Five Lectures on Dark Matter

Second Lecture

Università degli Studi di Padova

INFN Istituto Nazionale di Fisica Nucleare Sezione di Padova

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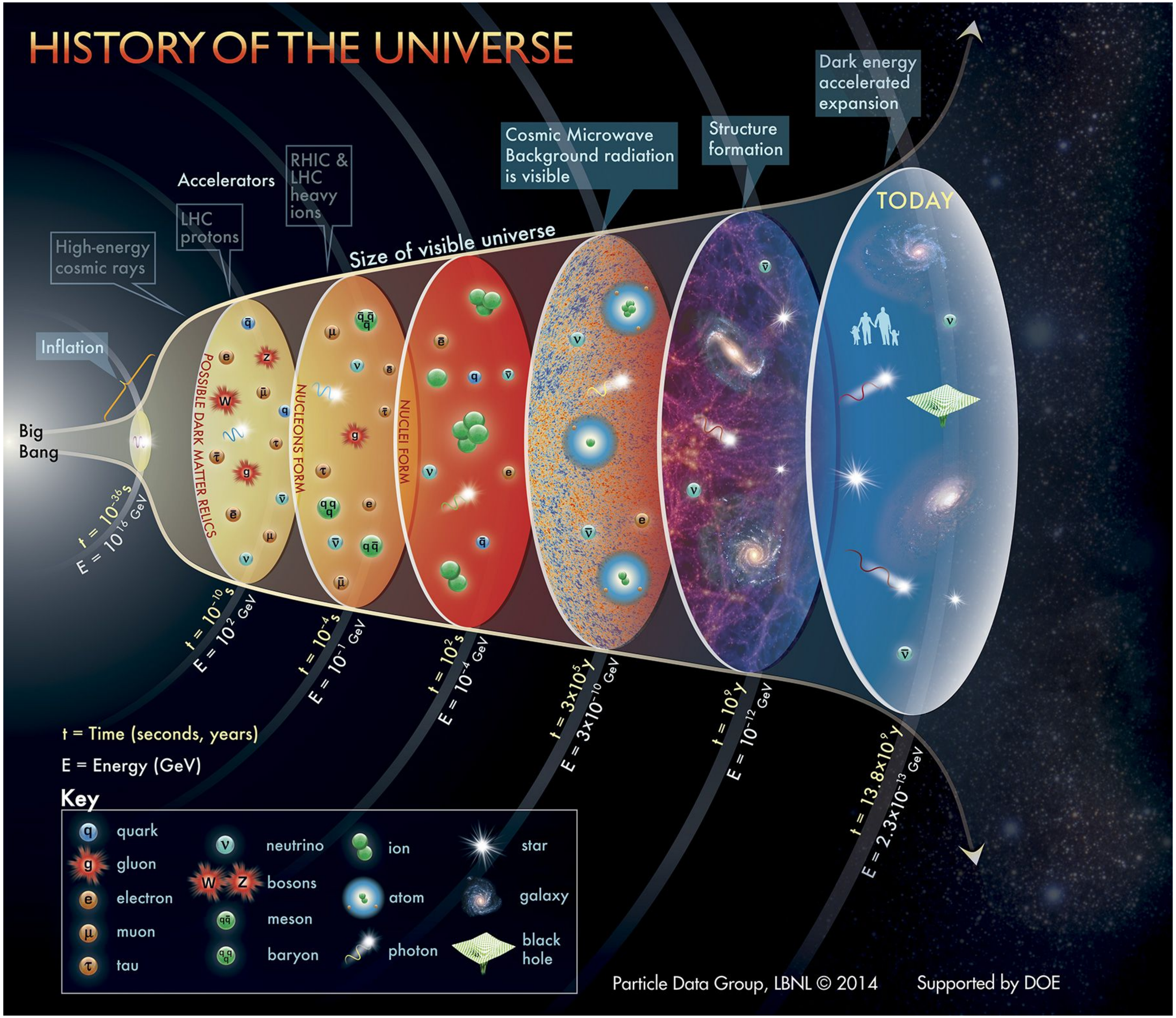
II Joint ICTP-Trieste/ICTP-SAIFR School on Particle Physics (June 22 — July 3, 2020)

Dark Matter Genesis



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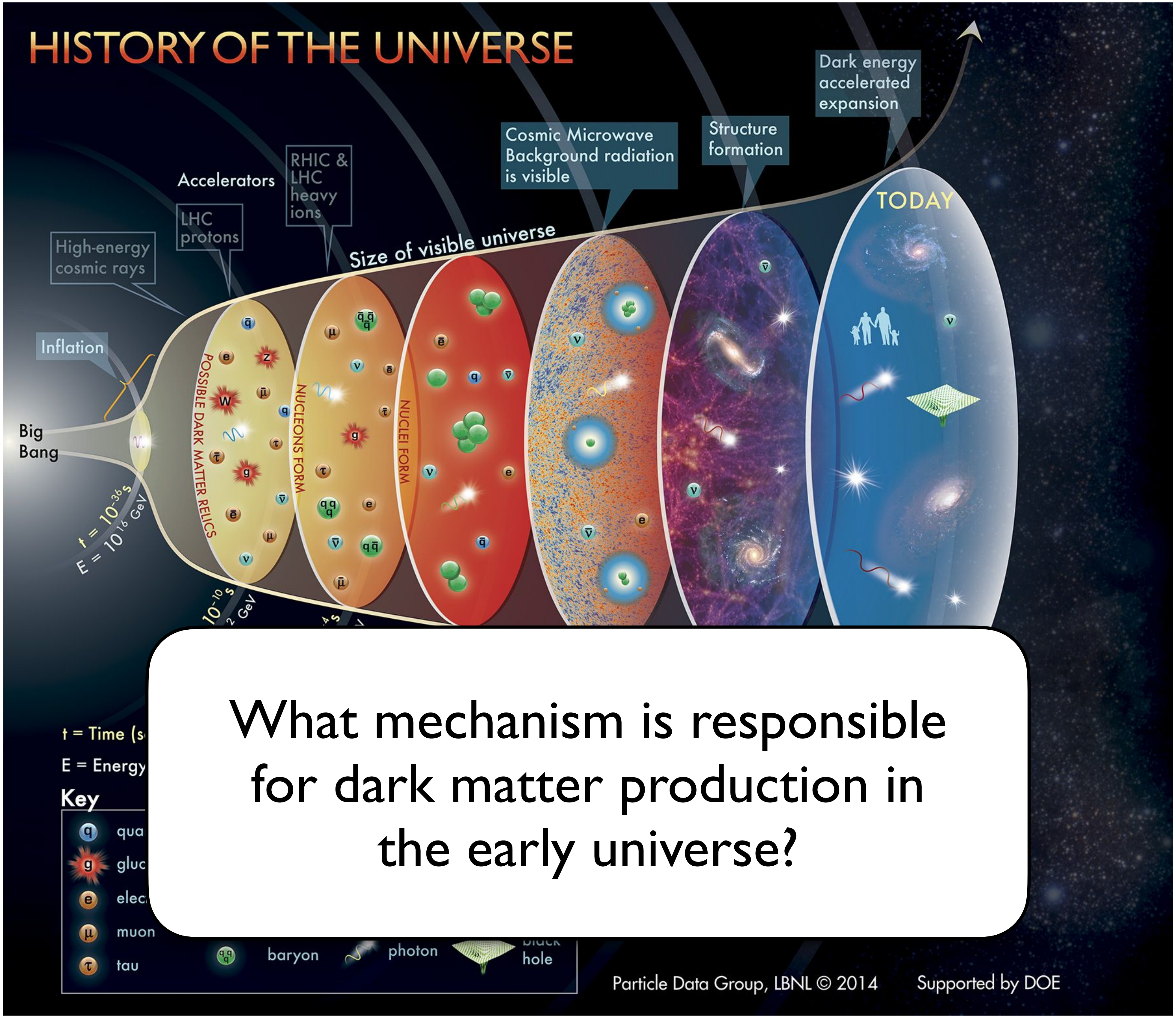
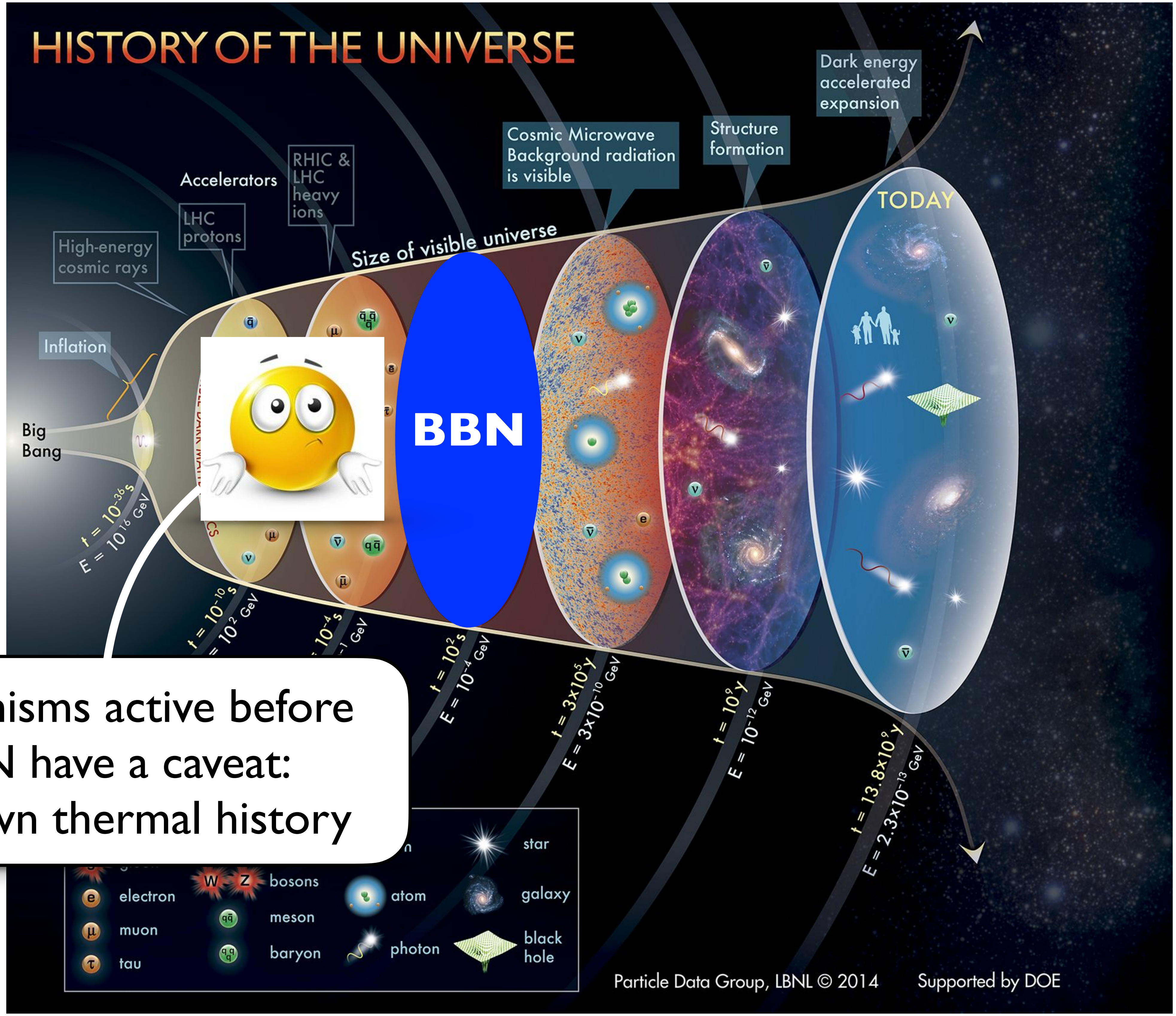


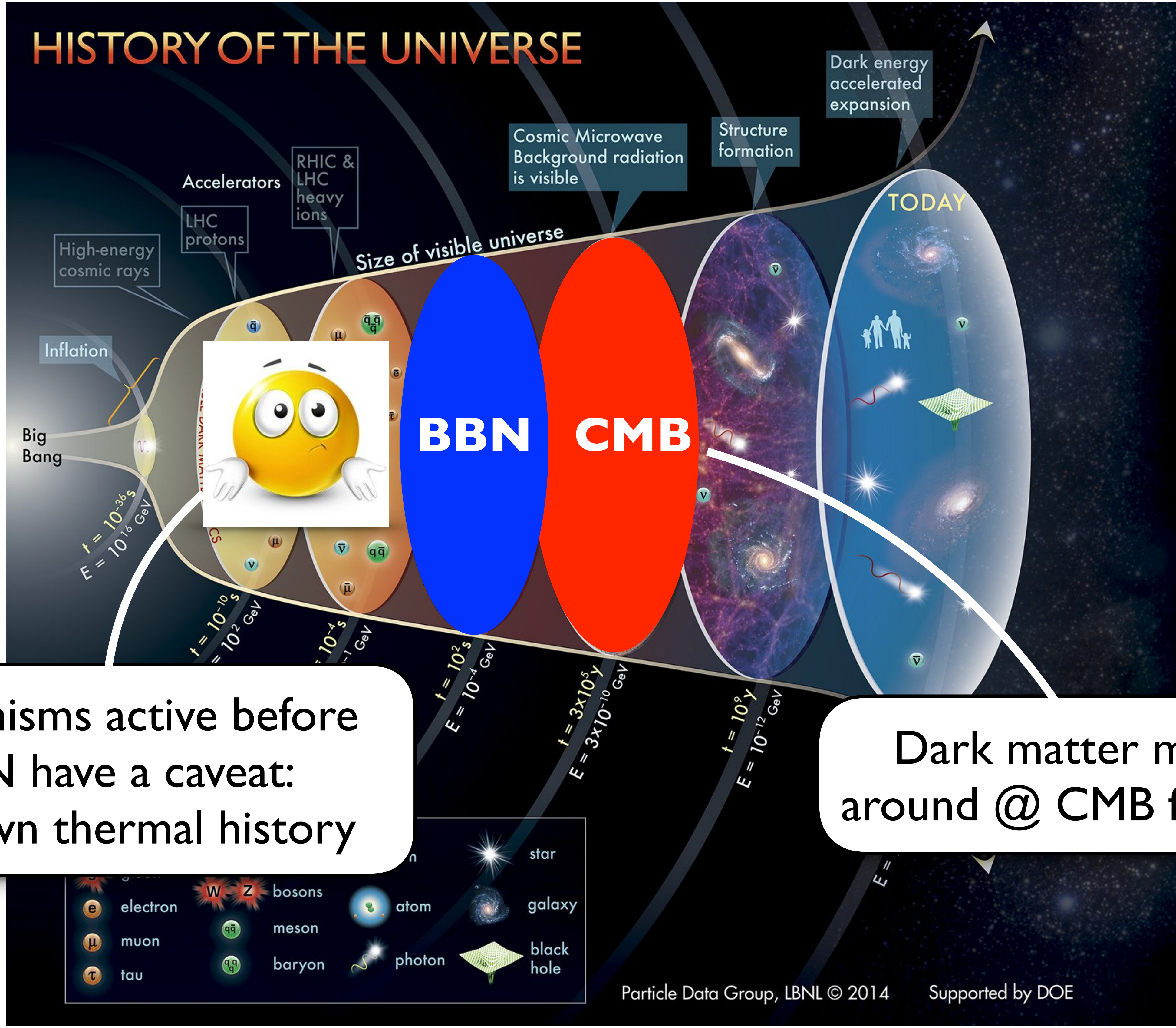
What mechanism is responsible for dark matter production in the early universe?

Dark Matter Genesis



Mechanisms active before BBN have a caveat: unknown thermal history

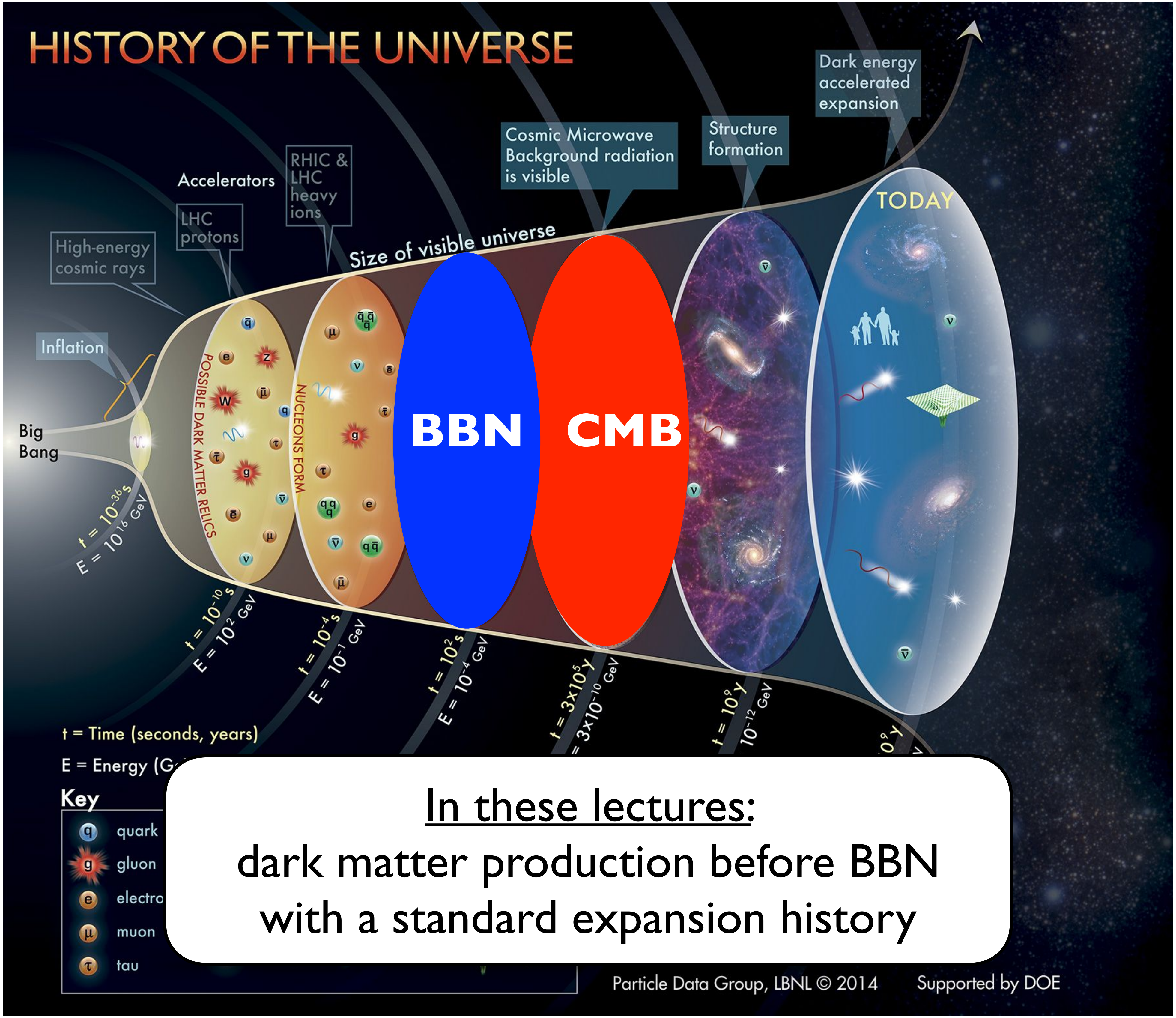
Dark Matter Genesis



Mechanisms active before BBN have a caveat: unknown thermal history

Dark matter must be around @ CMB formation

Dark Matter Genesis



<u>In these lectures:</u>
dark matter production before BBN
with a standard expansion history

Setting the Stage

Dark matter production takes place when the energy density is dominated by a gas of relativistic particles

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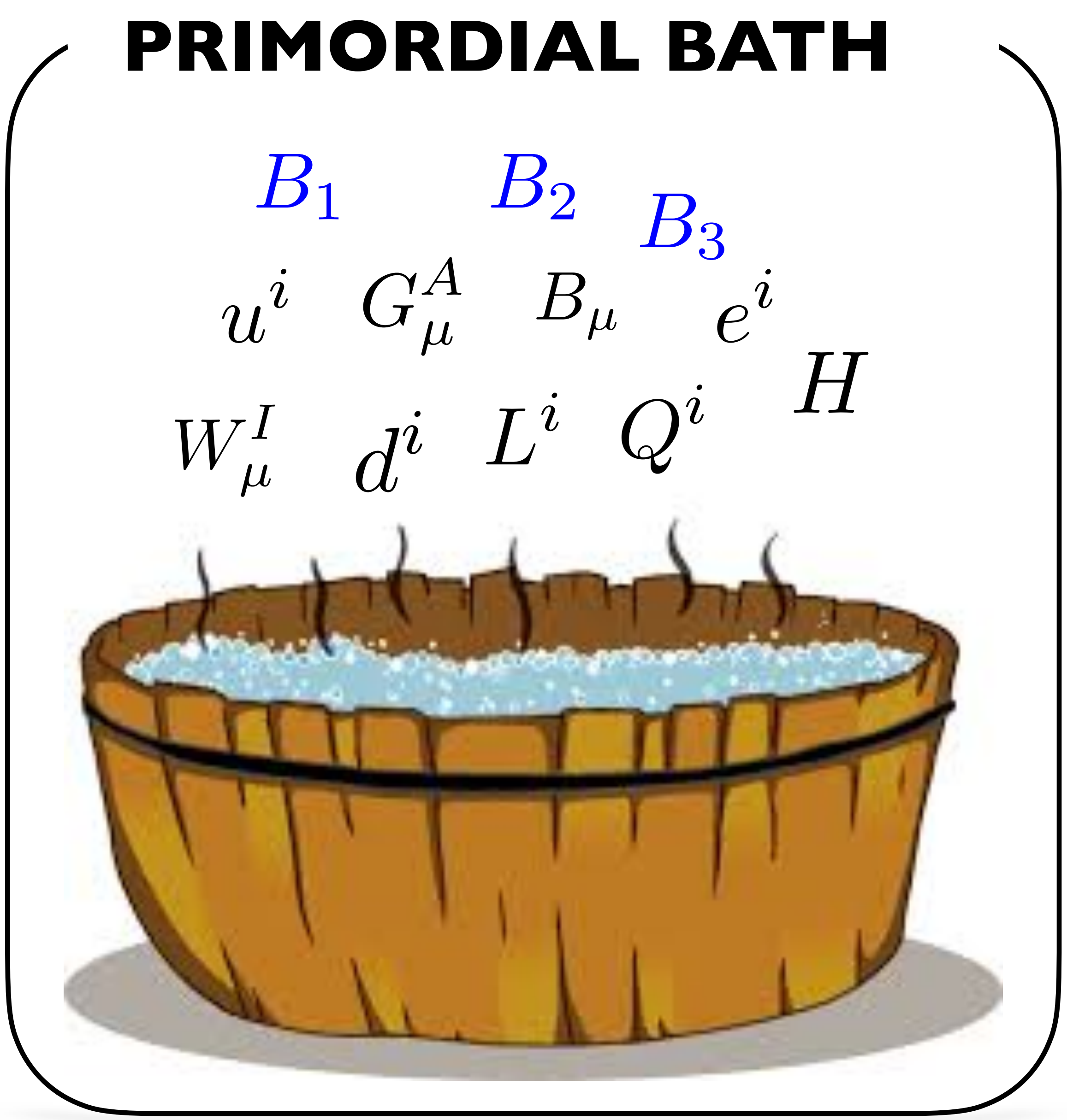
Primordial bath:
Standard Model particles and possibly additional bath particles, all sharing the same temperature T



Setting the Stage

Dark matter production takes place when the energy density is dominated by a gas of relativistic particles

NUMBER DENSITIES

$$n_i^{\mathrm{eq}} = \frac{g_i}{2\pi^2}\int_0^\infty dE \frac{E^2}{\exp\left[E/T\right] \pm 1}$$

RELATIVISTIC

$$n_i^{\mathrm{eq}} = g_i \frac{\zeta(3)}{\pi^2} T^3 \begin{cases} 1 & (\mathrm{BE}) \\ \frac{3}{4} & (\mathrm{FD}) \end{cases}$$

NON-RELATIVISTIC

$$n_i^{\mathrm{eq}} = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-\frac{(m_i - \mu_i)}{T}}$$

PRIMORDIAL BATH

B_1 B_2 B_3

u^i G_μ^A B_μ e^i H

W_μ^I d^i L^i Q^i

Setting the Stage

Dark matter production takes place when the energy density is dominated by a gas of relativistic particles

ENERGY DENSITIES

$$\rho_i^{\rm eq} = \frac{g_i}{2\pi^2}\int_0^\infty dE \frac{E^3}{\exp\left[E/T\right] \pm 1}$$

RELATIVISTIC

$$\rho_i^{\rm eq} = g_i \frac{\pi^2}{30} T^4 \begin{cases} 1 & ({\rm BE}) \\ \frac{7}{8} & ({\rm FD}) \end{cases}$$

NON-RELATIVISTIC

$$\rho_i^{\rm eq} = m_i n_i^{\rm eq}$$

PRIMORDIAL BATH

B_1 B_2 B_3

u^i G_μ^A B_μ e^i H

W_μ^I d^i L^i Q^i

Setting the Stage

Dark matter production takes place when the energy density is dominated by a gas of relativistic particles

RADIATION DOMINATION

$$\rho = \frac{\pi^2}{30} g_*(T) T^4$$

$$g_*(T) = \sum_{B\,(T>m_B)} g_B + \frac{7}{8} \sum_{F\,(T>m_F)} g_F$$

Setting the Stage

Dark matter production takes place when the energy density is dominated by a gas of relativistic particles

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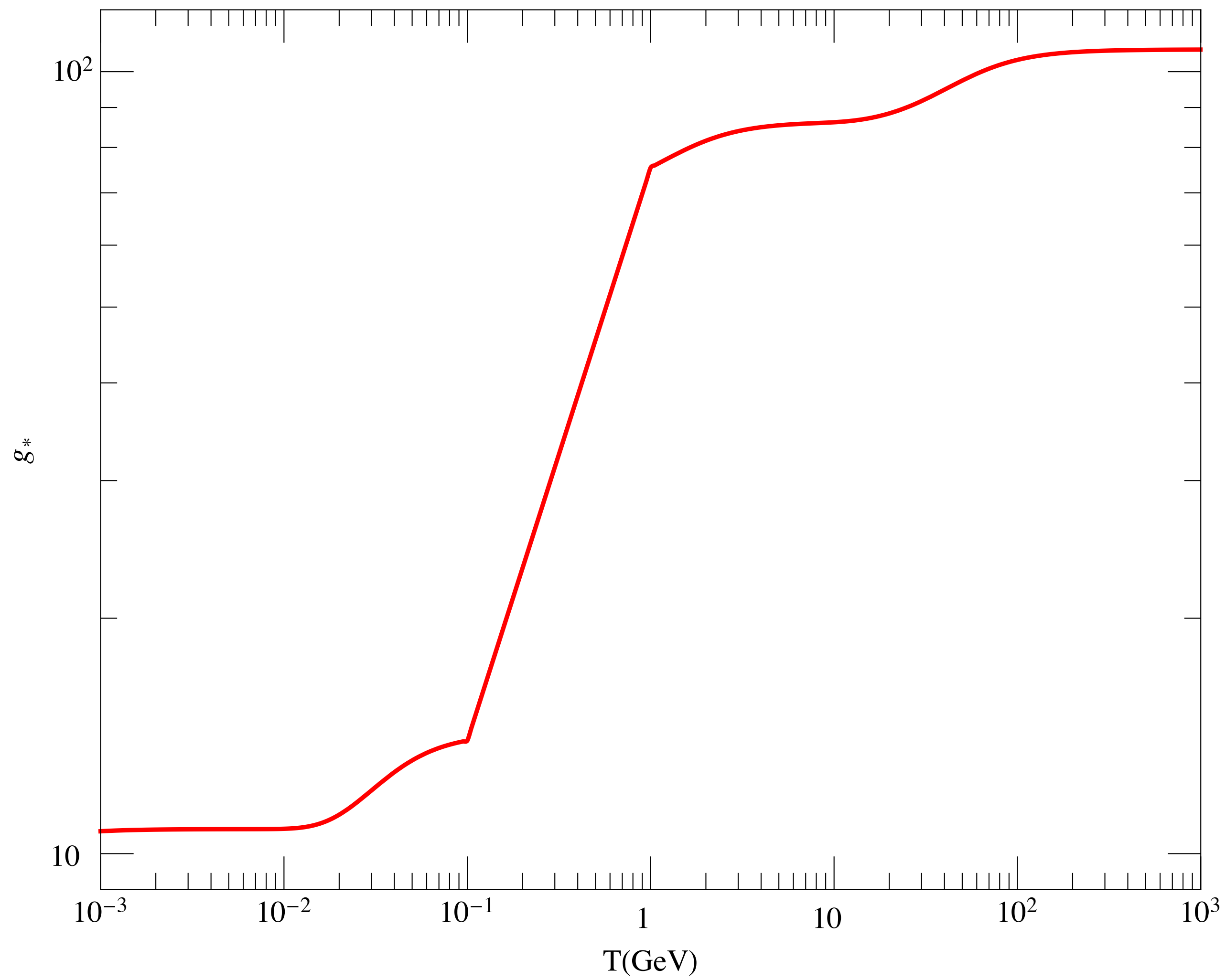

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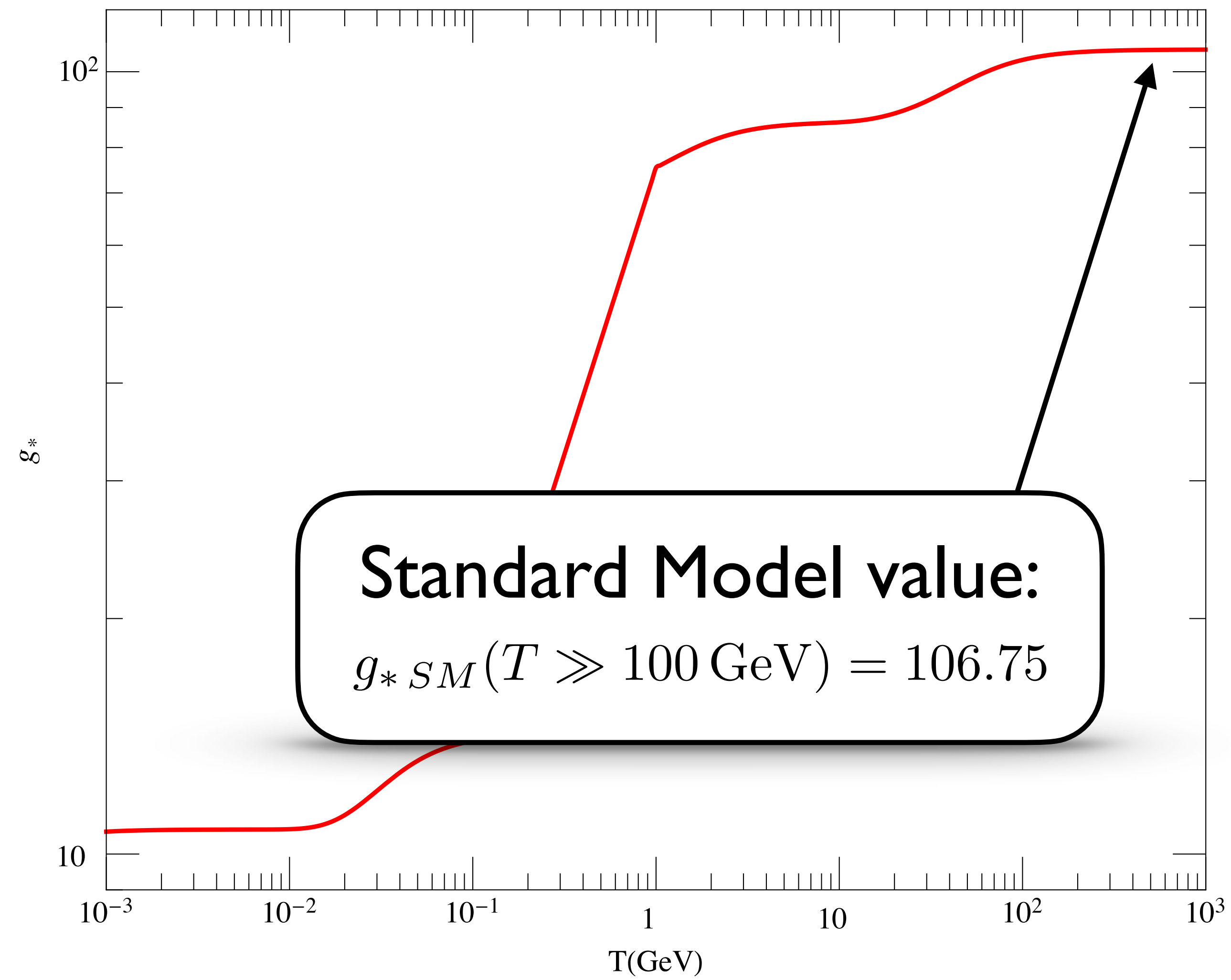

Setting the Stage

Dark matter production takes place when the energy density is dominated by a gas of relativistic particles

RADIATION DOMINATION

$$\text{Scale factor}: \qquad a(t) \propto t^{1/2}$$

$$\text{Friedmann equation}: \qquad H(t) = \frac{1}{2t} = \frac{\pi \, g_*(T)^{1/2}}{3\sqrt{10}} \frac{T^2}{M_{\rm Pl}}$$

$$M_{\rm Pl} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}\,\text{GeV}$$

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Time-temperature relation: $$\left(\frac{t}{1\,\text{sec}}\right)\left(\frac{T}{1\,\text{MeV}}\right)^2 \simeq 1$$

$$M_{\rm Pl} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}\,\text{GeV}$$

Entropy

Entropy density: $$s = \frac{2\pi^2}{45} g_{*s}(T) T^3$$

For the temperature range we are interested in: $g_*(T) \simeq g_{*s}(T)$

Entropy

Entropy density: $$s = \frac{2\pi^2}{45} g_{*s}(T) T^3$$

Within our working assumptions, the entropy is a comoving volume is conserved:

$$sa^3 = \text{const} \qquad \Rightarrow \qquad g_{*s}^{1/3}(T) T a = \text{const}$$

Entropy conservation (temperature vs scale factor)

For the temperature range we are interested in: $g_*(T) \simeq g_{*s}(T)$

Comoving densities

Number density of particles inside a volume that does not change with the expansion

$$Y_i \equiv \frac{n_i}{s}$$

Comoving densities

Number density of particles inside a volume that does not change with the expansion

$$Y_i \equiv \frac{n_i}{s}$$

Why is it useful?

It does not change in the absence of number changing processes

$$Y_i = \frac{n_i}{s} = \frac{n_i a^3}{s a^3} = \frac{\mathrm{const}_1}{\mathrm{const}_2} = \mathrm{const}$$

It scales out Hubble expansion

Comoving densities

Number density of particles inside a volume that does not change with the expansion

$$Y_i \equiv \frac{n_i}{s}$$

RELATIVISTIC

$$Y_i^{\rm eq} = \frac{g_{\rm eff}}{g_{*s}(T)} \frac{45\,\zeta(3)}{2\pi^4}$$

NON-RELATIVISTIC

$$Y_i^{\rm eq} = \frac{g_i}{g_{*s}(T)} \frac{45}{4\sqrt{2}\pi^{7/2}} \left(\frac{m_i}{T}\right)^{3/2} e^{\frac{\mu_i - m_i}{T}}$$

A General Classification

How do we classify dark matter production mechanisms?

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THERMAL

- Dark matter produced when energy budget was dominated by a radiation bath
- Dark matter particle at some point in thermal equilibrium with the primordial bath
- Departure from equilibrium is what set the DM abundance

A General Classification

How do we classify dark matter production mechanisms?

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- Dark matter produced when energy budget was dominated by a radiation bath
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- Departure from equilibrium is what set the DM abundance

NON-THERMAL

Anything else!

A General Classification

How do we classify dark matter production mechanisms?

THERMAL

WIMPs

SM neutrinos

NON-THERMAL

Asymmetric DM

Axions

Thermal Dark Matter

HOW DO THEY THERMALIZE?

Dark matter particles achieve thermal equilibrium with the primordial plasma via collisions

$$\chi\chi \to \mathrm{SM\,SM}$$

Thermal Dark Matter

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Dark matter particles achieve thermal equilibrium with the primordial plasma via collisions

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HOW DO THEY DECOUPLE?

The universe becomes colder and more diluted, not enough interactions to ensure equilibrium, Hubble expansion wins

$$\Gamma_{\rm ann}(T_{FO}) = n_\chi(T_{FO})\langle\sigma v_{\rm rel}\rangle_{T=T_{FO}} = H(T_{FO})$$

Freeze-out of dark matter number density

Thermal Dark Matter

EARLY TIMES

$$n_\chi(T) = n_\chi^{\rm eq}(T) \qquad (T > T_{FO})$$

At early times, much earlier than freeze-out, dark matter in thermal equilibrium

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LATE TIMES

$$n_\chi \propto a^{-3}$$

$$Y_\chi = \rm const$$

At late times, much later than freeze-out, dark matter diluted by Hubble expansion

Thermal Dark Matter

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How do we connect the two regimes?
The decoupling process is what sets the relic density

Two cases for decoupling

$$n_\chi(T_{FO})\langle\sigma v_{\rm rel}\rangle_{T=T_{FO}} = H(T_{FO})$$

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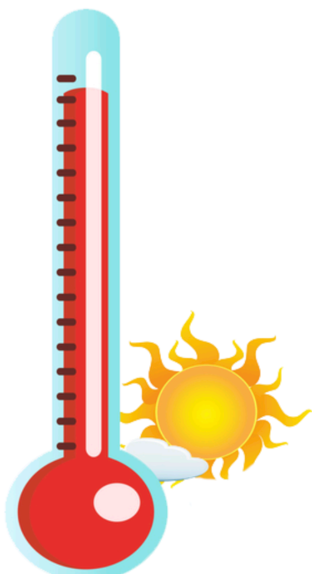
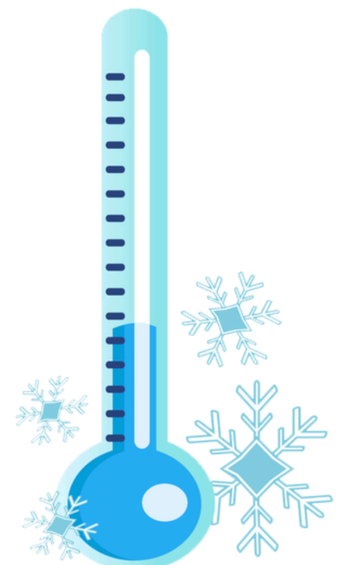


HOT RELICS

$$T_{FO} \gg m_\chi$$



COLD RELICS

$$T_{FO} \ll m_\chi$$

Two cases for decoupling

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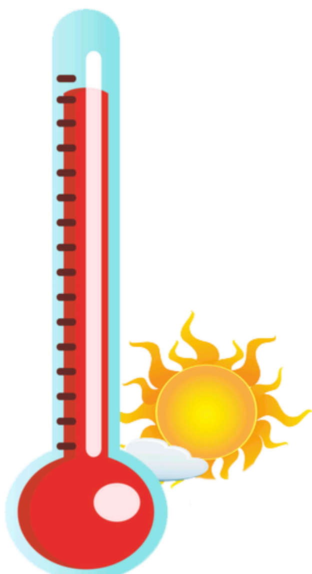

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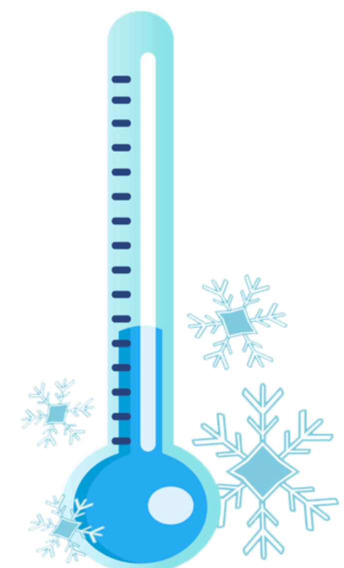
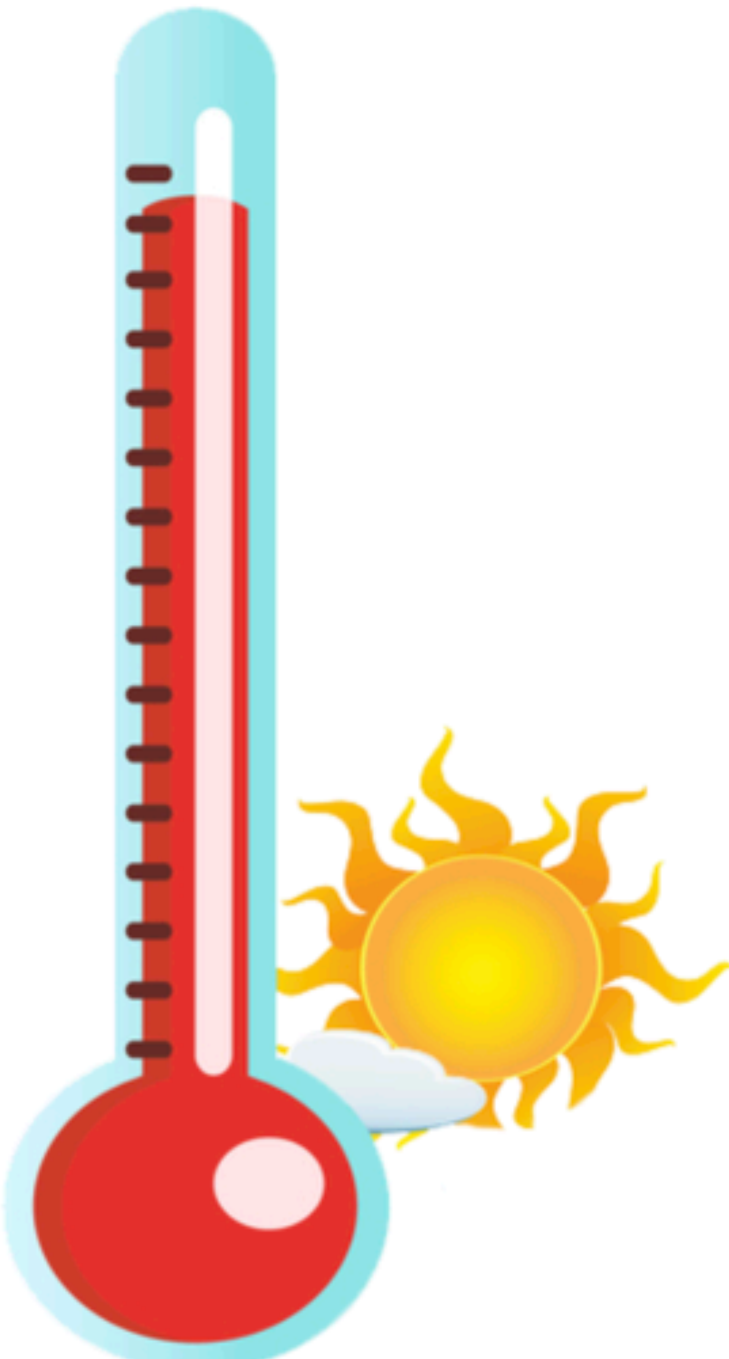


COLD RELICS

$$T_{FO} \ll m_\chi$$

$$n_\chi^{\rm eq}(T) = \begin{cases} g_{\rm eff}\,\frac{\zeta(3)}{\pi^2}T^3 & T \gg m_\chi \\ g_\chi\left(\frac{m_\chi T}{2\pi}\right)^{3/2}\exp(-m_\chi/T) & T \ll m_\chi \end{cases}$$



HOT RELICS

$$T_{FO} \gg m_\chi$$

Freeze-out Temperature

$$g_{\rm eff}\,\frac{\zeta(3)}{\pi^2}T_{FO}^3\,\langle\sigma v_{\rm rel}\rangle = \frac{\pi\,g_*^{1/2}(T_{\rm FO})}{3\sqrt{10}}\,\frac{T_{FO}^2}{M_{\rm Pl}}$$

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$$T_{FO} = \frac{\pi^3}{3\,\zeta(3)\,\sqrt{10}} \frac{g_*^{1/2}(T_{\rm FO})}{g_{\rm eff}} \frac{1}{\langle \sigma v_{\rm rel} \rangle\, M_{\rm Pl}}$$

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Consistency condition:

$$T_{FO} \gg m_\chi$$

We have hot relics for small dark matter mass and/or small annihilation cross section

Current Density for Hot Relics

After interactions stop being effective, number density diluted by Hubble expansion

$$Y_\chi(T_{FO}) = \frac{n_\chi^{\rm eq}(T_{FO})}{s(T_{FO})} = {\rm const} = Y_\chi(T_0)$$

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HOT RELICS

$$Y_\chi(T_{FO}) \simeq 0.0026\, g_{\rm eff} \left(\frac{106.75}{g_{*s}(T_{FO})} \right)$$

Cowsik-McClelland Bound

CURRENT MASS DENSITY

$$\rho_{\rm hot}(T_0) = m_\chi n_\chi(T_0) = m_\chi Y_\chi(T_{FO}) s(T_0)$$

$$\Omega_\chi h^2 \simeq 0.076 \left(\frac{g_{\rm eff}}{g_{*s}(T_{FO})}\right)\left(\frac{m_\chi}{\rm eV}\right)$$

Cowsik-McClelland Bound

CURRENT MASS DENSITY

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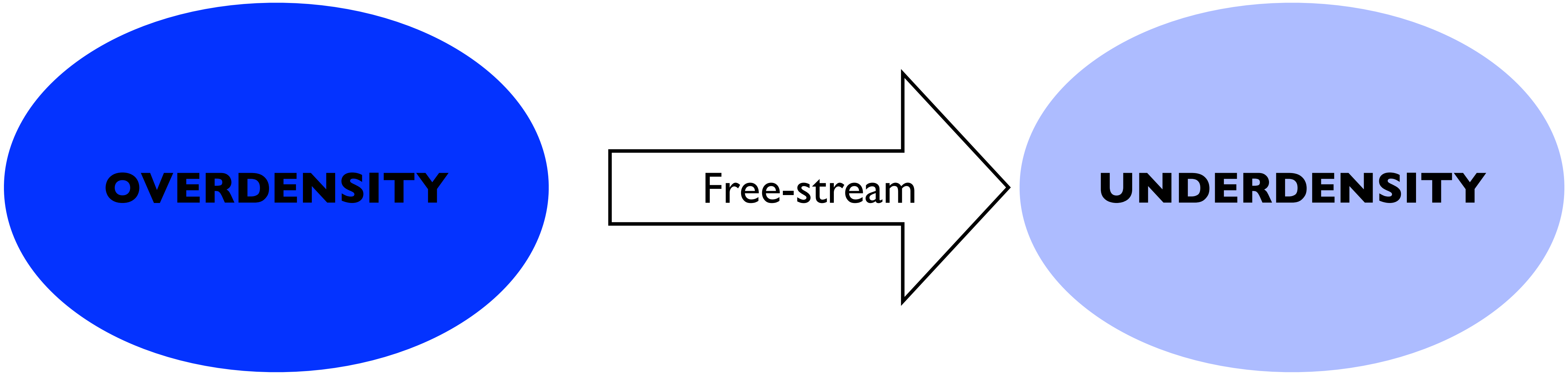
$$\Omega_\chi h^2 \simeq 0.076 \left(\frac{g_{\rm eff}}{g_{*s}(T_{FO})}\right)\left(\frac{m_\chi}{\rm eV}\right)$$

We cannot have more than what we observe!

$$m_\chi \lesssim 168\,{\rm eV}\,\frac{1}{g_{\rm eff}}\left(\frac{g_{*s}(T_{FO})}{106.75}\right)$$

Free-Streaming for Hot Relics

After decoupling, hot relics free-stream from overdense to underdense regions, erasing density perturbations



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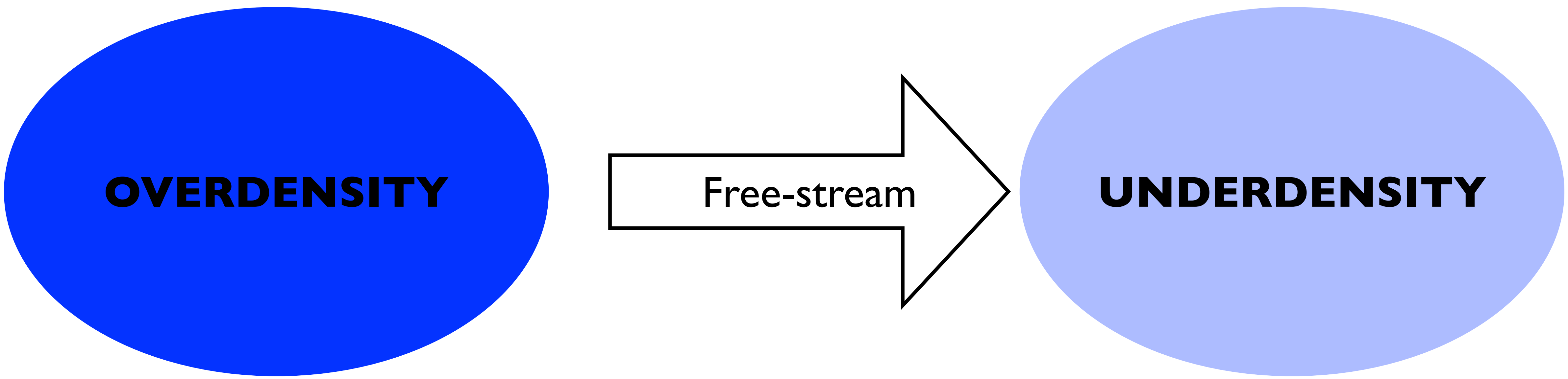


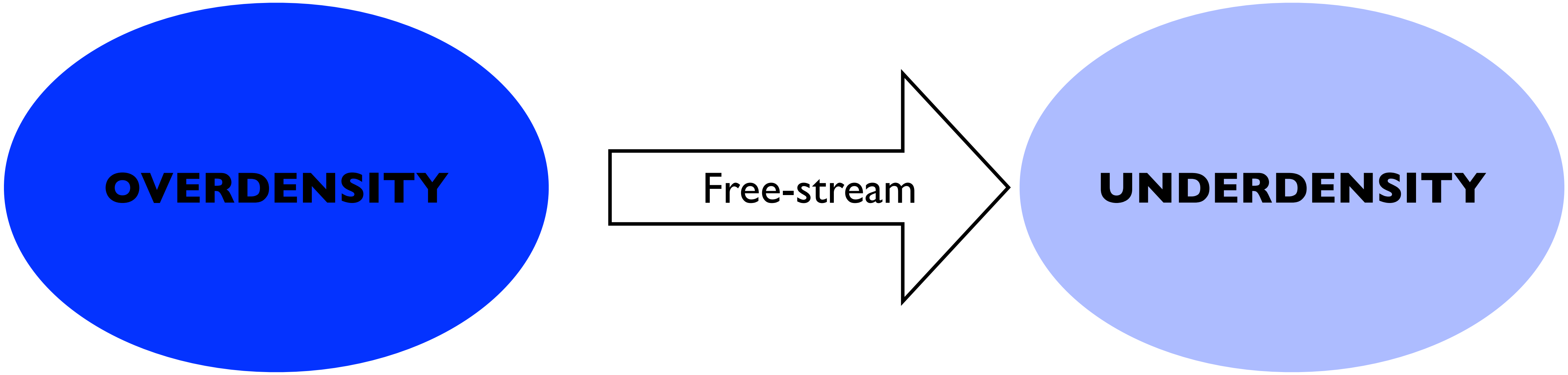
Free-streaming efficient until matter/radiation equality when gravitational collapse takes over

$$\frac{\delta\rho}{\rho} \propto \begin{cases} \ln(a) & \mathrm{RD} \\ a & \mathrm{MD} \end{cases}$$

From first lecture

Free-Streaming for Hot Relics

After decoupling, hot relics free-stream from overdense to underdense regions, erasing density perturbations



FREE-STREAMING SCALE

PROBLEM 1

$$\lambda_{\rm FS} = \frac{2\, t_{\rm NR}}{a(t_{\rm NR})} \left[1 + \ln \left(\frac{T_{\rm NR}}{T_{\rm eq}} \right) \right]$$

Free-Streaming for Hot Relics

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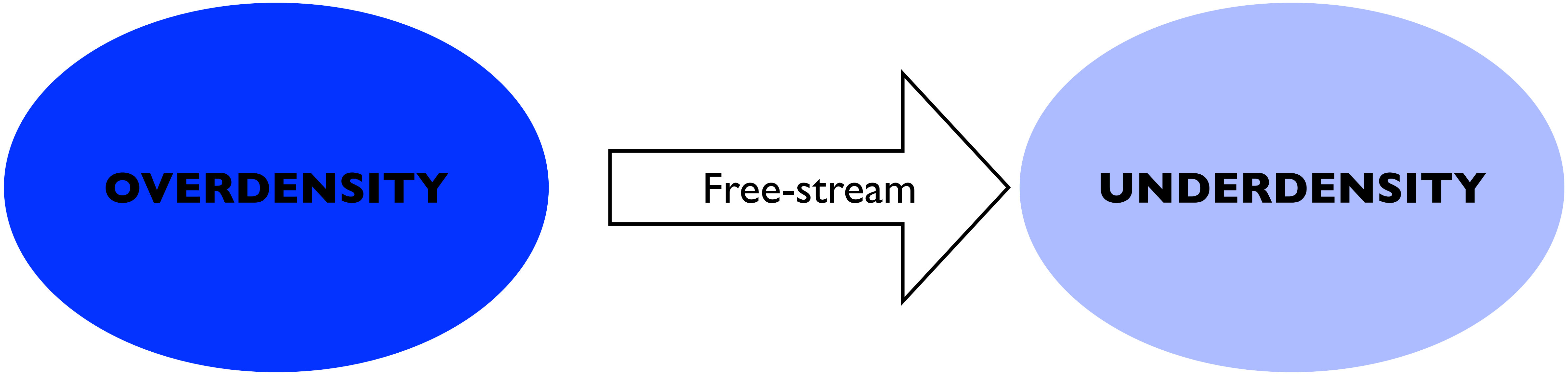

FREE-STREAMING MASS

PROBLEM 1

$$M_{\rm FS} \simeq 2.5 \times 10^{11}\, M_{\rm SUN} \left(\frac{1\,{\rm keV}}{m_X}\right)^2$$

Free-Streaming for Hot Relics

Hot relics lighter than keV erase density perturbations on scales greater than galactic halos

FREE-STREAMING MASS

PROBLEM I

$$M_{\mathrm{FS}} \simeq 2.5 \times 10^{11}\, M_{\mathrm{SUN}} \left(\frac{1\,\mathrm{keV}}{m_X}\right)^2$$

SM neutrinos do not work

We can now see why SM neutrinos are not viable dark matter candidates

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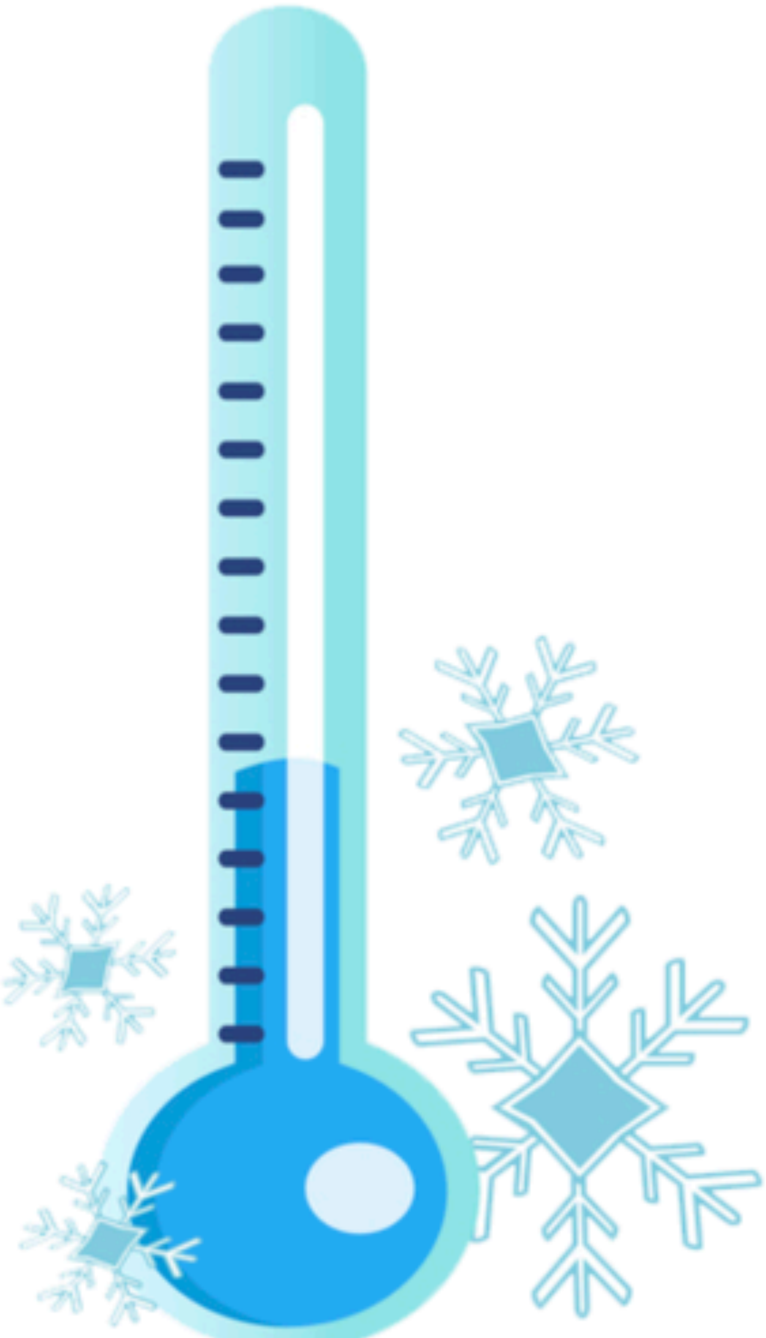
Terrestrial and cosmological bounds constrain their mass to be below eV:

- violation of the Tremaine-Gunn bound
- violation of the Cowsik-McClelland bound
- too much free-streaming

We need new physics beyond the Standard Model!



COLD RELICS

$$T_{FO} \ll m_\chi$$

WIMPs

Weakly Interacting Massive Particles (WIMPs): top-down motivated candidates

$$1\,\mathrm{GeV} \lesssim m_{\mathrm{WIMP}} \lesssim 10\,\mathrm{TeV}$$

$$\sigma_{\mathrm{WIMP}} \simeq 1\mathrm{pb}$$

WIMPs

Weakly Interacting Massive Particles (WIMPs):
top-down motivated candidates

$$1\,\mathrm{GeV} \lesssim m_{\mathrm{WIMP}} \lesssim 10\,\mathrm{TeV}$$

$$\sigma_{\mathrm{WIMP}} \simeq 1\mathrm{pb}$$

They decouple when they are non-relativistic

Partial wave expansion: $\sigma v = a + bv^2 + \ldots$

PROBLEM 2

WIMP Freeze-Out

$$g_\chi \left(\frac{m_\chi T_{FO}}{2\pi} \right)^{3/2} \exp(-m_\chi / T_{FO}) \langle \sigma v_{\rm rel} \rangle \simeq \frac{\pi \, g_*^{1/2}(T_{\rm FO})}{3\sqrt{10}} \, \frac{T_{FO}^2}{M_{\rm Pl}}$$

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$$x_{FO} \equiv \frac{m_\chi}{T_{FO}}$$

$$e^{x_{FO}}\, x_{FO}^{-1/2} \simeq \frac{3\sqrt{5}}{2\,\pi^{5/2}} \frac{g_\chi}{g_*^{1/2}(x_{\rm FO})} m_\chi\, M_{\rm Pl}\, \langle \sigma v_{\rm rel} \rangle$$

WIMP Freeze-Out

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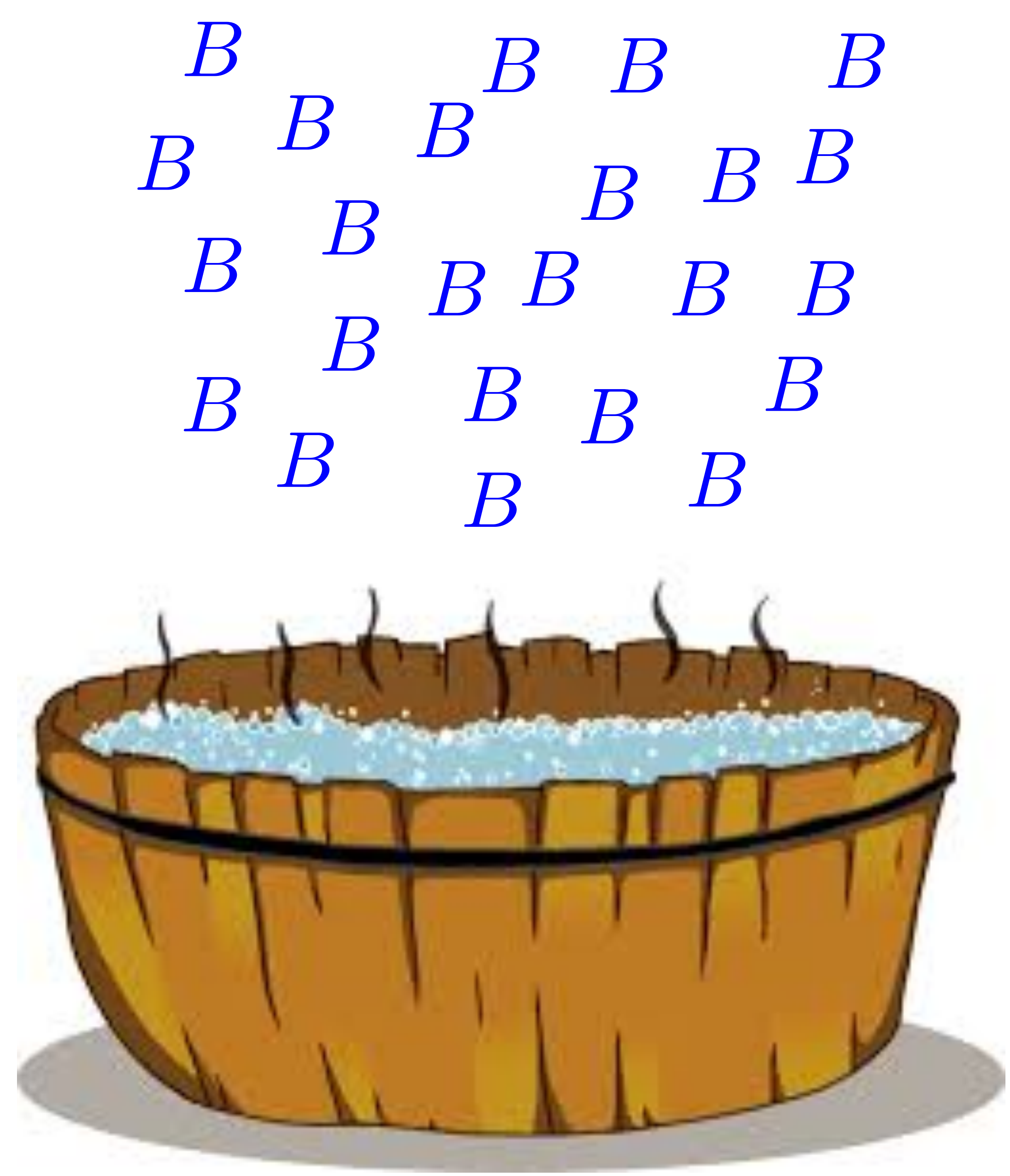
Weak (logarithmic) dependence
on mass and cross section

WIMPs

$$x_{FO} \equiv 20 - 25$$

WIMP Genesis

WIMP genesis takes place when the universe is dominated by a radiation bath (this is an assumption!)

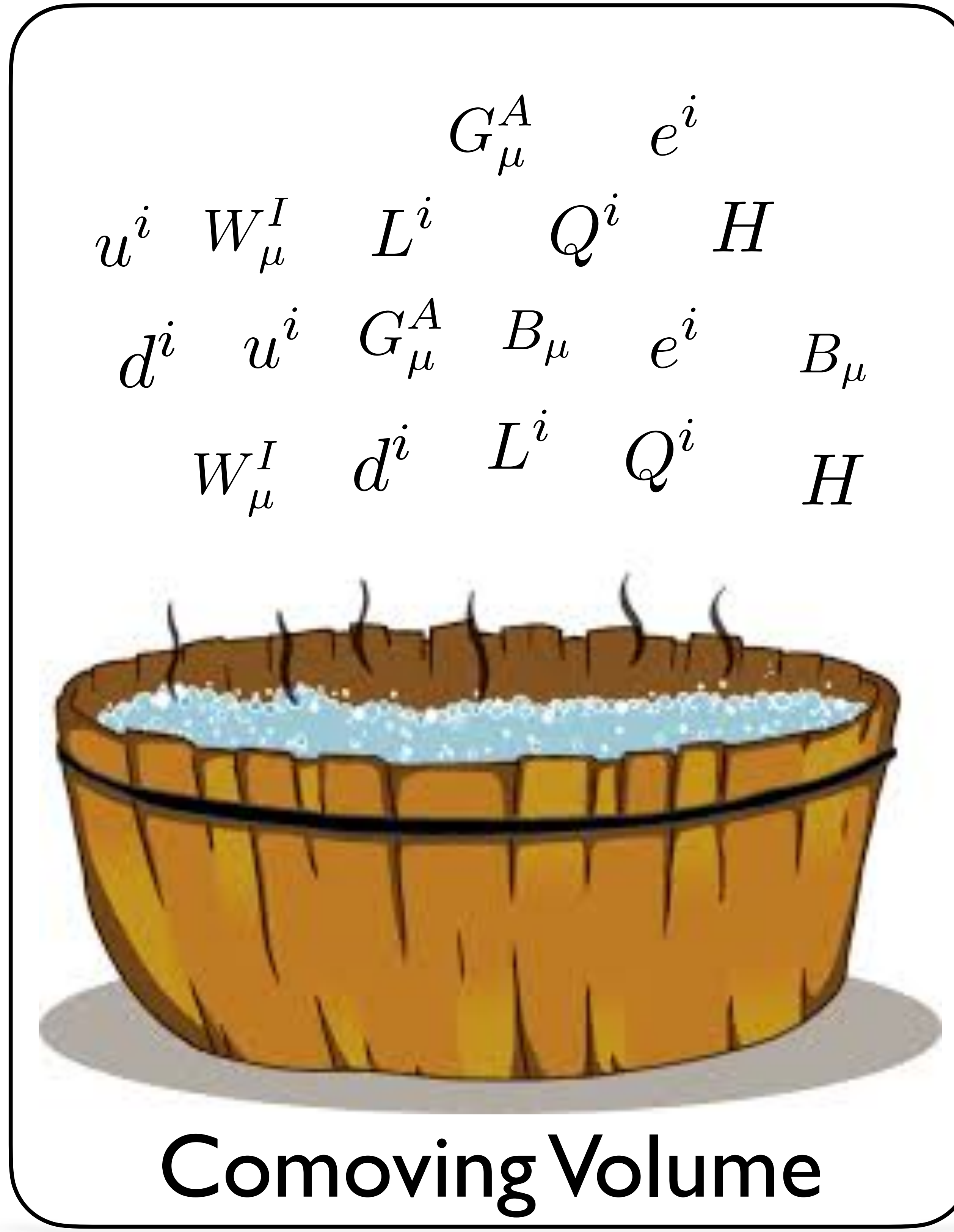


Comoving Volume

WIMP Genesis

WIMP genesis takes place when the universe is dominated by a radiation bath (this is an assumption!)

It is definitely filled by Standard Model particles

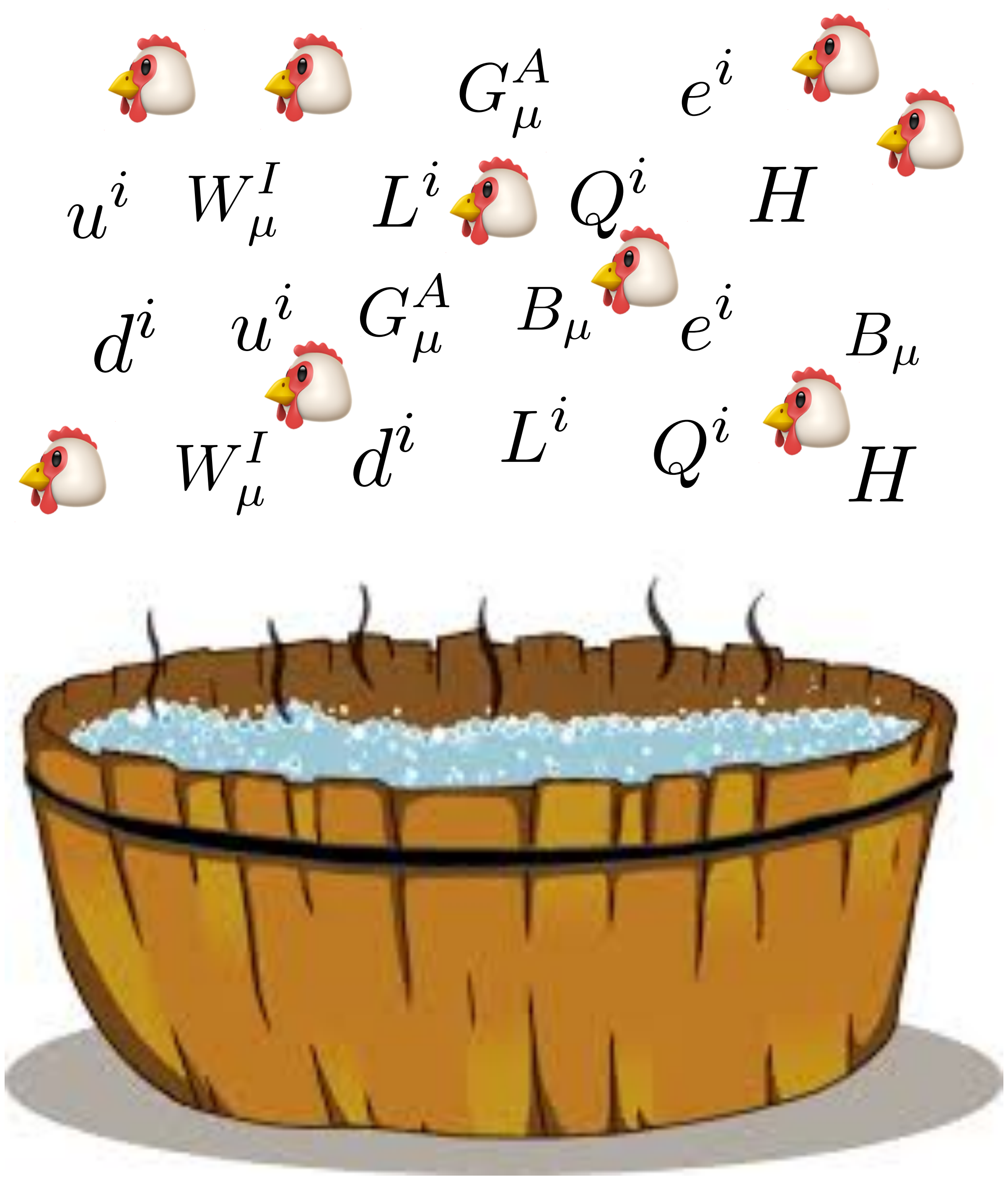


WIMP Genesis

WIMP genesis takes place when the universe is dominated by a radiation bath (this is an assumption!)

It is definitely filled by Standard Model particles

There could be also something else (as it is the case for several motivated theories), just a change in g_*

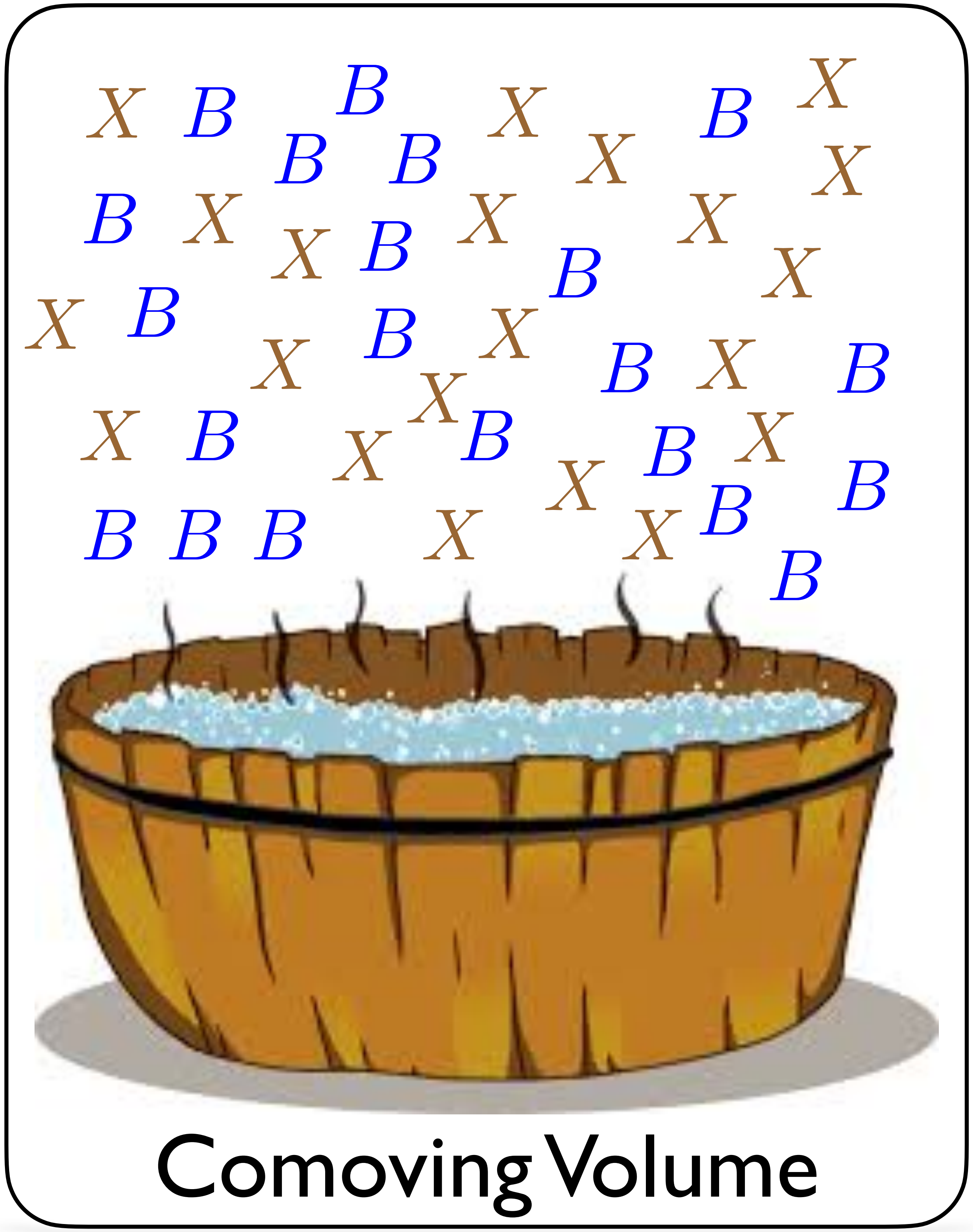


Comoving Volume

WIMP Genesis

<u>T ≫ dark matter mass:</u>

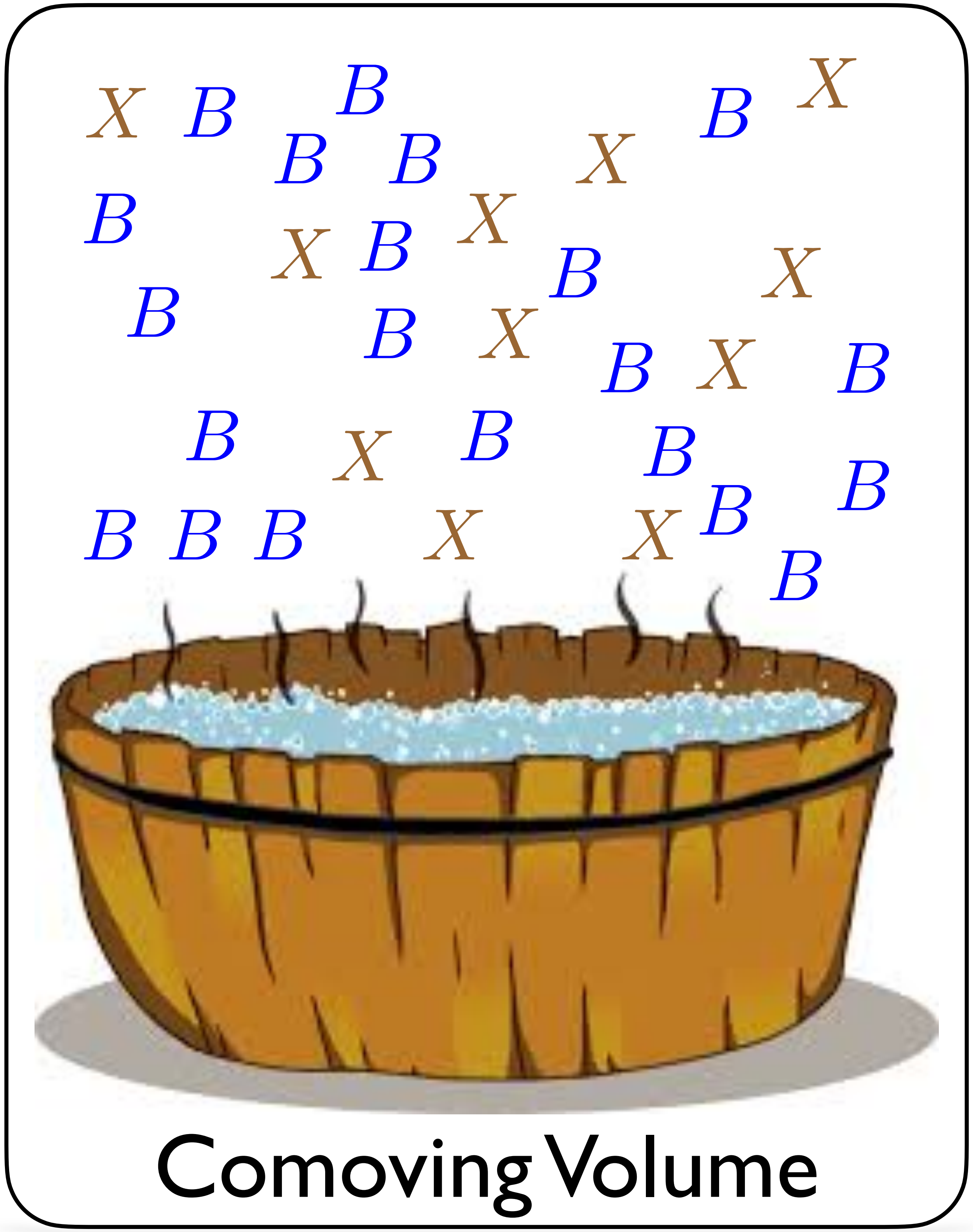
dark matter particles (X) in thermal equilibrium with the primordial bath particles (B)



WIMP Genesis

<u>T ~ dark matter mass:</u>

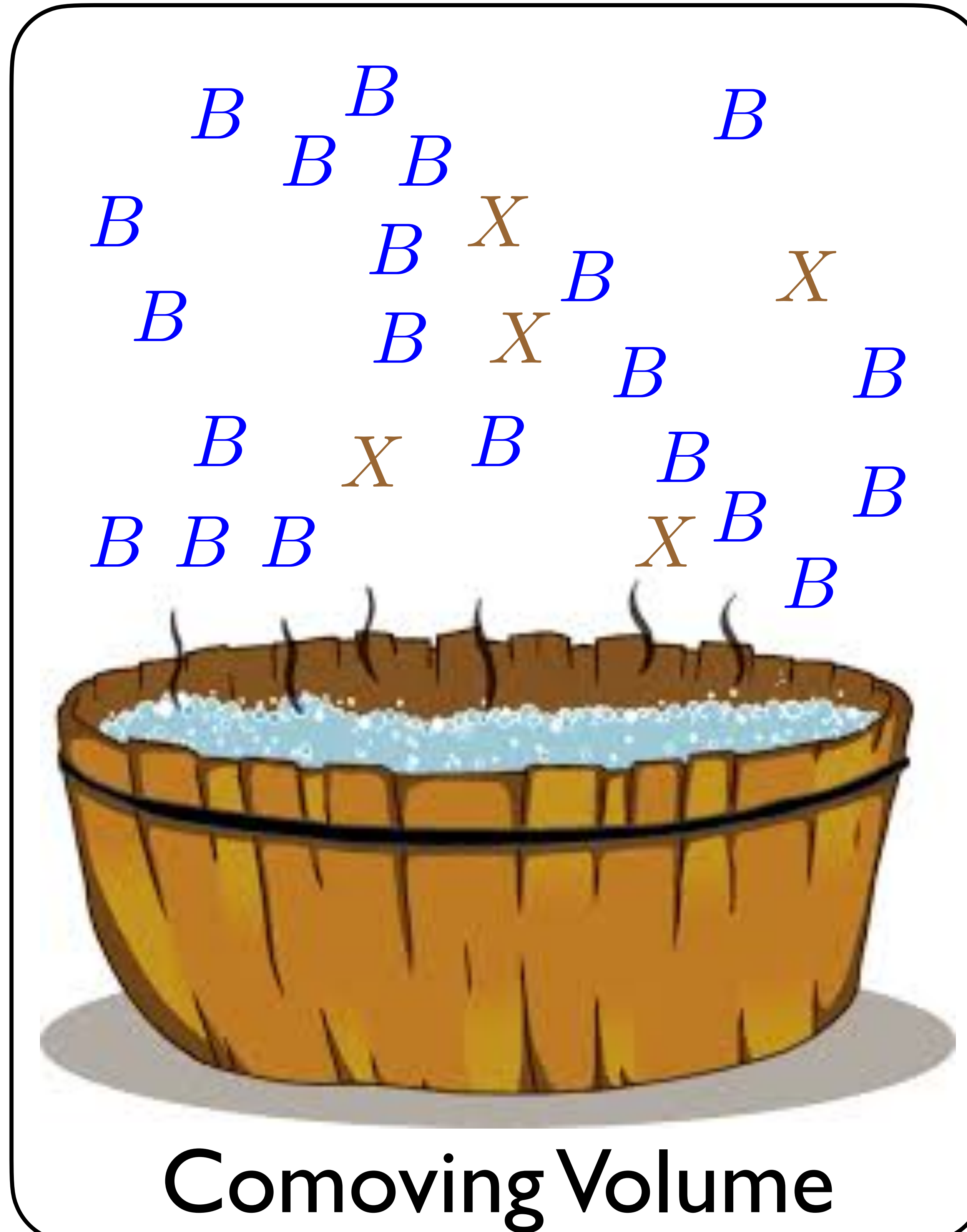
dark matter particles (X) starts to feel the Maxwell-Boltzmann suppression



WIMP Genesis

<u>T ~ dark matter mass / 25:</u>

dark matter particles (X) decouple from the plasma and their comoving density freezes-out

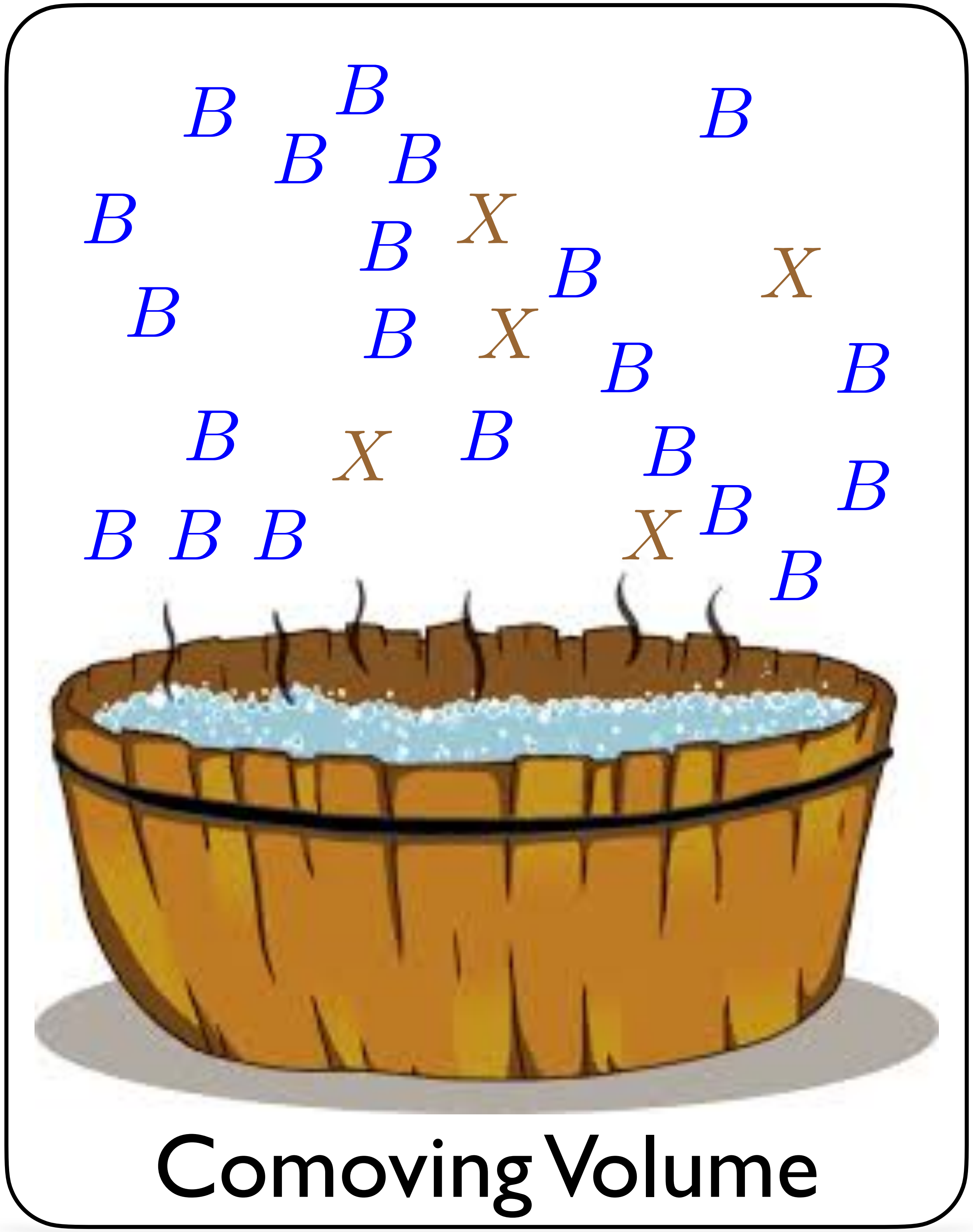


WIMP Genesis

<u>T ≪ dark matter mass / 25:</u>

dark matter particles (X) comoving density remains the same until today

We only need to compute the resulting relic density and compare it with observations!



WIMP Relic Density

$$Y_\chi(T_{FO}) = \frac{n_\chi(T_{FO})}{s(T_{FO})} \simeq \frac{H(T_{FO})/\langle\sigma v_{\rm rel}\rangle}{\frac{2\pi^2}{45} g_{*s}(T_{FO}) T_{FO}^3} \simeq \frac{3\sqrt{5}}{2\sqrt{2}\pi} \frac{g_*^{1/2}(T_{\rm FO})}{g_{*s}(T_{FO})} \frac{x_{FO}}{m_\chi\, M_{\rm Pl}\, \langle\sigma v_{\rm rel}\rangle}$$

WIMP Relic Density

$$Y_\chi(T_{FO}) = \frac{n_\chi(T_{FO})}{s(T_{FO})} \simeq \frac{H(T_{FO})/\langle\sigma v_{\rm rel}\rangle}{\frac{2\pi^2}{45} g_{*s}(T_{FO}) T_{FO}^3} \simeq \frac{3\sqrt{5}}{2\sqrt{2}\pi} \frac{g_*^{1/2}(T_{\rm FO})}{g_{*s}(T_{FO})} \frac{x_{FO}}{m_\chi\, M_{\rm Pl}\, \langle\sigma v_{\rm rel}\rangle}$$

$$\Omega_\chi h^2 = \frac{\rho_\chi}{\rho_{\rm cr}/h^2} = \frac{m_\chi Y_\chi(T_{FO})\, s_0}{\rho_{\rm cr}/h^2} \simeq 2.07 \times 10^8 \frac{g_*^{1/2}(T_{\rm FO})}{g_{*s}(T_{FO})} \frac{x_{\rm FO}\,{\rm GeV}^{-1}}{M_{\rm Pl}\langle\sigma v_{\rm rel}\rangle}$$

The relic density has a weak dependence on the dark matter mass (logarithmic) and it is proportional to the inverse annihilation cross section

(Tomorrow we will study this in more detail with the help of the Boltzmann equation)

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$$\Omega_\chi h^2 \simeq \frac{1}{M_{\rm Pl}\, T_{\rm eq}\, \langle \sigma v_{\rm rel}\rangle}$$

Not a miracle,
just a remarkable
numerical coincidence!