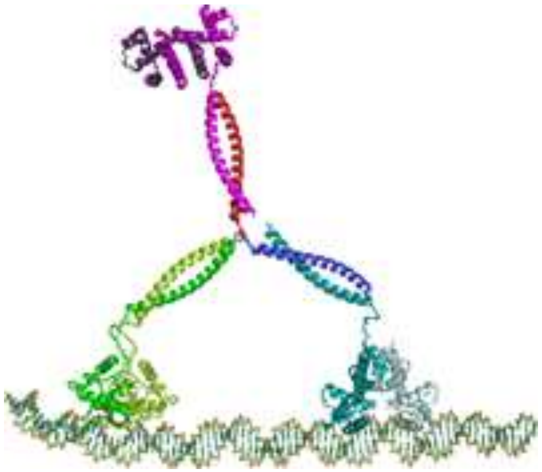
Active Matter

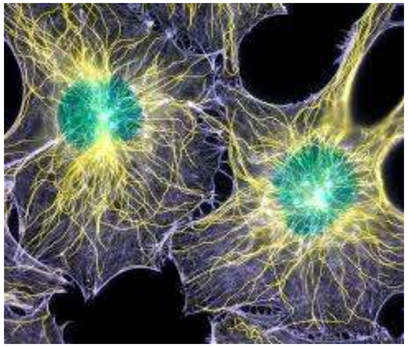
Julia Yeomans
University of Oxford

1. What is active matter and why is it interesting?
2. Background 1: low Reynolds number hydrodynamics
3. Background 2: nematic liquid crystals
4. Active nematics and active turbulence
5. Self-propelled topological defects
6. Confining active turbulence
7. Bacteria: the hare and the tortoise
8. Eukaryotic cells as an active system

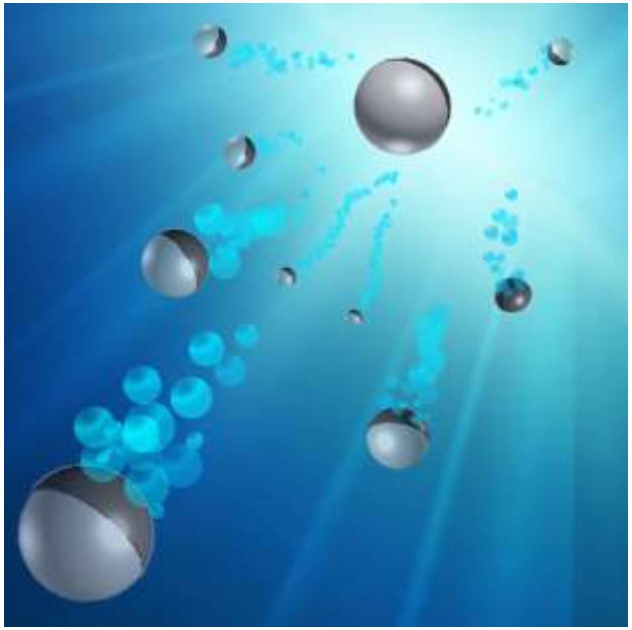
Active matter: takes energy from its surroundings on a single particle level and uses it to do work.

molecular motors





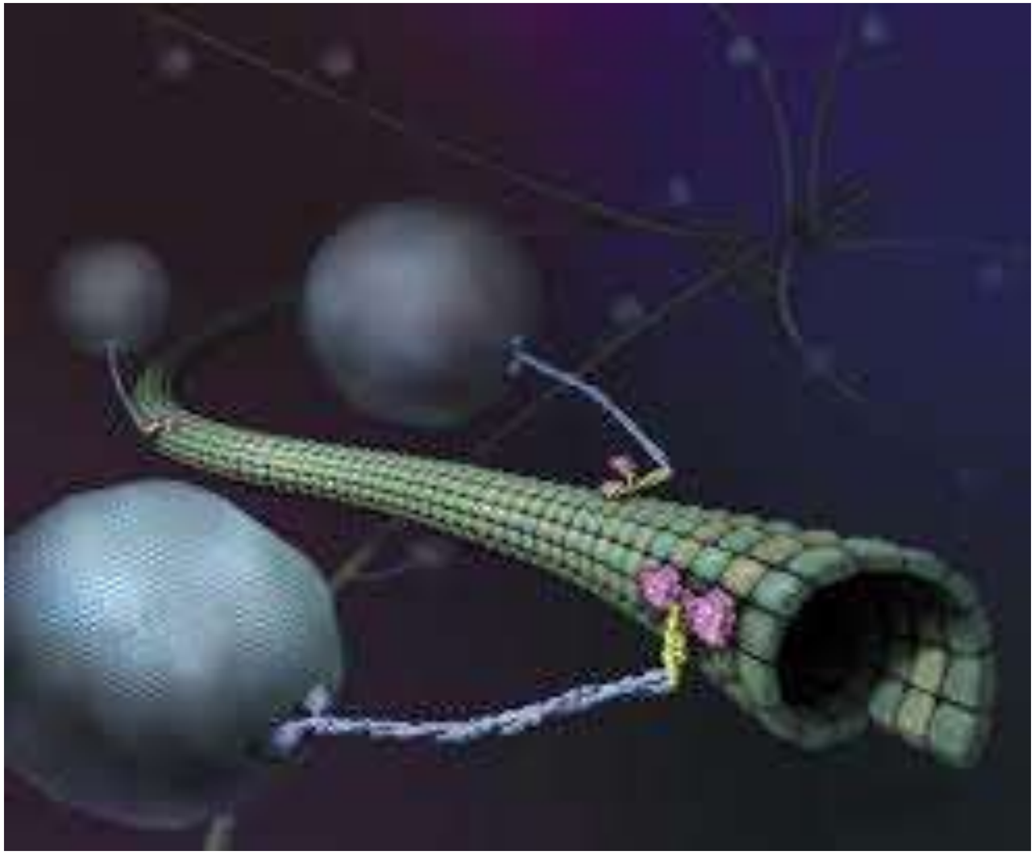
cells

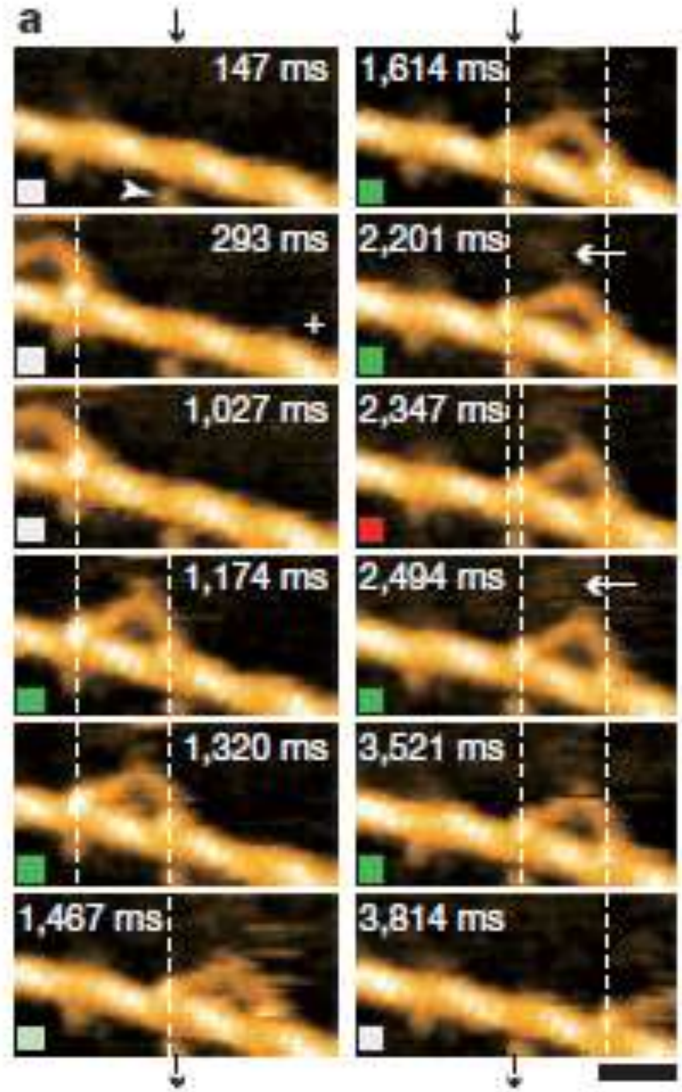


active colloids

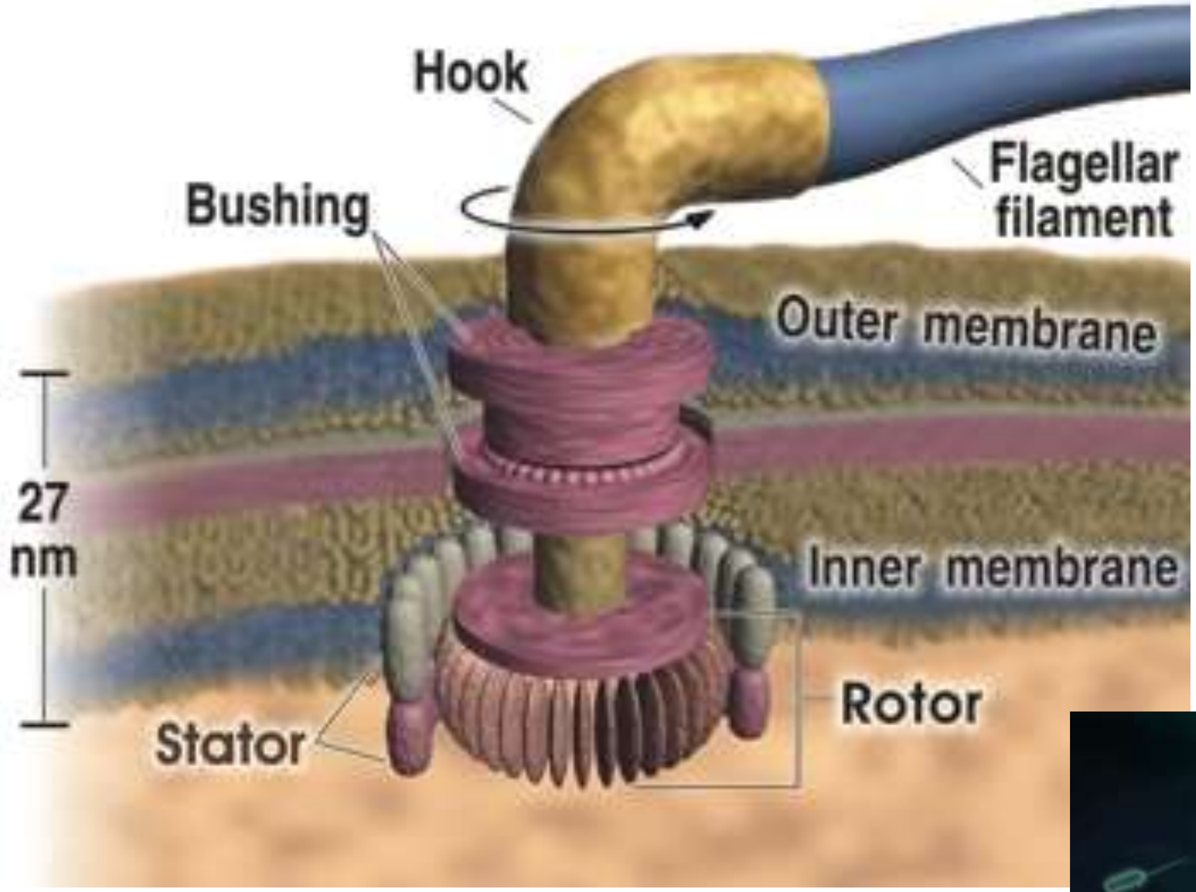
microswimmers

Kinesin walking, from Inner Life of a Cell

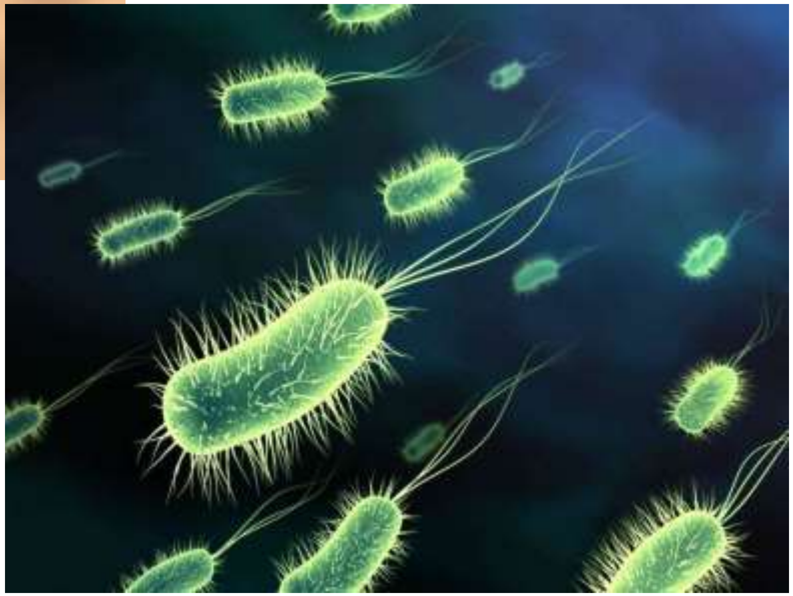


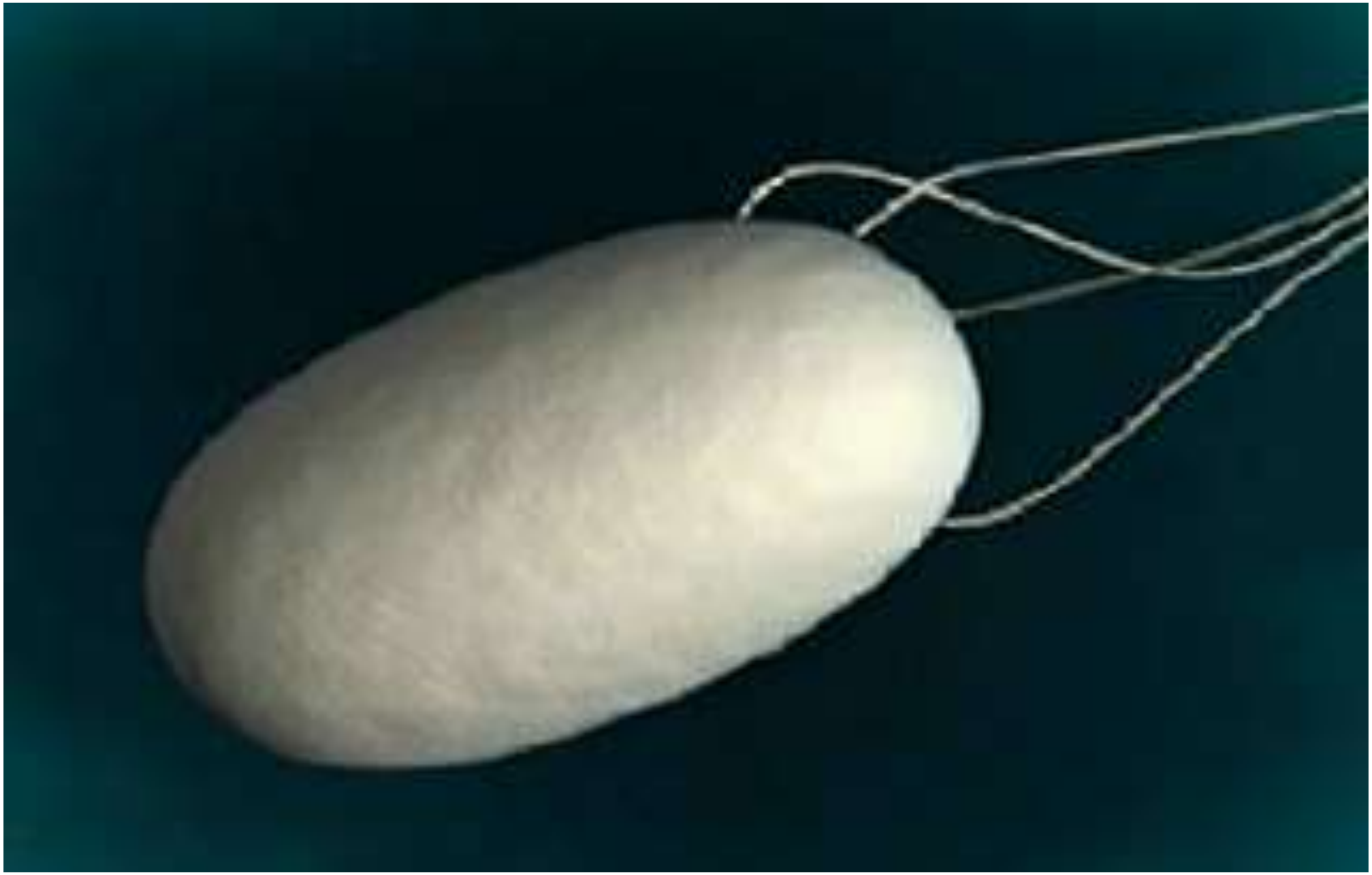


a
147 ms
293 ms
1,027 ms
1,174 ms
1,320 ms
1,467 ms
1,614 ms
2,201 ms
2,347 ms
2,494 ms
3,521 ms
3,814 ms

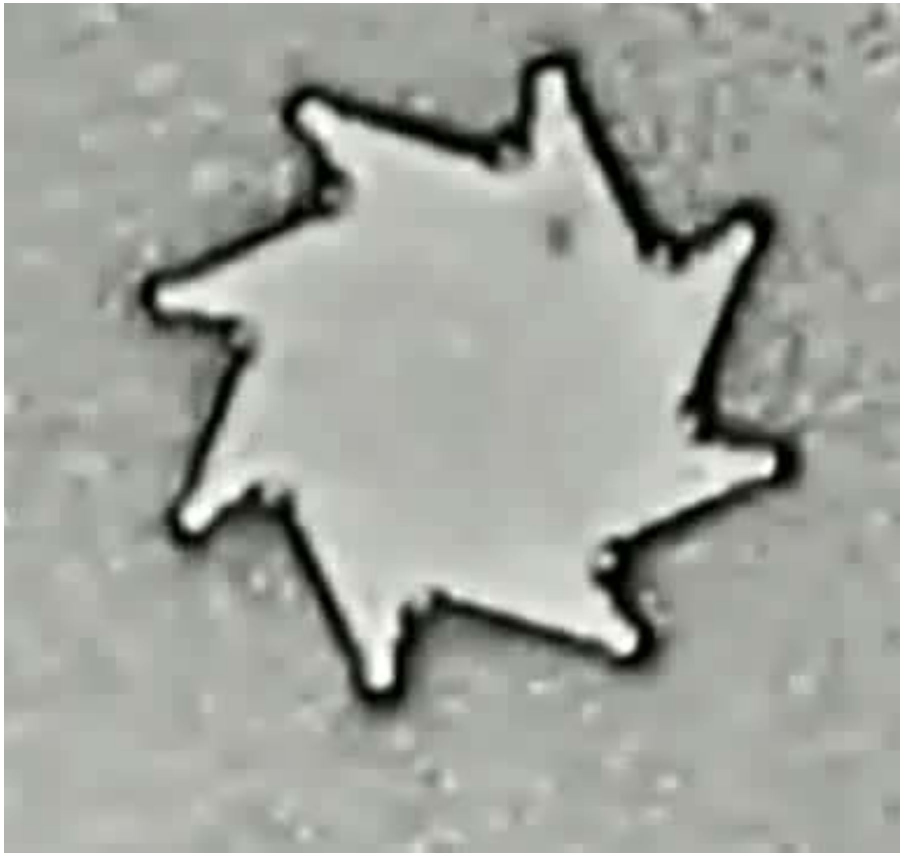


Bacterial flagellar motor

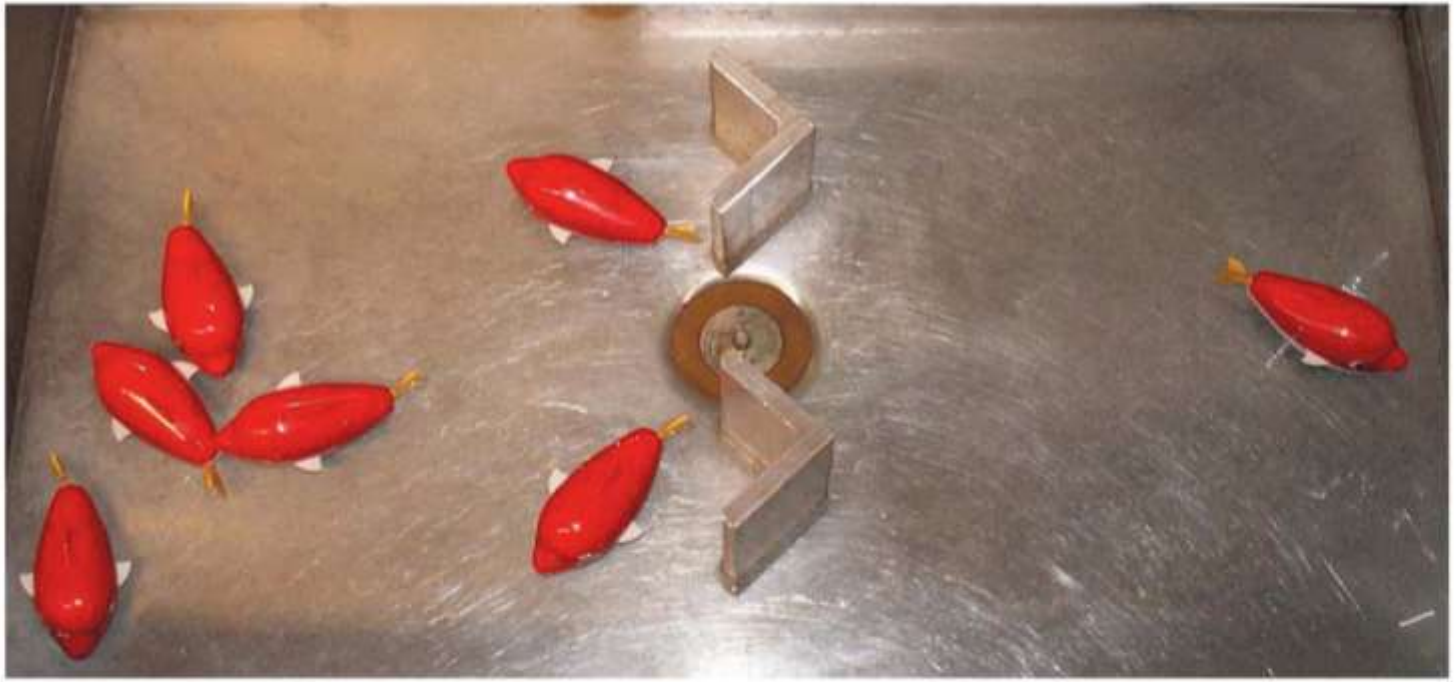


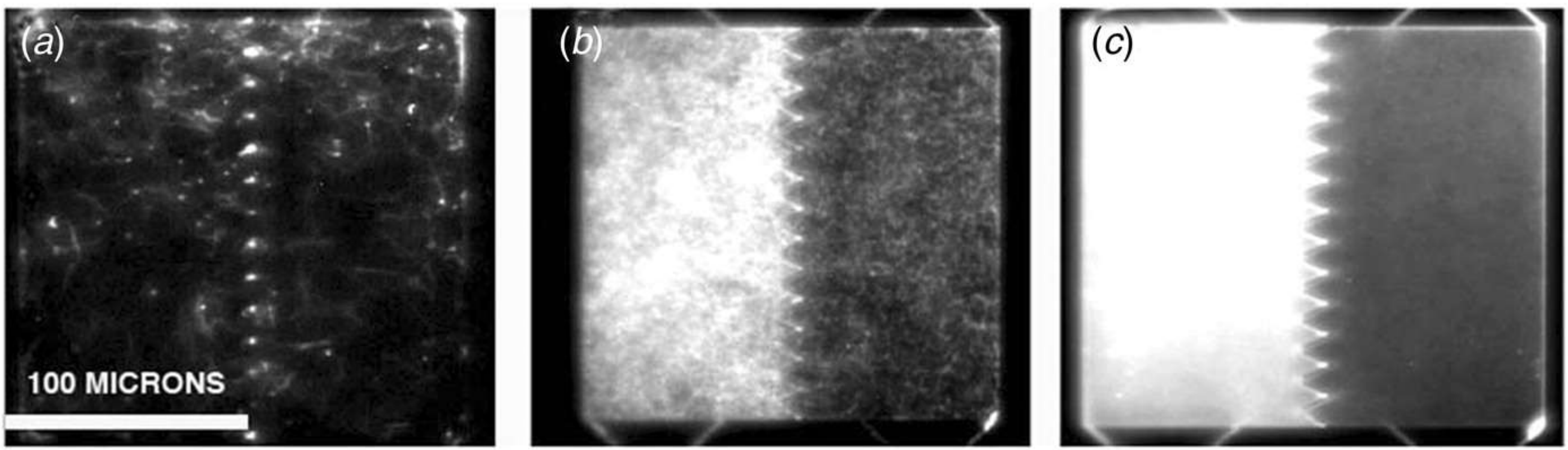


Active propulsion:

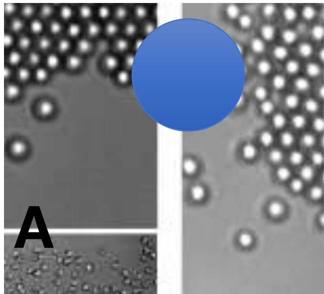


Di Leonardo, Sokolov,



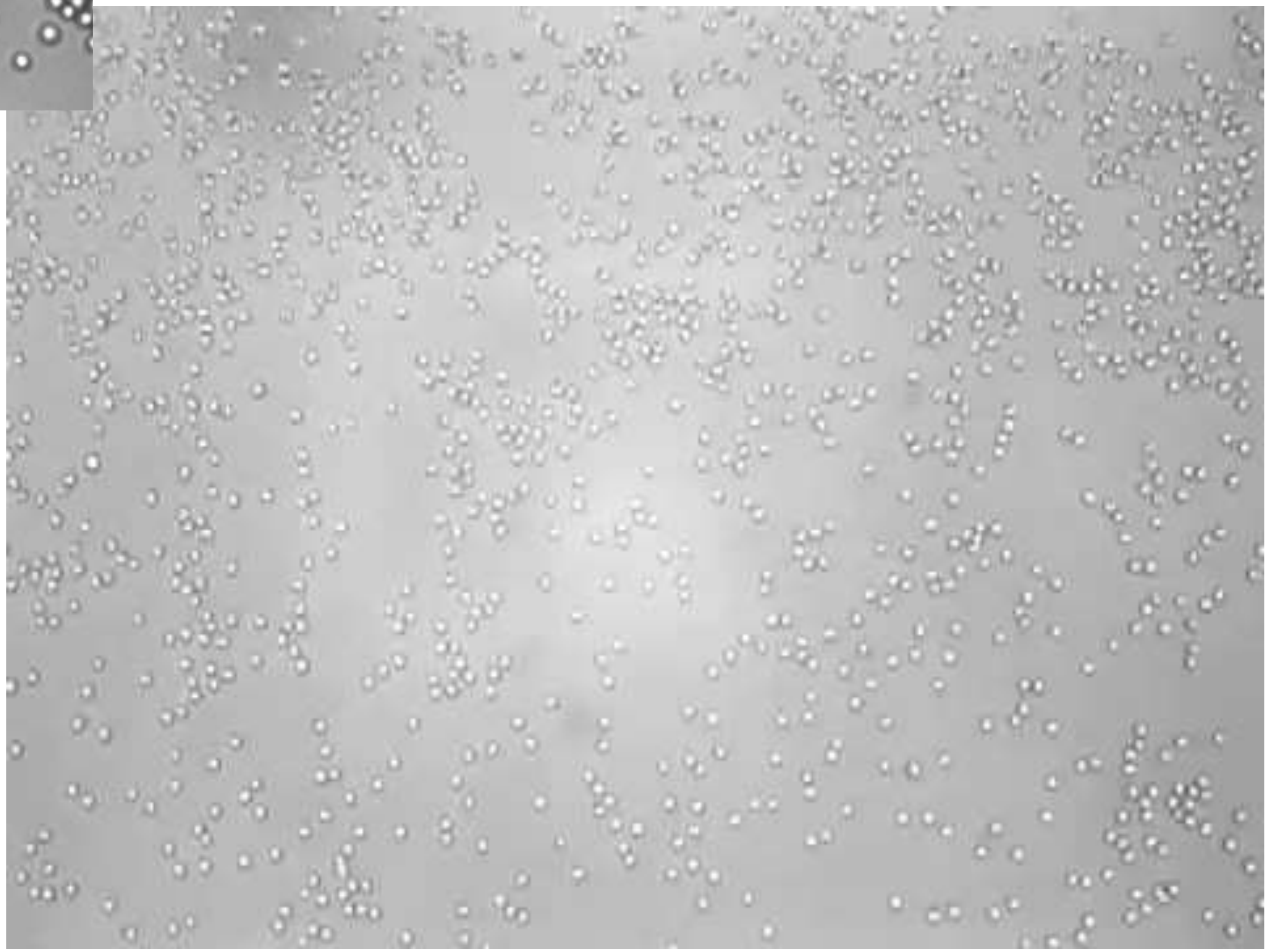


Galajda et al, J Modern Optics 2011



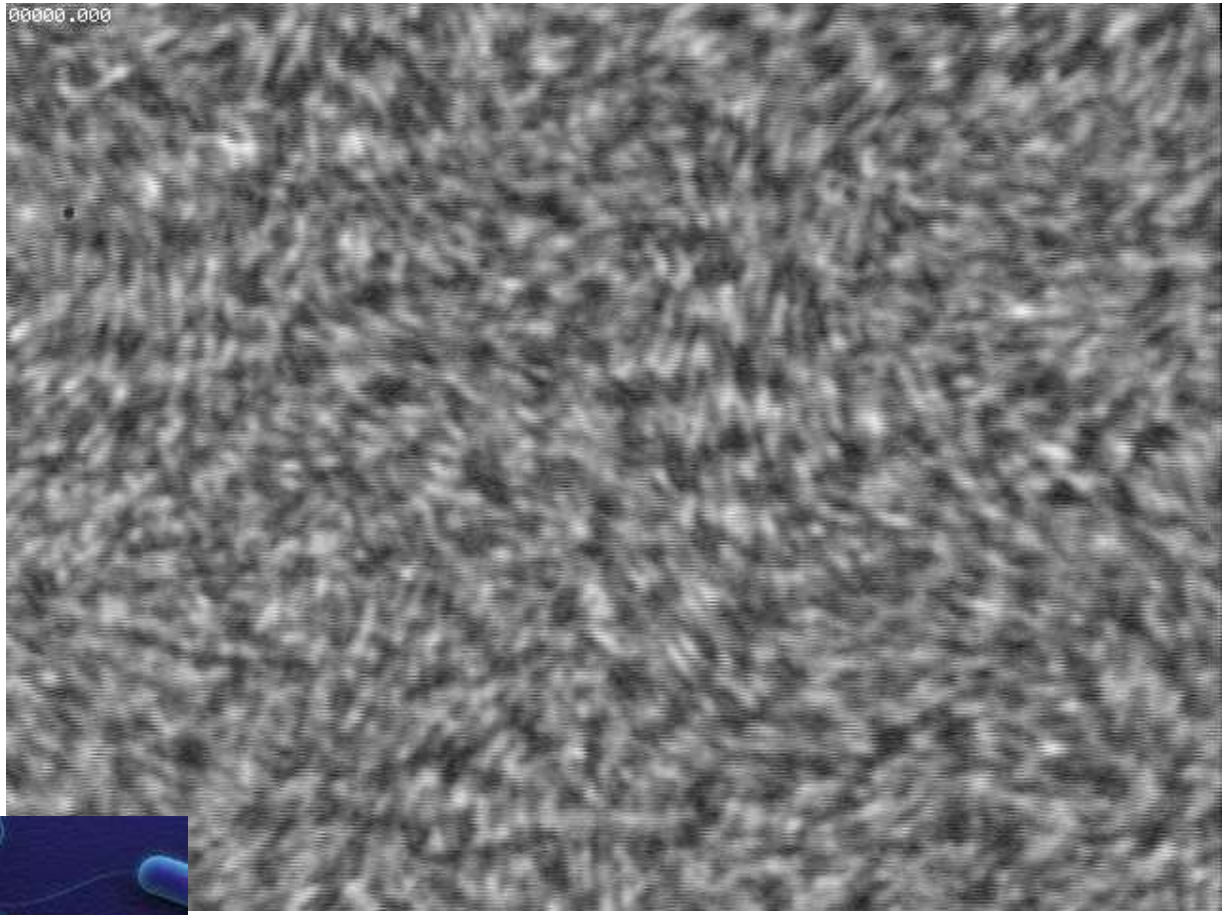
heamatite

hydrogen peroxide



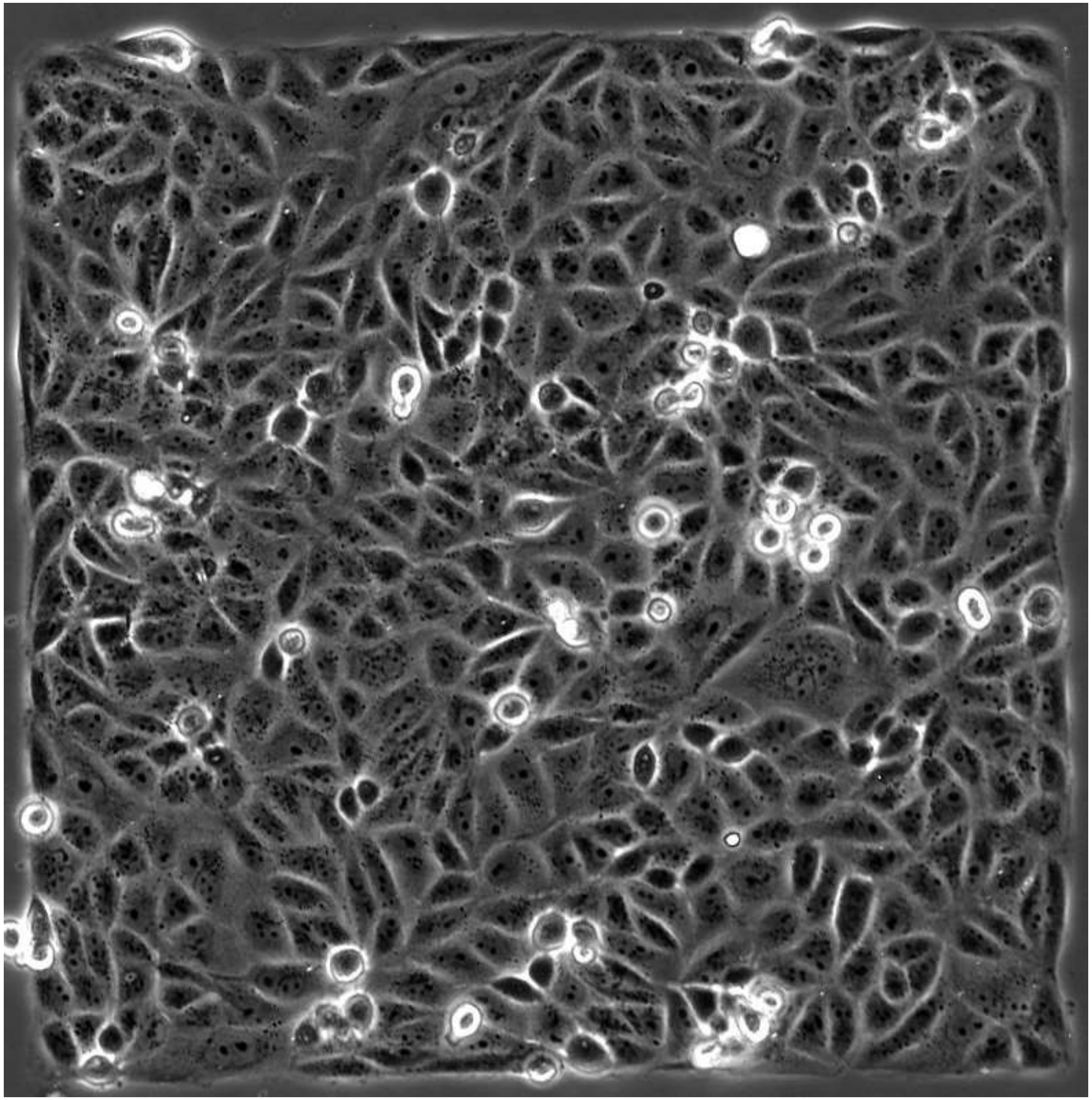
Pine group, New York

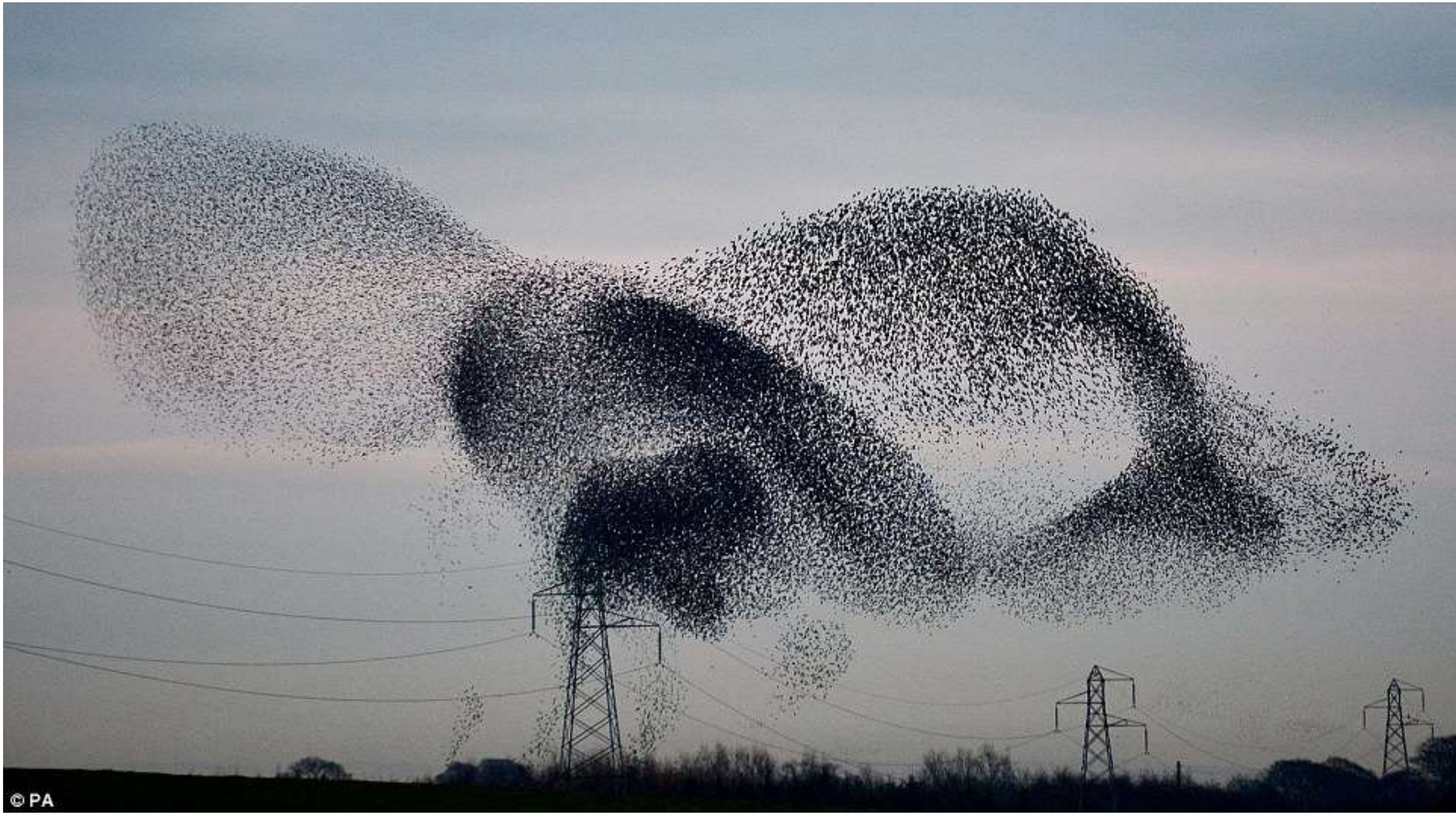
Active turbulence: bacteria



Dense suspension of microswimmers

Active turbulence: eukaryotic cells

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Why study active matter:

1. to understand biological systems: biomechanics and self-assembly

2. To create new types of micro-engines

3. As examples of non-equilibrium statistical physics

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$$\rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

$$\rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

inertial terms

viscous terms

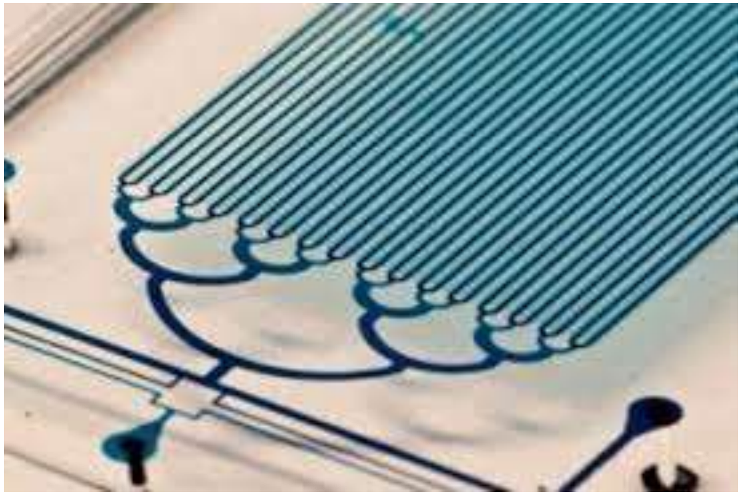
$$\tilde{v} = \frac{v}{V_0} \qquad \tilde{x} = \frac{x}{L_0}$$

$$\tilde{\nabla} = L_0 \nabla \qquad \tilde{t} = \frac{V_0}{L_0} t \qquad \frac{\partial}{\partial \tilde{t}} = \frac{L_0}{V_0} \frac{\partial}{\partial t}$$

$$\frac{V_0^2}{L_0}\rho\left\{\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde{t}}+(\tilde{\mathbf{v}}\cdot\tilde{\nabla})\tilde{\mathbf{v}}\right\}=-\nabla p+\frac{V_0}{L_0^2}\mu\tilde{\nabla}^2\tilde{\mathbf{v}}+\mathbf{f}$$

$$\left\{\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde{t}}+(\tilde{\mathbf{v}}\cdot\tilde{\nabla})\tilde{\mathbf{v}}\right\}=-\frac{L_0}{V_0^2\rho}\nabla p+\frac{\mu}{L_0V_0\rho}\tilde{\nabla}^2\tilde{\mathbf{v}}+\frac{L_0}{V_0^2\rho}\mathbf{f}$$

$$\mathrm{Re}=\frac{\text{inertial response}}{\text{viscous response}}\sim\frac{\rho L_0V_0}{\mu}$$



Low Re

High Re

$$\frac{V_0^2}{L_0}\rho\left\{\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde{t}}+(\tilde{\mathbf{v}}\cdot\tilde{\nabla})\tilde{\mathbf{v}}\right\}=-\nabla p+\frac{V_0}{L_0^2}\mu\tilde{\nabla}^2\tilde{\mathbf{v}}+\mathbf{f}$$

$$\left\{\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde{t}}+(\tilde{\mathbf{v}}\cdot\tilde{\nabla})\tilde{\mathbf{v}}\right\}=-\frac{L_0}{V_0^2\rho}\nabla p+\frac{\mu}{L_0V_0\rho}\tilde{\nabla}^2\tilde{\mathbf{v}}+\frac{L_0}{V_0^2\rho}\mathbf{f}$$

10^{-6}

10^{-6}

$$\mathrm{Re}=\frac{\text{inertial response}}{\text{viscous response}}\sim\frac{\rho L_0 V_0}{\mu}$$

10^{6}

$$\frac{V_0^2}{L_0}\rho\left\{\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde{t}}+(\tilde{\mathbf{v}}\cdot\tilde{\nabla})\tilde{\mathbf{v}}\right\}=-\nabla p+\frac{V_0}{L_0^2}\mu\tilde{\nabla}^2\tilde{\mathbf{v}}+\mathbf{f}$$

$$\left\{\frac{\partial\tilde{\mathbf{v}}}{\partial\tilde{t}}+(\tilde{\mathbf{v}}\cdot\tilde{\nabla})\tilde{\mathbf{v}}\right\}=-\frac{L_0}{V_0^2\rho}\nabla p+\frac{\mu}{L_0V_0\rho}\tilde{\nabla}^2\tilde{\mathbf{v}}+\frac{L_0}{V_0^2\rho}\mathbf{f}$$

10^{-6}

10^{-6}

$$\mathrm{Re}=\frac{\text{inertial response}}{\text{viscous response}}\sim\frac{\rho L_0V_0}{\mu}$$

10^{6}

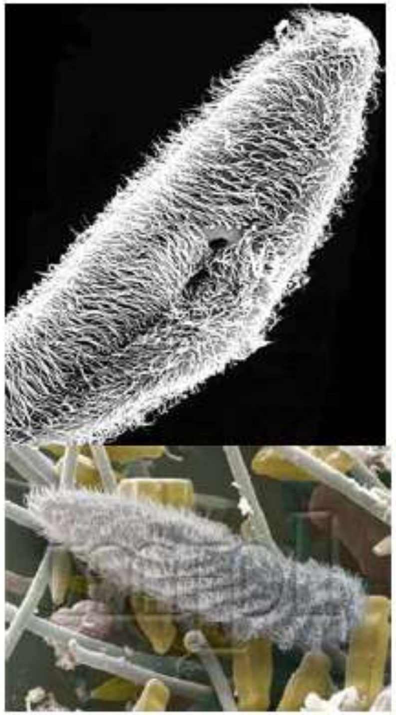
$$\boxed{\mathrm{Re}\sim 0}$$

Stokes equations

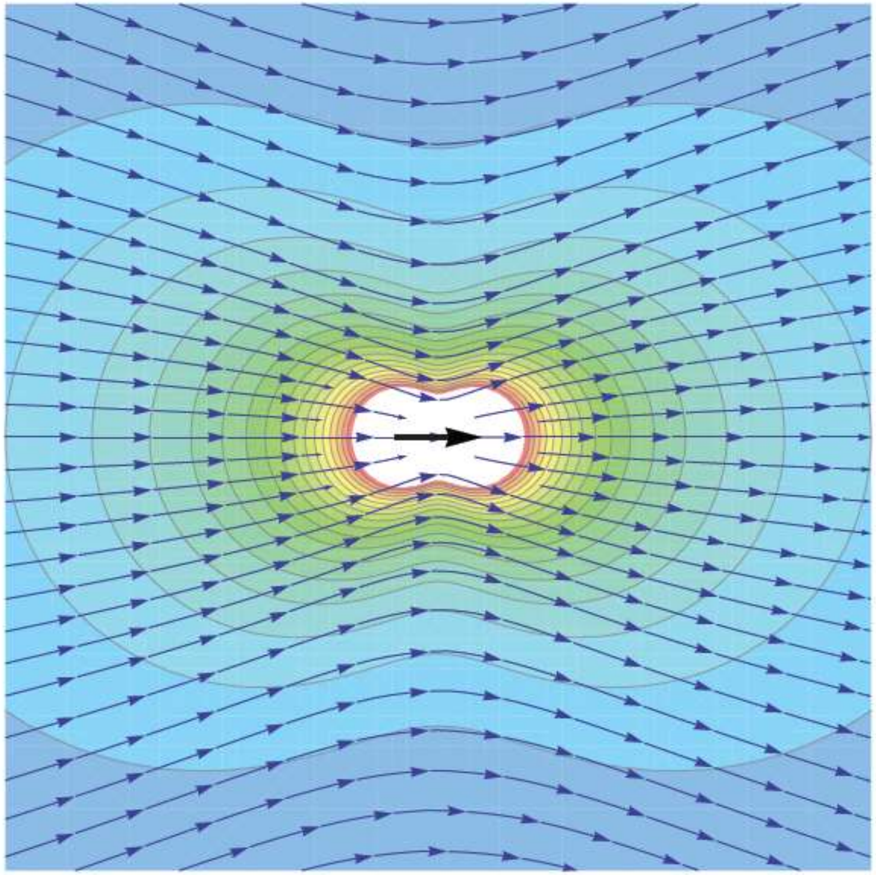
$$\nabla p = \mu \nabla^2 \mathbf{v} + \mathbf{f} \qquad\qquad \nabla \cdot \mathbf{v} = 0$$

Purcell's Scallop Theorem

A swimmer strokes must be non-invariant under time reversal



Green function of the Stokes equation (Stokeslet)



$$\mathbf{v}(\mathbf{r}) = \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r} + \frac{\mathbf{r}\mathbf{r}}{r^3} \right)$$

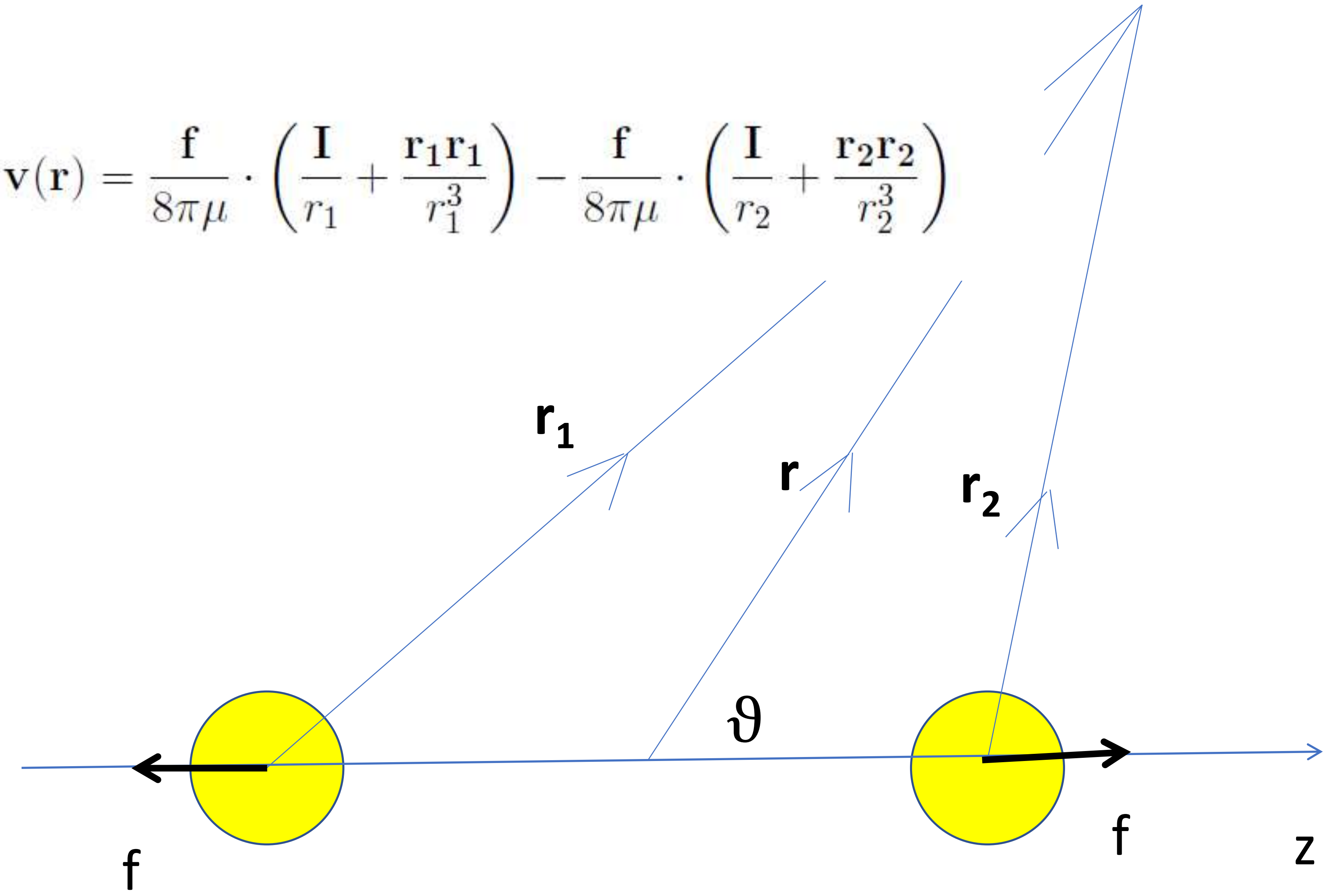
$$v_i(\mathbf{r}) = \frac{f_j}{8\pi\mu} \left(\frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3} \right) \equiv G_{ij}(\mathbf{r}) f_j$$

Far flow field of a swimmer

Swimmers have dipolar far flow fields because they have no net force acting on them

Dipolar far flow field

$$\mathbf{v}(\mathbf{r}) = \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r_1} + \frac{\mathbf{r}_1\mathbf{r}_1}{r_1^3} \right) - \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r_2} + \frac{\mathbf{r}_2\mathbf{r}_2}{r_2^3} \right)$$

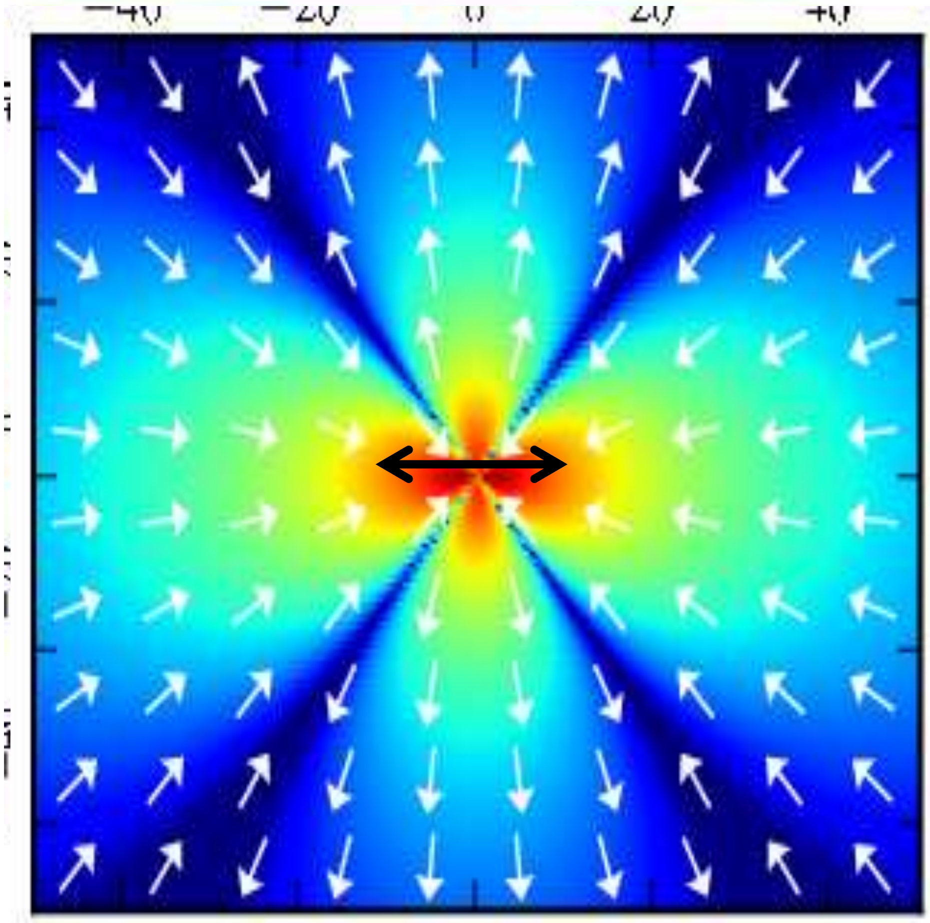


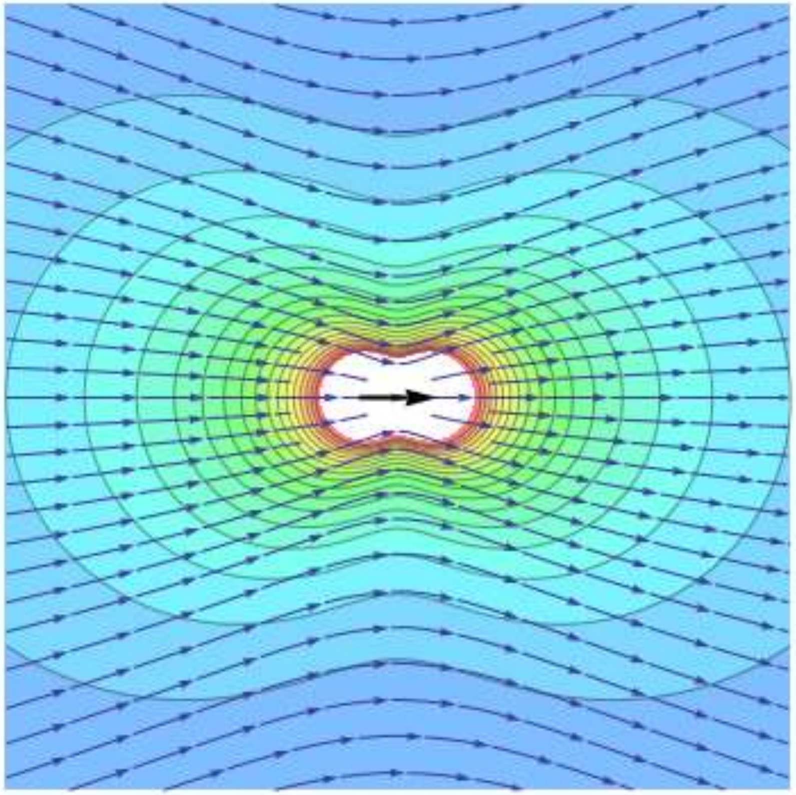
Far flow field of a swimmer

Swimmers have dipolar far flow fields because they have no net force acting on them

$$v_r = \frac{f}{4\pi\mu} \frac{L}{r^2} \left(3\cos^2\theta - 1\right)$$

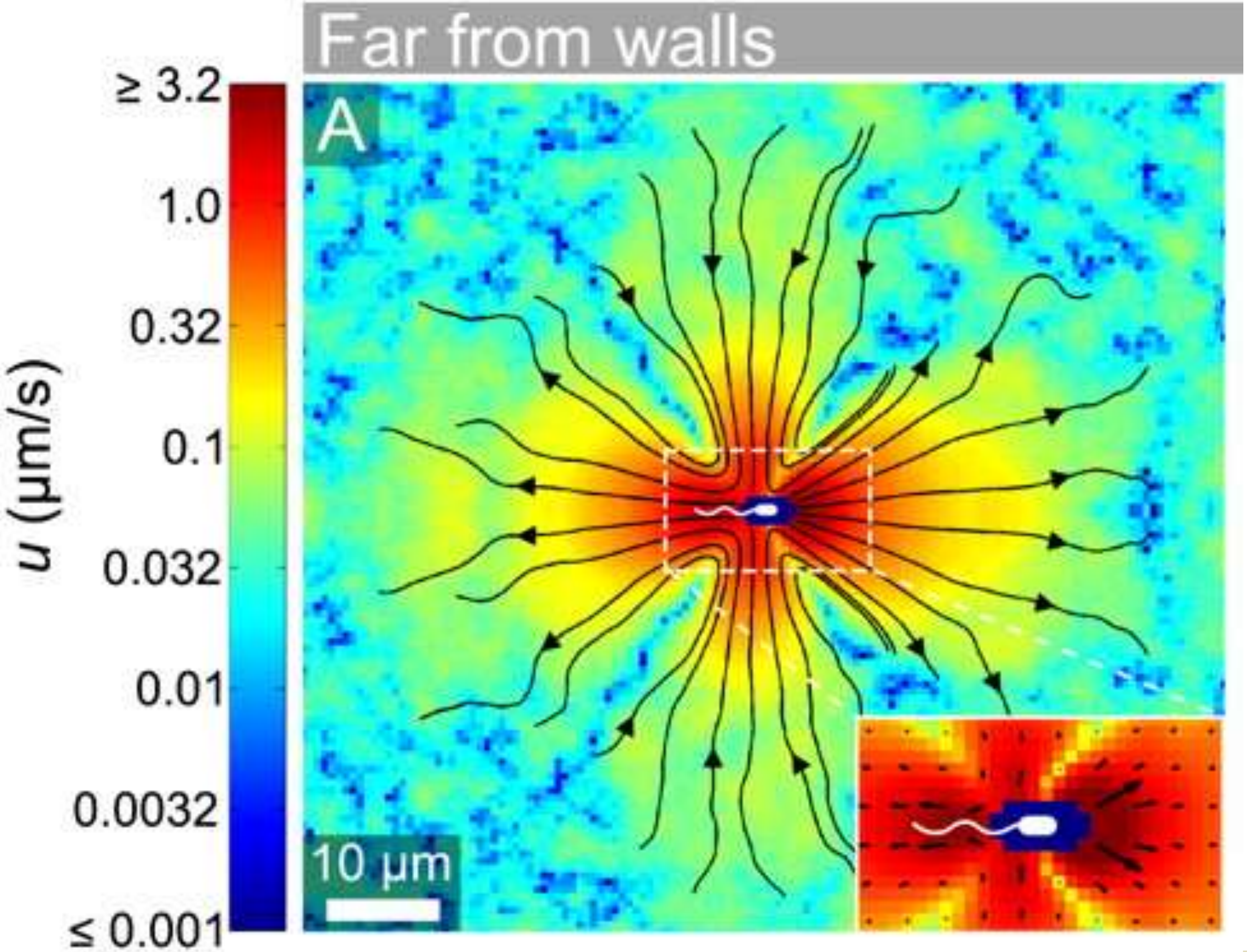
Swimmer and colloidal flow fields





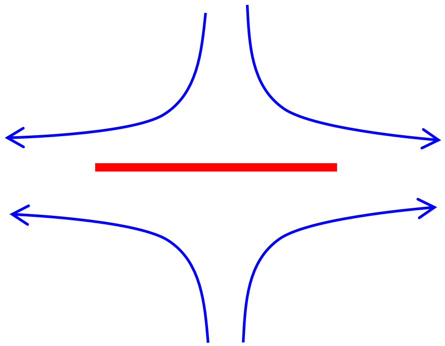
$$v \sim \frac{1}{r^2}$$

$$v \sim \frac{1}{r}$$



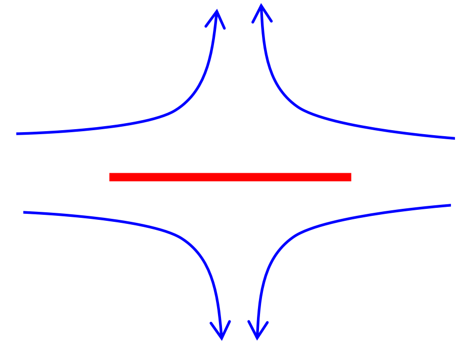
Goldstein group, Cambridge

E-coli



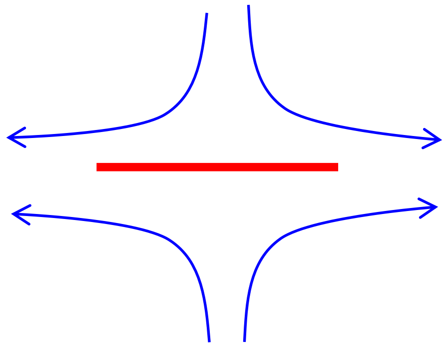
Extensile

Pusher



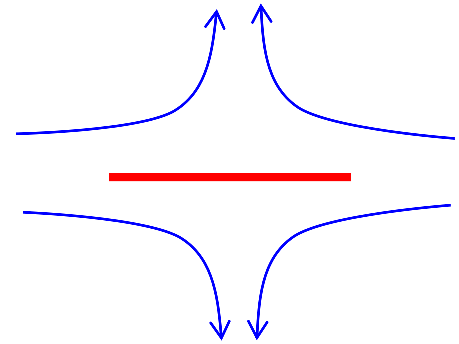
Contractile

Puller



Extensile

Pusher



Contractile

Puller

Flow field has nematic symmetry

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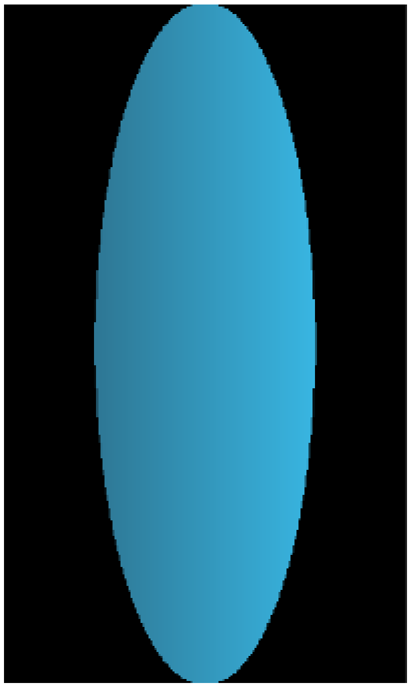
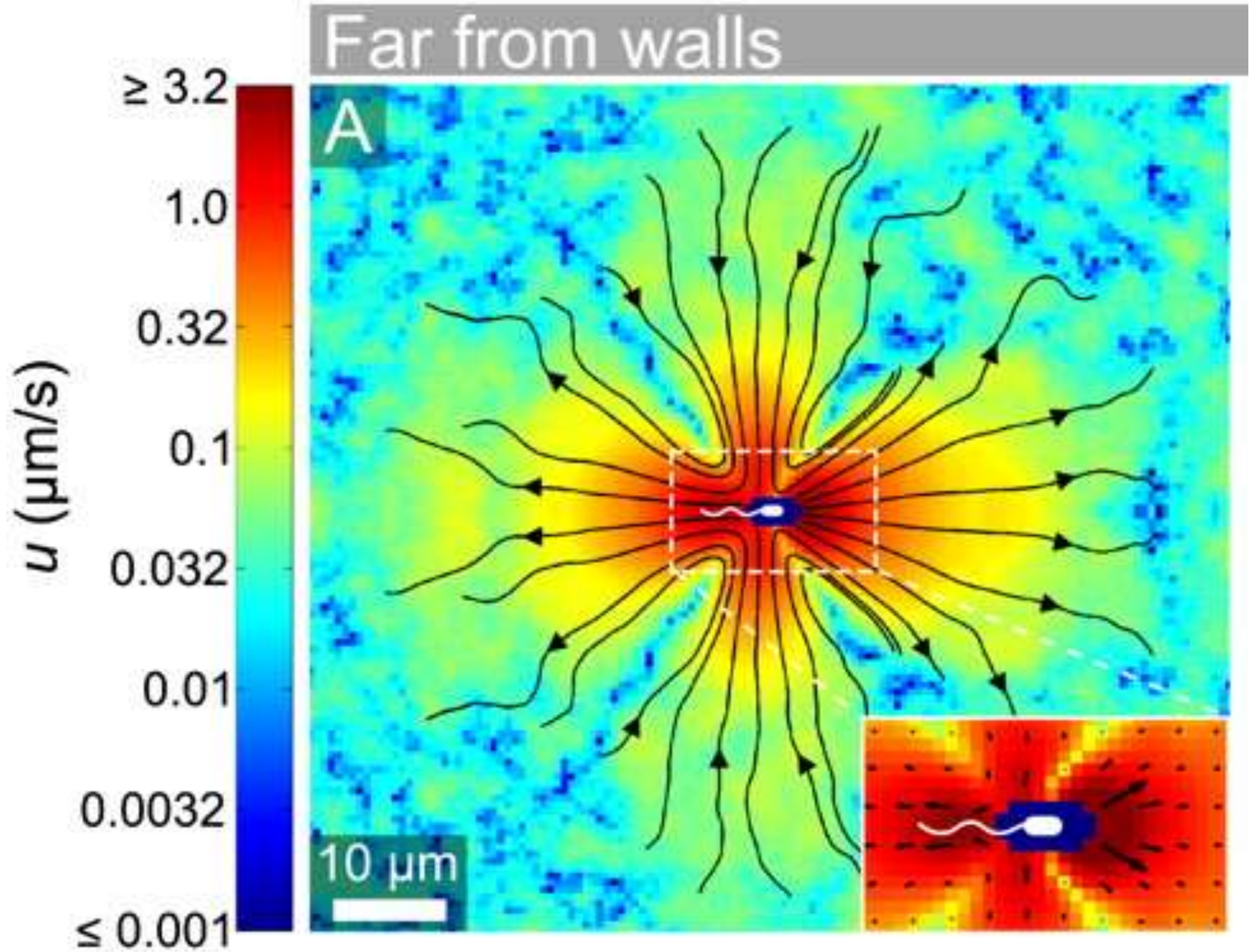
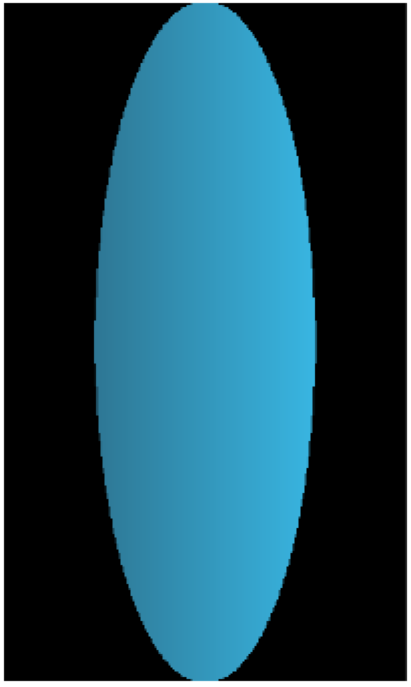
nematic symmetry

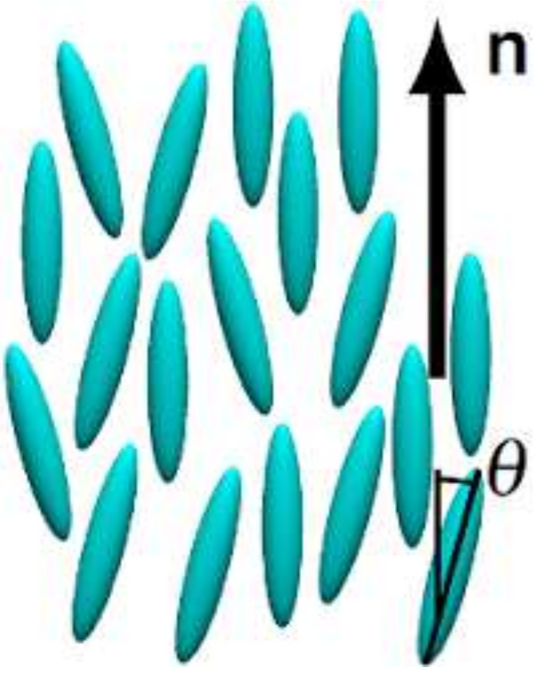




nematic symmetry



nematic phase



$$Q_{ij} = \langle n_i n_j - \frac{\delta_{ij}}{3} \rangle$$


alamy stock photo

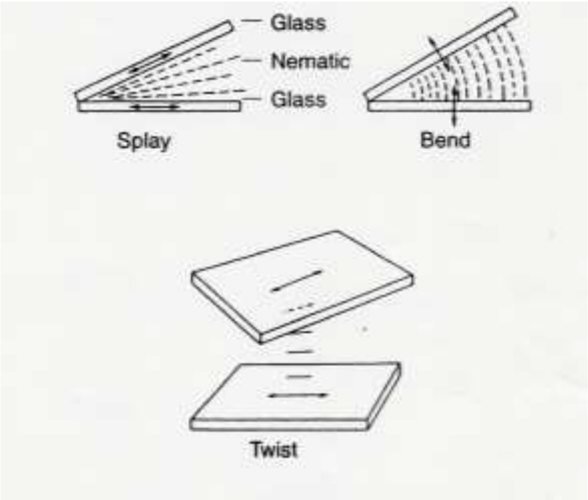
Tensor order parameter, Q

$$Q_{ij} = \langle n_i n_j - \frac{\delta_{ij}}{3} \rangle$$

Landau-de Gennes free energy

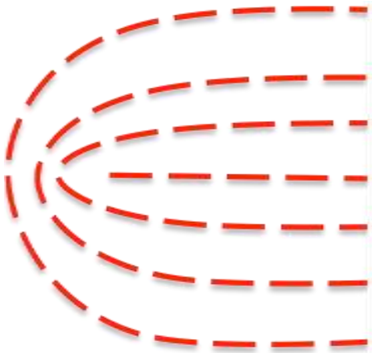
:

$$F = \frac{K}{2}(\partial_k Q_{ij})^2 + \frac{A}{2} Q_{ij} Q_{ji} + \frac{B}{3} Q_{ij} Q_{jk} Q_{ki} + \frac{C}{4}(Q_{ij} Q_{ji})^2$$

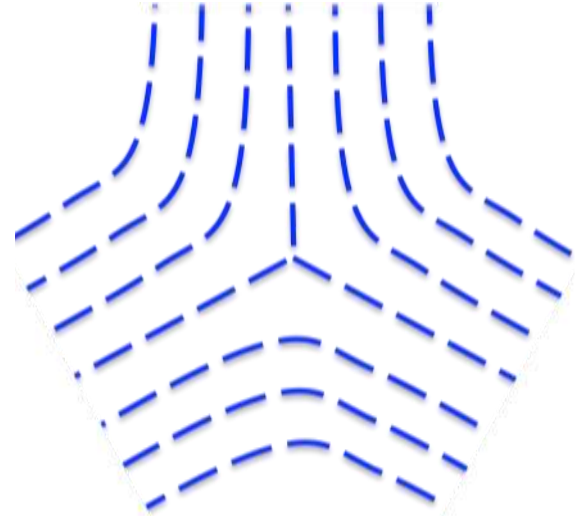


An ‘elastic liquid’

Topological defects in nematic liquid crystals



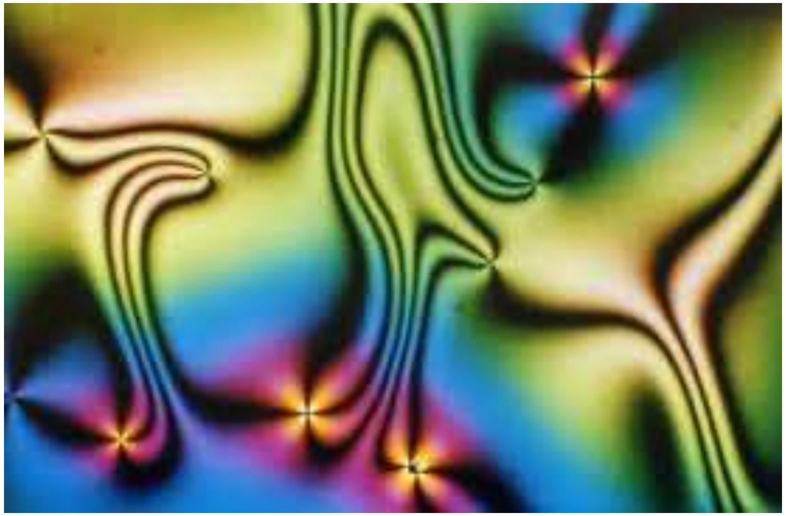
$m = +\frac{1}{2}$



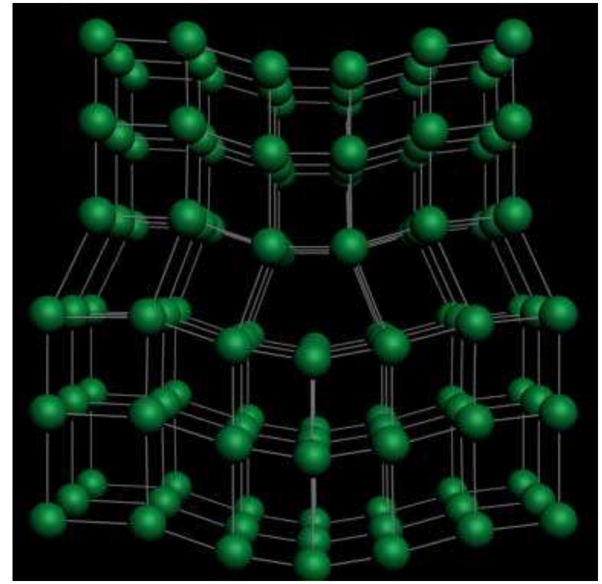
$m = -\frac{1}{2}$

topological charge

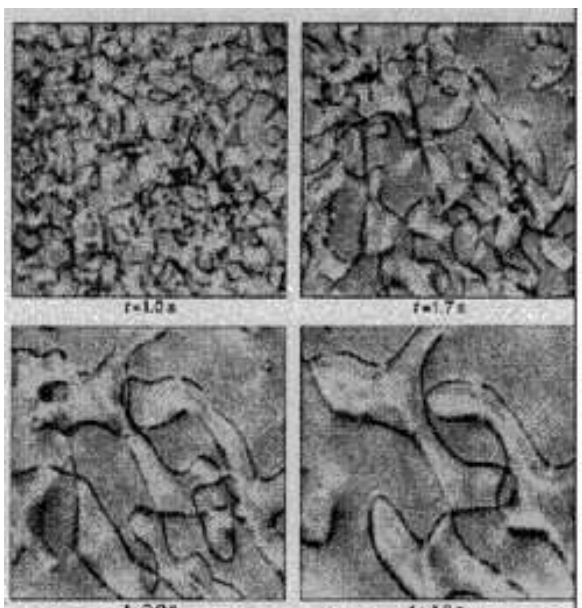
$$m = \frac{1}{2\pi}\int_{dS} d\theta$$



liquid crystals



crystal dislocations



cosmic strings in the early universe

magnetic monopoles in spin ice

topological insulators

quantum vortex in a superfluid