Baryakhtar
6/30/2020
p1

Scalar Dark Matter

(see arxiv paper by Khmelnitsky + Rubakov 1309.5888)

$$\phi = \phi_0 \cos(mt + \vec{k}\cdot\vec{x} + \beta)$$

assume plane wave scalar field w/ amplitude ϕ_0

The energy-momentum tensor is given by

$$T_{\mu\nu} = \partial_\mu\phi\, \partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\left((\partial\phi)^2 - m^2\phi^2 \right)$$

Calculating $T_{\mu\nu}$, we find

$$T_{00} \equiv \rho = \frac{1}{2} m^2 \phi_0^2 + \mathcal{O}(v^2 \langle\rho\rangle).$$

If we assume the scalar field makes up the local dark matter density, we can find its amplitude:

$$\frac{1}{2} m^2 \phi_0^2 = \rho_{DM} \rightarrow \phi_0 = \sqrt{\frac{2\rho_{DM}}{m^2}} \sim 10^{-9} M_{pl} \left(\frac{10^{-22}\,eV}{m}\right)$$

The spatial components T_{ij} are oscillatory at leading order

$$T_{ij} = -\frac{1}{2} m^2 \phi_0^2 \cos(2mt + 2kx + 2\beta)\, \delta_{ij} \equiv p\, \delta_{ij}$$

$\langle p_{ij} \rangle_t = 0$, average $\rightarrow 0$ on times long compared to oscillation time

$\rightarrow$ "pressureless dust"

We are interested in the <u>oscillation</u>.

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06/30/2020

Scalar dark matter

The oscillating pressure T_{ij} in turn produces oscillations in the metric. Consider the metric in Newtonian gauge,

$$ds^2 = (1 + 2\Phi(x,t))\,dt^2 - (1 - 2\Psi(x,t))\,\delta_{ij}\,dx^i dx^j$$

where $\Phi, \Psi \ll 1$.

Consider expanding Ψ in components with different frequencies,

$$\Psi(\vec{x},t) \simeq \Psi_0 + \Psi_c(\vec{x})\cos(2mt + 2kx) + \Psi_s(\vec{x})\sin(2mt+2kx)$$

The 00 component of the Einstein eqn gives

$$\nabla^2\Psi = 4\pi G \rho_{DM}$$

the ~~xxx~~ trace of the ij eqn gives

$$-6\ddot{\Psi} + 2\nabla^2(\Psi - \Phi) = 8\pi G T_{kk} = 24\pi G p(x,t)$$

Using the above, the time-independent gravitational potentials are equal: $\Psi_0 = \Phi_0$.

We also find $\Psi_s \simeq 0$

$$\Psi_c \simeq \frac{1}{2}\pi G \phi_0^2 = \frac{\pi G \rho_{DM}}{m^2}$$

Scalar dark matter

The observable in pulsar timing measurements is the change in arrival time as a function of time t, called the residuals:

$$R(t) = -\int_0^t \frac{\nu(t') - \nu_0}{\nu_0}\, dt'$$

where ν_0 is the emission frequency at the pulsar.

At leading order in the DM velocity, the fractional frequency change is just the difference in gravitational potential at the earth & at pulsar:

$$\frac{\nu(t) - \nu_0}{\nu_0} \simeq \Psi(x,t) - \Psi(x_p, t')$$

$$\simeq \Psi(x,t) - \Psi(x_p, t-D)$$

where D is the distance to the pulsar, taking into account the finite speed for propagation of the pulse.

The oscillatory component is then

$$R(t) = \int \Psi_c \Big[\cos(2mt + 2\vec{k}\vec{x}_e + 2\beta_e) - \cos(2m(t-D) + 2\vec{k}\cdot\vec{x}_p + 2\beta_p) \Big]$$

<u>signal</u>: $R(t) \sim \frac{\Psi_c}{m} \sin\left(mD + k(\vec{x}_e - \vec{x}_p) + \beta_e - \beta_p\right)$

$\times \cos\left(2mt + k(\vec{x}_e + \vec{x}_p) - mD\right)$

amplitude

time dependence

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Scalar Dark Matter

(see arxiv paper

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$\rightarrow$ "pressureless dust"

We are interested in the oscillation.

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06/30/2020
p2

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Baryakhtar
P3
06/30/2020

Scalar dark matter

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<u>signal</u>: $R(t) \sim \frac{\Psi_c}{m} \sin\left(mD + k(\vec{x}_e - \vec{x}_p) + \beta_e - \beta_p\right)$

$\times \cos(2mt + k(\vec{x}_e + \vec{x}_p) - D)$

amplitude

time dependence