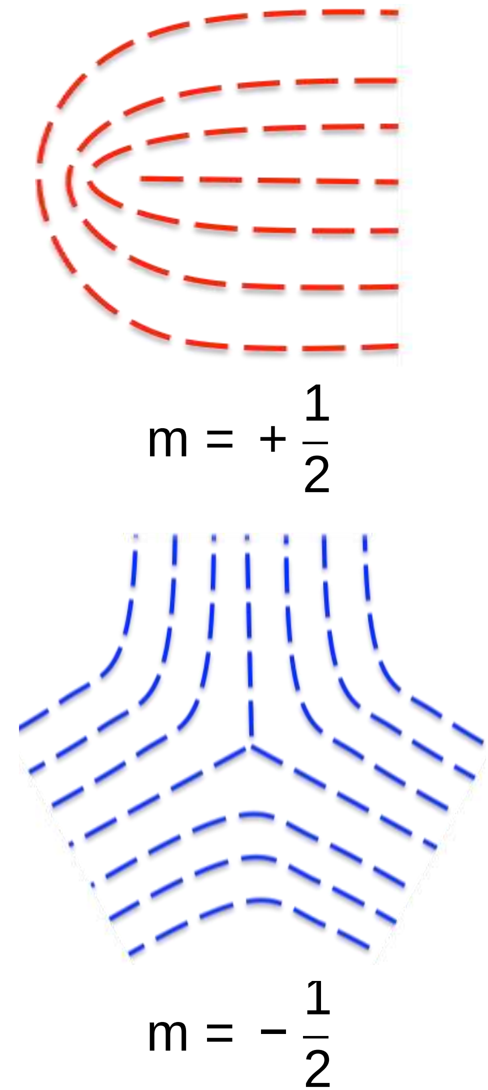
Active turbulence

Fluorescence Confocal Microscopy

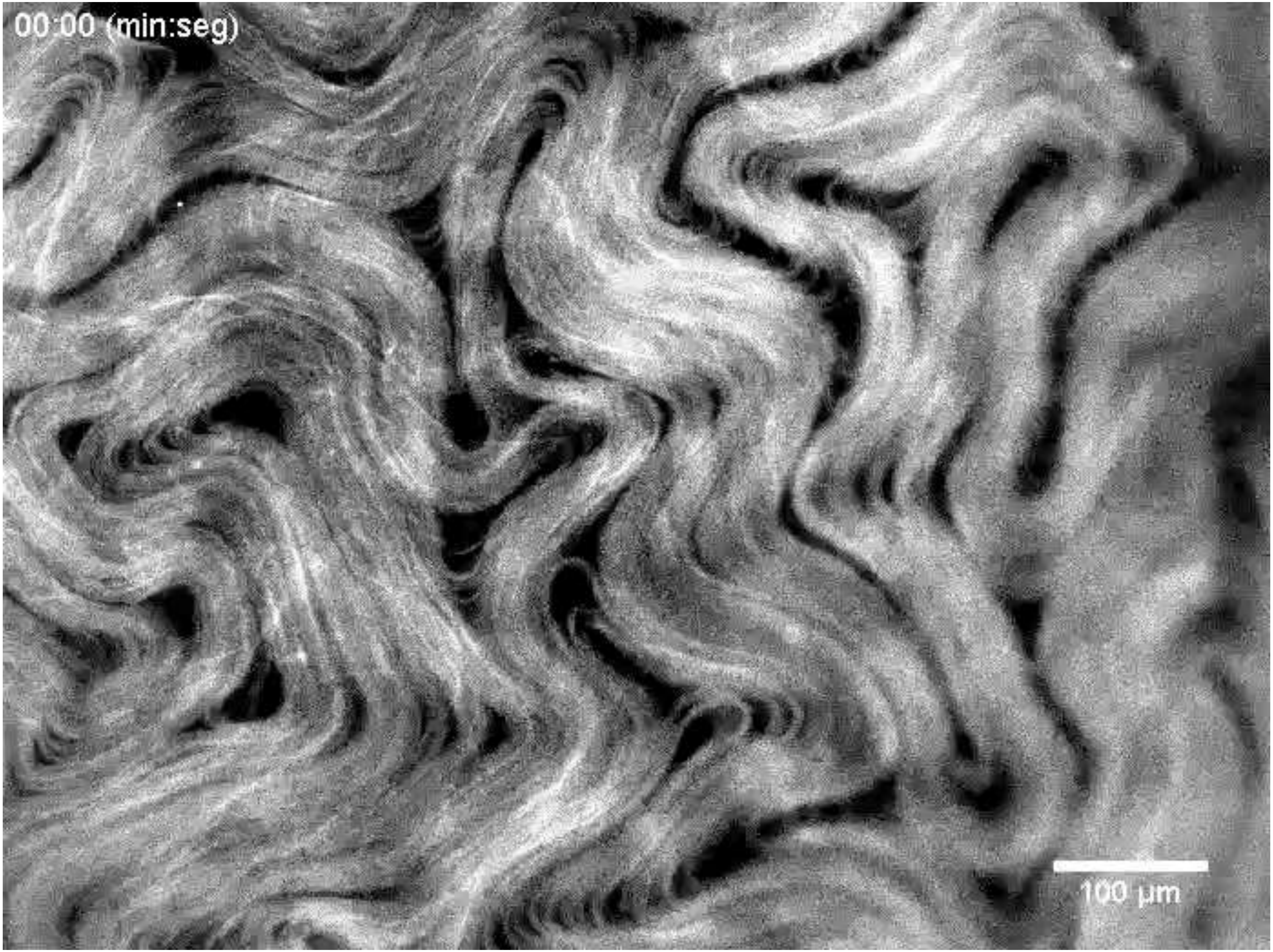








Sanchez, Chen, DeCamp, Heymann, Dogic, Nature 2012
L. Giomi, M.J. Bowick, Ma Xu, M.C. Marchetti, PRL 110, 228101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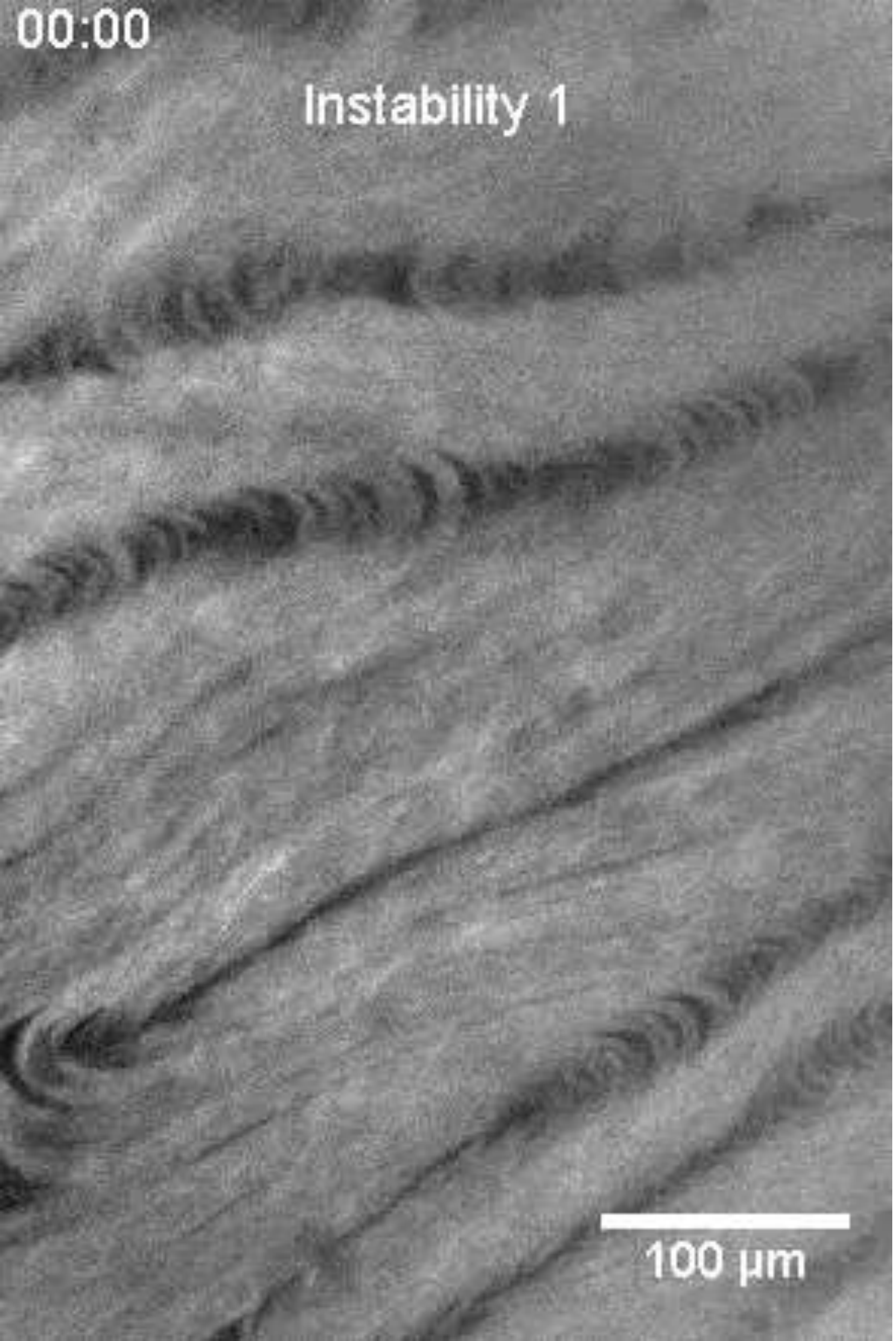
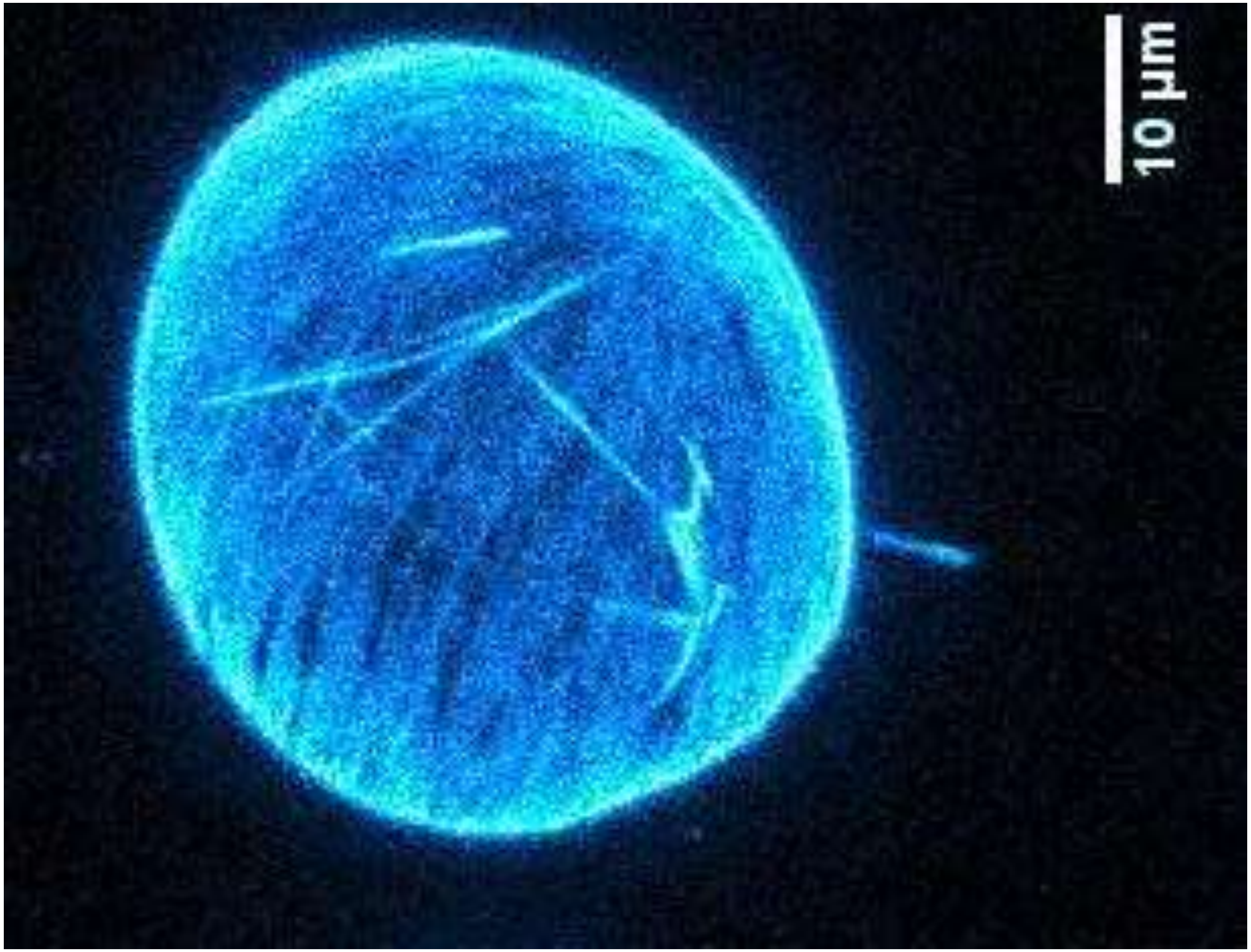


[Martínez-Prat](#) et al
Nature Physics **15**, 362 (2019)



FC Keber et al, Science, 2014

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Amin Doostmohammadi
Tyler Shendruk
Kristian Thyssen
Sumesh Thampi
Santhan Chandragiri
Jerome Hardouin
Rian Hughes

Justine Laurent
Jordi Ignes-Mullol
Teresa Lopex-Leon
Francesc Sagues

Why study confined active systems?

To control active turbulence

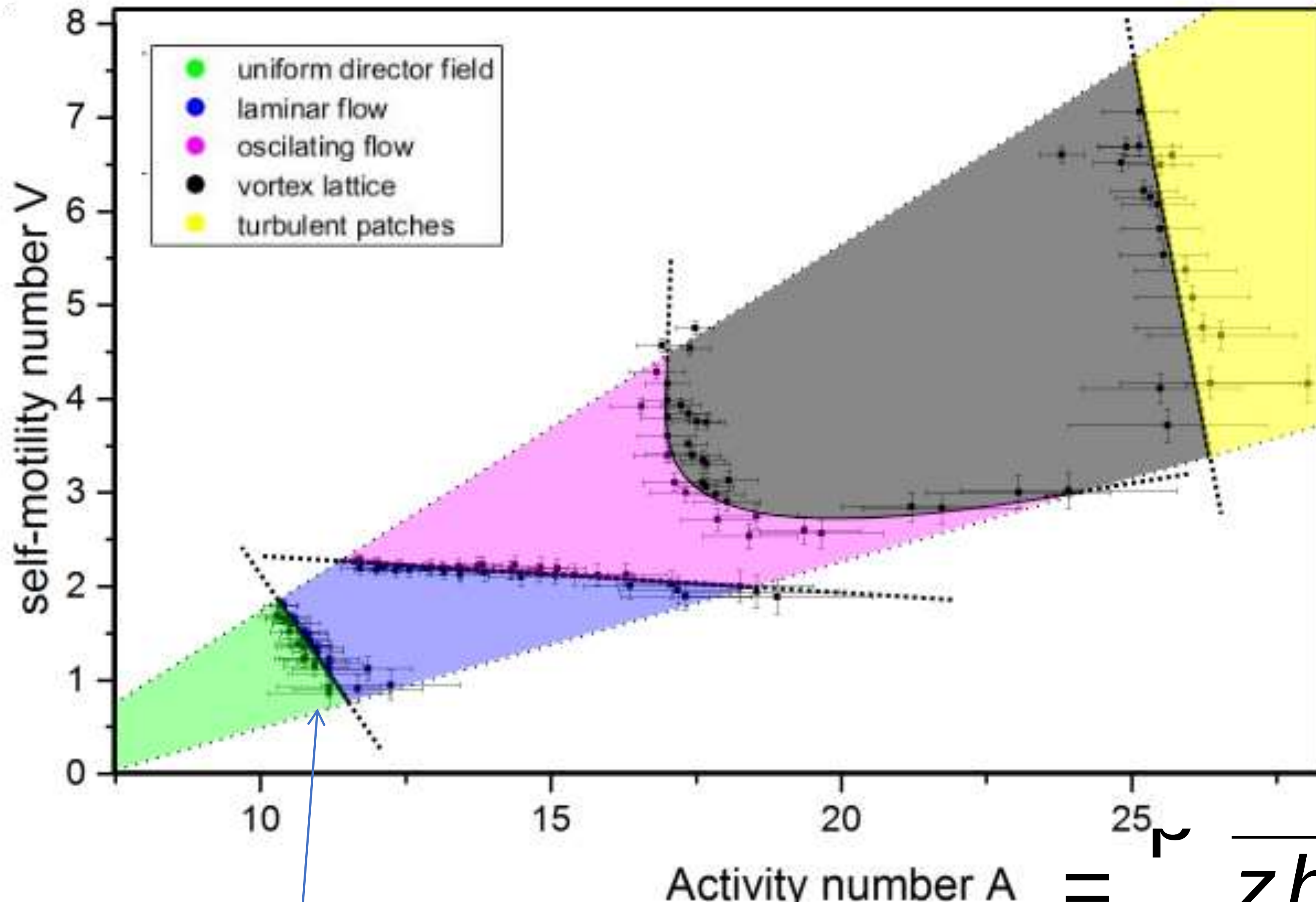
Many active systems are finite in extent – tissues,
organoids, tumours

The physics:

Two competing length scales

Motile topological defects

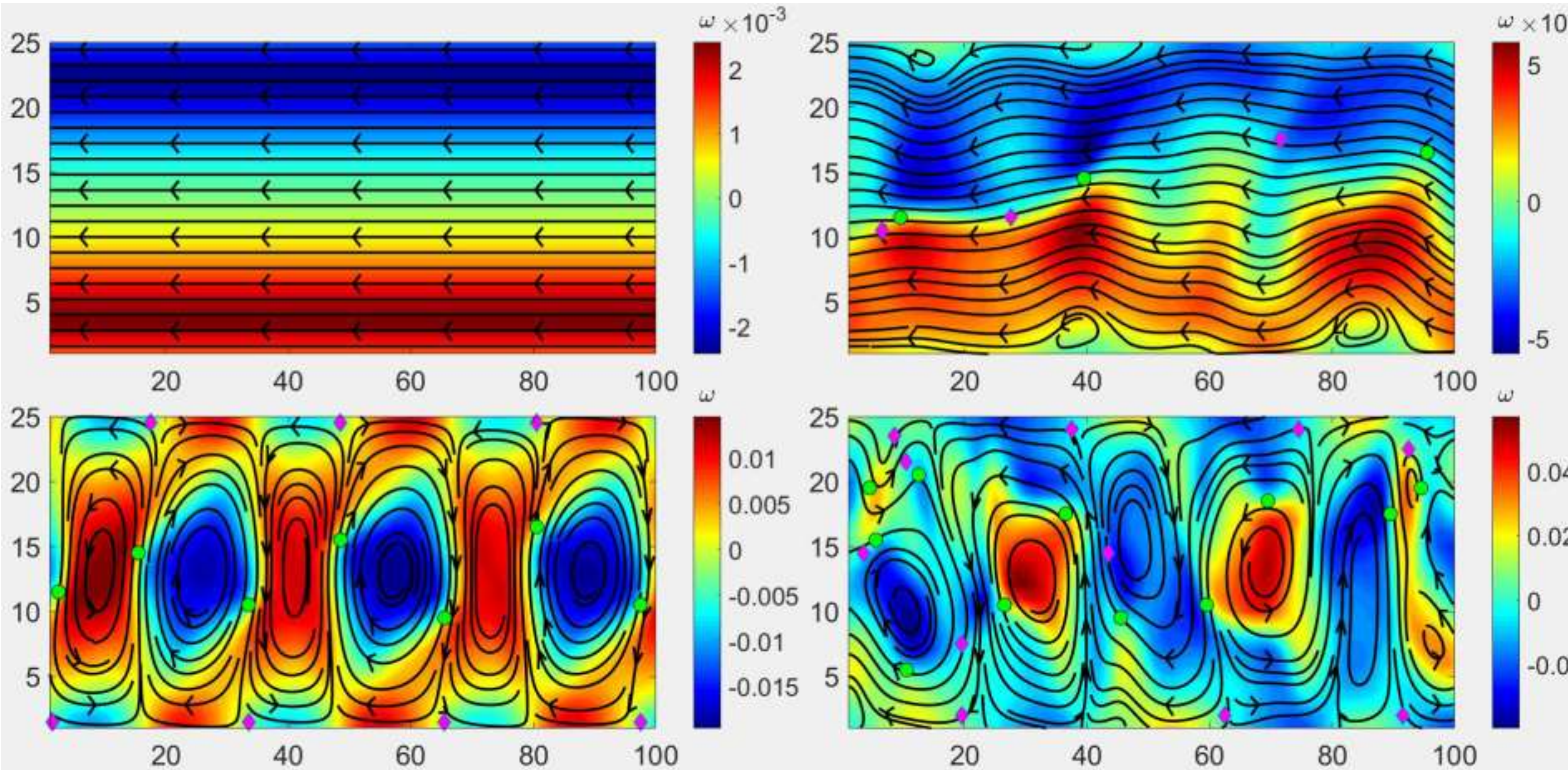
States of an Active Nematic in a Channel



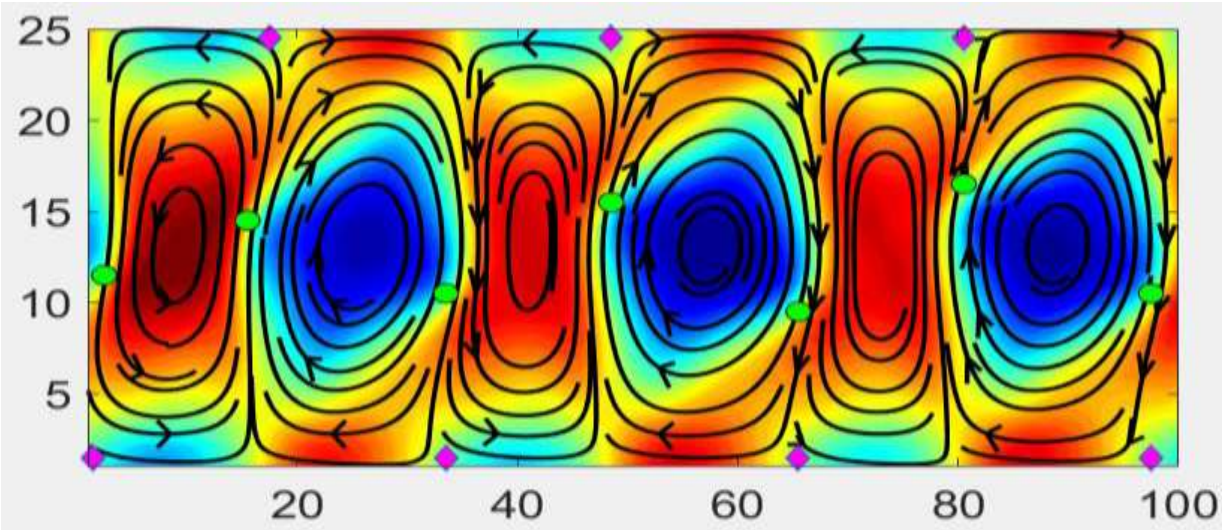
$= \sqrt{z h^2 / K}$

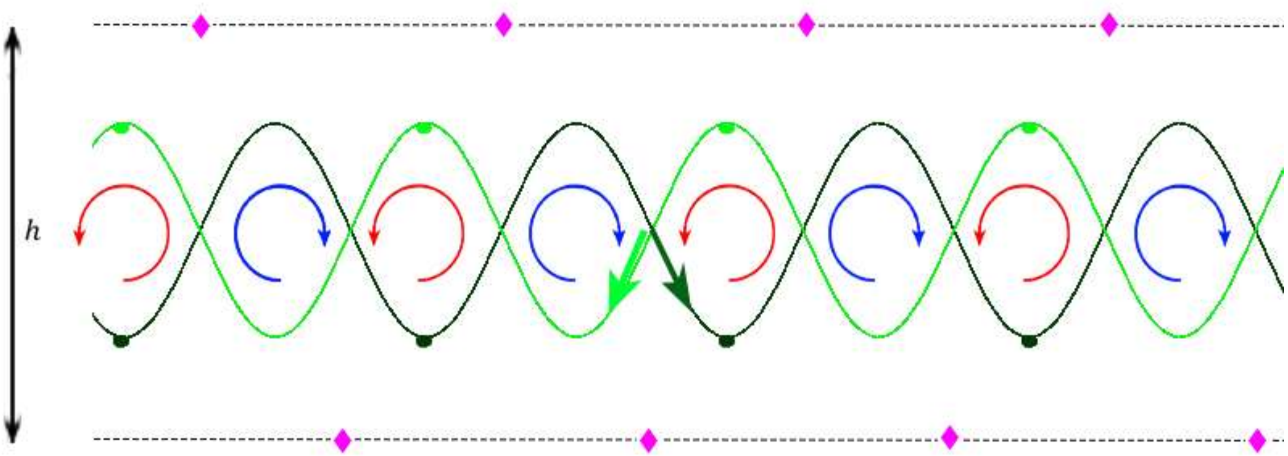
Voituriez et al
EPL (2005)

States of an Active Nematic in a Channel

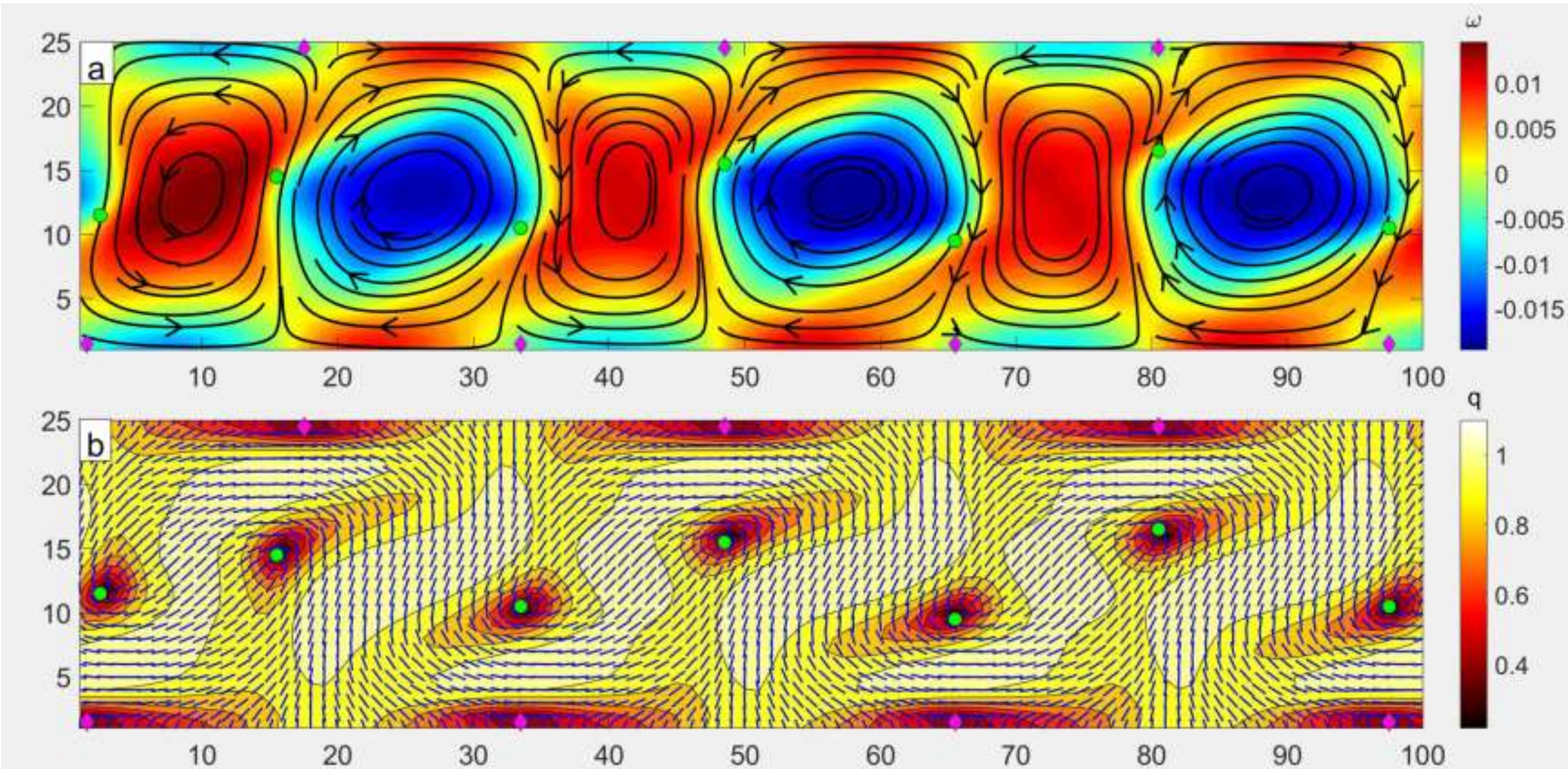


Ceilidh Dance





Vortex lattice and active topological microfluidics



No flow => laminar flow => the Ceilidh dance => active turbulence

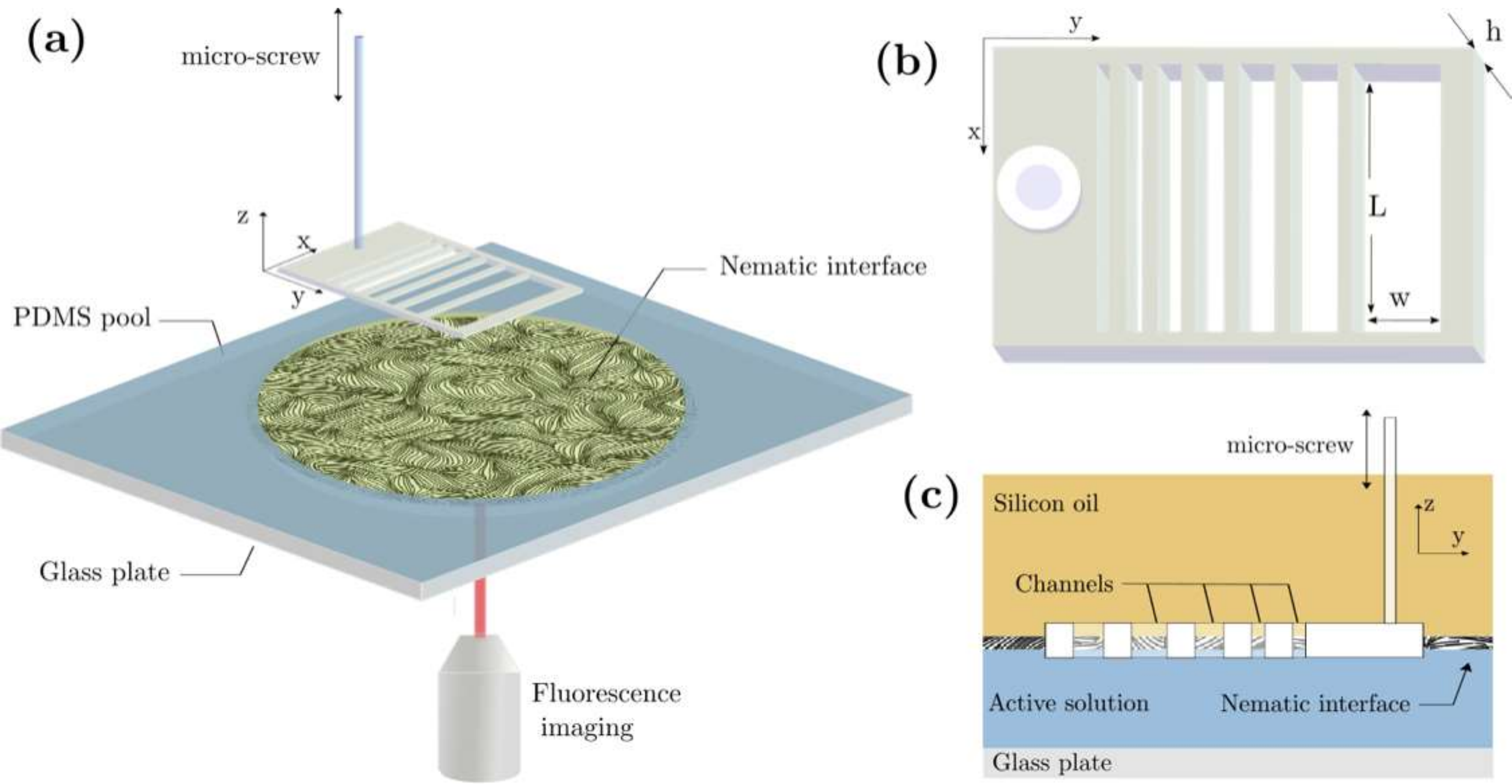
Increasing activity =>

<= Increasing confinement

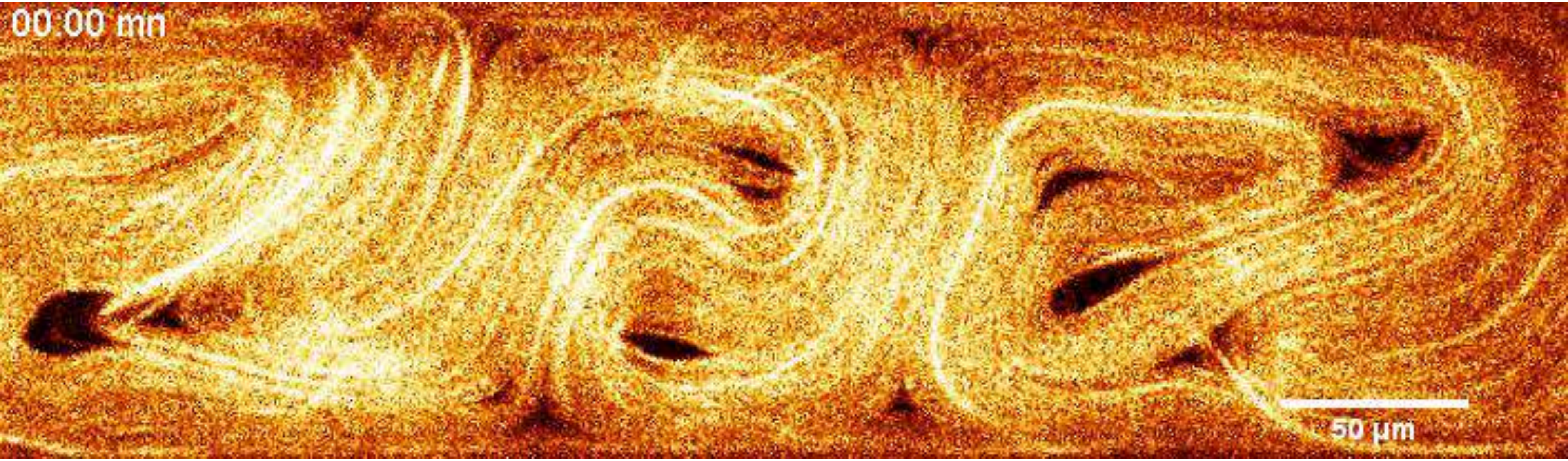
Microtubules and kinesin motors in channels

Hardouin et al, Communications Physics 2 (2019)

Widths 30 – 400 microns



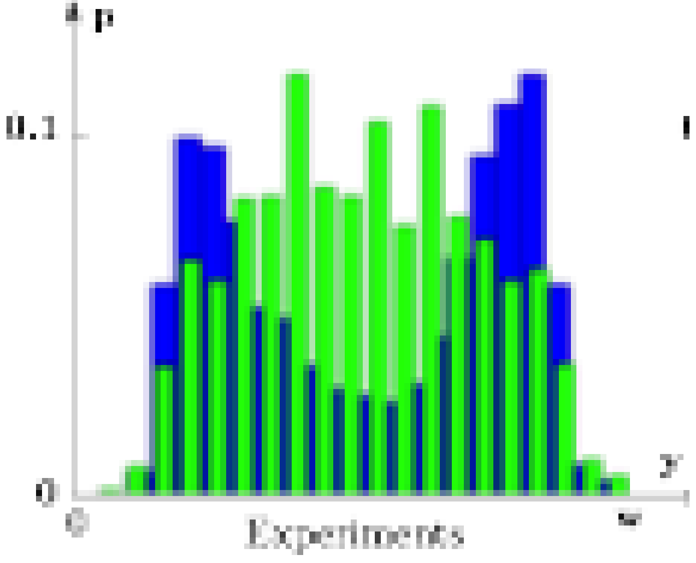
The dancing state in confined microtubule – kinesin mixtures

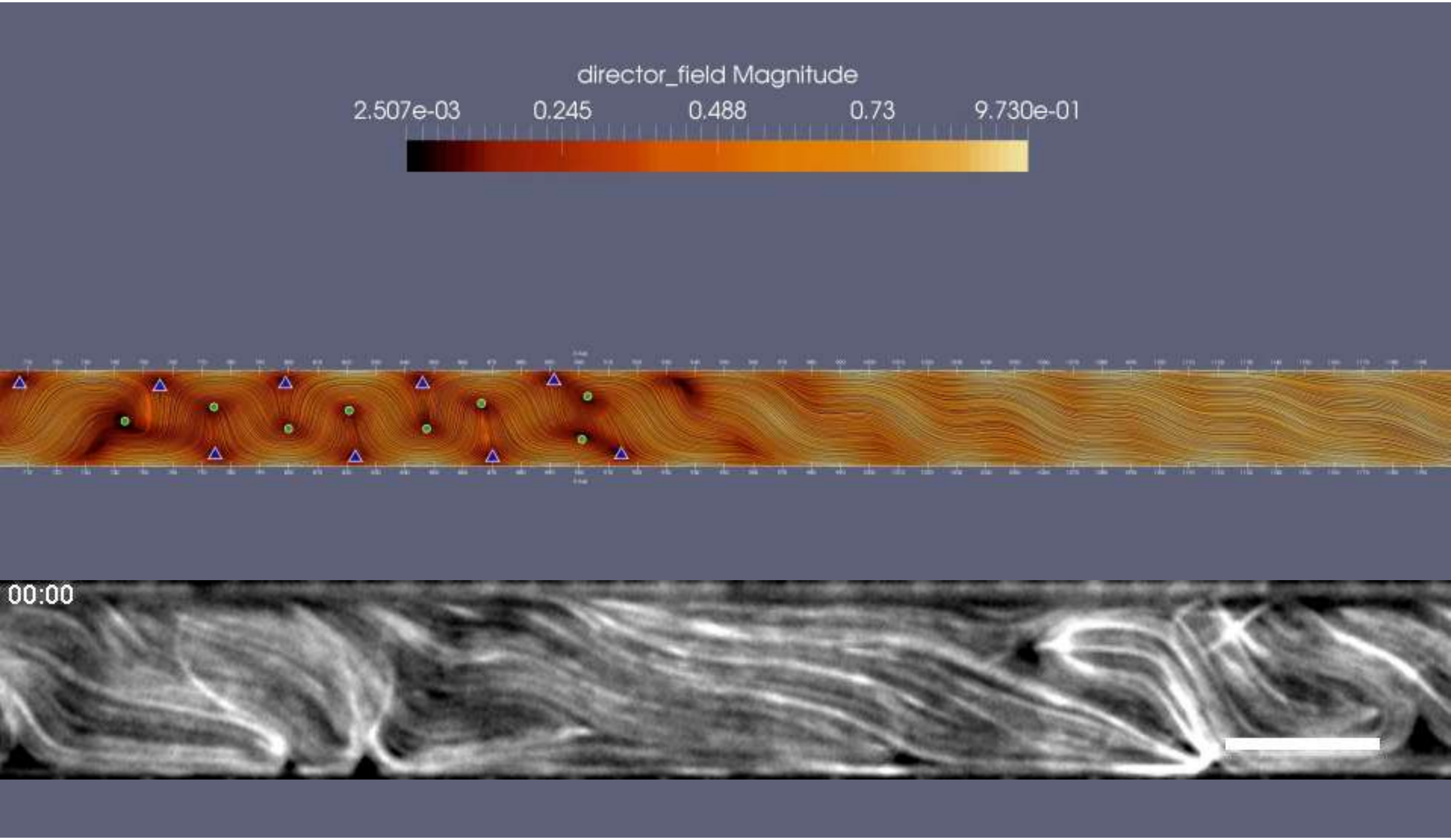


Distribution of defects across
the channel:

Blue -1/2

Green +1/2




director_field Magnitude
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0.488
0.73
9.730e-01
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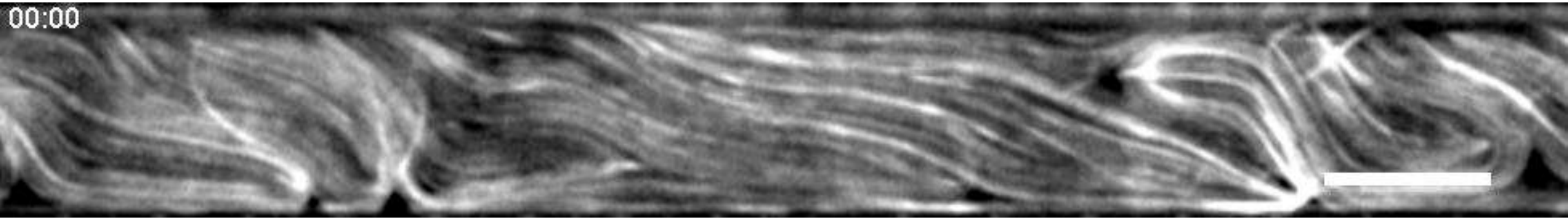
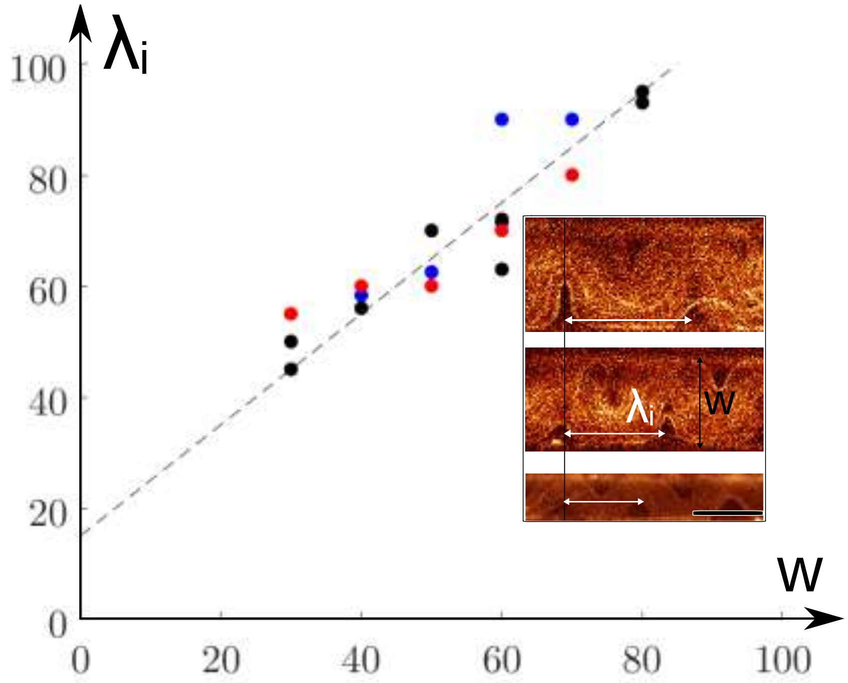
Shear + periodic bursts of defects

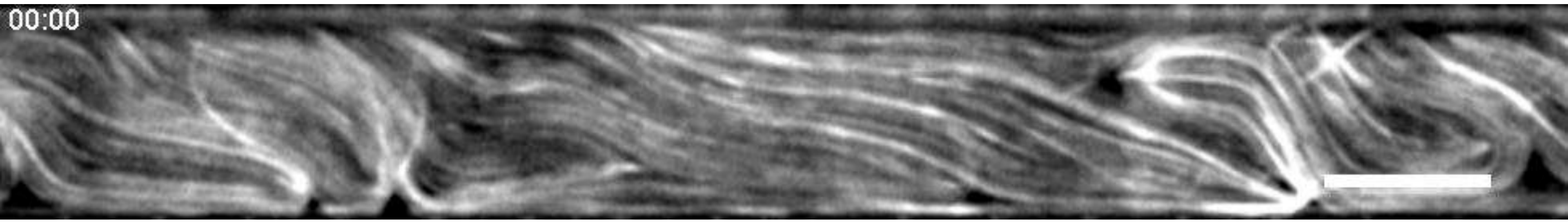


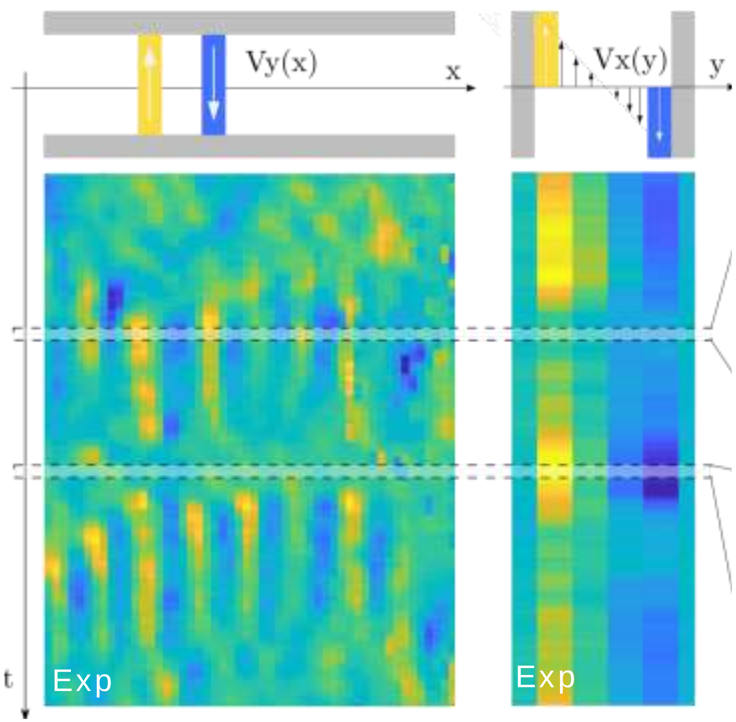


Distance between defects is set by the channel width

Shear + bursts of defects

Defect bursts are periodic





time