Active Matter
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1. What is active matter and why is it interesting?

2. Background 1: low Reynolds number hydrodynamics

3. Background 2: nematic liquid crystals

4. Active nematics and active turbulence

5. Self-propelled topological defects

6. Confining active turbulence

7. Bacteria: the hare and the tortoise

8. Eukaryotic cells as an active system
Active matter: takes energy from its surroundings on a single particle level and uses it to do work.
Kinesin walking, from Inner Life of a Cell
Bacterial flagellar motor
Active propulsion:
Pine group, New York

hematite

hydrogen peroxide
Active turbulence: bacteria

Dense suspension of microswimmers
Active turbulence: eukaryotic cells
Why study active matter:

1. to understand biological systems: biomechanics and self-assembly
2. To create new types of micro-engines
3. As examples of non-equilibrium statistical physics
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\[ \rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f} \]
\[ \rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f} \]

\textbf{inertial terms} \hspace{2cm} \textbf{viscous terms}

\[ \tilde{v} = \frac{v}{V_0} \hspace{2cm} \tilde{x} = \frac{x}{L_0} \]

\[ \tilde{\nabla} = L_0 \nabla \hspace{2cm} \tilde{t} = \frac{V_0}{L_0} t \hspace{2cm} \frac{\partial}{\partial \tilde{t}} = \frac{L_0}{V_0} \frac{\partial}{\partial t} \]
\[
\frac{V_0^2}{L_0 \rho} \left\{ \frac{\partial \tilde{v}}{\partial \tilde{t}} + (\tilde{v} \cdot \tilde{\nabla}) \tilde{v} \right\} = -\nabla p + \frac{V_0}{L_0^2 \mu} \tilde{\nabla}^2 \tilde{v} + \mathbf{f}
\]

\[
\left\{ \frac{\partial \tilde{v}}{\partial \tilde{t}} + (\tilde{v} \cdot \tilde{\nabla}) \tilde{v} \right\} = -\frac{L_0}{V_0^2 \rho} \nabla p + \frac{\mu}{L_0 V_0 \rho} \tilde{\nabla}^2 \tilde{v} + \frac{L_0}{V_0^2 \rho} \mathbf{f}
\]

\[
\text{Re} = \frac{\text{inertial response}}{\text{viscous response}} \sim \frac{\rho L_0 V_0}{\mu}
\]
\[
\frac{V_0^2}{L_0} \rho \left\{ \frac{\partial \tilde{v}}{\partial t} + (\tilde{v} \cdot \nabla) \tilde{v} \right\} = -\nabla p + \frac{V_0}{L_0^2} \mu \tilde{V}^2 \tilde{v} + f
\]

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\left\{ \frac{\partial \tilde{v}}{\partial \tilde{t}} + (\tilde{v} \cdot \tilde{\nabla}) \tilde{v} \right\} = -\frac{L_0}{V_0^2 \rho} \nabla p + \frac{\mu}{L_0 V_0 \rho} \tilde{\nabla}^2 \tilde{v} + \frac{L_0}{V_0^2 \rho} f
\]

\[
\text{Re} = \frac{\text{inertial response}}{\text{viscous response}} \sim \frac{\rho L_0 V_0}{\mu} \sim 10^{-6}
\]
\[
\frac{V_0^2}{L_0 \rho} \left\{ \frac{\partial \tilde{\mathbf{v}}}{\partial t} + (\tilde{\mathbf{v}} \cdot \nabla) \tilde{\mathbf{v}} \right\} = -\nabla p + \frac{V_0}{L_0^2 \mu} \tilde{\nabla}^2 \tilde{\mathbf{v}} + \mathbf{f}
\]

\[
\left\{ \frac{\partial \tilde{\mathbf{v}}}{\partial t} + (\tilde{\mathbf{v}} \cdot \nabla) \tilde{\mathbf{v}} \right\} = -\frac{L_0}{V_0^2 \rho} \nabla p + \frac{\mu}{L_0 V_0 \rho} \tilde{\nabla}^2 \tilde{\mathbf{v}} + \frac{L_0}{V_0^2 \rho} \mathbf{f}
\]

\[
\text{Re} = \frac{\text{inertial response}}{\text{viscous response}} \approx \frac{\rho L_0 V_0}{\mu}
\]

\[
\text{Re} \sim 0
\]
\[ \nabla p = \mu \nabla^2 v + f \]
\[ \nabla \cdot v = 0 \]
Purcell’s Scallop Theorem

A swimmer strokes must be non-invariant under time reversal
Green function of the Stokes equation (Stokeslet)

\[ \mathbf{v}(\mathbf{r}) = \frac{\mathbf{f}}{8\pi\mu} \cdot \left( \frac{\mathbf{I}}{r} + \frac{\mathbf{r}\mathbf{r}}{r^3} \right) \]

\[ v_i(\mathbf{r}) = \frac{f_j}{8\pi\mu} \left( \frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3} \right) \equiv G_{ij}(\mathbf{r}) f_j \]
Swimmers have dipolar far flow fields because they have no net force acting on them.
v(r) = \frac{f}{8\pi\mu} \cdot \left( \frac{I}{r_1} + \frac{r_1 r_1}{r_1^3} \right) - \frac{f}{8\pi\mu} \cdot \left( \frac{I}{r_2} + \frac{r_2 r_2}{r_2^3} \right)
Swimmers have dipolar far flow fields because they have no net force acting on them.

\[ v_r = \frac{f}{4\pi \mu} \frac{L}{r^2} \left(3\cos^2\theta - 1\right) \]
Swimmer and colloidal flow fields

\[ v \sim \frac{1}{r^2} \]

\[ v \sim \frac{1}{r} \]
Far from walls

Goldstein group, Cambridge

E-coli
Extensile Pusher

Contractile Puller
Extensile Pusher

Contractile Puller

Flow field has nematic symmetry
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nematic symmetry
nematic symmetry

nematic phase

\[ Q_{ij} = \langle n_i n_j - \frac{\delta_{ij}}{3} \rangle \]
Tensor order parameter, $Q$

$$Q_{ij} = \langle n_i n_j - \frac{\delta_{ij}}{3} \rangle$$

Landau-de Gennes free energy

$$F = \frac{K}{2} \left( \partial_k Q_{ij} \right)^2 + \frac{A}{2} Q_{ij} Q_{ji} + \frac{B}{3} Q_{ij} Q_{jk} Q_{ki} + \frac{C}{4} \left( Q_{ij} Q_{ji} \right)^2$$

An ‘elastic liquid’
Topological defects in nematic liquid crystals

\[ m = \mp \frac{1}{2} \]

\[ m = \frac{1}{2\pi} \int_{dS} d\theta \]

topological charge,
liquid crystals

crystal dislocations

magnetic monopoles in spin ice
topological insulators
quantum vortex in a superfluid

cosmic strings in the early universe