### TIME-SERIES FORECASTING -

#### Using Echo State Networks and Takens' Theorem

Laís de Souza Alves Physics' PhD Student March 2, 2020

Universidade de Brasília – UnB

- $\cdot$  Introduction
- · Time-Series and Takens' Theorem
  - · Lorenz System's Equations
  - · Representation of a State Space
  - · Embedded State Space
- · Echo State Networks
  - $\cdot\,$  Short-Term Predictions and Long-Term "Climate" Reproduction
  - Feed-Forward and Recurrent Neural Networks
  - · Echo State Network Workflow
- $\cdot$  Conclusion

#### $\cdot$ Introduction

- · Time-Series and Takens' Theorem
  - · Lorenz System's Equations
  - · Representation of a State Space
  - · Embedded State Space
- · Echo State Networks
  - $\cdot\,$  Short-Term Predictions and Long-Term "Climate" Reproduction
  - Feed-Forward and Recurrent Neural Networks
  - · Echo State Network Workflow
- $\cdot$  Conclusion

### INTRODUCTION

- · Real time-series forecasting using Artificial Neural Networks:
  - We don't need to know the underlying equations of the system's dynamics, only past data is needed.



### INTRODUCTION

- $\cdot\,$  Takens' Theorem and Attractor Reconstruction
  - $\cdot\,$  It is possible to have "access" to other dimensions of your system;
  - $\cdot$  Through only one dimensional measurement.



 $\cdot$  Introduction

#### · Time-Series and Takens' Theorem

- · Lorenz System's Equations
- · Representation of a State Space
- · Embedded State Space
- · Echo State Networks
  - $\cdot\,$  Short-Term Predictions and Long-Term "Climate" Reproduction
  - $\cdot\,$  Feed-Forward and Recurrent Neural Networks
  - · Echo State Network Workflow
- $\cdot$  Conclusion

Lorenz's System Equations:

$$\frac{dx}{dt} = \sigma(y - x);$$
$$\frac{dy}{dt} = \rho x - y - xz;$$
$$\frac{dz}{dt} = xy - \beta z.$$

Parameters:

$$\beta = \frac{8}{3}$$
$$\sigma = 10$$
$$\rho = 28$$

## TIME-SERIES AND TAKENS' THEOREM

· State Space Representation of Lorenz's Equations:



### **TIME-SERIES AND TAKENS' THEOREM**

· Lorenz's x(t) "measurement":



#### · Takens' Embedding Theorem:

Let **M** be a compact manifold of dimension d,  $\phi$  a smooth vector field and X a smooth function on **M**. It is a generic property that

$$\Phi_{(\phi X)}(m): \mathbf{M} \to \mathbb{R}^{2d+1}$$

is a embedding, where  $\phi$  is the flow on **M** and

$$\Phi_{(\phi X)}(m) = \left\langle X(m), X(\phi(m)), X(\phi^2(m)), \cdots, X(\phi^{2d}(m)) \right\rangle.$$

Source: Takens, F., "Detecting strange attractors in turbulence." (1981)

## TIME-SERIES AND TAKENS' THEOREM

#### • Takens' Embedding Theorem (more digestible form):

For the right  $\tau$  ( $\tau > 0$ ) and enough dimensions 2d + 1 for the Takens' Vectors, the embedded dynamics are diffeomorphic to the original state space dynamics:



One of the methods used for choosing the appropriate  $\tau$ :

 Mutual information – You can choose the first or the second minima, or even the first maxima. (Note: this choice is heuristic)



And for choosing the appropriate dimension:

- $\cdot$  False Nearest Neighbors (the chosen threshold is also heuristic); or
- · Correlation (Grassberger-Procaccia) dimension:



Some caveats to be aware of:

- $\cdot\,$  Noise in the data;
- · Limited length of data;
- · Variation on the temporal sampling;
- $\cdot$  And non-stationarity (dynamics changes) on the data;

- $\cdot$  Introduction
- · Time-Series and Takens' Theorem
  - · Lorenz System's Equations
  - · Representation of a State Space
  - · Embedded State Space
- Echo State Networks
  - $\cdot\,$  Short-Term Predictions and Long-Term "Climate" Reproduction
  - Feed-Forward and Recurrent Neural Networks
  - · Echo State Network Workflow
- $\cdot$  Conclusion

## **ECHO STATE NETWORKS**

- · Forecast of a time-series:
  - It is possible to use Recurrent Neural Networks (RNNs) to make short-term forecasts and long-term reproduction of the system's "climate":



· Prediction formula:

$$\hat{y} = \Theta\left(\sum_{i} x_i w_i + b\right)$$



Source: URL=<https://medium.com/ensina-ai/redes-neurais-perceptron-multicamadas-e-o-algoritmo-backpropagation-eaf89778f5b8>

Representation of a Recurrent Neural Network (RNN):



Workflow For a input on the form (N, T, V)

- · Choose the Reservoir size and the matrix sparsity (of connections);
- Generate the input weights **W**<sub>in</sub>, from a binomial random distribution;
- $\cdot$  Generate the Reservoir weights  $\mathbf{W}_r$  from a uniform distribution

· Calculate the hidden state h(t):

$$\mathbf{h}(t) = f(\mathbf{W}_{in}\mathbf{x}(t) + \mathbf{W}_r\mathbf{h}(t-1))$$

· Create an new input representation: A matrix with all the collected hidden states, or with roots and slopes, or a mean value of h(t - 1)

## ECHO STATE NETWORK





• Connect this new input representation to the output layer (can be a simple RNN with one or two layers)



- · Advantages of ESNs:
  - The training phase is faster and doesn't suffer from backpropagation instabilities of Vanilla RNNs;
  - $\cdot$  They are well suited for chaotic systems;
  - $\cdot\,$  They are capable of "learning" short-term and long-term correlations;
- · Disadvantages of ESNs:
  - $\cdot\,$  Selection of tuning parameters  $\rightarrow$  trial and error;
  - · Limited size of Reservoir: its size is dependent of computer's memory capacity;

• So what does Takens' Embedding have to do with Echo State Networks?



- 1 T.F.M. Rocha and P.M.M. Rocha. **"Inefficiency of the Brazilian Stock Market: the IBOVESPA Future Contracts."** arXiv:1904.09214 (2019)
- 2 Takens, F. "Detecting strange attractors in turbulence." (1981)
- 3 Fraser, A. M., and Swinney, H. L. "Independent coordinates for strange attractors from mutual information." Physical review A, 33(2):1134. (1986)
- 4 Kennel, M. B., Brown, R. and Abarbanel, H. D. I. **"Determining** embedding dimension for phase-space reconstruction using a geometrical construction." Physical review A 45(6):3403 (1992)
- 5 Hilborn, Robert C. **"Chaos and nonlinear dynamics: an introduction for scientists and engineers."** (2000)

Feel free to reach me by: laisouzalves@gmail.com

# THANK YOU!

### ANY QUESTIONS?