

Scattering of Massive States in Pure Spinor

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Introduction

- Pure Spinor formulation is a manifestly super-Poincare covariant worldsheet theory of superstrings.
Berkovits
- Tremendously successful in computing of scattering of massless string states – at loop levels surpasses the well-known RNS formalism approach.
Berkovits, Chandia, Mafra, Gomez, Schlotterer,...
- The massive states remained comparatively unexplored. Unintegrated vertex for 1st massive state was constructed by Berkovits & Chandia ([hep-th/0204121](#)). DDF-like construction was given by Jusinskas ([1406.1902](#)).
- General arguments established equivalence between PS and RNS/GS formalism. Explicit equivalence known for massless states.
Berkovits; Berkovits, Mafra...
- Explicit equivalence for massive states?

1-slide summary of PS formalism

- The worldsheet action is $S = \frac{1}{\pi\alpha'} \int d^2z \ \partial X^m \bar{\partial} X_m + p_\alpha \partial \theta^\alpha + w_\alpha \partial \lambda^\alpha$
- The commuting Majorana-Weyl spinor λ satisfies the pure spinor constraint

$$\lambda \gamma^m \lambda = 0$$

- Spacetime SUSY is made manifest in terms of the GS variables Π^m, d_α
- The PS constraint implies a gauge invariance $\delta w_\alpha = \Lambda_m (\gamma^m \lambda)_\alpha$, which is dealt with by working with gauge invariant currents

$$N^{mn} = w \gamma^{mn} \lambda \quad J = w_\alpha \lambda^\alpha$$

- Our ordering convention for any composite object is $\{\Pi^m, d_\alpha, \partial \theta^\alpha, N^{mn}, J, \lambda^\alpha\}$
- BRST operator takes a simple form

$$Q = \oint dz \lambda^\alpha d_\alpha$$

Vertex operators in PS

- All vertex operators are of the form

$$V \sim \sum_i O_i \ S_i \xrightarrow{\hspace{1cm}} \text{Appropriate superfields}$$

\downarrow

$$\{\Pi^m, d_\alpha, \partial\theta^\alpha, N^{mn}, J, \lambda^\alpha\}$$

- For n-th massive state, the unintegrated vertex V must have conformal weight = n and ghost number = 1.
- $\{\Pi^m, d_\alpha, \partial\theta^\alpha, N^{mn}, J\}$ All have conformal weight = 1, ghost no. = 0.
- λ^α has conformal weight = 0 but ghost number = 1.

Unintegrated vertex

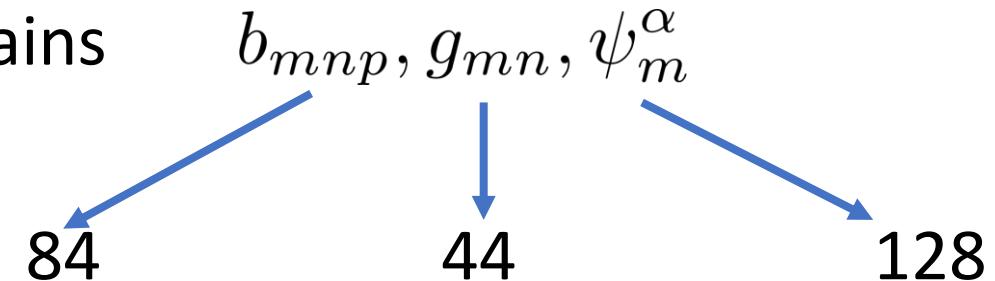
- Constructing the vertex requires determining the superfields S_i in terms of the physical superfield representing the supermultiplet such that the BRST condition is satisfied.

$$QV = 0 \longrightarrow \sum_i (QO_i)S_i + O_i(QS_i) = 0 \implies \sum_i \tilde{O}_i \tilde{S}_i = 0$$

- Ideally, we would like to set all $\tilde{S}_i = 0$ and solve for the Superfields.
- However, due to pure spinor constraint, not all of the \tilde{O}_i are actually independent. So one needs to take care of that.

1st Massive states in open superstrings

- The first massive supermultiplet contains



- These physical fields satisfy

$$k^m b_{mnp} = k^m g_{mn} = k^m \psi_m^\alpha = 0$$

$$k^m k_m = -\frac{1}{\alpha'}$$

$$(\gamma^m \psi_m)_\alpha = 0 \qquad \eta^{mn} g_{mn} = 0$$

- Berkovits-Chandia constructed the unintegrated vertex for this supermultiplet. All superfields must be expressed in terms of the physical on-shell superfield B_{mnp} and its super-covariant derivative.

hep-th/0204121

Berkovits-Chandia construction

$$V = \partial\lambda^\alpha A_\alpha(X, \theta) + : \partial\theta^\beta \lambda^\alpha B_{\alpha\beta}(X, \theta) : + : d_\beta \lambda^\alpha C_\alpha^\beta(X, \theta) : + : \Pi^m \lambda^\alpha H_{m\alpha}(X, \theta) : + : J\lambda^\alpha E_\alpha(X, \theta) : + : N^{mn} \lambda^\alpha F_{\alpha mn}(X, \theta) :$$

$$V \sim \sum_i O_i S_i$$

$$B_{\alpha\beta} = (\gamma^{mnp})_{\alpha\beta} B_{mnp} , \quad C^\alpha_\beta = (\gamma^{mnpq})^\alpha_\beta C_{mnpq}$$

$$\gamma^{m\alpha\beta} F_{\beta mn} = 0 , \quad (\gamma^m H_m)_\alpha = 0$$

$\delta V = Q\Lambda$ and field redefinition

recall that all operators were not independent

$$: N^{mn} \lambda^\alpha : (\gamma_m)_{\alpha\beta} - \frac{1}{2} : J\lambda^\alpha : (\gamma^n)_{\alpha\beta} = \alpha' \partial\lambda^\alpha (\gamma^n)_{\alpha\beta} \longrightarrow \text{follows from pure spinor constraint}$$

↓

$$-2 : N_{st} \lambda^\alpha \lambda^\beta : (\gamma^{vwxy} \gamma^{[s})_{\alpha\beta} K^{t]}_{vwxy} + : J\lambda^\alpha \lambda^\beta : (\gamma^{vwxy} \gamma_s)_{\alpha\beta} K^s_{vwxy} + \alpha' \lambda^\alpha \partial\lambda^\beta \left[2(\gamma^{vwxys})_{\alpha\beta} \eta_{st} K^t_{vwxy} + 16(\gamma^{wxy})_{\alpha\beta} K^s_{wxy s} \right] = 0$$

introduce lagrange multiplier superfield K^s_{mnpq}

Berkovits-Chandia construction

- QV + (Lagrange multiplier term) = 0 gives \longrightarrow $(\gamma_{mnpqr})^{\alpha\beta} [D_\alpha B_{\beta\sigma} - \gamma_{\alpha\sigma}^s H_{s\beta}] = 0$
- Formidable set of equations, but it was solved! $(\gamma_{mnpqr})^{\alpha\beta} [D_\alpha H_{s\beta} - \gamma_{s\alpha\sigma} C^\sigma{}_\beta] = 0$
- All Superfields were expressed in terms of a Single superfield B_{mnp} and its supercovariant derivative $D_\alpha B_{mnp}$. $(\gamma_{mnpqr})^{\alpha\beta} \left[D_\alpha C^\sigma{}_\beta + \delta^\sigma{}_\alpha E_\beta + \frac{1}{2} (\gamma^{st})^\sigma{}_\alpha F_{\beta st} \right] = 0$
- Rest frame analysis showed this superfield B_{mnp} contains all the 128 bosonic + 128 fermionic d.o.f $(\gamma_{mnpqr})^{\alpha\beta} \left[D_\alpha A_\beta + B_{\alpha\beta} + \alpha' \gamma_{\beta\sigma}^s \partial_s C^\sigma{}_\alpha - \frac{\alpha'}{2} D_\beta E_\alpha + \frac{\alpha'}{4} (\gamma^{st} D)_\beta F_{\alpha st} \right]$
 $= 2\alpha' \gamma_{mnpqr}^{\alpha\beta} \gamma_{\alpha\beta}^{vwxy} \eta_{st} K^t{}_{vwxy}$
 $(\gamma_{mnp})^{\alpha\beta} \left[D_\alpha A_\beta + B_{\alpha\beta} + \alpha' \gamma_{\beta\sigma}^s \partial_s C^\sigma{}_\alpha - \frac{\alpha'}{2} D_\beta E_\alpha + \frac{\alpha'}{4} (\gamma^{st} D)_\beta F_{\alpha st} \right]$
 $= 16\alpha' \gamma_{mnp}^{\alpha\beta} \gamma_{\alpha\beta}^{wxy} \eta_{st} K^s{}_{wxy}$
- $(\gamma_{mnpqr})^{\alpha\beta} D_\alpha E_\beta = (\gamma_{mnpqr} \gamma^{vwxy} \gamma_s)^\alpha{}_\alpha K^s{}_{vwxy}$
- $(\gamma_{mnpqr})^{\alpha\beta} D_\alpha F_\beta^{st} = -2(\gamma_{mnpqr} \gamma^{vwxy} \gamma^{[s})^\alpha{}_\alpha K^{t]}{}_{vwxy}$

What more do we want?

- Vertex operators are used to compute scattering amplitudes. For this one needs the full covariant θ expansion of the vertex.
- The BC construction only allows one to perform a θ expansion in *rest frame*.
- We also need a general prescription to repeat this feat for integrated vertex and for all higher massive states.
- Goals-
 1. Give a systematic procedure to construct massive vertices in PS .
 2. The procedure must also allow one to perform the full covariant θ expansion of the vertex.
 3. Compute scattering of massive states, compare with RNS and extend the explicit equivalence to include massive states.

A systematic procedure to construct massive vertex

- Step 1: For the n-th massive state, write down the most general operator of form

$$V \sim \sum_i O_i S_i \xrightarrow{\text{unknown superfield with proper indices}}$$

↓

conf wt = n and gh no. = 1 operator

- *Example:* For n=1,

$$\begin{aligned} V = & \partial\lambda^\alpha A_\alpha(X, \theta) + : \partial\theta^\beta \lambda^\alpha B_{\alpha\beta}(X, \theta) : + : d_\beta \lambda^\alpha C_\alpha^\beta(X, \theta) : + : \Pi^m \lambda^\alpha H_{m\alpha}(X, \theta) : \\ & + : J\lambda^\alpha E_\alpha(X, \theta) : + : N^{mn} \lambda^\alpha F_{\alpha mn}(X, \theta) : \end{aligned}$$

- Step 2: For each physical field in the supermultiplet, introduce a superfield defined as

$$\begin{aligned} B_i \Big|_{\theta=0} &\equiv b_i \quad , \quad \forall i = 1(1)n_B \\ F_i \Big|_{\theta=0} &\equiv f_i \quad , \quad \forall i = 1(1)n_F . \end{aligned}$$

and promote all algebraic conditions on the physical fields to superfields. We will call these Superfields ***physical Superfields.***

Example: For n=1, $B_{mnp} \Big|_{\theta=0} \equiv b_{mnp} \quad , \quad G_{mn} \Big|_{\theta=0} = g_{mn} \quad , \quad \Psi_{m\alpha} \Big|_{\theta=0} = \psi_{m\alpha}.$

$$k^m \psi_{m\alpha} = 0 \rightarrow k^m \Psi_{m\alpha} \equiv 0 \quad , \quad (\gamma^m \psi_{(m)})_\alpha = 0 \rightarrow (\gamma^m \Psi_{(m)})_\alpha \equiv 0$$

$$k^m b_{mnp} = 0 \rightarrow k^m B_{mnp} \equiv 0 \quad , \quad k^m g_{mn} = 0 \rightarrow k^m G_{mn} \equiv 0$$

- Step 3: Derive all constraints between operators O_i at conformal weight n and ghost no. 2. These constraints are either due to pure spinor constraint or due the OPE of various constituent objects.
- Handle the constraints either by introducing Lagrange multiplier Superfields or by solving them to eliminate some operators in favor of others.
- Incorporate them in the BRST condition $QV=0$.
- *Example: for n=1,*

$$\begin{aligned}
 & -2 : N_{st} \lambda^\alpha \lambda^\beta : (\gamma^{vwxy} \gamma^{[s})_{\alpha\beta} K^t]_{vwxy} + : J \lambda^\alpha \lambda^\beta : (\gamma^{vwxy} \gamma_s)_{\alpha\beta} K^s_{vwxy} \\
 & + \alpha' \lambda^\alpha \partial \lambda^\beta \left[2(\gamma^{vwxys})_{\alpha\beta} \eta_{st} K^t_{vwxy} + 16(\gamma^{wxy})_{\alpha\beta} K^s_{wxy s} \right] = 0
 \end{aligned}$$

- Step 4: Use representation theory of the little group $\text{SO}(9)$ to write down ansatz for each unknown Superfields S_i as a linear combination of the physical Superfields B_i, F_i .
- Example: For $n=1$, we had the superfield $F_{\alpha mn}$ appearing in the vertex. We can decompose it as

$$F_{\alpha mn} \Rightarrow \begin{cases} F_{\alpha 0a} \\ F_{\alpha ab} \end{cases}$$

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$$F_{\alpha mn} \Rightarrow \left\{ \begin{array}{l} F_{\alpha 0a} \longrightarrow 16 \otimes 9 \rightarrow 16 \oplus 128 \\ F_{\alpha ab} \end{array} \right.$$

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$$F_{\alpha mn} \Rightarrow \left\{ \begin{array}{l} F_{\alpha 0a} \xrightarrow{\hspace{1cm}} 16 \otimes 9 \rightarrow 16 \oplus 128 \xrightarrow{\hspace{1cm}} \text{physical irrep} \\ F_{\alpha ab} \end{array} \right.$$

unphysical irrep

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$$F_{\alpha mn} \Rightarrow \left\{ \begin{array}{l} F_{\alpha 0a} \xrightarrow{\quad} \mathbf{16} \otimes \mathbf{9} \rightarrow \mathbf{16} \oplus \mathbf{128} \xrightarrow{\quad} \text{physical irrep} \\ F_{\alpha ab} \downarrow \qquad \qquad \qquad \downarrow \text{unphysical irrep} \\ \mathbf{16} \otimes \mathbf{36} = \mathbf{16} \oplus \mathbf{128} \oplus \mathbf{432} \end{array} \right.$$

- *Therefore there are 2 independent tensor structures linear in the physical superfield Ψ_α^m in terms of which $F_{\alpha mn}$ can be expressed.*

$$F_{\alpha mn} = a k_{[m} \Psi_{n]\alpha} + b k^s (\gamma_s [m] \Psi_{n]})_\alpha$$

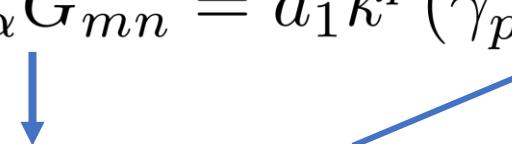
- Step 5: Write down similar ansatz for supercovariant derivative of the physical fields using representation theory.

$$D_\alpha F_i = \sum_j s_j^{(i)} B_j \quad \forall i = 1(1)n_F$$

$$D_\alpha B_i = \sum_j \hat{s}_j^{(i)} F_j \quad \forall i = 1(1)n_B$$

- Example: For $n=1$,

$$D_\alpha G_{mn} = a_1 k^p (\gamma_{p(m} \Psi_{n)})_\alpha$$



 $16 \otimes 44 = 128 \oplus 576 \longrightarrow$ unphysical irrep

- We will see soon that these relations are the ones which will allow us to perform completely covariant θ expansion of the vertex.

- Step 6: Solve for the unknown co-efficients in the ansatz by requiring
 1. The ansatz solves $QV=0$.
 2. The ansatz are consistent with the definitions of the physical superfields.
 3. All the ansatz are mutually consistent.
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New result consistent with BC vertex!

3. *Readily extended to integrated vertex which was constructed for the first time for $n=1$.* (see Sitender's talk right after this!)
4. *For 1st massive states, this vertex gives same result as RNS.*

θ - expansion

- After fixing the unknown coefficients, we get

$$D_\alpha \Psi_{s\beta} = \frac{1}{16} G_{sm} \gamma_{\alpha\beta}^m + \frac{i}{24} k_m B_{nps} (\gamma^{mnp})_{\alpha\beta} - \frac{i}{144} k^m B^{npq} (\gamma_{smnpq})_{\alpha\beta}$$

$$D_\alpha B_{mnp} = 12(\gamma_{[mn} \Psi_{p]})_\alpha + 24\alpha' k^t k_{[m} (\gamma_{|t|n} \Psi_{p]})_\alpha$$

$$k^{\alpha\beta} D_\alpha D_\beta \propto k^2$$

$$D_\alpha G_{sm} = 16ik^p (\gamma_{p(s} \Psi_{m)})_\alpha$$

- Schematically these gives a set of recursion relations which can be leveraged to get the θ expansion

$$D^{(\ell+1)} \Psi_{s\beta} \sim D^\ell G_{sm} + D^\ell B_{mnp}$$

$$D^\ell B_{mnp} \sim D^{(\ell-1)} \Psi_{s\beta} \quad D \equiv \theta^\alpha D_\alpha$$

$$D^\ell G_{mn} \sim D^{(\ell-1)} \Psi_{s\beta}$$

θ - expansion

$\Psi_{s\beta}$

$$\begin{aligned}
 &= \psi_{s\beta} + \frac{1}{16}(\gamma^m \theta)_\beta g_{sm} - \frac{i}{24}(\gamma^{mnp} \theta)_\beta k_m b_{nps} - \frac{i}{144}(\gamma_s{}^{npqr} \theta)_\beta k_n b_{pqr} \\
 &\quad - \frac{i}{2}k^p(\gamma^m \theta)_\beta (\psi_{(m} \gamma_{s)p} \theta) - \frac{i}{4}k_m(\gamma^{mnp} \theta)_\beta (\psi_{[s} \gamma_{np]} \theta) - \frac{i}{24}(\gamma_s{}^{mnpq} \theta)_\beta k_m (\psi_q \gamma_{np} \theta) \\
 &\quad - \frac{i}{6}\alpha' k_m k^r k_s (\gamma^{mnp} \theta)_\beta (\psi_p \gamma_{rn} \theta) + \frac{i}{288}\alpha' (\gamma^{mnp} \theta)_\beta k_m k^r k_s (\theta \gamma^q{}_{nr} \theta) g_{pq} \\
 &\quad - \frac{i}{192}(\gamma^{mnp} \theta)_\beta k_m (\theta \gamma^q{}_{[np} \theta) g_{s]q} - \frac{i}{1152}(\gamma_{smnpq} \theta)_\beta k^m (\theta \gamma_{npt} \theta) g^{qt} \\
 &\quad - \frac{i}{96}k^p(\gamma^m \theta)_\beta (\theta \gamma_{pq(s} \theta) g_{m)q} - \frac{1}{1728}(\gamma^{mnp} \theta)_\beta k_m (\theta \gamma^{tuvw}{}_{nps} \theta) k_t b_{uvw} \\
 &\quad - \frac{1}{864\alpha'}(\gamma_s \theta)_\beta (\theta \gamma^{npq} \theta) b_{npq} - \frac{1}{10368}(\gamma_s{}^{mnpq} \theta)_\beta k_m (\theta \gamma_{tuvw npq} \theta) k^t b^{uvw} \\
 &\quad - \frac{1}{864}(\gamma^m \theta)_\beta (\theta \gamma^{npq} \theta) b_{npq} k_m k_s - \frac{1}{576}(\gamma_{smnpq} \theta)_\beta k^m (\theta \gamma^{tun} \theta) b_u{}^{pq} k_t \\
 &\quad - \frac{1}{96\alpha'}(\gamma^m \theta)_\beta (\theta \gamma^{qr}{}_{(s} \theta) b_{m)r} + \frac{1}{96}(\gamma^m \theta)_\beta (\theta \gamma^{nqr} \theta) k_n k_{(s} b_{m)qr} \\
 &\quad + \frac{1}{96}(\gamma^{mnp} \theta)_\beta k_m (\theta \gamma^r{}_{q[n} \theta) b_{ps]r} k^q + O(\theta^4)
 \end{aligned}$$

Tree level amplitudes in PS

- N-point amplitudes

$$\mathcal{A}_N = \langle V_1 V_2 V_3 \int U_4 \cdots \int U_N \rangle$$

focus on 3-pt $\implies \langle V_1 V_2 V_3 \rangle$

- Factorizes in two parts

$$\mathcal{A}_3 \sim \langle V_1 V_2 V_3 \rangle_{PSS} \langle e^{ik_1 \cdot X} \cdots \rangle$$

- The PSS measure is normalized as

$$\langle \lambda^3 \theta^5 \rangle_{PSS} = 1$$

Tree level amplitudes in PS

- We need to evaluate $\langle V_1 V_2 V_3 \rangle_{PSS}$ subjected to $\langle \lambda^3 \theta^5 \rangle_{PSS} = 1$.
- Each unintegrated vertex is at ghost no. 1, so $V_1 V_2 V_3 \sim \lambda^3$ is automatic.
- We need to just θ expand all 3 vertices and keep only those terms from the product which has 5 θ 's.

$$V_1 V_2 V_3 \rightarrow V_1 V_2 V_3 \Big|_{\theta^5}$$

- Taking into account all such terms and incorporating plane wave part, we get the full answer.

Equivalence to RNS for massive states

- We now have all the ingredients to compute scattering amplitudes of massless and 1st massive states using PS formalism.
- This allows us to directly compare them with corresponding RNS formalism and establish explicit equivalence.
- Consider all *massless-massless-massive* 3-point functions for a *fixed order*.
- Let the relative normalization be defined as

$$\mathcal{N}_{RNS} = g_{field} \mathcal{N}_{PS}$$

- We keep $\mathcal{N}_{PS} = 1$ and keep the relative normalization g_{field} in RNS vertex operators.

$a = \text{gluon}$, $\chi = \text{gluino}$

| | Correlator | RNS | PS |
|------------------------|------------------------------|---|-----------------------|
| all massless | $\langle aaa \rangle$ | $-g_a^3 \sqrt{2\alpha'}$ | $\frac{i}{180}$ |
| | $\langle a\chi\chi \rangle$ | $\frac{1}{\sqrt{2}} g_a g_\chi^2$ | $\frac{1}{360}$ |
| 2-massless & 1 massive | $\langle aab \rangle$ | $6g_a^2 g_b \sqrt{2\alpha'}$ | $\frac{i}{20}$ |
| | $\langle \chi\chi b \rangle$ | $-\frac{1}{2\sqrt{2}} g_\chi^2 g_b$ | $\frac{1}{480}$ |
| | $\langle aag \rangle$ | $-g_a^2 g_g$ | $-\frac{1}{80}$ |
| | $\langle \chi\chi g \rangle$ | $\sqrt{\alpha'} g_\chi^2 g_g$ | $\frac{i\alpha'}{80}$ |
| | $\langle a\chi\psi \rangle$ | $-\frac{16\alpha'}{\sqrt{2}} g_a g_\chi g_\psi$ | $\frac{1}{5}$ |
| | $\langle \chi a\psi \rangle$ | $-\frac{16\alpha'}{\sqrt{2}} g_a g_\chi g_\psi$ | $\frac{1}{5}$ |

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Exactly matches! \rightarrow non-trivial consistency check

$\mathcal{PS} = \mathcal{RNS}$

$$(g_a)^3 = \frac{-i}{180\sqrt{2\alpha'}} \quad , \quad (g_\chi)^2 = \frac{\sqrt{2}}{360 g_a}.$$

$$g_b = \frac{i}{120\sqrt{2\alpha'} g_a^2} \quad , \quad g_g = \frac{1}{80 g_a^2} \quad , \quad g_\psi = -\frac{\sqrt{2}}{80\alpha' g_a g_\chi}$$

With these choices of relative normalizations, we have extended the explicit equivalence between PS & RNS amplitudes to include 1st massive states as well.

Summary & Outlook

- We have given a systematic procedure to construct massive vertices in PS formalism that also gives the covariant θ expansion to all orders.
- We have explicitly shown the equivalence between RNS & PS amplitudes for 1st massive states.
- Can this construction be adapted to construct massive vertices in AdS backgrounds?
- N-point amplitudes for massive states? Higher Loop level?
[Schlotterer's Talk]
- First principle derivation of this prescription? Off-shell states in PS?

SFT Website

- Please check out the following website

<http://string-field-theory.org>

- Associated Zulip chat page

<https://sft.zulipchat.com>

- Comments, suggestions, questions... - Harold Erbin & S.C

*Thank you for listening
&
Stay Safe!*