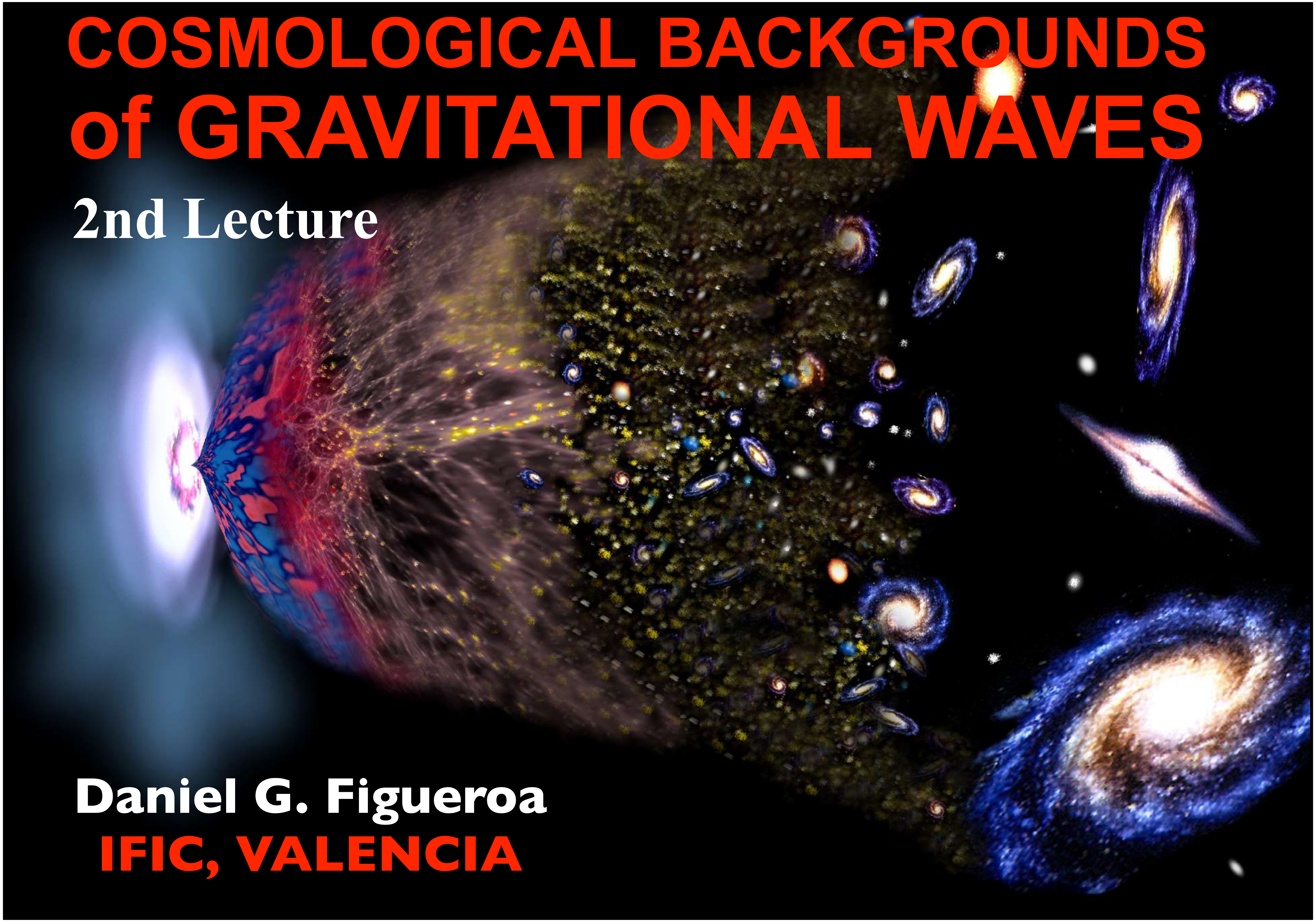


COSMOLOGICAL BACKGROUNDS of GRAVITATIONAL WAVES

2nd Lecture

Daniel G. Figueroa
IFIC, VALENCIA

II Joint ICTP-Trieste/ICTP-SAIFR School on Particle Physics (June 22 - July 3 2020)

COSMOLOGICAL GRAVITATIONAL WAVES

OUTLINE (~ 4.5 h)

✓ 1) Gravitational Waves definition

1st lecture (~ 1.5 h)

Early Universe

2) GWs from Inflation

3) GWs from Preheating

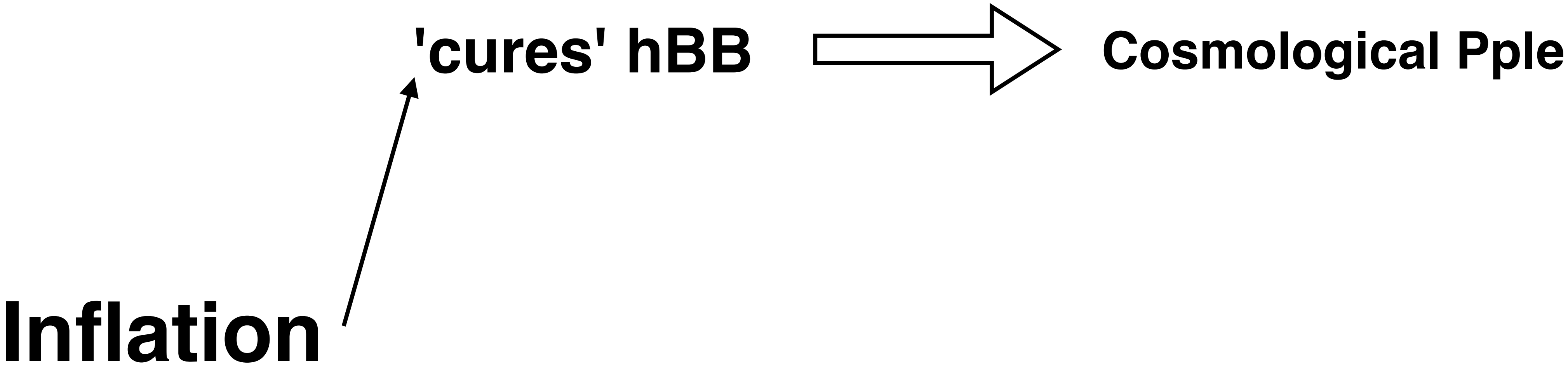
2nd lecture (~ 1.5 h)

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

A primer
on Inflation

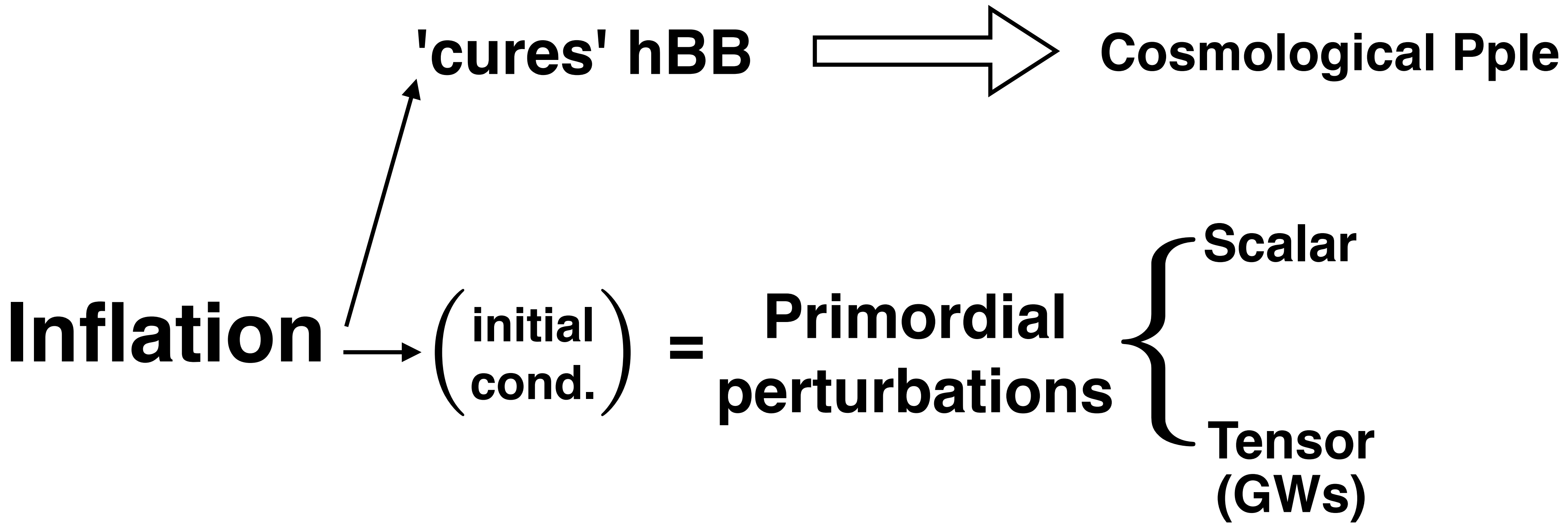
INFLATIONARY COSMOLOGY



'cures' hBB ⇒ Cosmological Pple

Inflation

INFLATIONARY COSMOLOGY

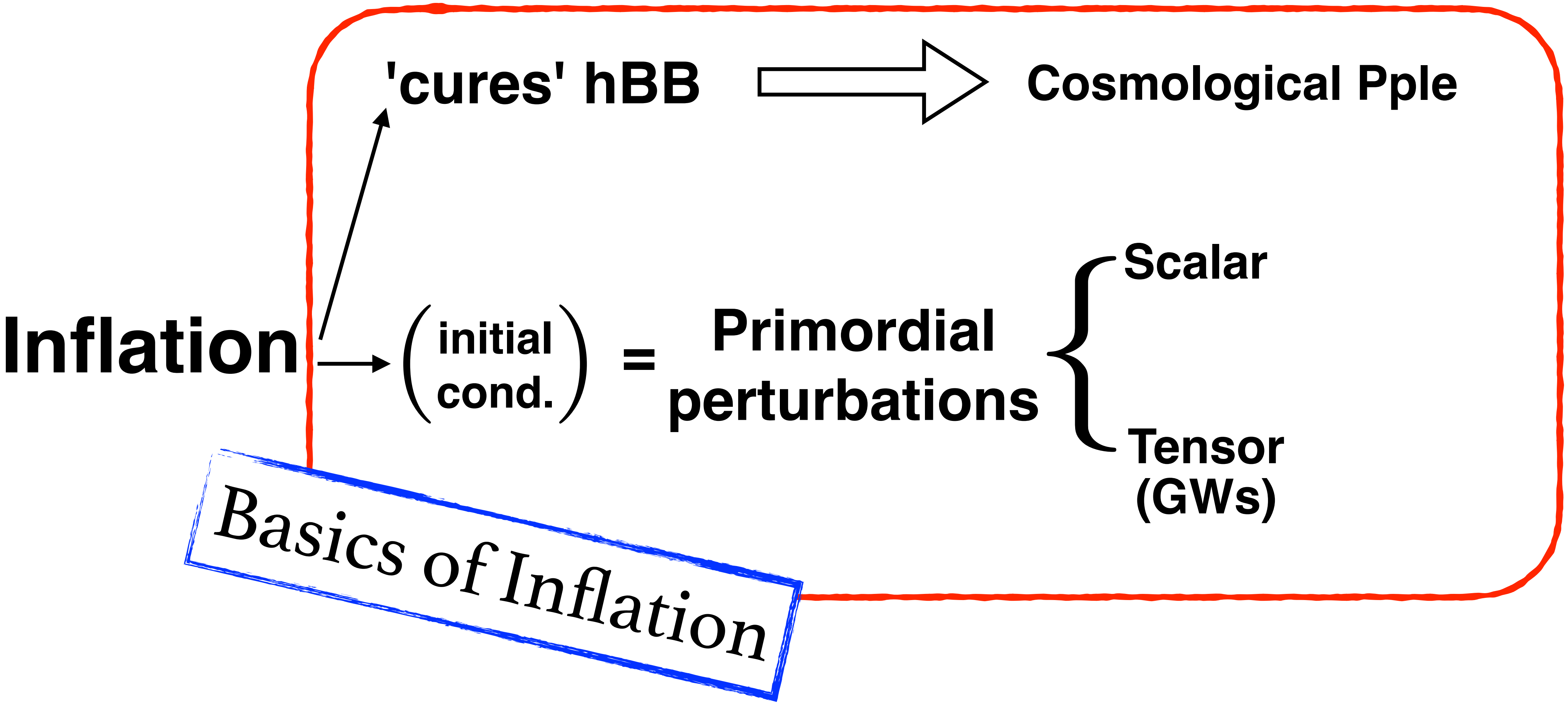


'cures' hBB ⇒ Cosmological Pple

Inflation → (initial cond.) = Primordial perturbations

- Scalar
- Tensor (GWs)

INFLATIONARY COSMOLOGY



Inflation

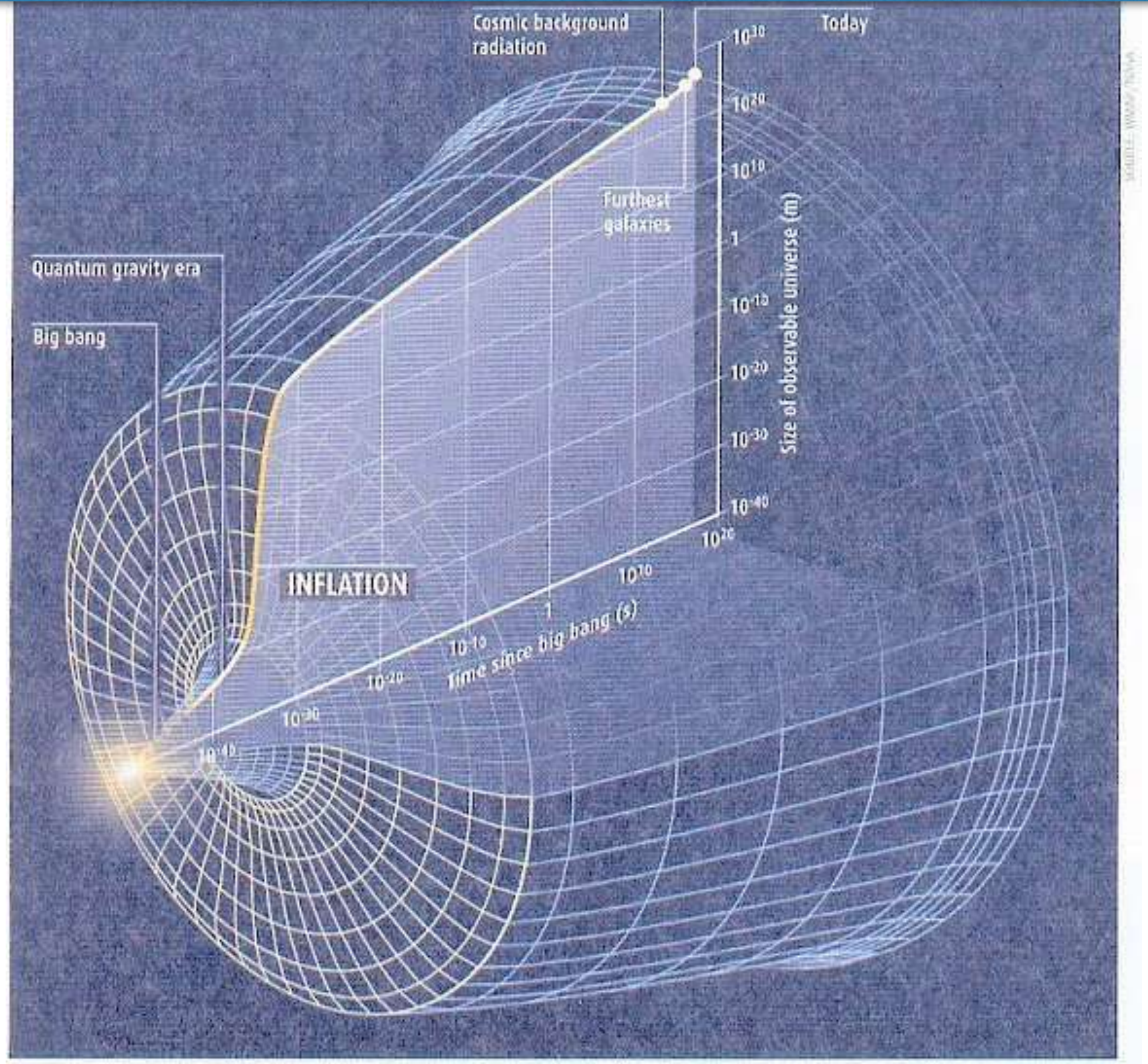
'cures' hBB $\Longrightarrow$ Cosmological Pple

$\rightarrow \begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations $\begin{cases}\text{Scalar}\\ \text{Tensor (GWs)}\end{cases}$

Basics of Inflation

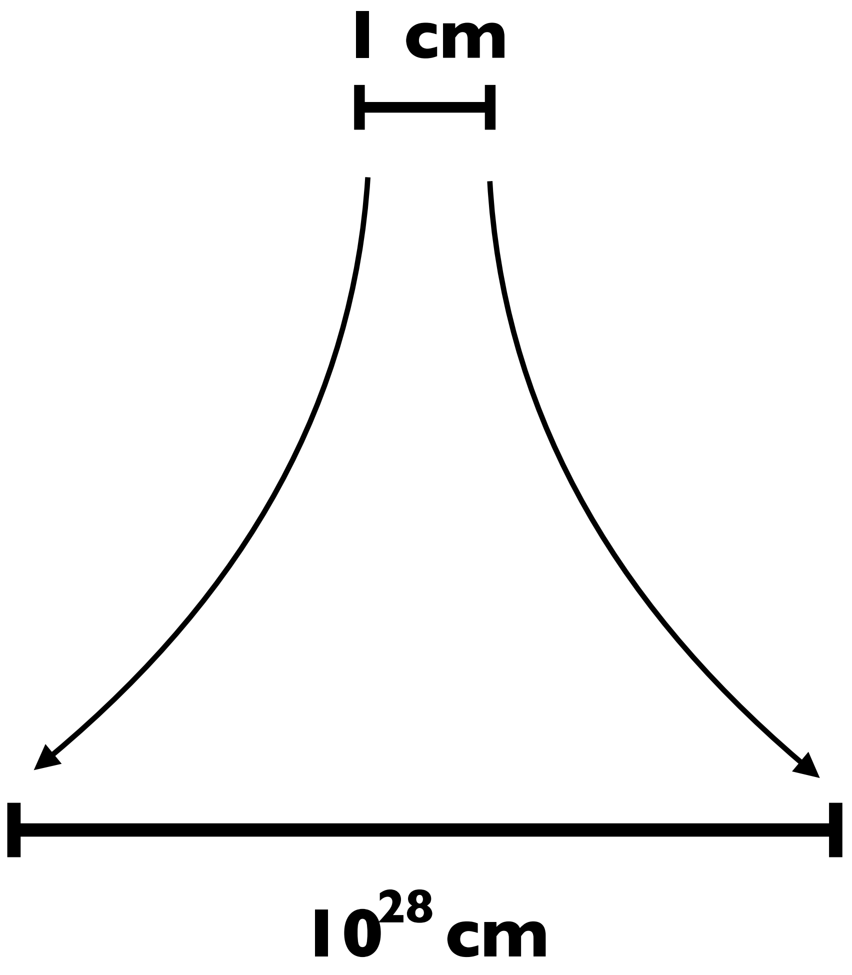
Inflation (basics)

COSMIC INFLATION



Needed for Consistency of the Big Bang theory

$$\boxed{a \sim e^{H_* t} \gtrsim e^{60}}$$



1 cm

10^{28} cm

Inflation: Definition + Implementation

* **Definition:**

$$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$

INF

Inflation: Definition + Implementation

INF

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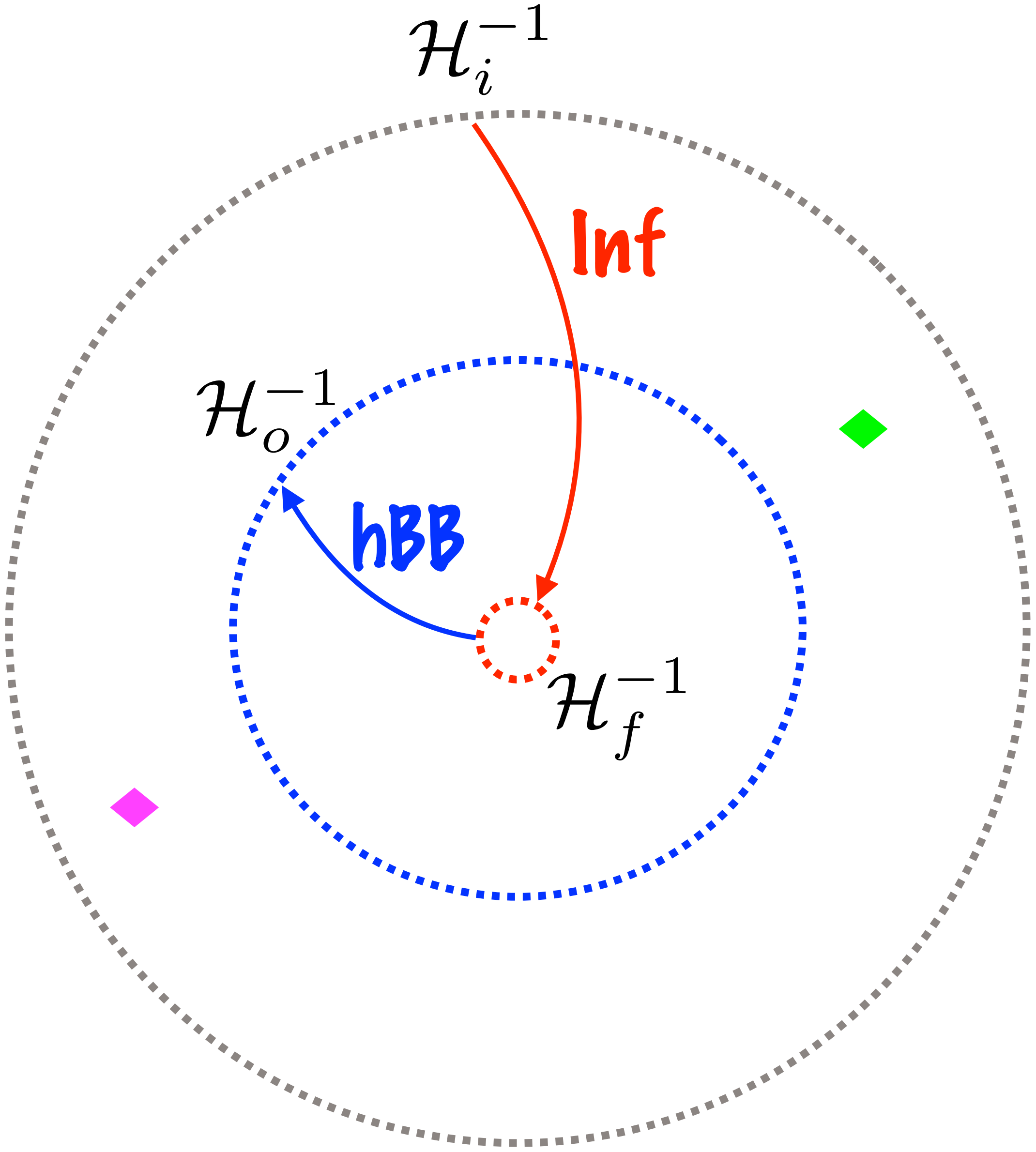
$$\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0$$



$$\frac{a_f}{a_i} \equiv e^N \quad (\#\ \text{e-folds})$$

$$\begin{aligned} \boxed{N} &\geq \log(\mathcal{H}_f/\mathcal{H}_o) = \log(E_f/E_o) \\ &\gtrsim \boxed{60} + \log(E_f[GeV]/10^{16}) \end{aligned}$$

Inflation: Definition + Implementation

INF

- **Definition:** $$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$

- **Consequences:** If $N \gtrsim 60$ → Horizon Problem Solved !

  → Bonus: Null Curvature

$$\boxed{\left(\left|\frac{K}{\mathcal{H}^2}\right| \sim |K/H_i^2|e^{-2N} = |K/H_i^2|e^{-120} \ll 1\right)}$$

Inflation: Definition + Implementation

INF

∗ **Definition:**

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$$\boxed{\left(\left|\frac{K}{\mathcal{H}^2}\right| \sim |K/H_i^2|e^{-2N} = |K/H_i^2|e^{-120} \ll 1\right)}$$

∗ **Implementation:**

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$$

(ϕ Inflaton)

IF $\boxed{V(\phi) \gg \frac{1}{2}\dot{\phi}^2, \frac{1}{2}(\nabla\phi)^2}$

$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\cancel{\frac{1}{2}\dot{\phi}^2} - \cancel{\frac{1}{6a^2}(\nabla\phi)^2} - V(\phi)}{\cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2a^2}(\nabla\phi)^2} + V(\phi)} \simeq \frac{-V(\phi)}{V(\phi)} \simeq -1$$

Inflation: Definition + Implementation

* **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

Slow Roll (SR) Regime:

$$\boxed{\epsilon = \frac{\dot{\phi}^2}{2m_p^2 H^2} \ll 1 \; ; \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1}$$

Inflation: Definition + Implementation

*** Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

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$$\boxed{\begin{aligned} \epsilon_V &\equiv \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2 \\ \eta_V &\equiv m_p^2\left(\frac{V''}{V}\right) \end{aligned}}$$

$(\epsilon \simeq \epsilon_V \, , \quad \eta \simeq \eta_V - \epsilon_V)$

Inflation: Definition + Implementation

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$(\epsilon \simeq \epsilon_V\ , \quad \eta \simeq \eta_V - \epsilon_V)$

➡

$$\boxed{\text{If } \epsilon_V\,,\eta_V \ll 1 \ \Rightarrow\ \epsilon, \eta \ll 1}$$

$$\text{SR} \Longrightarrow \underbrace{\text{quasi dS}}_{\displaystyle |||} \text{ for } \Delta N = 60$$

$$\boxed{a(t) \simeq a_i e^{\int_t H(\phi)dt'}}$$

Inflation: Definition + Implementation

- **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

Case of Study: $\boxed{V(\phi) = \frac{1}{2}m_\phi^2\phi^2}$

$$\epsilon_V(\phi) = \eta_V(\phi) = 2(m_p/\phi)^2$$

$$N(\phi) = (\phi/2m_p)^2 - 1/2$$

Inflation: Definition + Implementation

- **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

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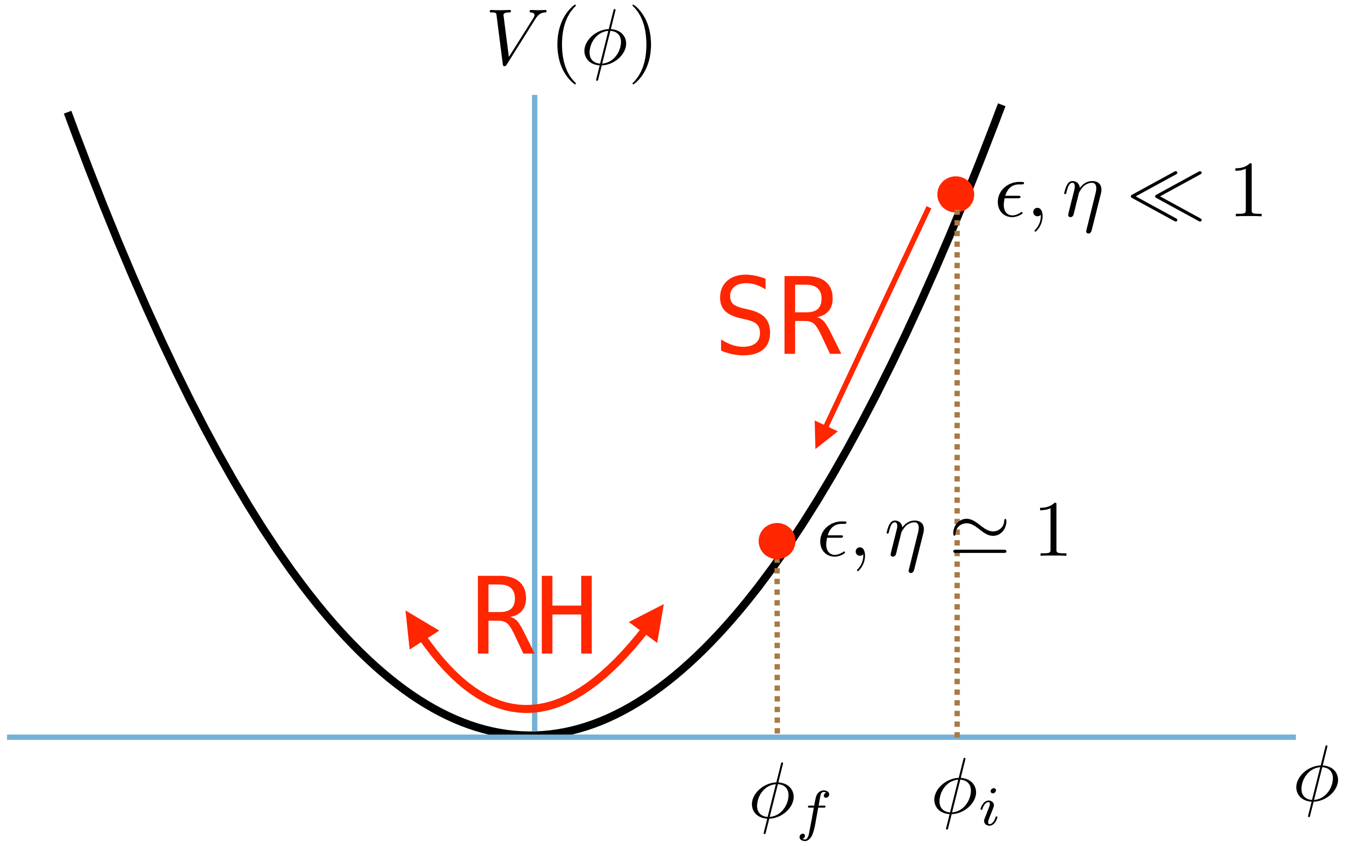
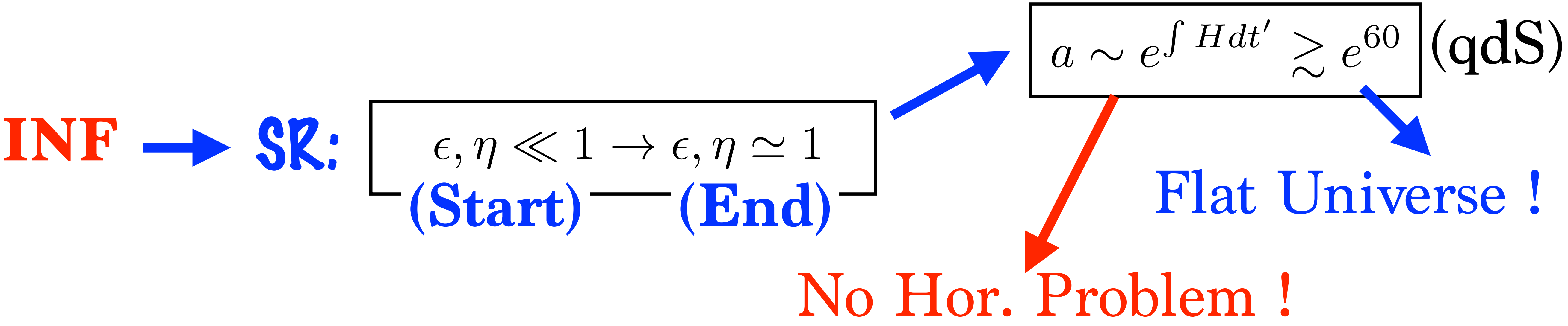


'Inflating ´ is easy with any potential of the type $V(\phi) \propto \phi^p$

Inflation & Primordial Perturbations



INF → **SR:** $\boxed{\epsilon, \eta \ll 1 \to \epsilon, \eta \simeq 1}$

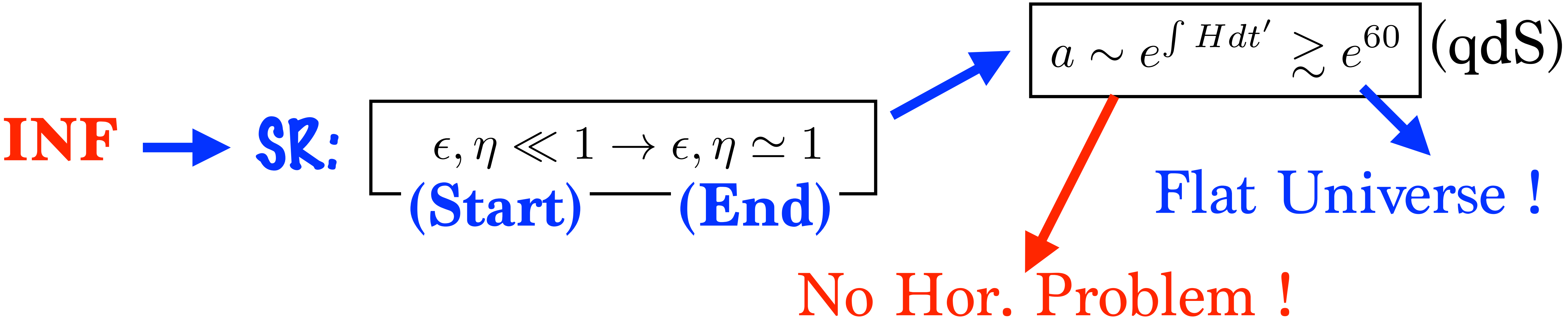
(Start) **(End)**

$\boxed{a \sim e^{\int H dt'} \gtrsim e^{60}}$ (qdS)

Flat Universe !

No Hor. Problem !

Inflation & Primordial Perturbations



INF → **SR:** $\boxed{\epsilon,\eta \ll 1 \to \epsilon,\eta \simeq 1}$

(Start) —— **(End)**

$\boxed{a \sim e^{\int H dt'} \gtrsim e^{60}}$ (qdS)

Flat Universe !

No Hor. Problem !

* **Is that ALL?** **NO!**

$$\boxed{\begin{matrix} \phi(t) \\ \\ g_{\mu\nu}(t) \end{matrix}}$$

(Background)

Inflation & Primordial Perturbations

INF → **SR:** $\epsilon, \eta \ll 1 \to \epsilon, \eta \simeq 1$

(Start) — **(End)**

$a \sim e^{\int H dt'} \gtrsim e^{60}$ (qdS)

Flat Universe !

No Hor. Problem !

* **Is that ALL?** **NO!**

$\phi(t)$ → $\phi(t) + \delta\phi(\vec{x}, t)$

$g_{\mu\nu}(t)$ → $g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$

(Background) **(Fluctuations)**

INF

↓

Primordial fluctuations !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\phi(t) \longrightarrow \phi(t) + \delta\phi(\vec{x}, t)$$

$$g_{\mu\nu}(t) \longrightarrow g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$$

(Background) **(Fluctuations)**

but WHY fluctuations ?

Quantum Mechanics !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\phi(t) \longrightarrow \phi(t) + \delta\phi(\vec{x},t)$$

$$g_{\mu\nu}(t) \longrightarrow g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

(Background) **(Fluctuations)**

but WHY fluctuations ?

Quantum Mechanics !

VeV

Vacuum Quam. Fluct.

QM:

$$\hat{\phi}(\vec{x},t) \;\rightarrow\; \langle\hat{\phi}(\vec{x},t)\rangle = \phi(t) \quad\Rightarrow\quad \boxed{\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t)}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\begin{tikzcd}
\phi(t) \arrow[r] & \phi(t) + \delta\phi(\vec{x},t) \\
g_{\mu\nu}(t) \arrow[r] & g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)
\end{tikzcd}$$

(Background) **(Fluctuations)**

but WHY fluctuations ?

Quantum Mechanics !

QM:

$$\begin{cases} \hat{\phi}(\vec{x},t) \;\rightarrow\; \langle \hat{\phi}(\vec{x},t) \rangle = \phi(t) \quad \Rightarrow \boxed{\hat{\phi}(\vec{x},t) = \overset{\text{VeV}}{\phi(t)} + \overset{\text{Vacuum Quam. Fluct.}}{\hat{\delta\phi}(\vec{x},t)}} \\ \\ \langle \hat{\delta\phi}(\vec{x},t) \rangle = 0 \quad \text{but...} \quad \left\langle\, [\hat{\delta\phi}(\vec{x},t)]^2 \,\right\rangle \neq 0 \end{cases}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\begin{array}{c}\phi(t)\\[2em] g_{\mu\nu}(t)\end{array}}\;\begin{array}{c}\longrightarrow\\[2em]\longrightarrow\end{array}\;\boxed{\begin{array}{c}\phi(t)+\delta\phi(\vec{x},t)\\[2em] g_{\mu\nu}(t)+\delta g_{\mu\nu}(\vec{x},t)\end{array}}$$

(Background) **(Fluctuations)**

but WHY fluctuations ?

Quantum Mechanics !

VeV

Vacuum Quam. Fluct.

QM:

$$\left\{\begin{array}{l}\hat{\phi}(\vec{x},t)\;\rightarrow\;\langle\hat{\phi}(\vec{x},t)\rangle=\phi(t)\quad\Rightarrow\;\boxed{\hat{\phi}(\vec{x},t)=\phi(t)+\hat{\delta\phi}(\vec{x},t)}\\[3em] \langle\hat{\delta\phi}(\vec{x},t)\rangle=0\end{array}\right.$$

but... $\left\langle\,[\hat{\delta\phi}(\vec{x},t)]^2\,\right\rangle\neq 0$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\hat{\phi}(\vec{x},t) = \phi(t) + \delta\hat{\phi}(\vec{x},t) \quad \rightarrow \quad \langle \delta\hat{\phi}^2(\vec{x},t) \rangle \neq 0}$$

but … ~~Minkowski~~ → Curved Space: (quasi)dS

Inflation & Primordial Perturbations

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but … ~~Minkowski~~ → Curved Space: (quasi)dS

$$\begin{tikzcd} & \phi(t) + \delta\phi(\vec{x},t) \arrow[dd, bend left, <->] \\ S = \frac{m_p^2}{2} \int d^4x \sqrt{-g} \{ R - (\partial\phi)^2 - 2V(\phi) \} \arrow[ur] \arrow[dr] & \\ & g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t) \end{tikzcd}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t) \quad \rightarrow \quad \langle \hat{\delta\phi}^2(\vec{x},t)\rangle \neq 0$$

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$$\begin{tikzcd}
 & \phi(t) + \delta\phi(\vec{x},t) \arrow[dd, bend left, leftrightarrow] \\
S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\} \arrow[ur] \arrow[dr] & \\
 & g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)
\end{tikzcd}$$

$$ds^2 \quad = \quad g^{\text{tot}}_{\mu\nu}dx^\mu dx^\nu = (g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t))dx^\mu dx^\nu$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\begin{tikzcd}
 & \phi(t) + \delta\phi(\vec{x},t) \arrow[d, bend left] \\
S = \dfrac{m_p^2}{2} \displaystyle\int d^4x \sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\} \arrow[ur] \arrow[dr] & \\
 & g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t) \arrow[u, bend right]
\end{tikzcd}$$

$$\begin{aligned}
ds^2 &= g^{\rm tot}_{\mu\nu} dx^\mu dx^\nu = (g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)) dx^\mu dx^\nu \\
&= -(1 + 2\underset{\uparrow}{\Phi}) dt^2 + 2\underset{\uparrow}{B_i} dx^i dt + a^2[(1 - 2\underset{\uparrow}{\Psi})\delta_{ij} + \underset{\uparrow}{E_{ij}}] dx^i dx^j
\end{aligned}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$B_i = \partial_i B - S_i$$

$$E_{ij} = 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$B_i = \partial_i B - \cancel{S_i}$$

$$E_{ij} = 2\partial_{ij}E + \cancel{2\partial_{(i}F_{j)}} + h_{ij}$$

Expanding U. $\longrightarrow$ **Vector Perturbations** $S_i, F_i \propto \dfrac{1}{a}$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$B_i = \partial_i B \cancel{- S_i}$$

$$E_{ij} = 2\partial_{ij}E + \cancel{2\partial_{(i}F_{j)}} + h_{ij}$$

$$\partial_i h_{ij} = h_{ii} = 0$$

(tensors = GWs)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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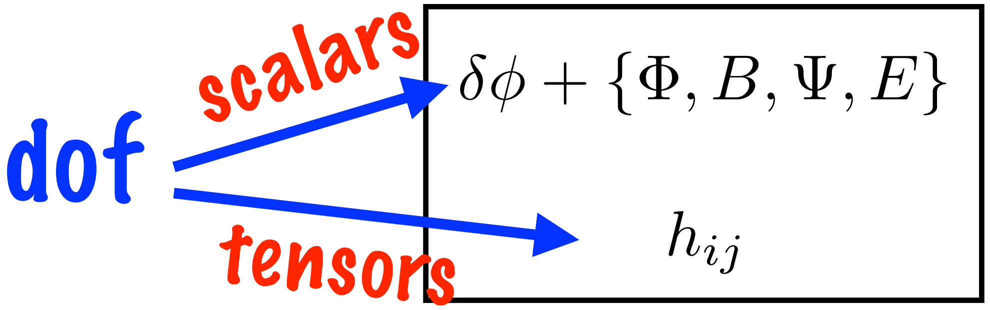
$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$



$\delta\phi + \{\Phi, B, \Psi, E\}$

h_{ij}

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

dof

scalars

tensors

$$\delta\phi + \{\Phi, B, \Psi, E\} \xrightarrow{\text{Diff.}} \delta\phi' + \{\Phi', B', \Psi', E'\}$$

~~Gauge Inv.~~

$$h_{ij} \xrightarrow{\text{Diff.}} h_{ij}$$

Gauge Inv. !

Diff.: $\boxed{x^\mu \;\rightarrow\; x^\mu + \xi^\mu}$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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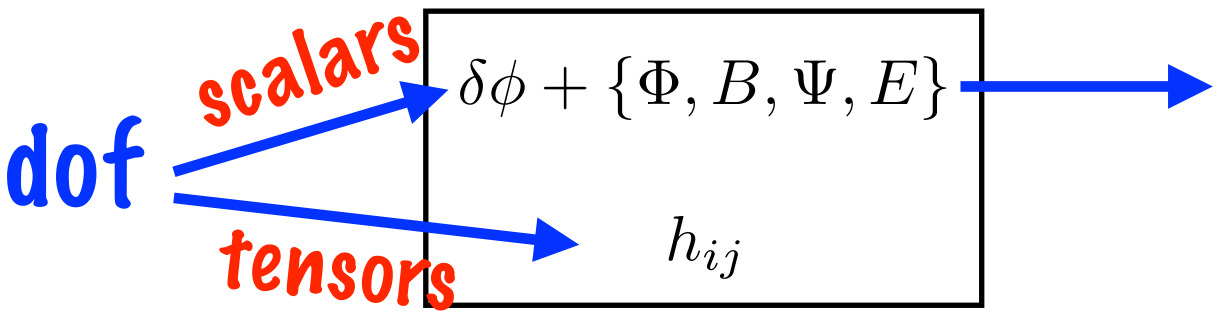

$$\delta\phi + \{\Phi, B, \Psi, E\}$$

$$h_{ij}$$

$$\zeta \equiv -[\Psi + (H/\dot{\rho})\delta\rho_\phi] \xrightarrow{\text{Diff.}} \zeta$$

$$\mathcal{R} \equiv [\Psi + (H/\dot{\phi})\delta\phi] \xrightarrow{\text{Diff.}} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot{\phi}/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

All Gauge Inv. !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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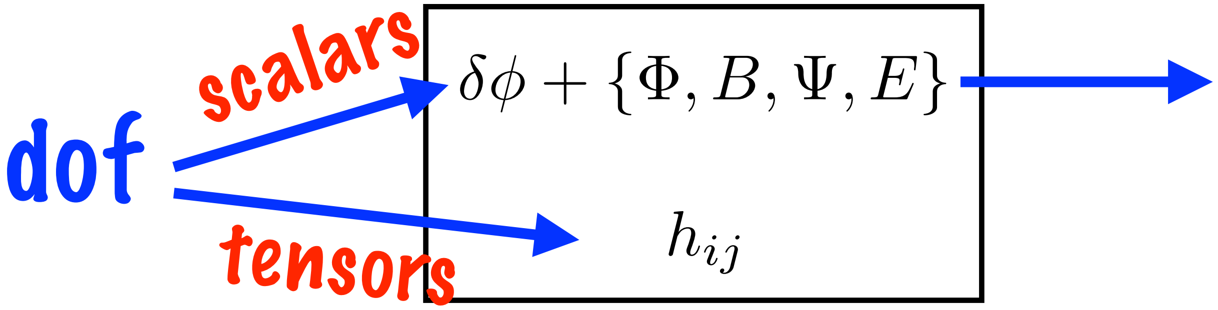

$$\delta\phi + \{\Phi, B, \Psi, E\}$$

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$$\zeta \equiv -[\Psi + (H/\dot{\rho})\delta\rho_\phi] \xrightarrow{\text{Diff.}} \zeta$$

$$\mathcal{R} \equiv [\Psi + (H/\dot{\phi})\delta\phi] \xrightarrow{\text{Diff.}} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot{\phi}/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

All Gauge Inv. !

Curvature Pert.

Tensor Pert. (GW)

Fixing Gauge: e.g. $E, \delta\phi = 0 \Rightarrow g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

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Inflation & Primordial Perturbations

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$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$$

$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\} \Rightarrow$$

$$\Rightarrow \quad S = S_{(0)} + S^{(s)}_{(2)} + S^{(t)}_{(2)}$$

$$S^{(s)}_{(2)} = \tfrac{1}{2}\int d^4x\ a^3\tfrac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]$$

$$S^{(t)}_{(2)} = \tfrac{m_p^2}{8}\int dtdx^3 a^3\left[(\dot h_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]$$

Background
Inflationary dynamics

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\ a^3\frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \tfrac{1}{2}\int d^4x\; a^3\,\tfrac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]}$$

$$d\tau \equiv dt/a(t)$$

$$= \boxed{\frac{1}{2}\int d\tau dx^3\left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]}$$

$$\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\tfrac{\dot{\phi}}{H}\right]$$ (Mukhanov variable)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\; a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]} = \boxed{\frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z} v^2\right]} \Rightarrow$$

(F.T.: $v(\mathbf{x},t) = \int d\mathbf{k}\, e^{-i\mathbf{k}\mathbf{x}}\, v_{\mathbf{k}}(t)$)

$$\Rightarrow \boxed{v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0}$$

with

$$\boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right) \;,\quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Inflation & Primordial Perturbations

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Scalar Fluctuations:

$$S^{(s)}_{(2)} = \frac{1}{2}\int d^4x \; a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right] = \frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z} v^2\right] \Rightarrow$$

$$\Rightarrow \quad v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0$$

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$$\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right) \; , \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta$$

Quantization:

$$v_{\vec{k}}(t) \rightarrow v_k(t)\hat{a}_{\vec{k}} + v^*_k(t)\hat{a}^\dagger_{-\vec{k}} \; , \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}') \quad \Rightarrow$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$S^{(s)}_{(2)} = \frac{1}{2}\int d^4x \; a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right] = \frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z} v^2\right] \Rightarrow$$

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Quantization: $v_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v^*_k(t)\hat{a}^\dagger_{-\vec{k}} \; , \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}')$ →

→ 2 linearly independent solutions (Hankel functions)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \tfrac{1}{2}\int d^4x\ a^3\tfrac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]} = \boxed{\frac{1}{2}\int d\tau dx^3\left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]} \Rightarrow$$

$$\Rightarrow \boxed{v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0}$$

with

$$\boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\ ,\quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Quantization:

$$\boxed{v_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v^*_k(t)\hat{a}^\dagger_{-\vec{k}}\ ,\quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}')} \rightarrow$$

$$\rightarrow \boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2}e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2}H^{(1)}_\nu(-k\tau)}$$

(we keep only one, $\hat{H}v_k = +kv_k$)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\, a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]} = \boxed{\frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right]} \Rightarrow$$

$$\Rightarrow \boxed{v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0} \quad \text{with} \quad \boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right) \ , \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Quantization: $\boxed{v_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v^*_k(t)\hat{a}^\dagger_{-\vec{k}} \ , \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}')}$ $\rightarrow$

$$\rightarrow \boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2}e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2}H^{(1)}_\nu(-k\tau)} \xrightarrow[\text{(sub-Hubble)}]{-k\tau \gg 1} \frac{1}{\sqrt{2k}}e^{-ik\tau}$$

(we keep only one, $\hat{H}v_k = +kv_k$)

Positive define freq

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$S^{(s)}_{(2)} = \tfrac{1}{2}\int d^4x \; a^3 \tfrac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right] = \frac{1}{2}\int d\tau dx^3 \left[(v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right] \Rightarrow$$

$$\Rightarrow \quad v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0$$

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(Bunch-Davies)
Vacuum Fluct.

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluct:

$$\boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H_\nu^{(1)}(-k\tau)}$$

$$\boxed{\hat{v}_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v_k^*(t)\hat{a}^\dagger_{-\vec{k}}}$$

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(Bunch-Davies) Vacuum Fluct.

$$\boxed{\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right]}$$

➡

$$\boxed{\left\langle \hat{\mathcal{R}}_{\vec{k}}\hat{\mathcal{R}}_{\vec{k}'}\right\rangle \equiv \frac{1}{z^2}\left\langle \hat{v}_{\vec{k}}\hat{v}_{\vec{k}'}\right\rangle \equiv (2\pi)^3 \boxed{\frac{H^2}{a^2\dot{\phi}^2}\left|v_k(\eta)\right|^2}\delta(\vec{k}+\vec{k}')}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluct:

$$v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H_\nu^{(1)}(-k\tau)$$

$$\hat{v}_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v_k^*(t)\hat{a}^\dagger_{-\vec{k}}$$

(Bunch-Davies) Vacuum Fluct.

$$\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right] \Longrightarrow$$

$$\left\langle \hat{\mathcal{R}}_{\vec{k}}\hat{\mathcal{R}}_{\vec{k}'}\right\rangle \equiv \frac{1}{z^2}\left\langle \hat{v}_{\vec{k}}\hat{v}_{\vec{k}'}\right\rangle \equiv (2\pi)^3 \boxed{\frac{H^2}{a^2\dot{\phi}^2}\left|v_k(\eta)\right|^2}\,\delta(\vec{k}+\vec{k}')$$

$$\boxed{\equiv P_{\mathcal{R}}(k,\eta)}$$

Scalar Power Spectrum

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluct:

$$\boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2} (-\tau)^{1/2} H_\nu^{(1)}(-k\tau)}$$

$$\boxed{\hat{v}_{\vec{k}}(t) \rightarrow v_k(t)\hat{a}_{\vec{k}} + v_k^*(t)\hat{a}^\dagger_{-\vec{k}}}$$

(Bunch-Davies) Vacuum Fluct.

$$\boxed{\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right]} \Rightarrow \boxed{\left\langle \hat{\mathcal{R}}_{\vec{k}}\hat{\mathcal{R}}_{\vec{k}'} \right\rangle \equiv \frac{1}{z^2}\left\langle \hat{v}_{\vec{k}}\hat{v}_{\vec{k}'} \right\rangle \equiv (2\pi)^3 \boxed{\frac{H^2}{a^2\dot{\phi}^2}|v_k(\eta)|^2} \delta(\vec{k}+\vec{k}')}$$

$$\boxed{\equiv P_{\mathcal{R}}(k,\eta)}$$

Scalar Power Spectrum

$$\boxed{\Delta^2_{\mathcal{R}}(k,\tau) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k,\tau)} \longrightarrow \boxed{\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{2\eta-4\epsilon}}$$

Dimensionless Scalar PS

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dtdx^3a^3\left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$d\tau \equiv dt/a(t)$$

$$\boxed{S^{(t)}_{(2)} = \frac{m_p^2}{8} \int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2} (\partial_l h_{ij})^2 \right]} = \boxed{\sum_s \frac{1}{2} \int \mathrm{d}\tau \mathrm{d}^3\mathbf{k} \left[(v^{s\prime}_{\mathbf{k}})^2 - \left(k^2 - \frac{a''}{a} \right) (v^s_{\mathbf{k}})^2 \right]}$$

$$h_{ij}(\vec{k}, \tau) = \epsilon^{(s)}_{ij} h^{(s)}_{\vec{k}} \longrightarrow v^{(s)} \equiv \frac{a}{2} m_p h^{(s)}_{\vec{k}}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$\boxed{S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]} = \boxed{\sum_s \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3\mathbf{k}\left[(v^{s\,\prime}_{\mathbf{k}})^2 - \left(k^2 - \frac{a''}{a}\right)(v^s_{\mathbf{k}})^2\right]} \Rightarrow$$

$\Rightarrow$ [Same Procedure as with Scalar Pert.

Quantize → Bunch-Davies → Power Spectrum]

Quantization of Gravity dof !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right] = \sum_s \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3\mathbf{k}\left[(v^{s\,\prime}_{\mathbf{k}})^2 - \left(k^2 - \frac{a''}{a}\right)(v^s_{\mathbf{k}})^2\right] \Rightarrow$$

$\Rightarrow$ [Same Procedure as with Scalar Pert.

Quantize → Bunch-Davies → Power Spectrum]

Quantization of Gravity dof !

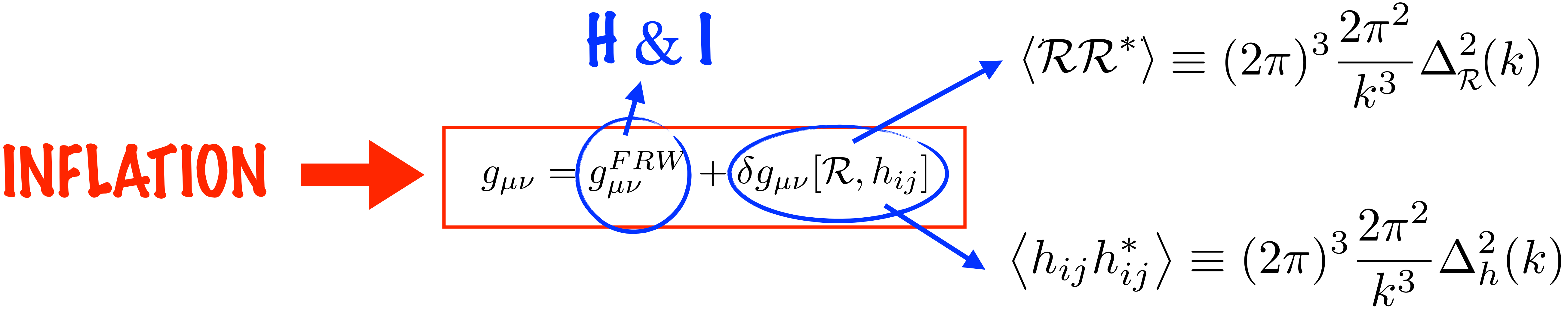
$$\Delta_h^2(k,\tau) \equiv \frac{k^3}{2\pi^2}P_h(k,\tau) \longrightarrow \Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}$$

Inflation & Primordial Perturbations

INFLATION ➡

$$\boxed{g_{\mu\nu} = \overset{\text{H \& I}}{\overset{\uparrow}{g^{FRW}_{\mu\nu}}} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]}$$

Inflation & Primordial Perturbations



INFLATION

H & I

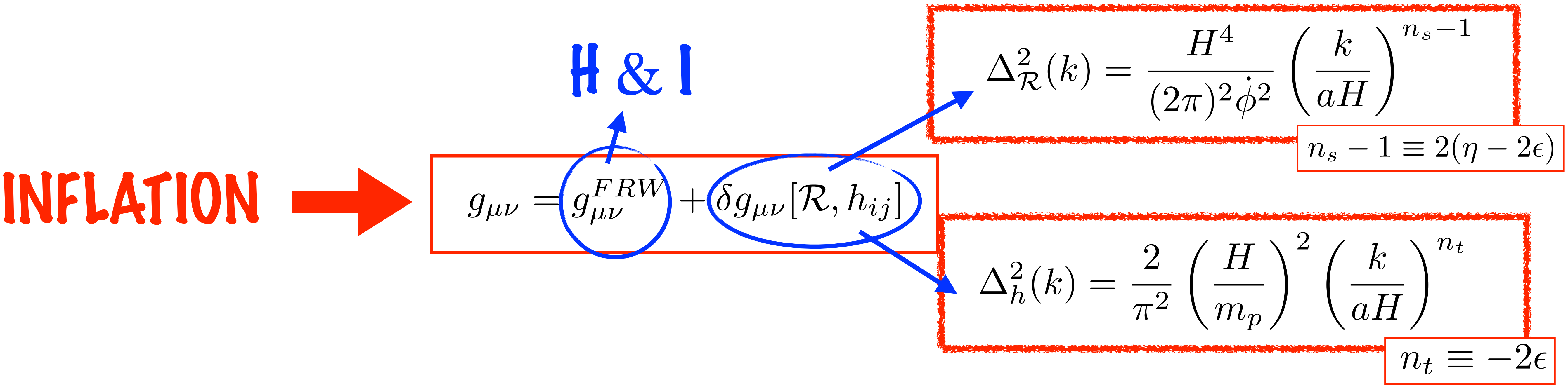
$$\boxed{g_{\mu\nu} = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]}$$

$$\langle \mathcal{R}\mathcal{R}^* \rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta^2_{\mathcal{R}}(k)$$

$$\langle h_{ij} h^*_{ij} \rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta^2_{h}(k)$$

Quantum fluctuations !

Inflation & Primordial Perturbations



INFLATION

H & I

$$\boxed{g_{\mu\nu} = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]}$$

$$\boxed{\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}}$$

$$\boxed{n_s - 1 \equiv 2(\eta - 2\epsilon)}$$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

Inflation & Primordial Perturbations

INFLATION →

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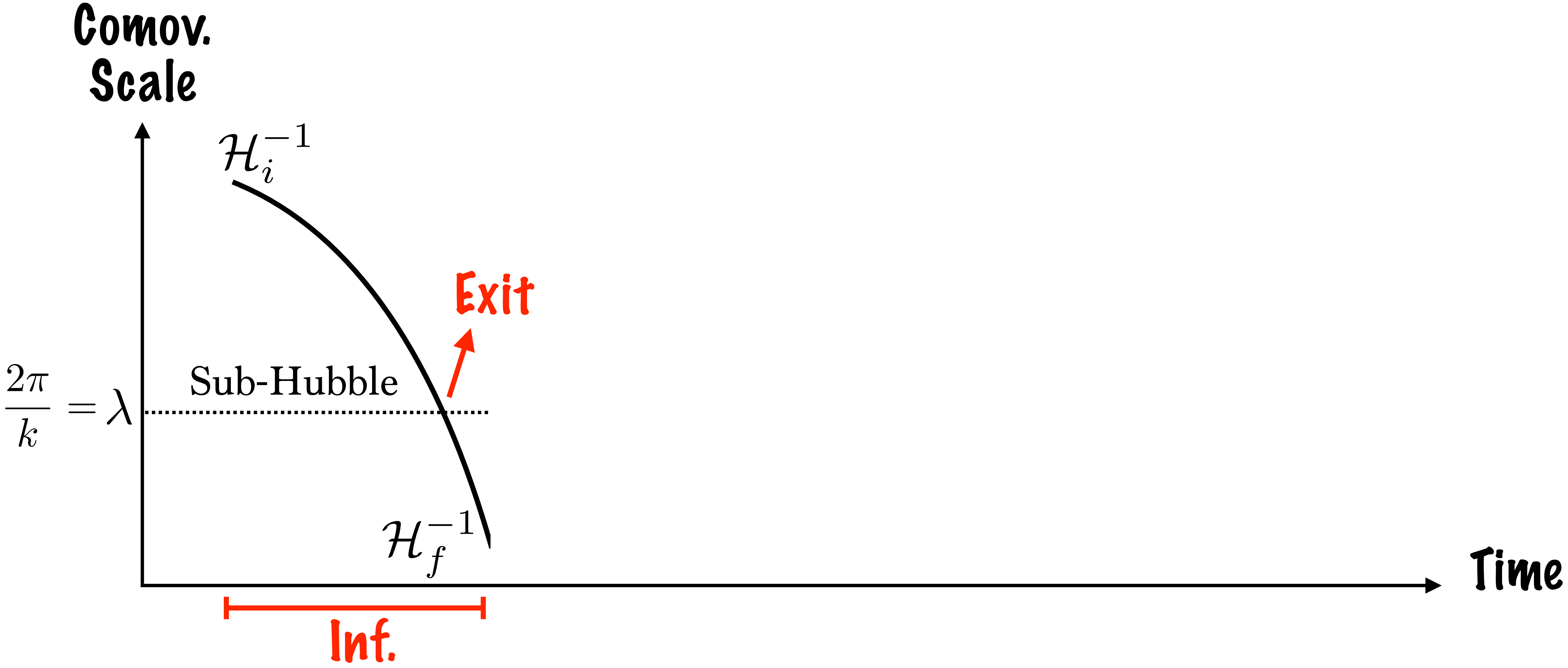

Inflation & Primordial Perturbations

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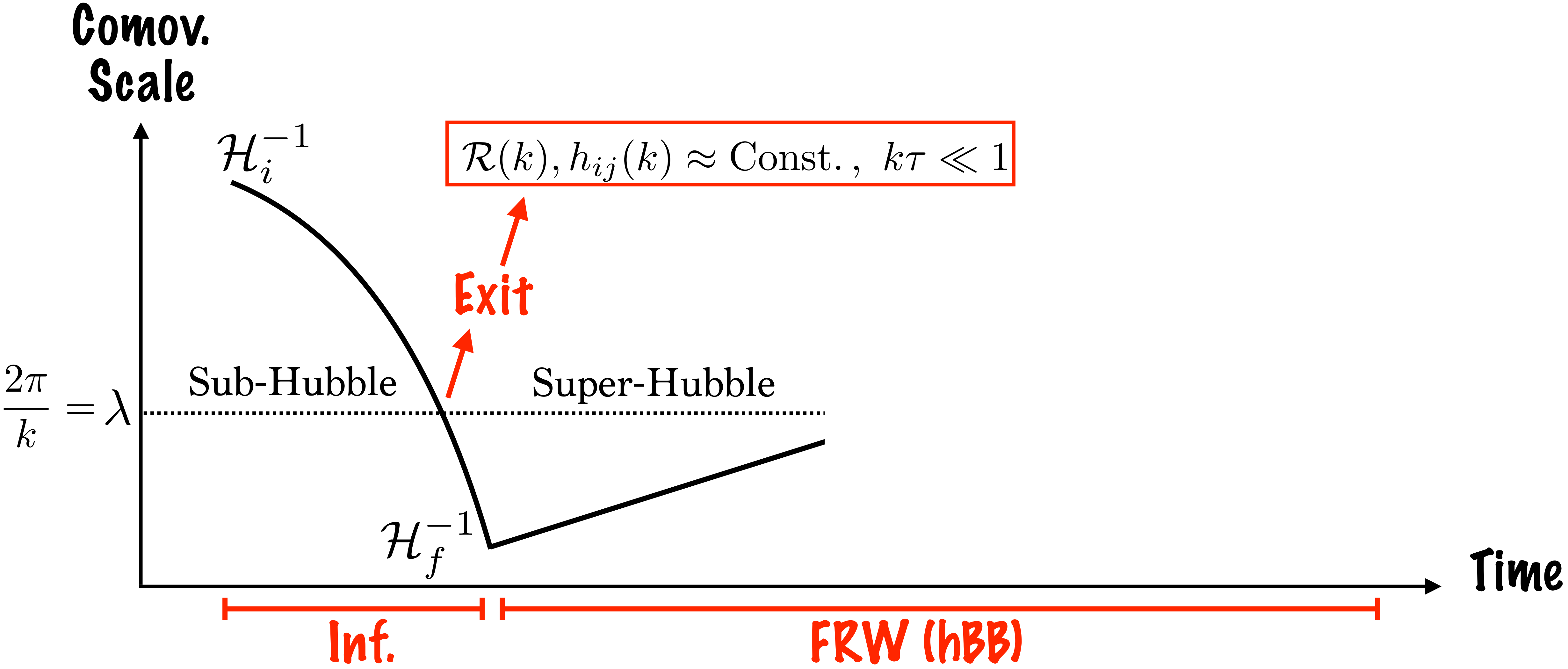

$$\mathcal{R}(k), h_{ij}(k) \approx \text{Const.}, \quad k\tau \ll 1$$

Inflation & Primordial Perturbations

INFLATION →

$$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

H & I

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

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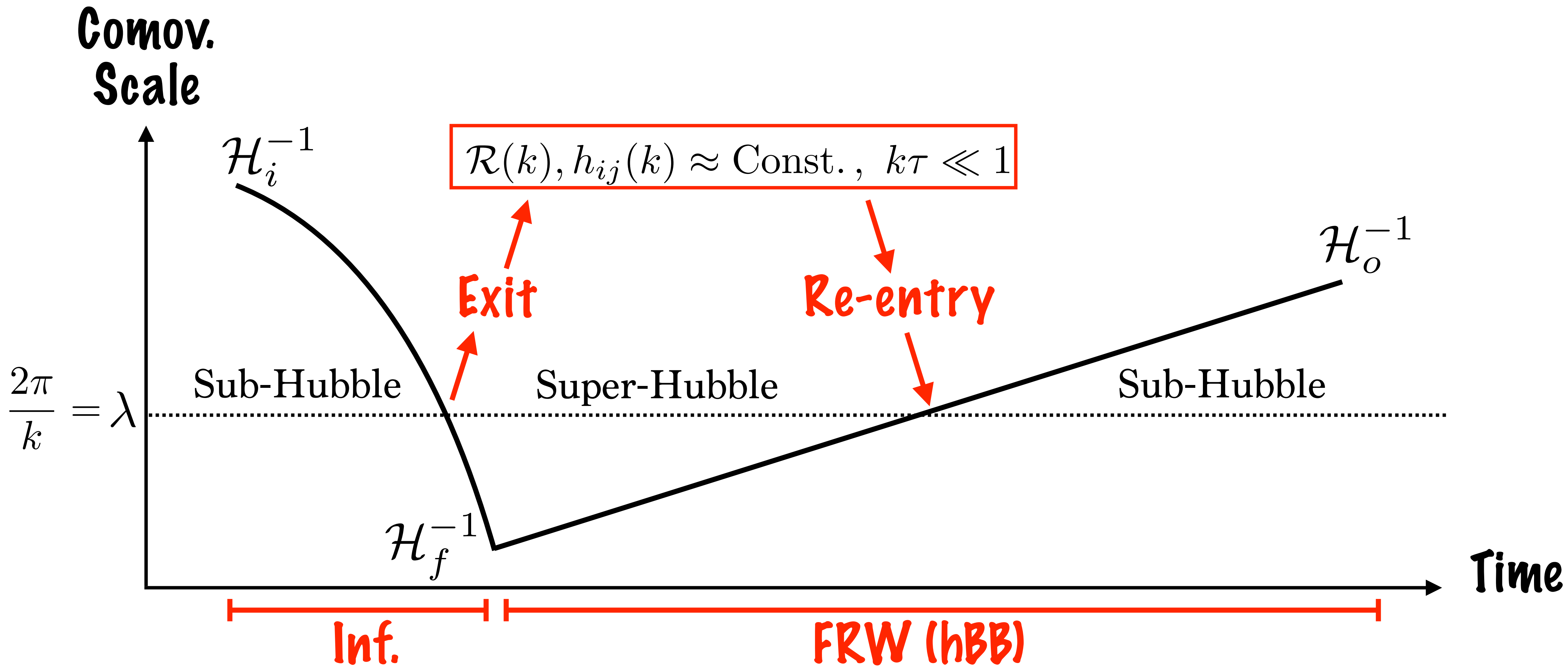
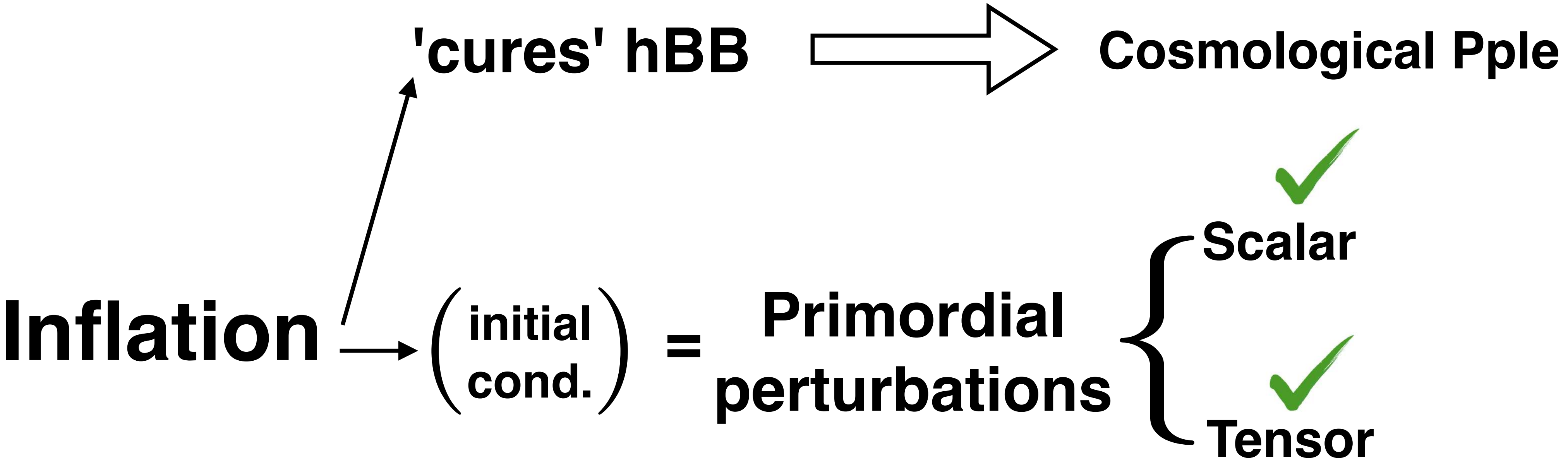


End primer
on Inflation

INFLATIONARY COSMOLOGY

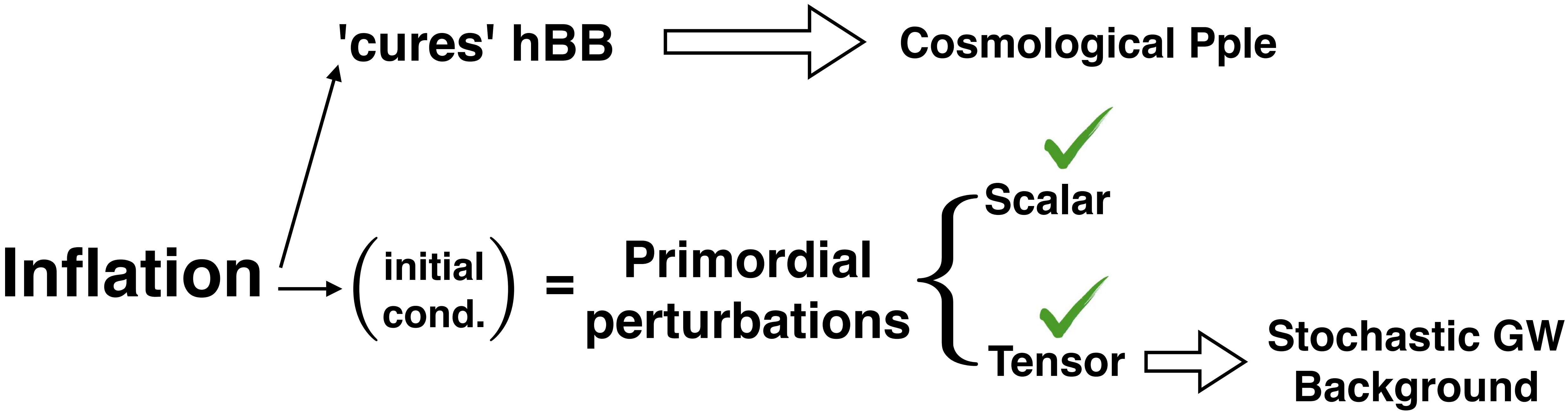


'cures' hBB ⇒ Cosmological Pple

Inflation → (initial cond.) = Primordial perturbations

- Scalar ✓
- Tensor ✓

INFLATIONARY COSMOLOGY



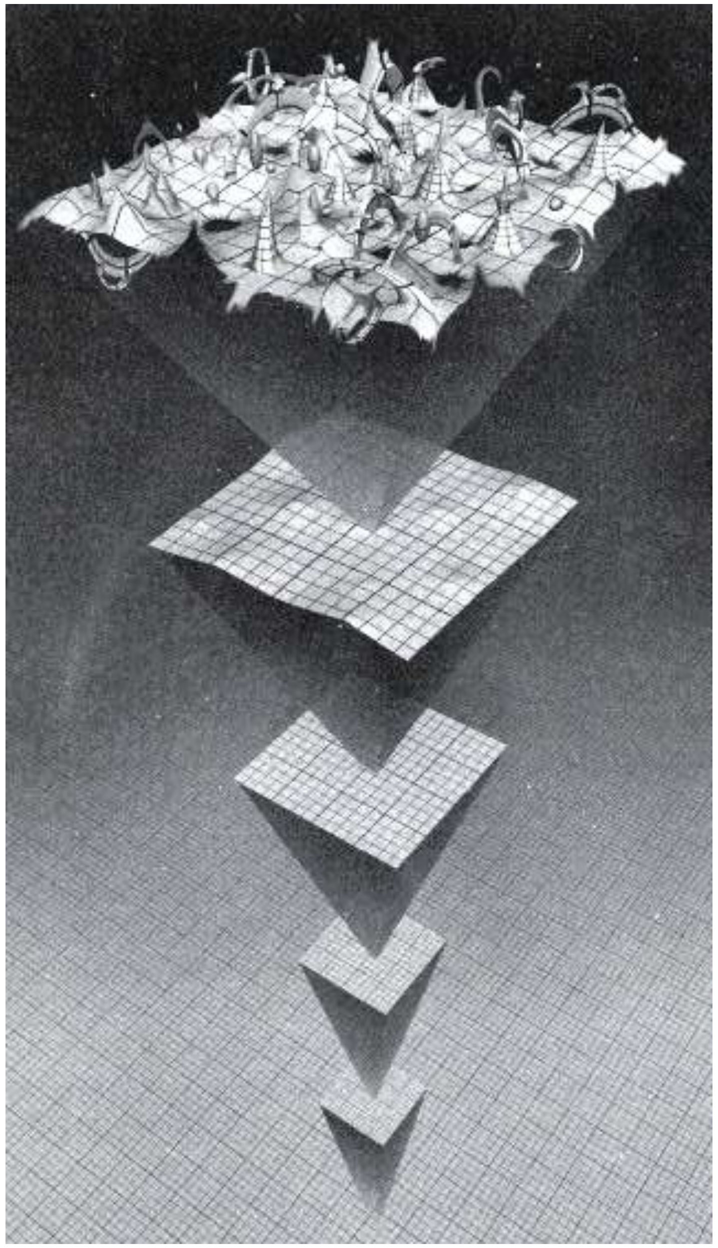
Inflation → 'cures' hBB ⇒ Cosmological Pple

Inflation → (initial cond.) = Primordial perturbations

- ✓ Scalar
- ✓ Tensor ⇒ Stochastic GW Background

Irreducible GW background from Inflation

$$g_{\mu\nu} = g^{(\mathrm{B})}_{\mu\nu} + \delta g_{\mu\nu} \qquad ; \qquad [\delta g_{\mu\nu}]^{\mathrm{TT}} = h_{ij} \ , \ \begin{cases} h_{ii} = 0 \\ \partial_i h_{ij} = 0 \end{cases}$$



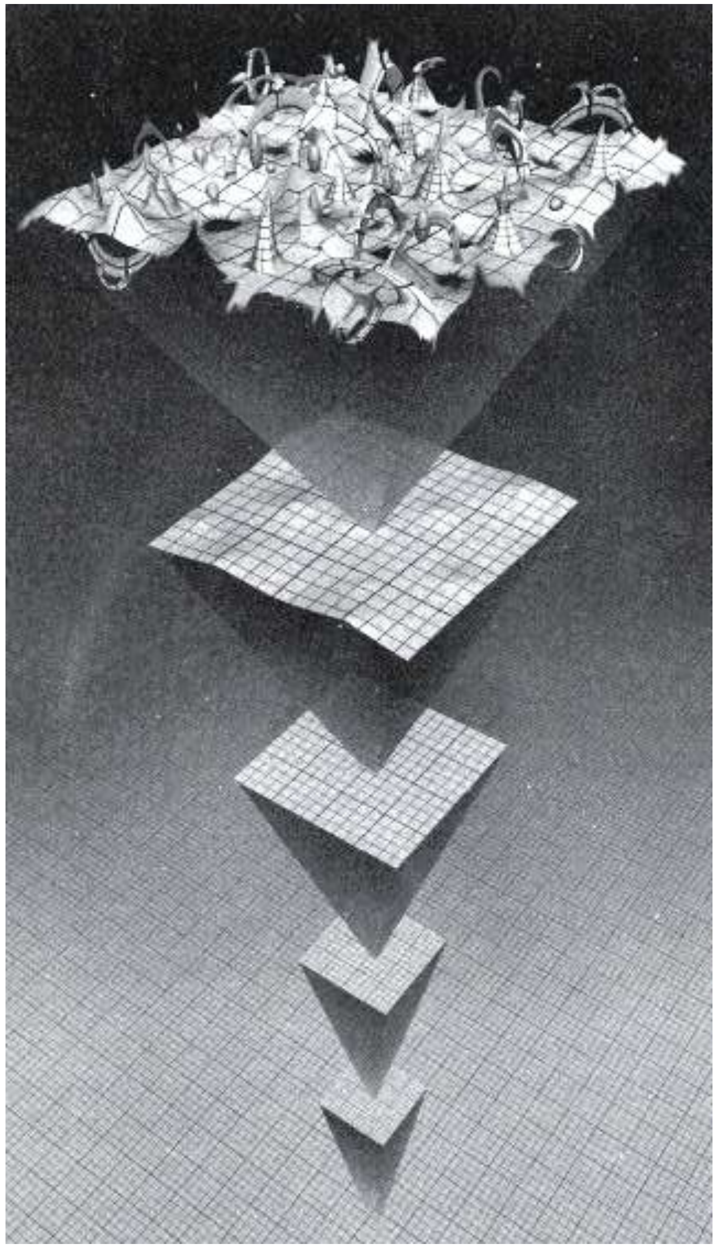
$$\left\langle h_{ij}(\vec{k},t) \right\rangle = 0$$

Quantum Fluctuations

$$\left\langle h_{ij}(\vec{k},t) h^*_{ij}(\vec{k}',t) \right\rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta^2_h(k) \delta(\vec{k} - \vec{k}')$$

Irreducible GW background from Inflation

$$g_{\mu\nu} = g^{(\mathrm{B})}_{\mu\nu} + \delta g_{\mu\nu} \quad ; \quad [\delta g_{\mu\nu}]^{\mathrm{TT}} = h_{ij} \;, \; \begin{cases} h_{ii} = 0 \\ \partial_i h_{ij} = 0 \end{cases}$$



Quantum Fluctuations

$$\left\langle h_{ij}(\vec{k},t) \right\rangle = 0$$

$$\left\langle h_{ij}(\vec{k},t) h^*_{ij}(\vec{k}',t) \right\rangle \equiv (2\pi)^3 \frac{2\pi^2}{k^3} \Delta^2_h(k) \delta(\vec{k} - \vec{k}')$$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

energy scale

Irreducible GW background from Inflation

$$\boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

energy scale

$$\boxed{n_t \equiv -2\epsilon}$$

Irreducible GW background from Inflation

$$\boxed{\Omega^{(o)}_{\rm GW}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \underbrace{\frac{\Omega^{(o)}_{\rm Rad}}{24}}_{}\Delta^2_{h_*}(k)}$$

Transfer Funct.: $T(k) \propto k^0 (\mathrm{RD})$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

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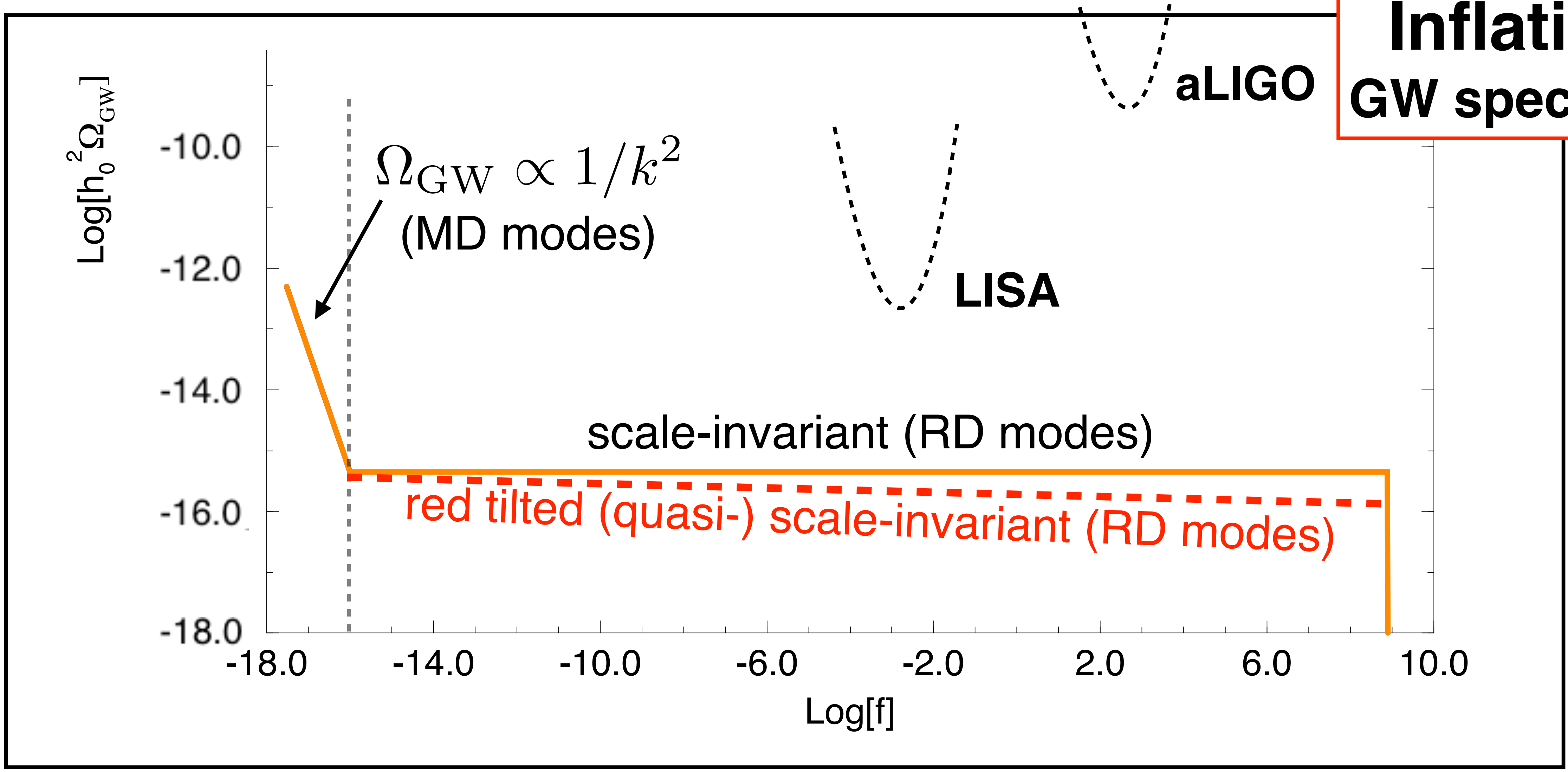
energy scale

Inflation GW spectrum



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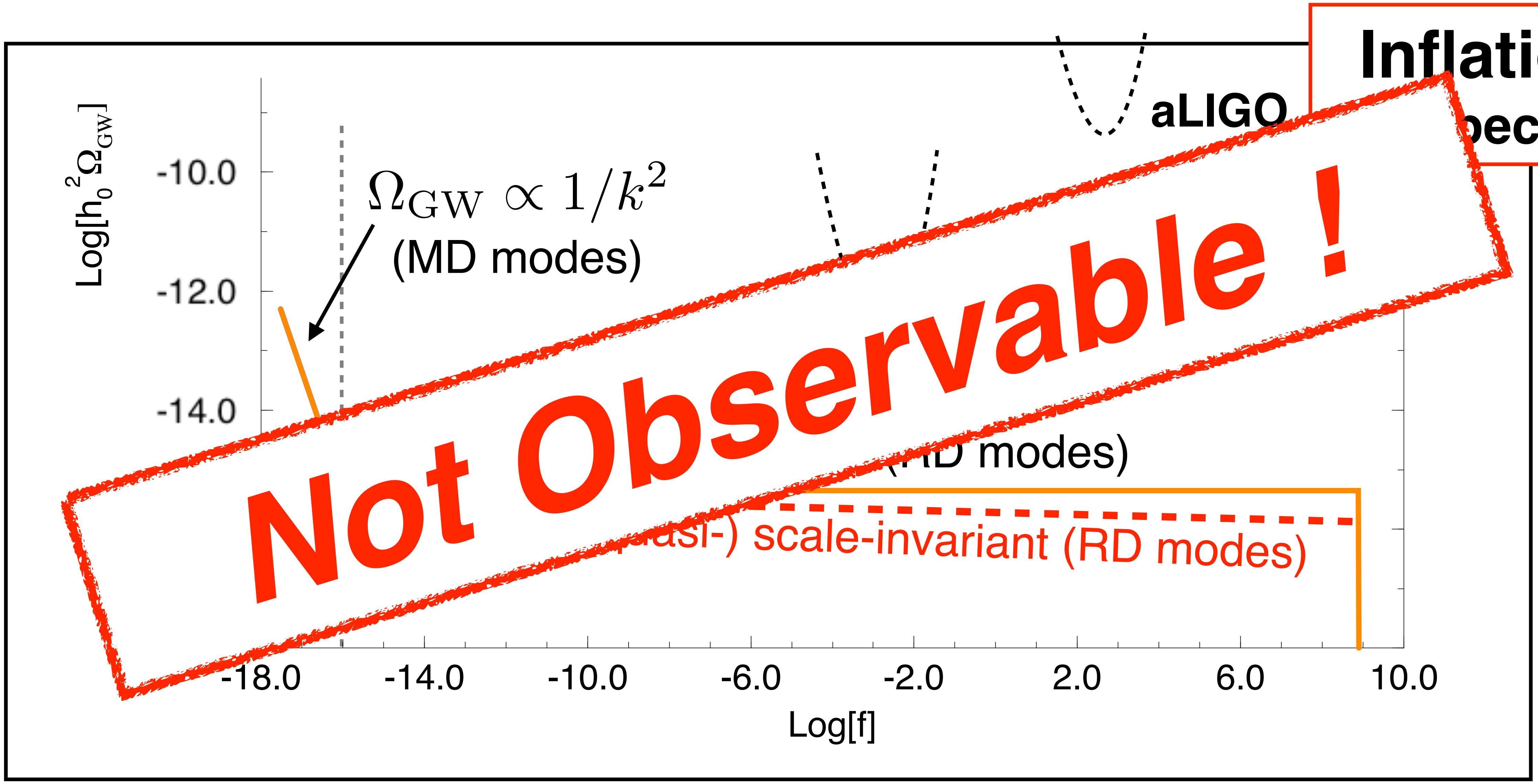
$\boxed{n_t \equiv -2\epsilon}$

energy scale

Inflation Spectrum



Not Observable !

Irreducible GW background from Inflation

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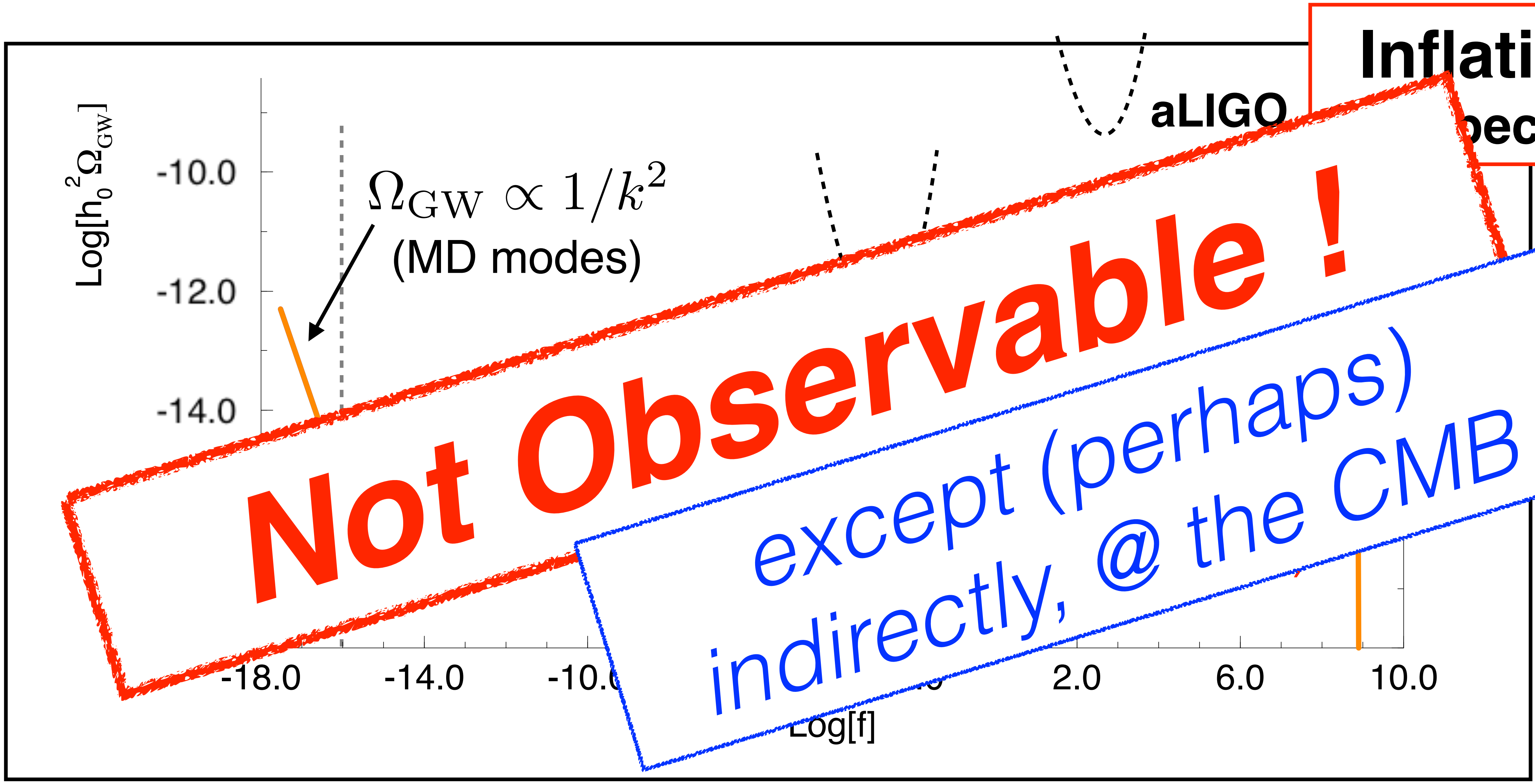
energy scale



Inflation
pectrum

Not Observable !

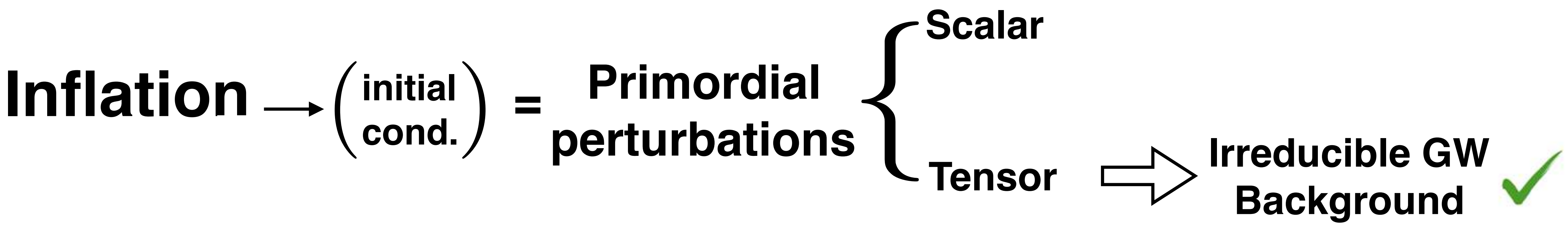
except (perhaps) indirectly, @ the CMB

Shall we
coffee break ?

Let’s continue …

Inflating !

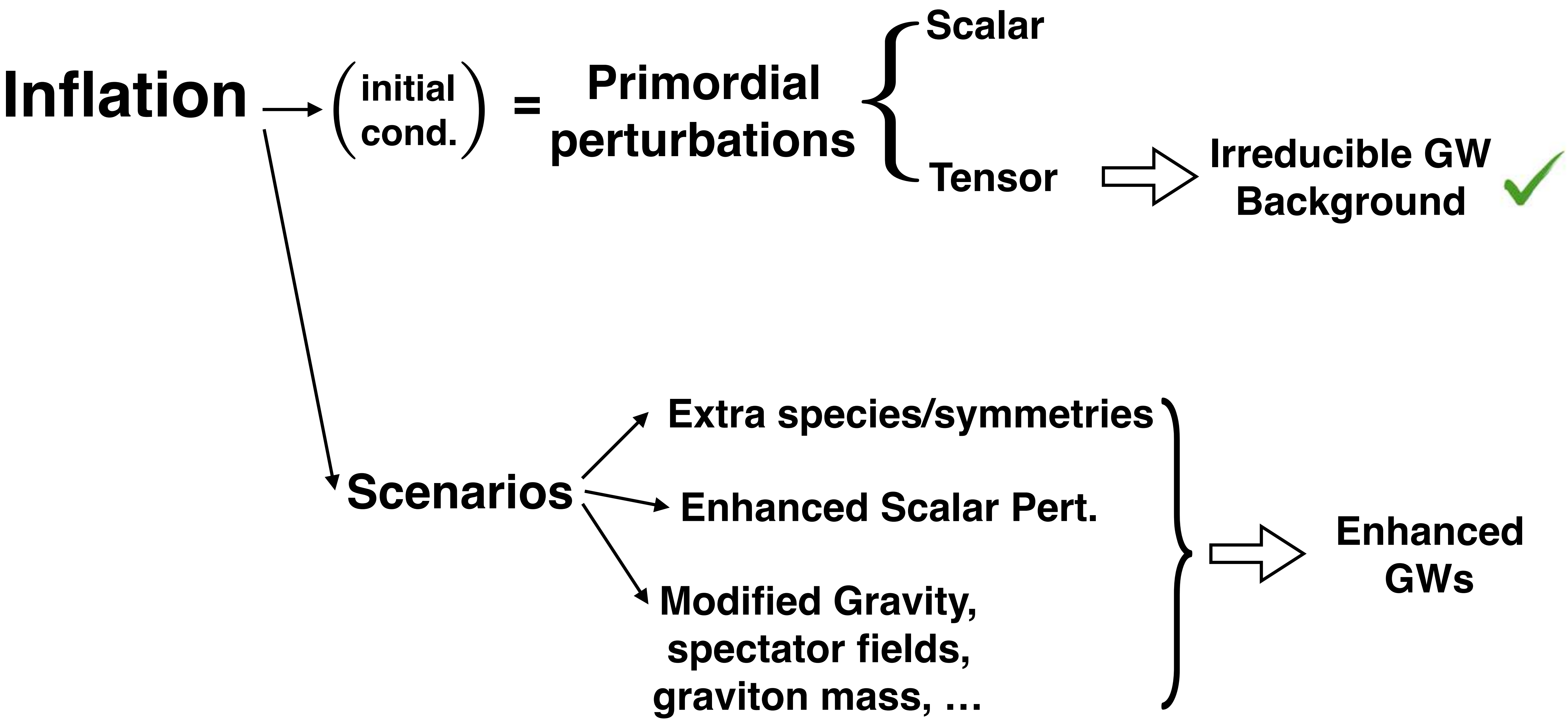
INFLATIONARY COSMOLOGY



Inflation → (initial cond.) = **Primordial perturbations**

- **Scalar**
- **Tensor** ⇒ **Irreducible GW Background** ✓

INFLATIONARY COSMOLOGY



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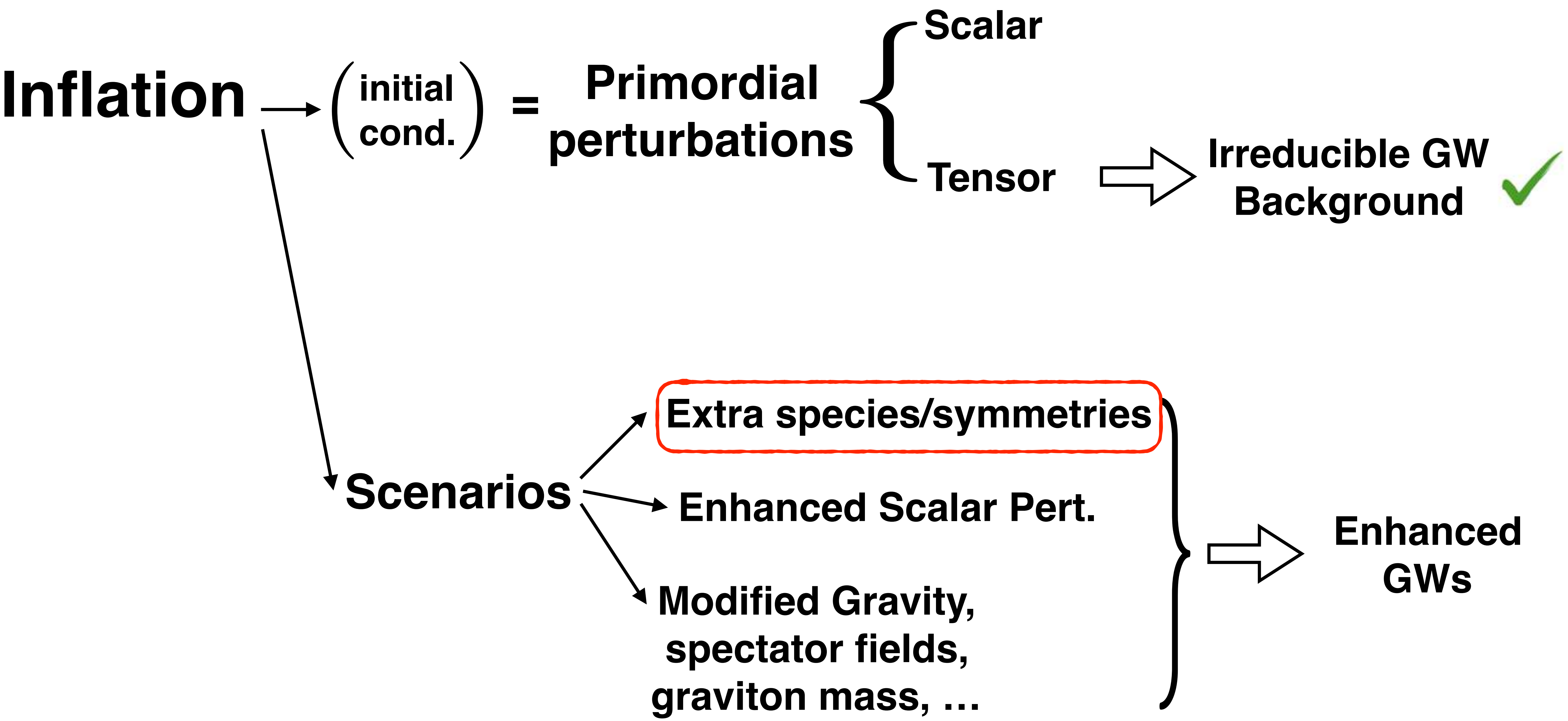
Scenarios

- Extra species/symmetries
- Enhanced Scalar Pert.
- Modified Gravity, spectator fields, graviton mass, …

⇒ Enhanced GWs

INFLATIONARY COSMOLOGY



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INFLATIONARY MODELS

Axion-Inflation

Freese, Frieman, Olinto ’90; . . .

Shift symmetry $\varphi \to \varphi + const.$

$$V(\varphi) + \boxed{\frac{\varphi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}}$$

inflaton φ = pseudo-scalar axion

INFLATIONARY MODELS

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[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

Photon:
2 helicities

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 \pm \frac{2k\xi}{\tau}\right] A_\pm(\tau, k) = 0,$$

$$\boxed{\xi \equiv \frac{\dot{\varphi}}{2fH}}$$

Chiral instability

$$A_+ \propto e^{\pi\xi}\,, \quad |A_-| \ll |A_+|$$

A+ exponentially amplified,

INFLATIONARY MODELS

Axion-Inflation

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---

chiral GWs !

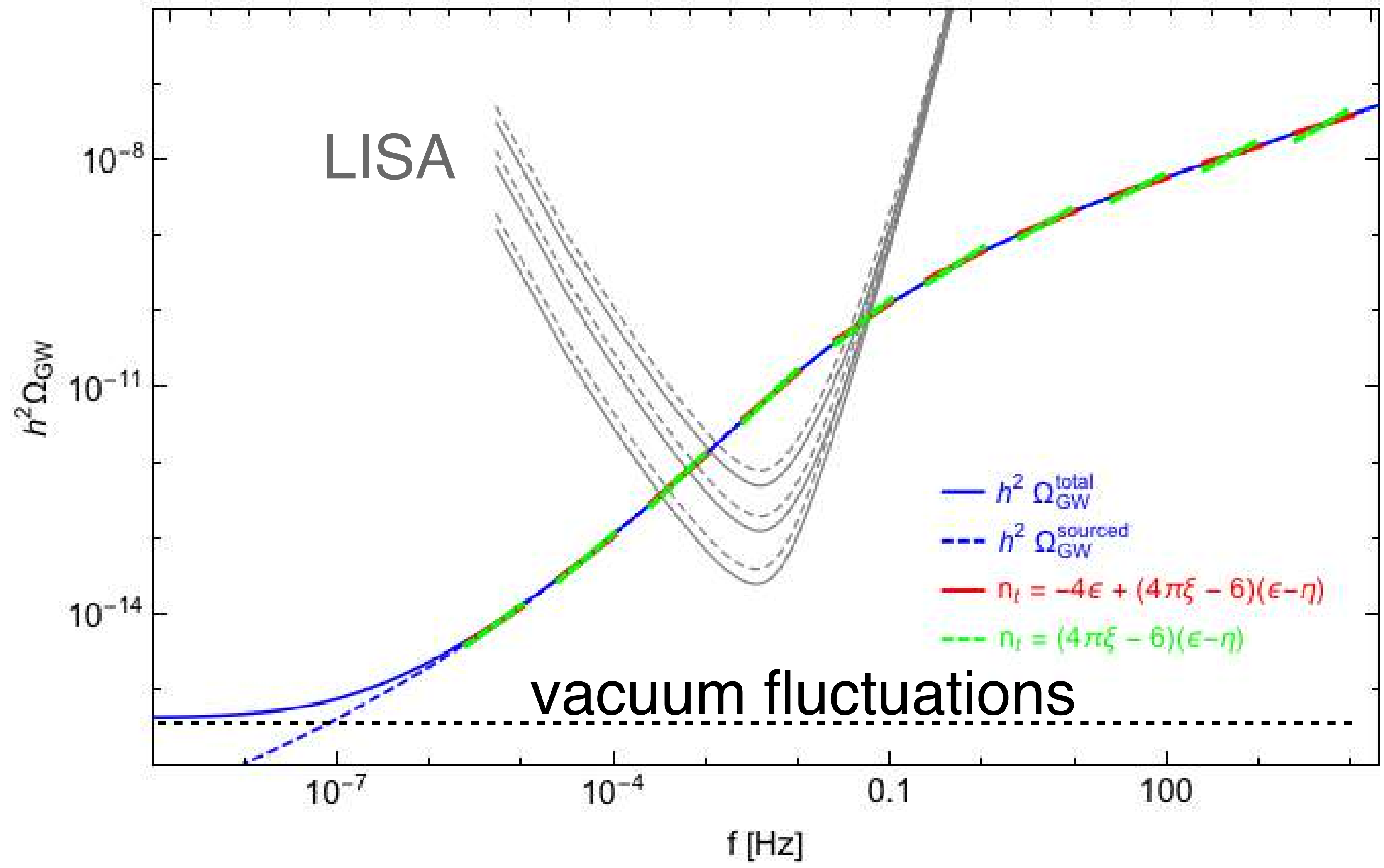
$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\rm TT}_{ij} \propto \{E_iE_j + B_iB_j\}^{TT}$$

GW left-chirality only ! ⇐ A_μ **Chiral**

INFLATIONARY MODELS

Axion-Inflation

GW energy spectrum today



Gauge fields
source a
blue tilted
Non-Gaussian,
& Chiral
GW Background

Bartolo et al ’16, 1610.06481

INFLATIONARY MODELS

Axion-Inflation

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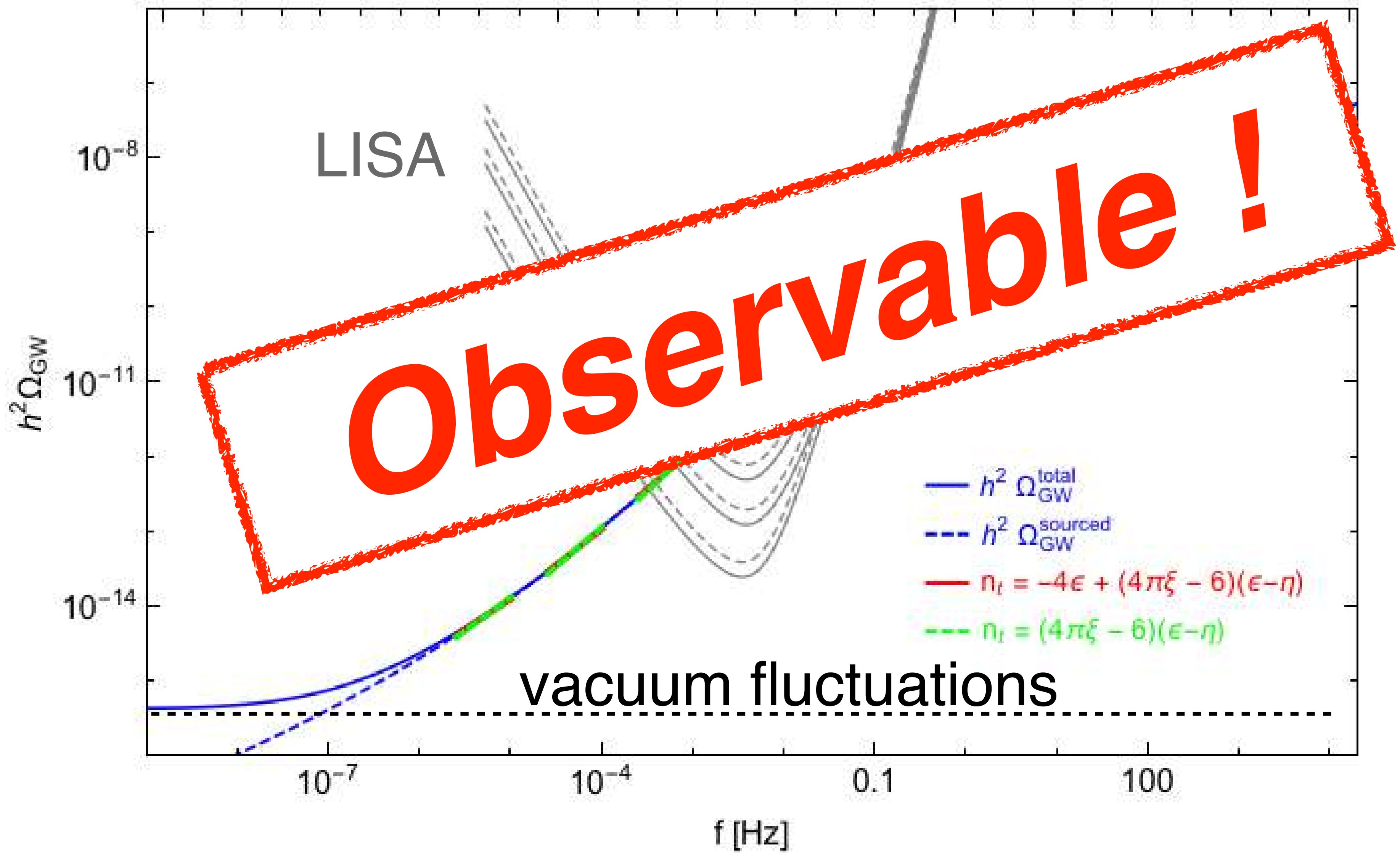

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INFLATIONARY MODELS

What if there are arbitrary fields coupled to the inflaton ?
(i.e. no need of extra symmetry)

→

large excitation of fields !?
will they create GWs?

$$\text{inflaton} \;\; \phi \longrightarrow V(\phi)$$

$$-\mathcal{L}_\chi = (\partial\chi)^2/2 + g^2(\phi-\phi_0)^2\chi^2/2$$ **Scalar Fld**

$$-\mathcal{L}_\psi = \bar{\psi}\gamma^\mu\partial_\mu\psi + g(\phi-\phi_0)\bar{\psi}\psi$$ **Fermion Fld**

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - gA_\mu)\Phi)|^2 - V(\Phi^\dagger\Phi)$$ **Gauge Fld ($\Phi = \phi e^{i\theta}$)**

INFLATIONARY MODELS

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(i.e. no need of extra symmetry)

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large excitation of fields !?
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$$\text{inflaton } \phi \longrightarrow V(\phi)$$

All 3 cases:

$$m = g(\phi(t) - \phi_0) \xRightarrow{\text{non-adiabatic}} \dot{m} \gg m^2 \text{ during } \Delta t_{\rm na} \sim 1/\mu\,, \qquad \boxed{\mu^2 \equiv g\dot{\phi}_0}$$

$$\boxed{n_k = {\rm Exp}\{-\pi(k/\mu)^2\}}$$

Non-adiabatic field excitation (particle creation)

⇓

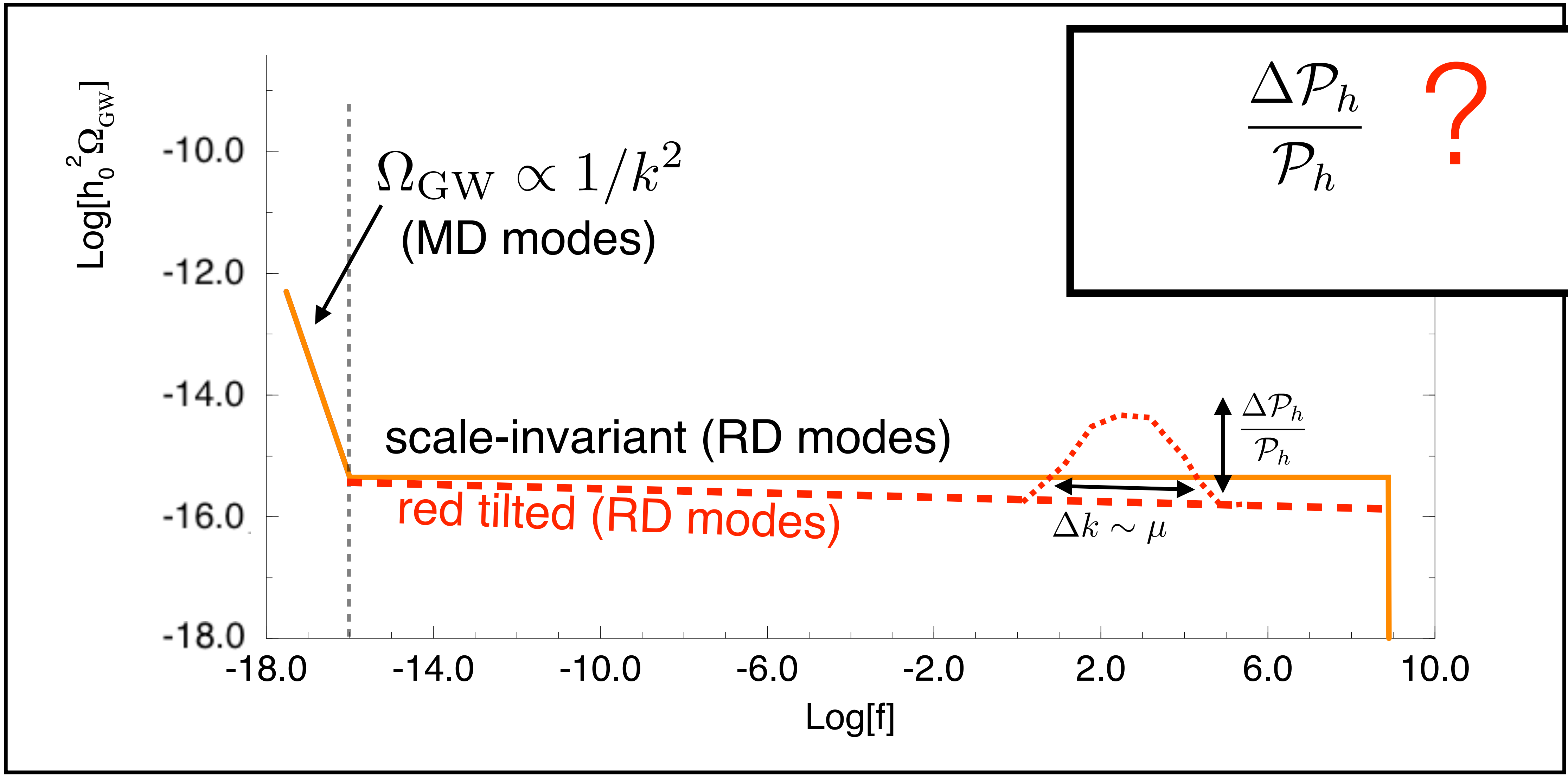
GW

INFLATIONARY MODELS

$$\frac{\Delta\mathcal{P}_h}{\mathcal{P}_h} \equiv \frac{\mathcal{P}_h^{(\mathrm{tot})} - \mathcal{P}_h^{(\mathrm{vac})}}{\mathcal{P}_h^{(\mathrm{vac})}} \equiv \frac{\mathcal{P}_h^{(\mathrm{pp})}}{\mathcal{P}_h^{(\mathrm{vac})}} \sim few \times \mathcal{O}(10^{-4})\frac{H^2}{m_{\mathrm{pl}}^2}W(k\tau_0)\left(\frac{\mu}{H}\right)^3 \ln^2(\mu/H)$$

$$\mu^2 \equiv g\dot{\phi}_0$$

(Sorbo et al 2011, Peloso et al 2012-2013, Caprini & DGF 2018)



$$\frac{\Delta\mathcal{P}_h}{\mathcal{P}_h} \;?$$

INFLATIONARY MODELS

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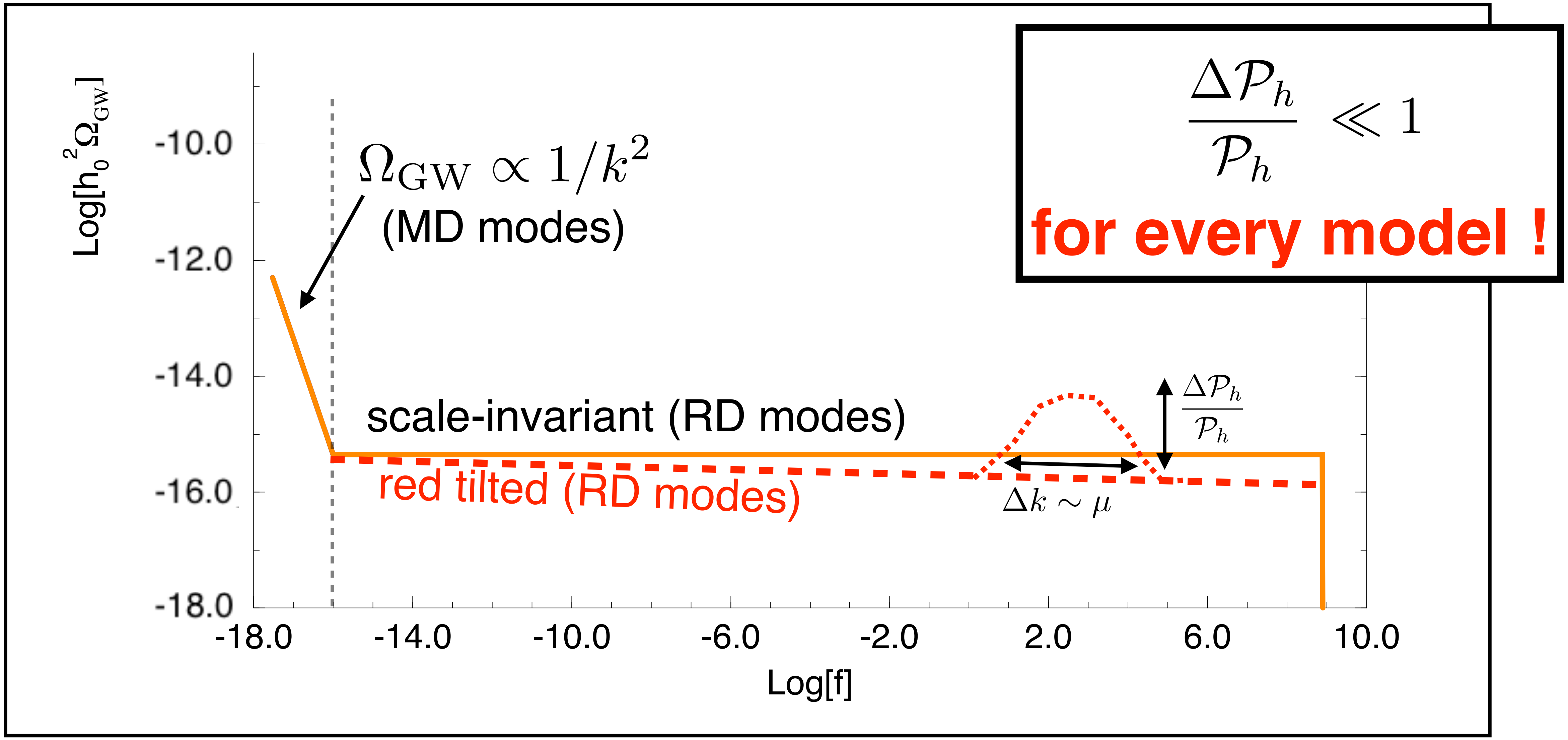
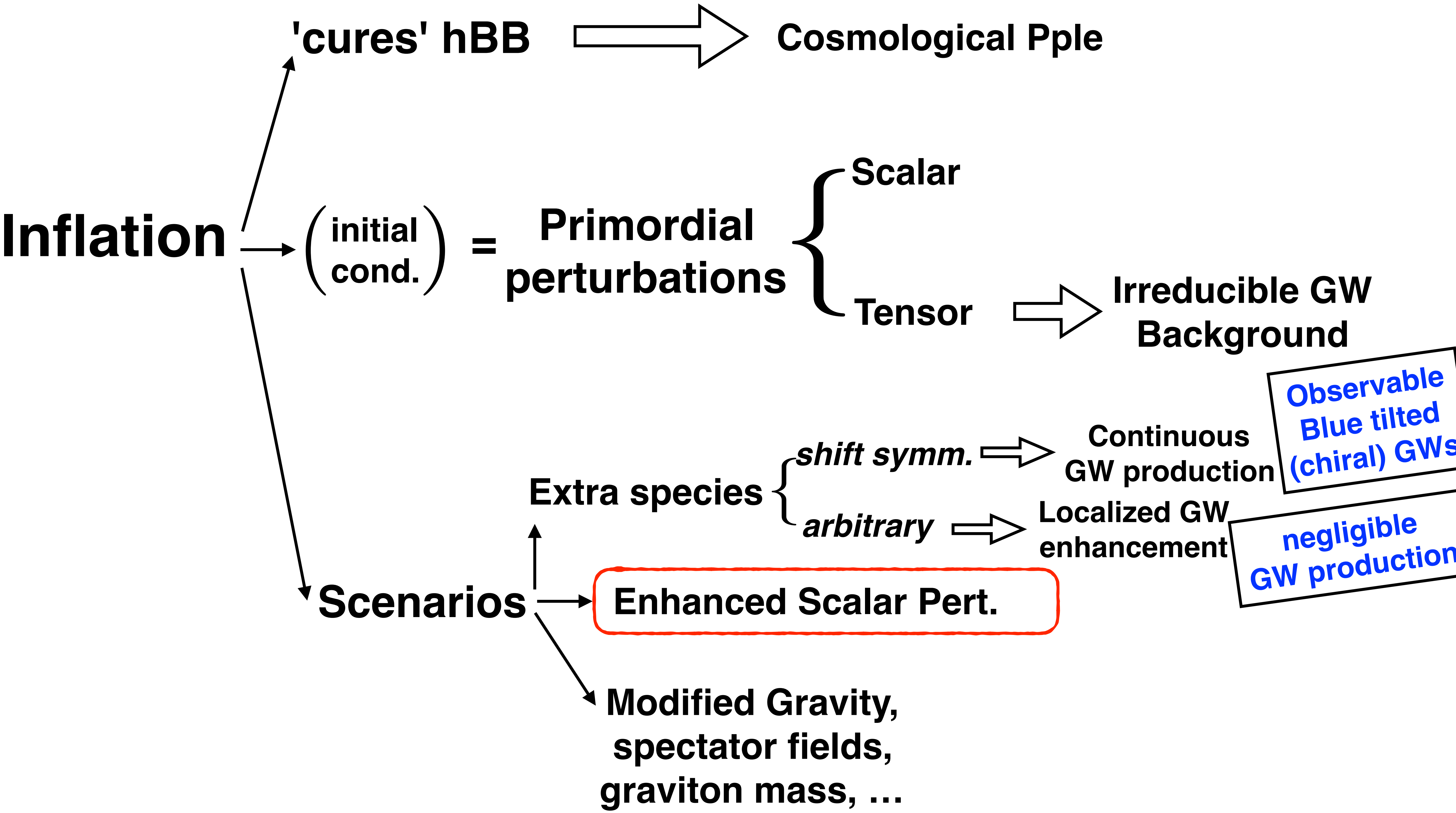


$$\frac{\Delta\mathcal{P}_h}{\mathcal{P}_h} \ll 1$$

for every model !

INFLATIONARY COSMOLOGY



Inflation

'cures' hBB

Cosmological Pple

(initial cond.) = Primordial perturbations

Scalar

Tensor

Irreducible GW Background

Scenarios

Extra species

shift symm.

Continuous GW production

Observable Blue tilted (chiral) GWs

arbitrary

Localized GW enhancement

negligible GW production

Enhanced Scalar Pert.

Modified Gravity, spectator fields, graviton mass, …

INFLATIONARY MODELS

INFLATION → **IF** $\left\{\begin{array}{l}\text{non-monotonic}\\ \text{multi-field}\end{array}\right\}$ ⇒ **possible to enhance** $\Delta^2_{\mathcal{R}}$ **(at small scales)**

Let us suppose $\boxed{\Delta^2_{\mathcal{R}} \gg \Delta^2_{\mathcal{R}}\Big|_{\rm CMB} \sim 3\cdot 10^{-9}}$, @ small scales

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + [(1-2\Psi)\delta_{ij} + 2F_{(i,j)} + h_{ij}]dx^i dx^j]$$

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$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}} \sim \Phi * \Phi$$ **(2nd Order Pert.)**

$$\begin{aligned} S_{ij} = \;& 2\Phi\partial_i\partial_j\Phi - 2\Psi\partial_i\partial_j\Phi + 4\Psi\partial_i\partial_j\Psi + \partial_i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi) \\ & - \frac{2c_s^2}{3w\mathcal{H}}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi) \end{aligned}$$

D. Wands et al, 2006-2010
Baumann et al, 2007
Peloso et al, 2018

INFLATIONARY MODELS

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BBN $\Omega_{gw,0} < 1.5 \times 10^{-5} \longrightarrow \triangle^2_{\mathcal{R}} < 0.1$

LIGO $\Omega_{gw,0} < 6.9 \times 10^{-6} \longrightarrow \triangle^2_{\mathcal{R}} < 0.07$

PTA $\Omega_{gw,0} < 4 \times 10^{-8} \longrightarrow \triangle^2_{\mathcal{R}} < 5 \times 10^{-3}$

LISA $\Omega_{gw,0} < 10^{-13} \longrightarrow \triangle^2_{\mathcal{R}} < 1 \times 10^{-5}$

BBO $\Omega_{gw,0} < 10^{-17} \longrightarrow \triangle^2_{\mathcal{R}} < 3 \times 10^{-7}$

Phys.Rev. D81 (2010) 023527

INFLATIONARY MODELS

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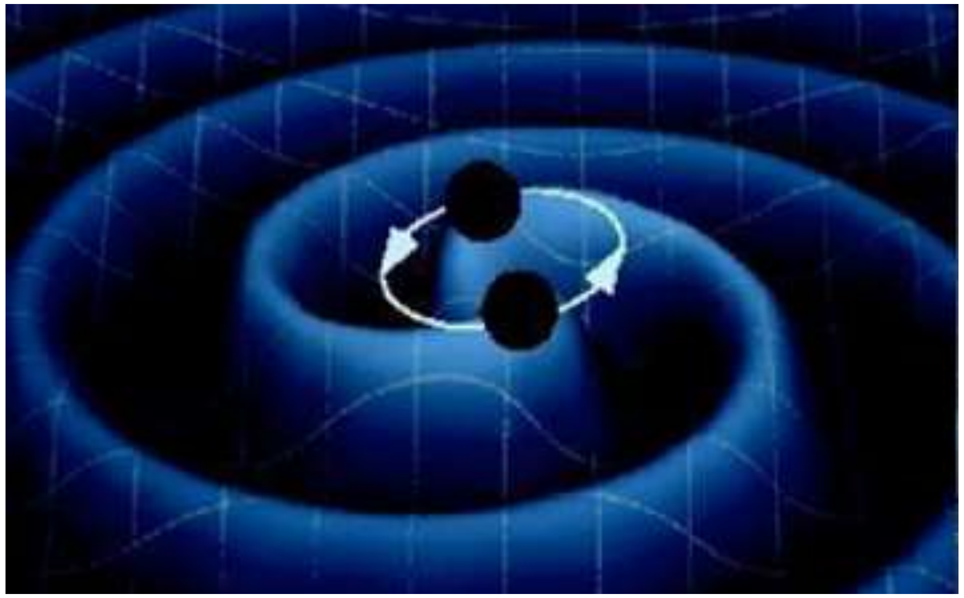
IF $\Delta^2_{\mathcal{R}}$ very enhanced → **Primordial Black Holes (PBH) may be produced!**

Has LIGO detected PBH's ?



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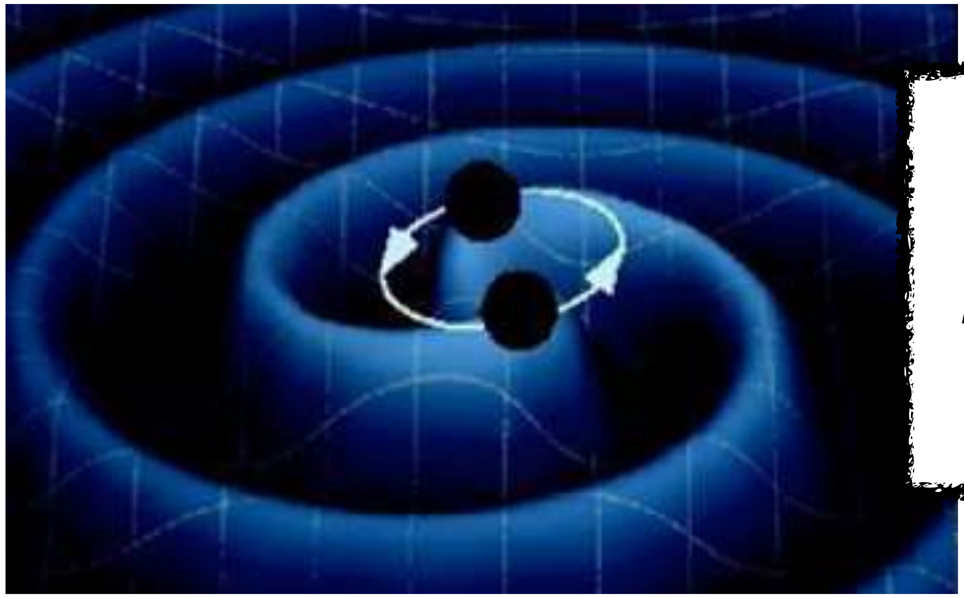

'We will know soon, determining mass/spin distributions'
(M. Fishbach (LIGO), Moriond'19)

INFLATIONARY MODELS

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PBH candidate for Dark Matter ?

Clesse & Garcia-Bellido, 2015-2017
Ali-Haimoud et al 2016-2017
Window is very narrow

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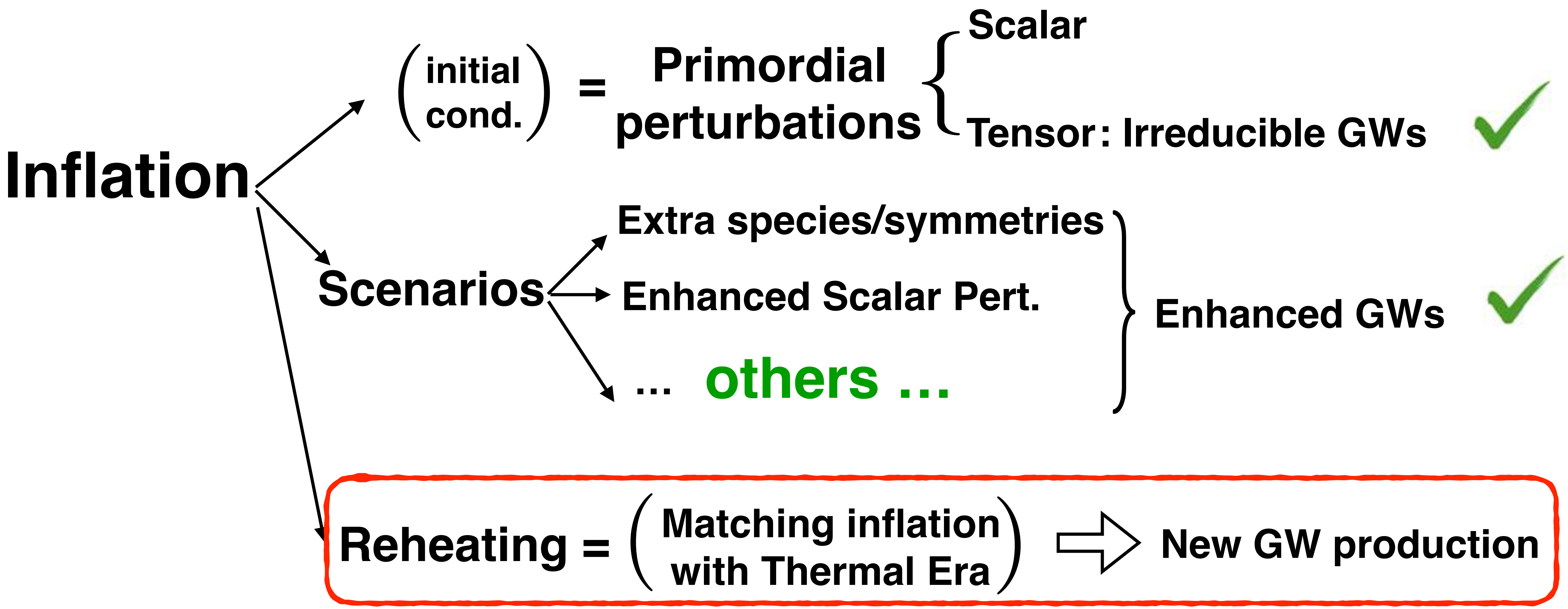
* If PBH are the DM, what is the GW from 2nd O(Φ)? **Bartolo et al, '18**

* If GW from from 2nd O(Φ) PBH, then Non-Gaussianity? **Bartolo et al, '19**

* If GW from from 2nd O(Φ) PBH, then Anisotropies? **Bartolo et al, '19**

INFLATIONARY COSMOLOGY



Inflation

(initial cond.) = Primordial perturbations

Scalar

Tensor : Irreducible GWs ✓

Scenarios

Extra species/symmetries

Enhanced Scalar Pert.

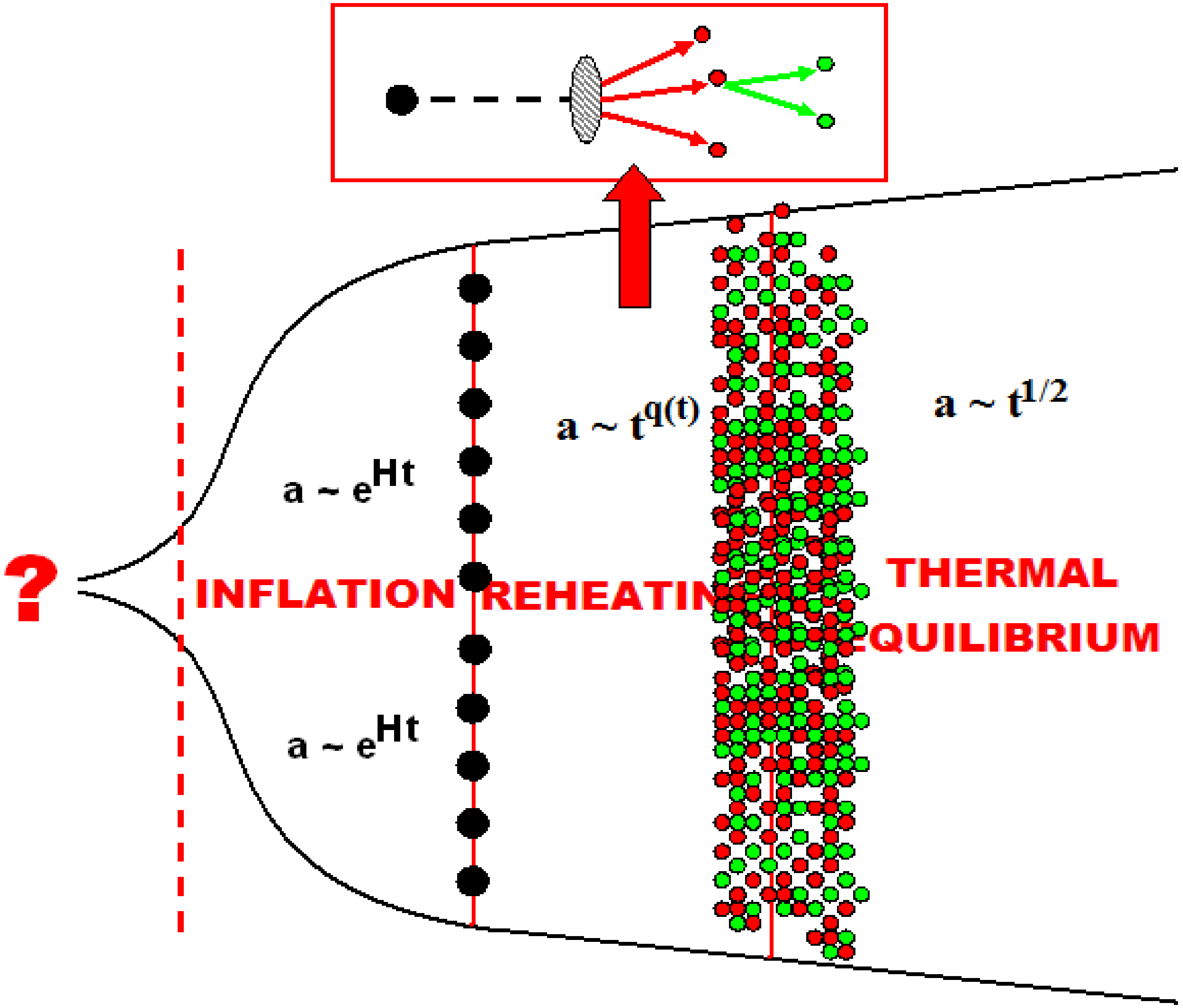
... others ...

Enhanced GWs ✓

Reheating = (Matching inflation with Thermal Era) ⇨ New GW production

GWs from Reheating

INFLATION ⟶ REHEATING ⟶ BIG BANG THEORY



SCALAR REHEATING

1) $V(\phi, \chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2$ (Chaotic)

2) $V(\phi, \chi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2}g^2\phi^2\chi^2$ (Hybrid)

INFLATON **MATTER** **COUPLING**

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INFLATON **MATTER** **COUPLING**

$$\begin{cases} \ddot{\phi}(t) + 3H\dot{\phi} + V'(\phi) = 0 & (\textbf{Inflaton Zero-Mode} \ : \text{Damped Oscillator}) \\ \Box\phi_k + F(\int dq\phi_q\chi_{|k-q|})\phi_k + ... = 0 & (\textbf{Inflaton Fluctuations}) \\ \Box\chi_k + F(\int dq\chi_q, \phi_{|k-q|})\chi_k + ... = 0 & (\textbf{Matter Fluctuations}) \end{cases}$$

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DYNAMICS:

Non-Linear, Non-Perturbative & Far-From-Equilibrium

$$\mathbf{k}_i \pm \Delta\mathbf{k}_i \rightarrow \varphi_k(t), n_k(t) \sim exp\{\mu_k\, t\}$$

SCALAR (P)REHEATING

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DYNAMICS:

Non-Linear, Non-Perturbative & Far-From-Equilibrium

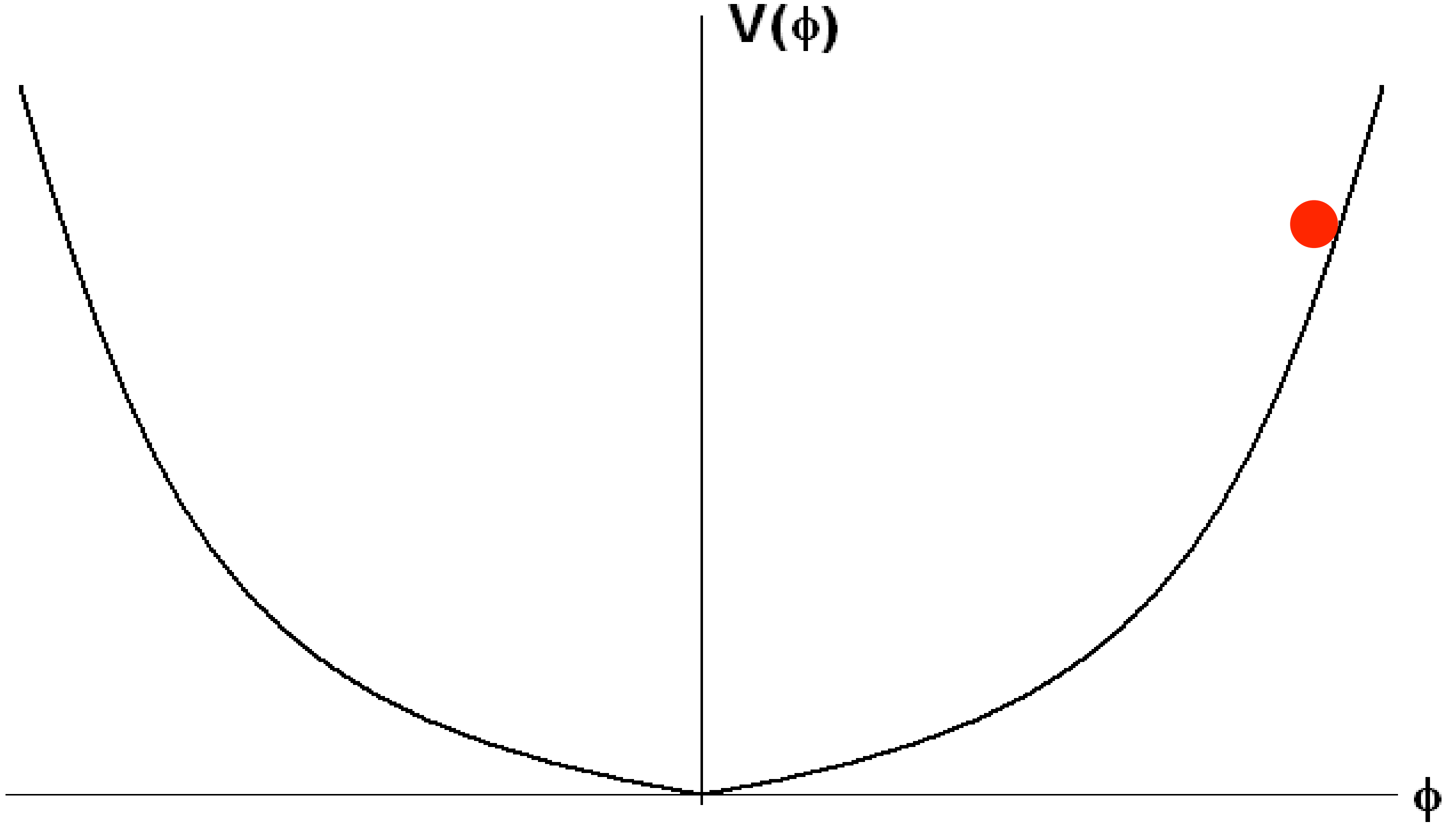
$\mathbf{k}_i \pm \Delta\mathbf{k}_i \rightarrow \varphi_k(t), n_k(t) \sim exp\{\mu_k\, t\} \rightarrow \boxed{\text{PREHEATING}}$

SCALAR (P)REHEATING

1) Chaotic Scenarios: PARAMETRIC RESONANCE

$$V(\phi,\chi) = \quad V(\phi) \quad + \quad \frac{1}{2}m_\chi^2\chi^2 \quad + \quad \frac{1}{2}g^2\phi^2\chi^2$$ (Chaotic Models)

$$X_k'' + [\kappa^2 + m^2(\phi)]X_k = 0$$ (Fluctuations of Matter)



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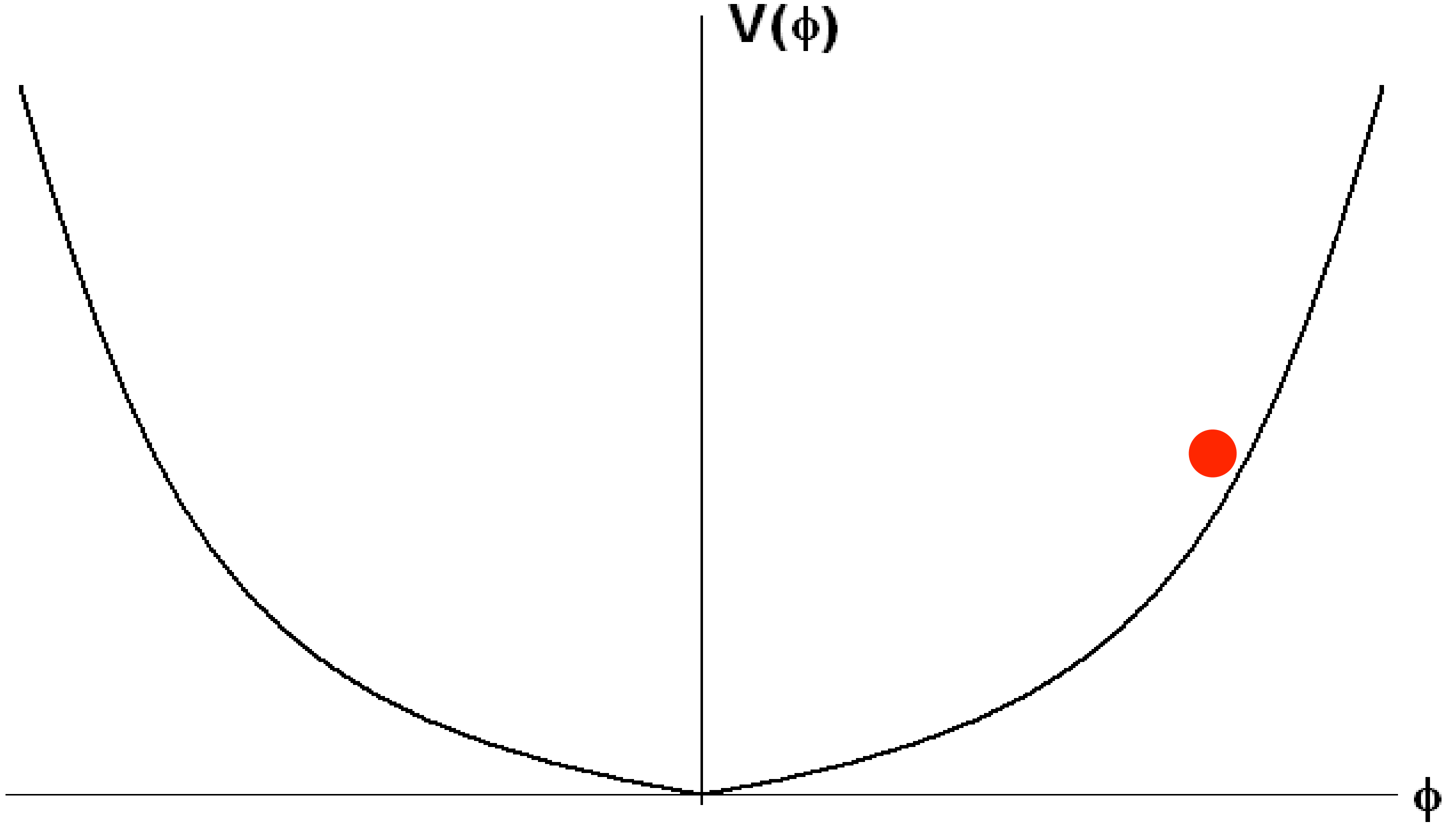

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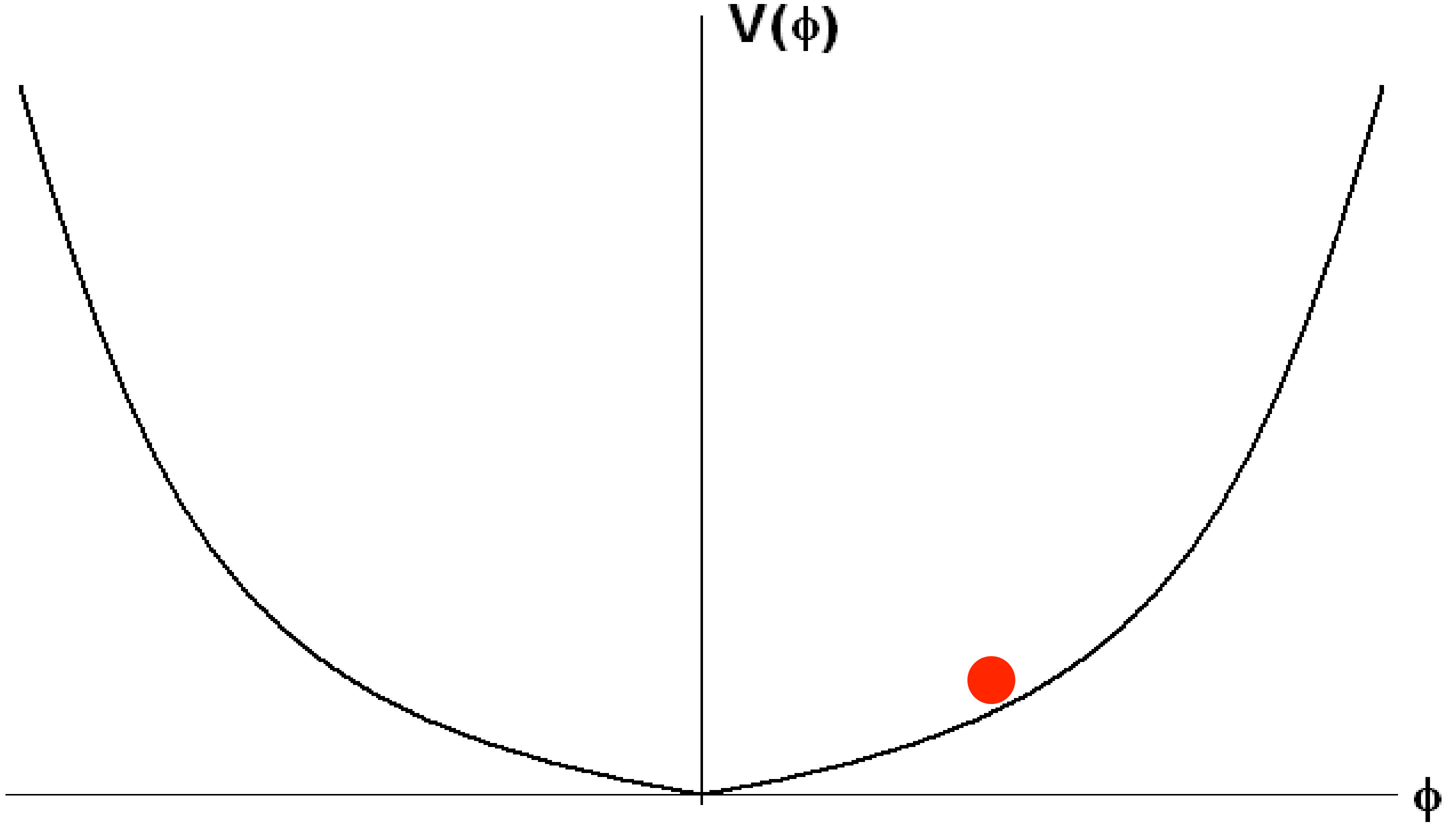

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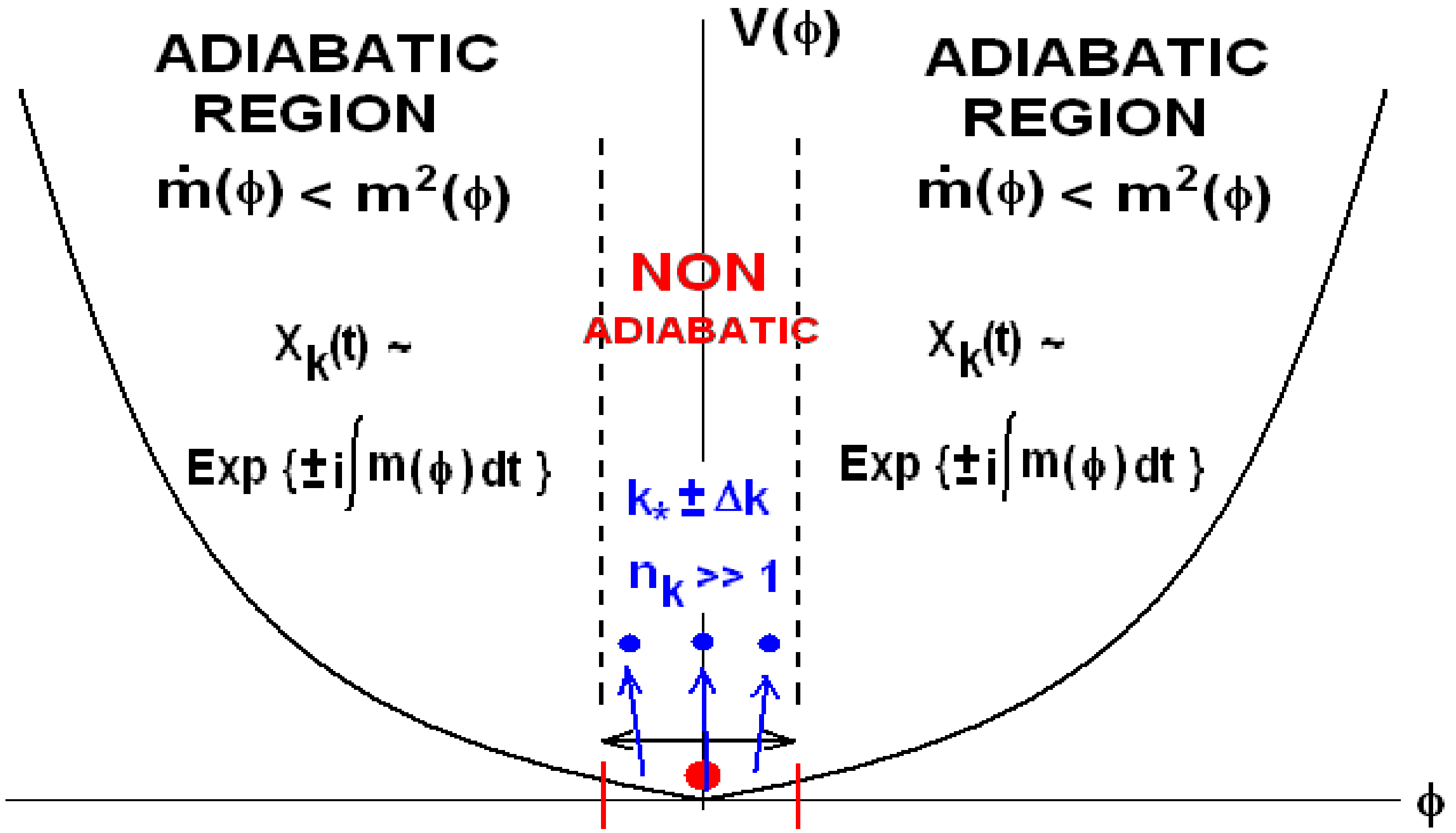

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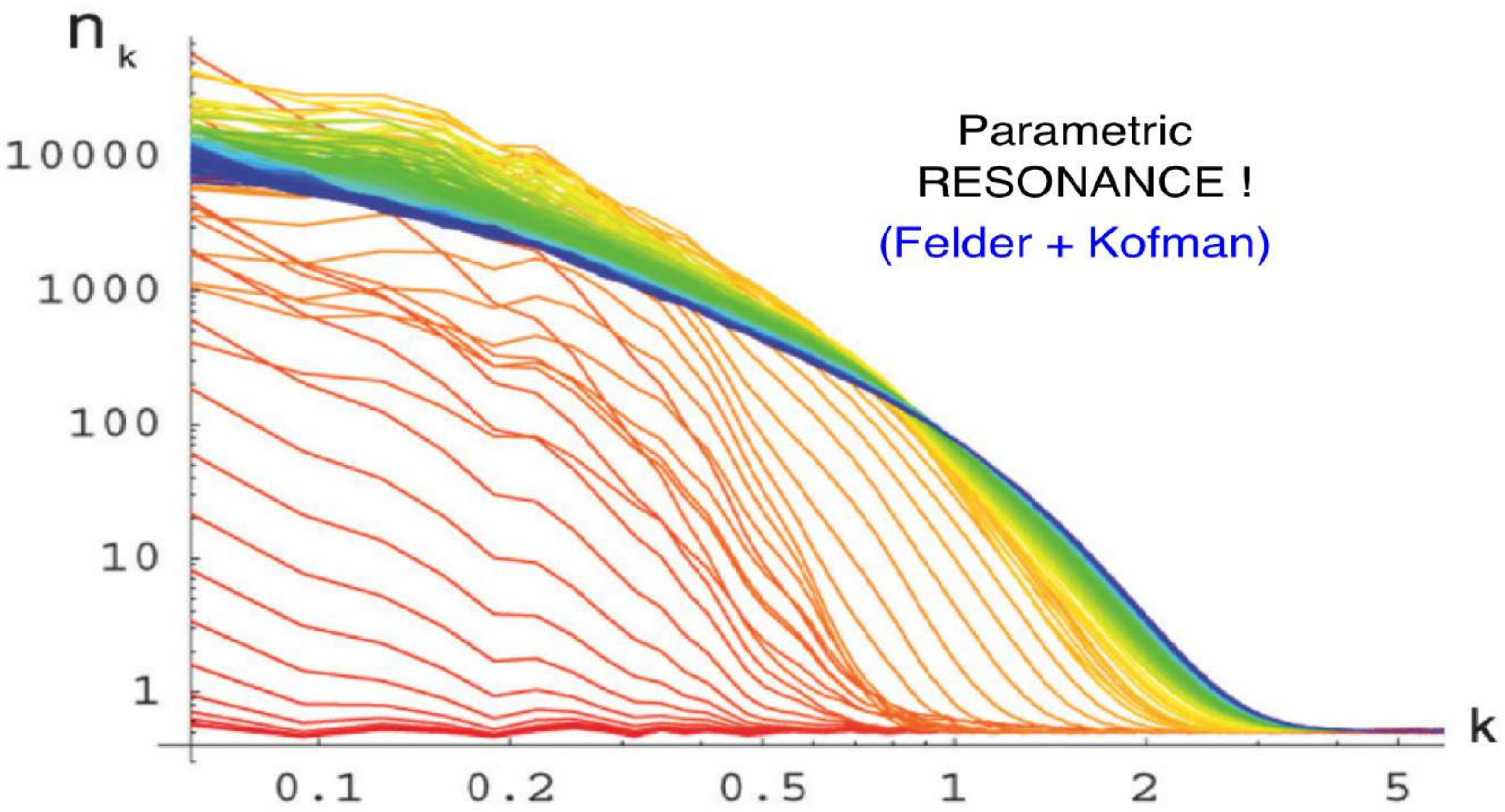
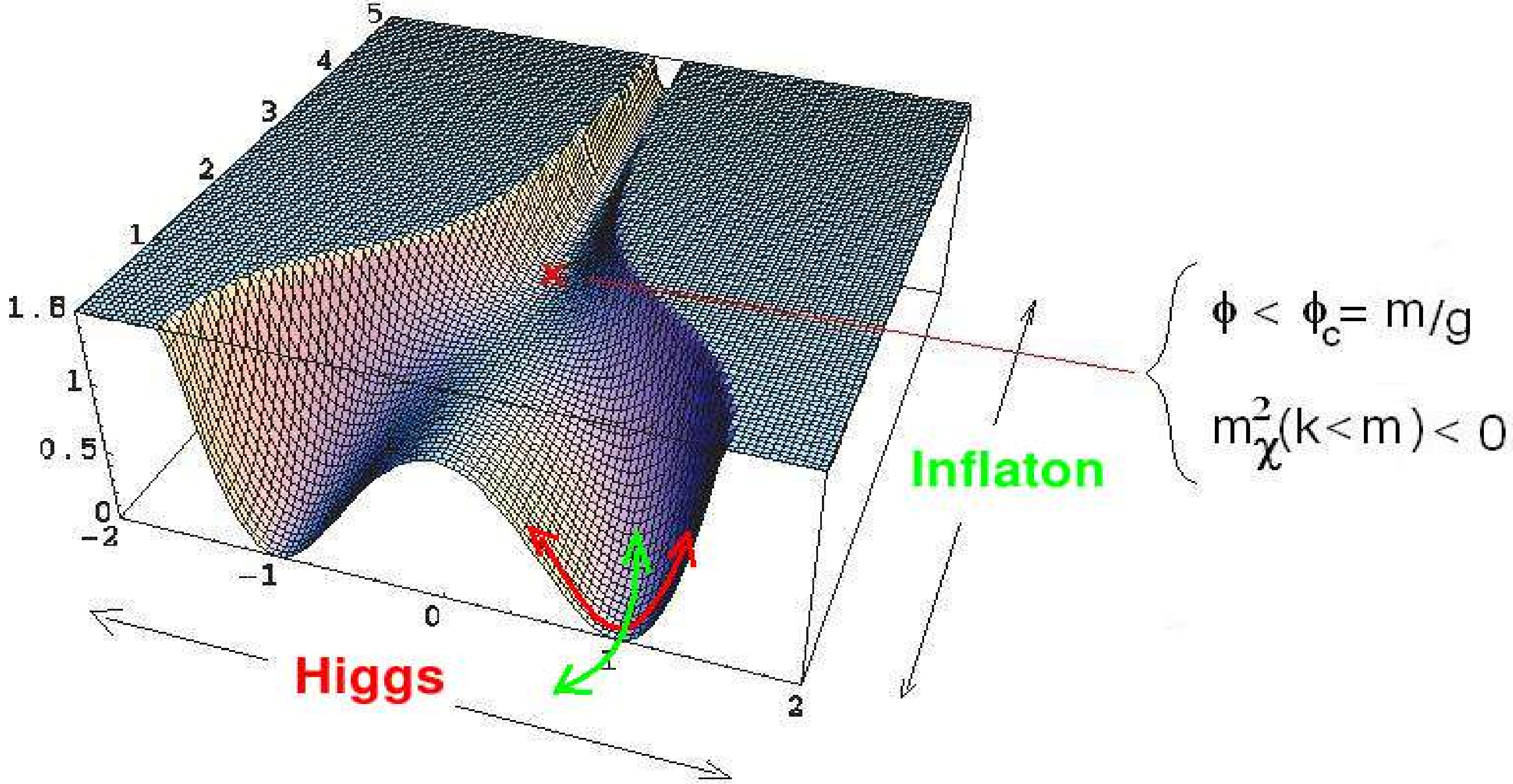


SCALAR (P)REHEATING

2) Hybrid Scenarios : SPINODAL INSTABILITY

$$\left.\begin{aligned} &\ddot{\phi}(t) + (\mu^2 + g^2|\chi|^2)\phi(t) = 0 \\ &\ddot{\chi}_k + \left(k^2 + m^2\left(\frac{\phi^2}{\phi_c^2} - 1\right) + \lambda|\chi|^2\right)\chi_k = 0 \end{aligned}\right\}$$

Hybrid Preheating



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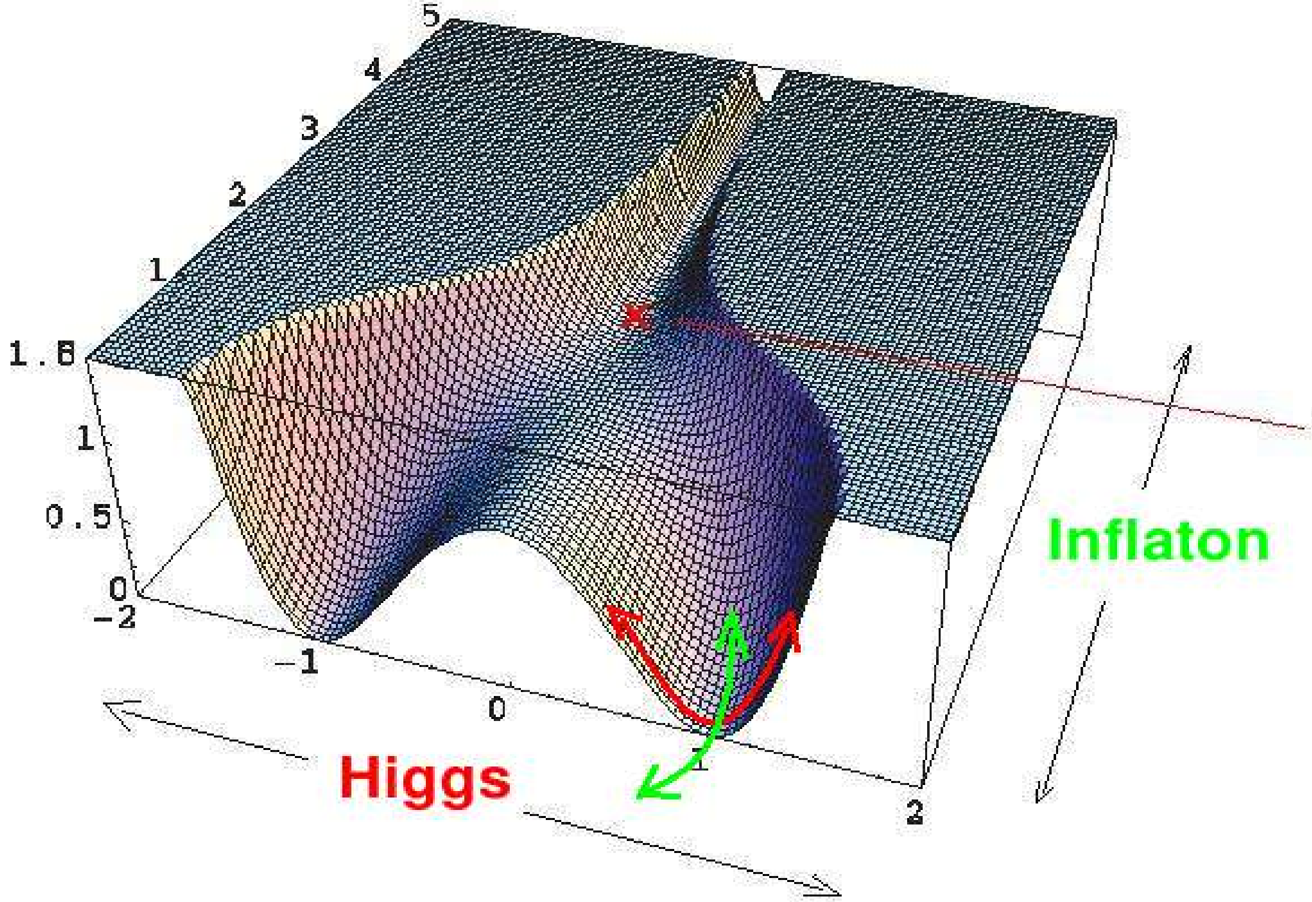
$$\boxed{\begin{gathered}(k < m = \sqrt{\lambda}v)\\ \chi_k, n_k \sim e^{\sqrt{m^2-k^2}t}\end{gathered}}$$

Hybrid Preheating



$\phi < \phi_c = m/g$

$m_\chi^2(k<m) < 0$

SCALAR (P)REHEATING

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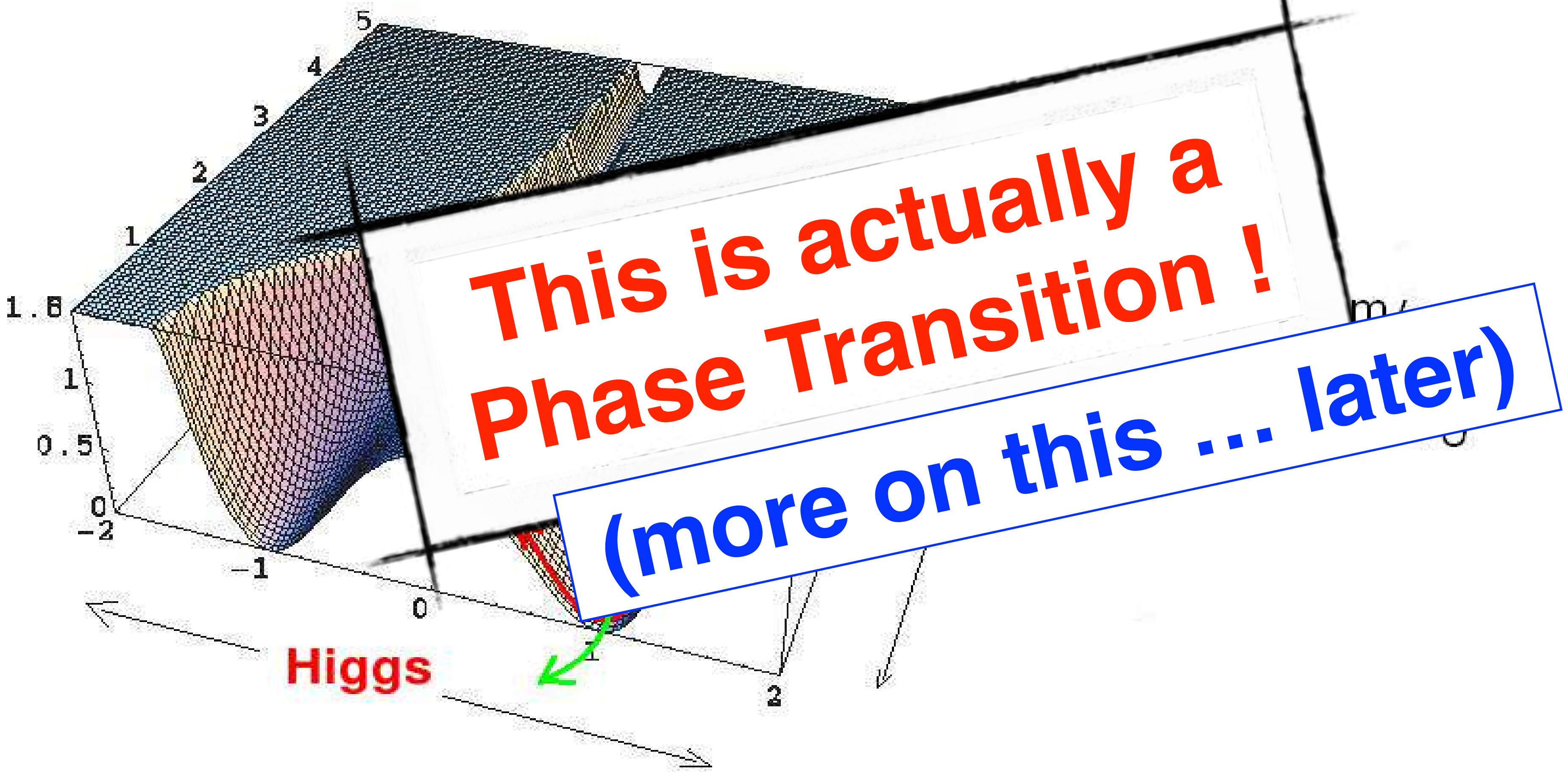
Hybrid Preheating



This is actually a Phase Transition !

(more on this … later)

INFLATIONARY PREHEATING

Physics of (p)REHEATING: $\ddot{\varphi}_k + \omega^2(k,t)\varphi_k = 0$

$$\left\{ \begin{array}{lll} \textbf{Hybrid Preheating}: & \omega^2 = k^2 + m^2(1 - V\,t) < 0 & \text{(Tachyonic)} \\ \textbf{Chaotic Preheating}: & \omega^2 = k^2 + \Phi^2(t)\sin^2\mu t & \text{(Periodic)} \end{array} \right.$$

At $\mathbf{k}_i$: $\varphi_{k_i}, n_{k_i} \sim e^{\mu(k,t)t} \Rightarrow$ Inhomogeneities: $\left\{ \begin{array}{c} L_i \sim 1/k_i \\ \delta\rho/\rho \gtrsim 1 \\ v \approx c \end{array} \right.$

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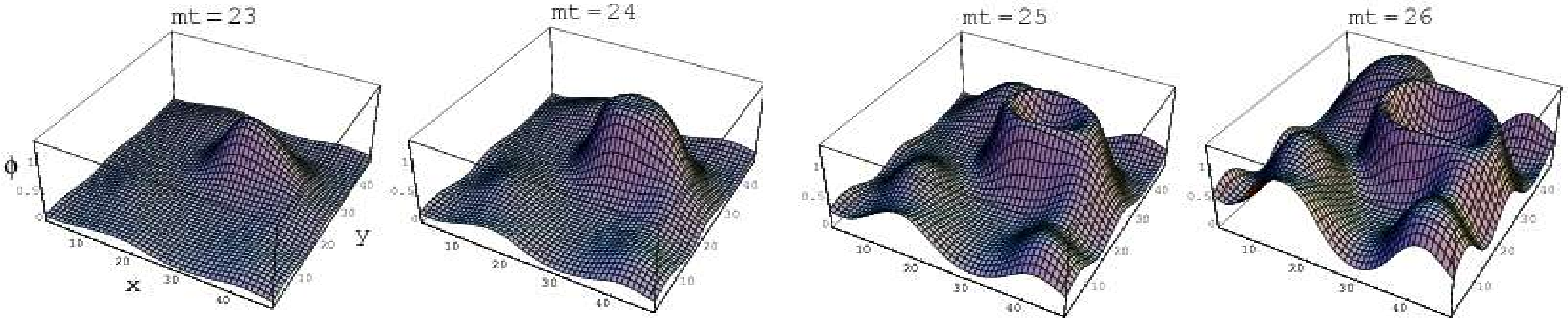

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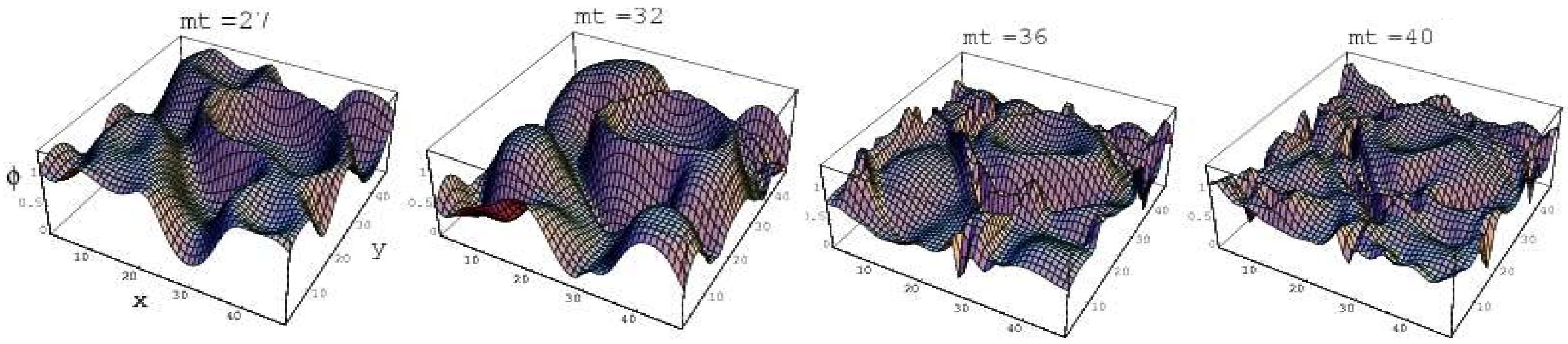

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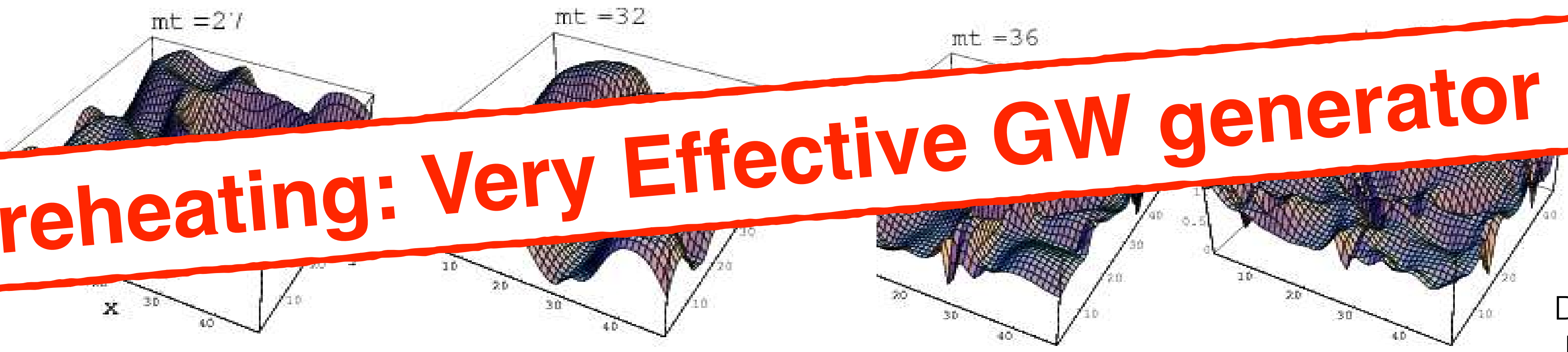

Preheating: Very Effective GW generator !

Easther, Giblin, Lim '06-'08
DGF, Ga-Bellido, et al '07-'10
Kofman, Dufaux et al '07-'09

INFLATIONARY PREHEATING

Non - linear dynamics

↓

Lattice Simulations

Let’s look at an example …

INFLATIONARY (p)REHEATING (pRH)

Lattice Simulations: Dynamics → **non-linear**
→ **out-Eq**

INFLATIONARY (p)REHEATING (pRH)

Lattice Simulations: Dynamics → **non-linear**, **out-Eq**

- Scalars ($n_k \gg 1$): $\Box\phi + V_{,\phi} = 0,\ \Box\chi_a + V_{,\chi_a} = 0$

Semi-classical regime $\pi_k \approx \kappa\phi_k + ...$ **(Squeezed States)**

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- FLRW: $H^2 = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$, $\left\{ \begin{array}{l} \rho = \langle \rho_\phi + \rho_\chi + ... \rangle \\ p = \langle p_\phi + p_\chi + ... \rangle \end{array} \right.$

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- GW: $h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi_{ij}^{\rm TT}, \qquad \Pi_{ij}^{TT} = \{\partial_i\chi^a\partial_j\chi^a\}^{TT}$

$$ds^2 = a^2(-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \qquad {\rm TT} : \begin{cases} h_{ii} = 0 \\ h_{ij\,,j} = 0 \end{cases}$$

INFLATIONARY PREHEATING

Lattice Simulations: Dynamics → **non-linear**
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INFLATIONARY PREHEATING

Lattice Simulations: Dynamics → non-linear
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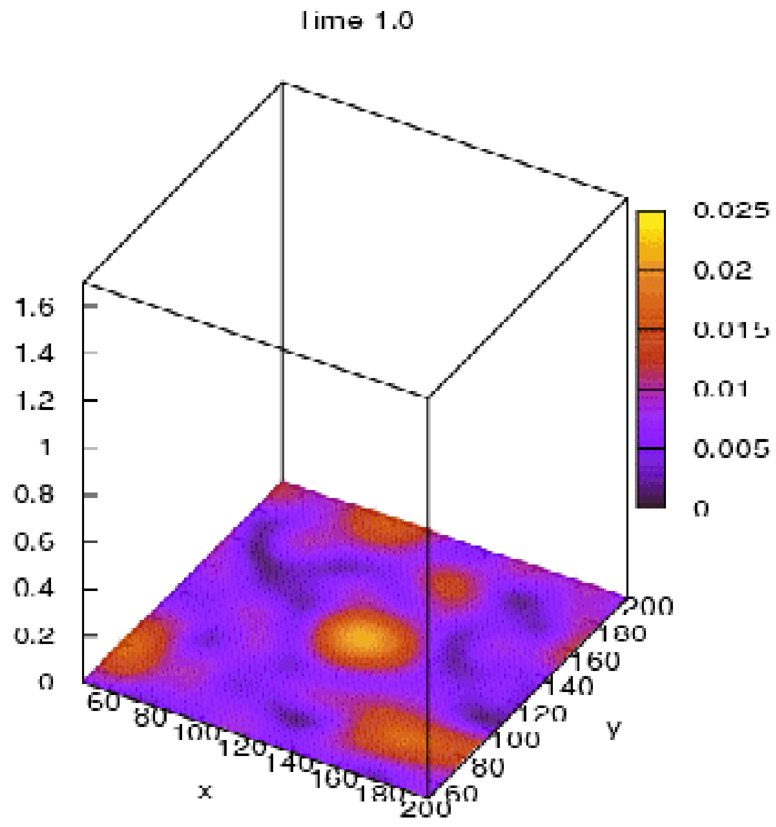
$$V(\phi,\chi) = \frac{\lambda}{4}(|\chi|^2 - v^2)^2 + \frac{1}{2}|\chi|^2\phi^2 + V(\phi)$$



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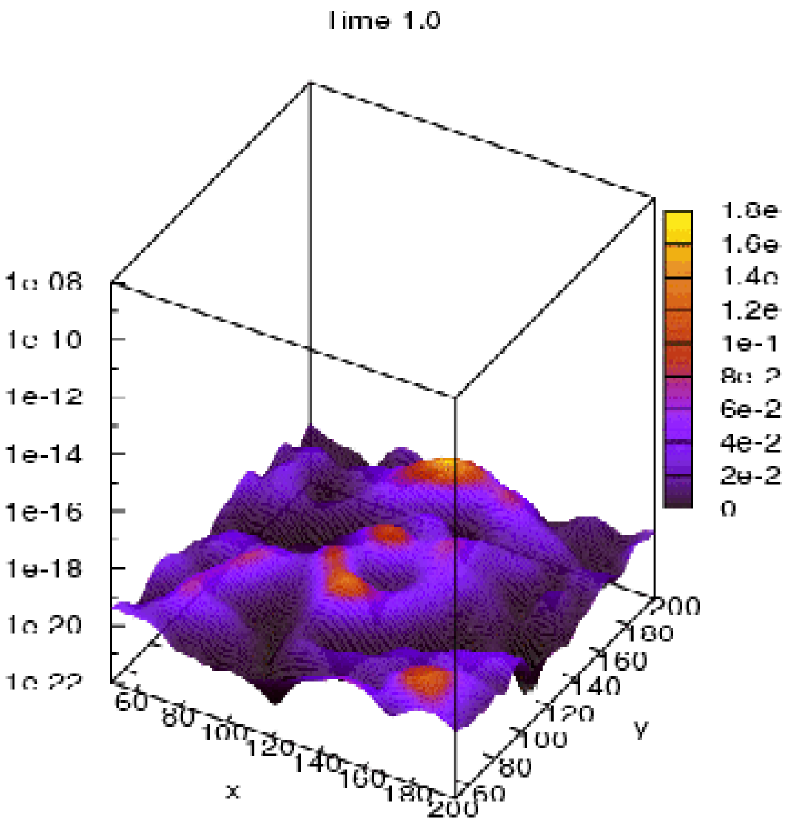

GW (Energy density)

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Lattice Simulations: Dynamics → non-linear → out-Eq

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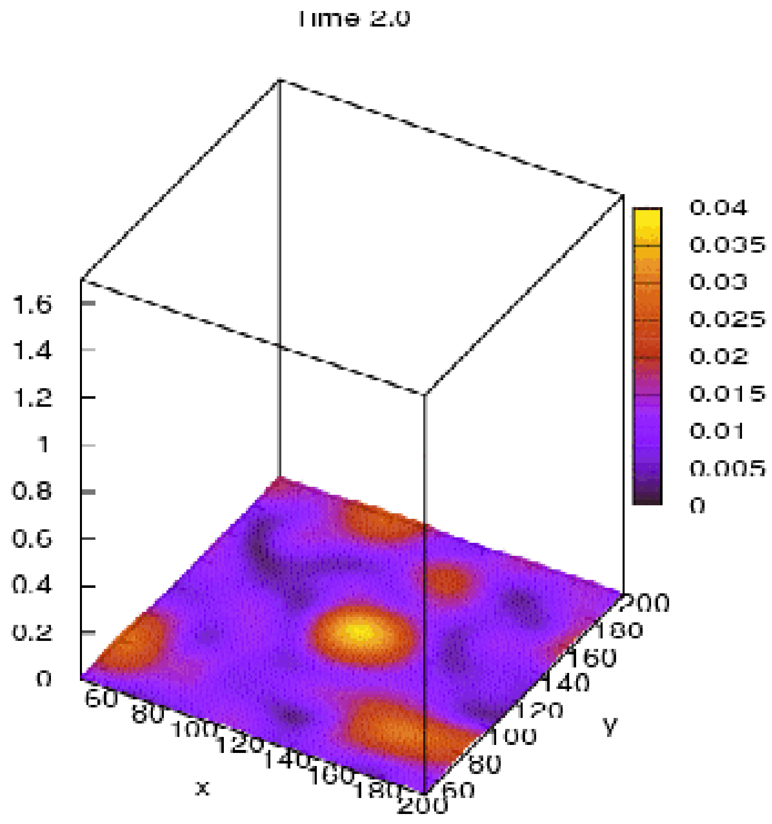

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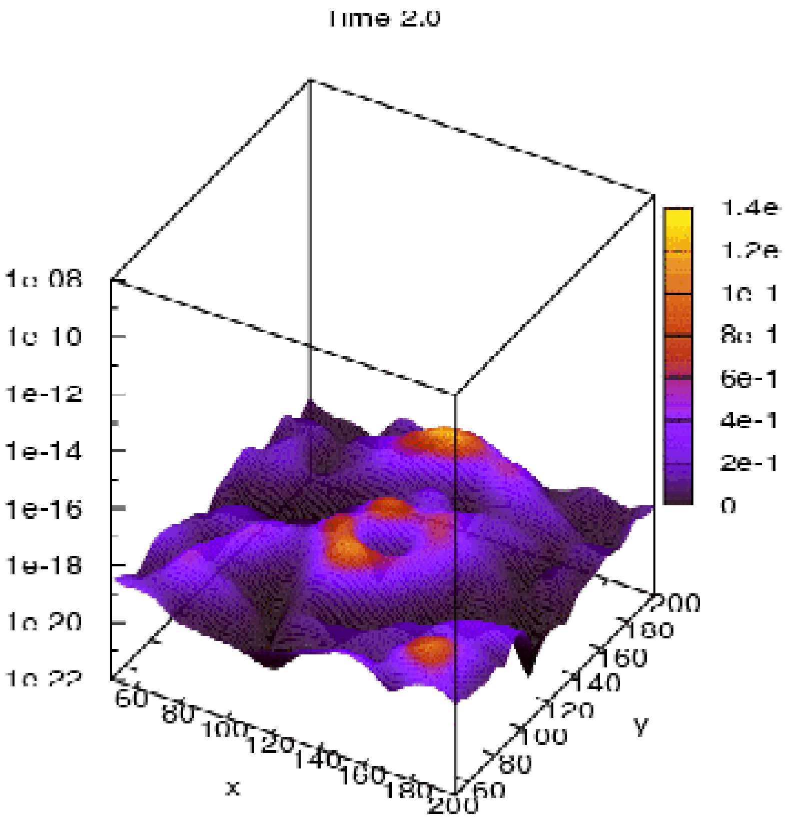

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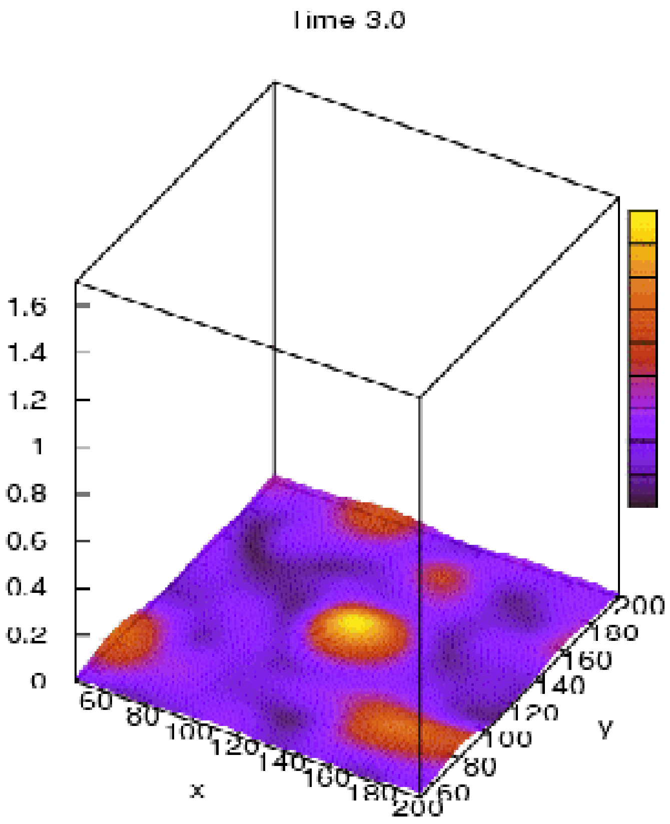

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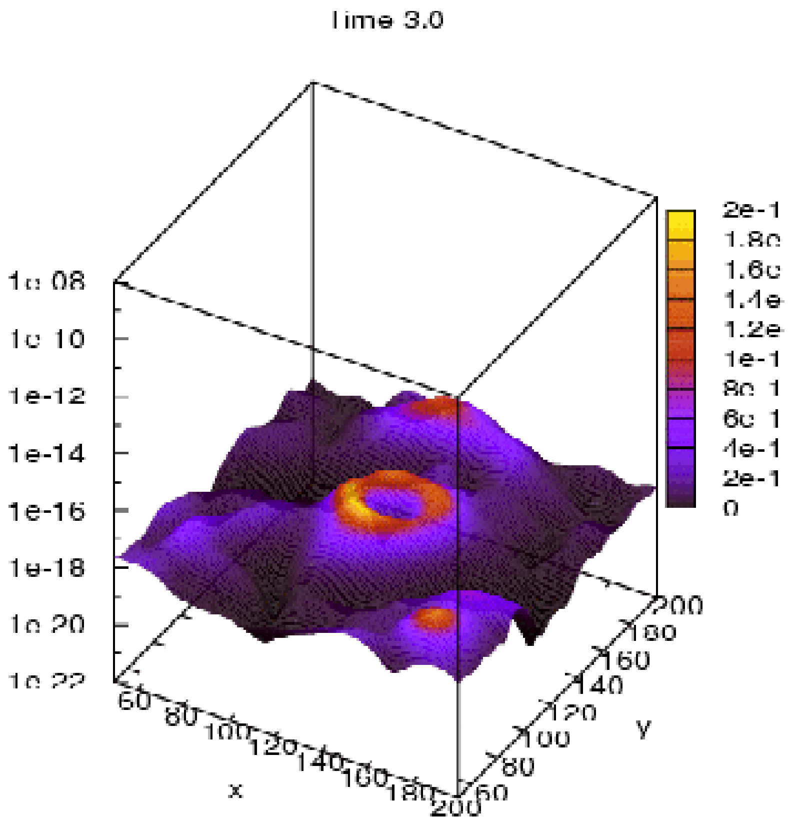

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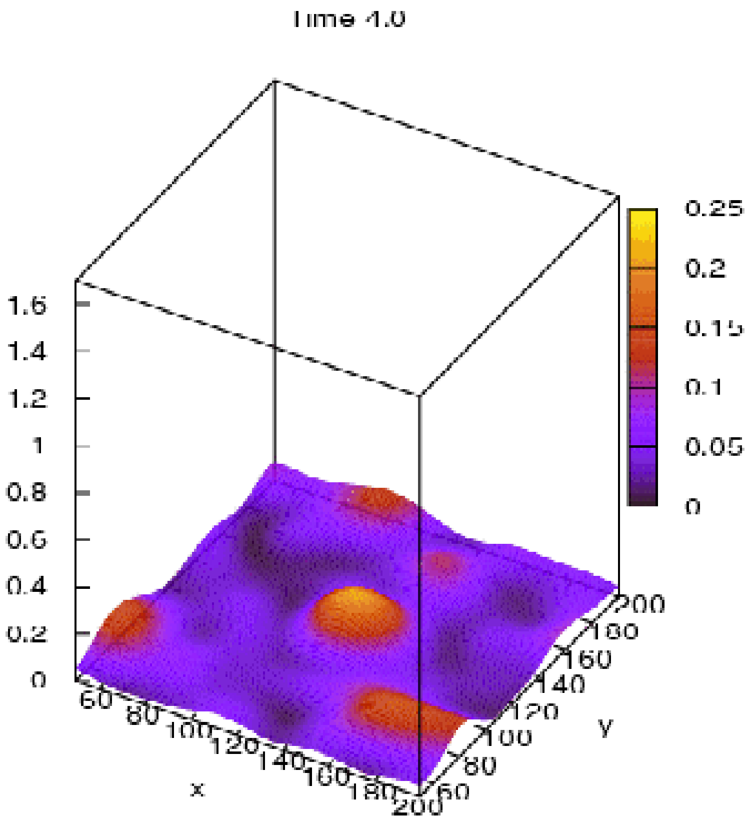

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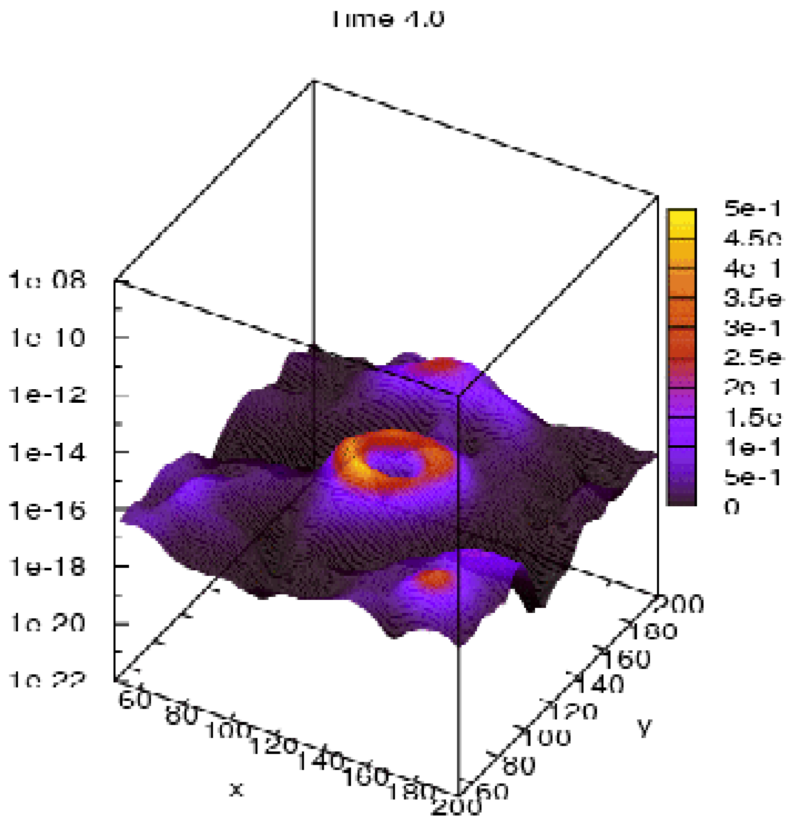

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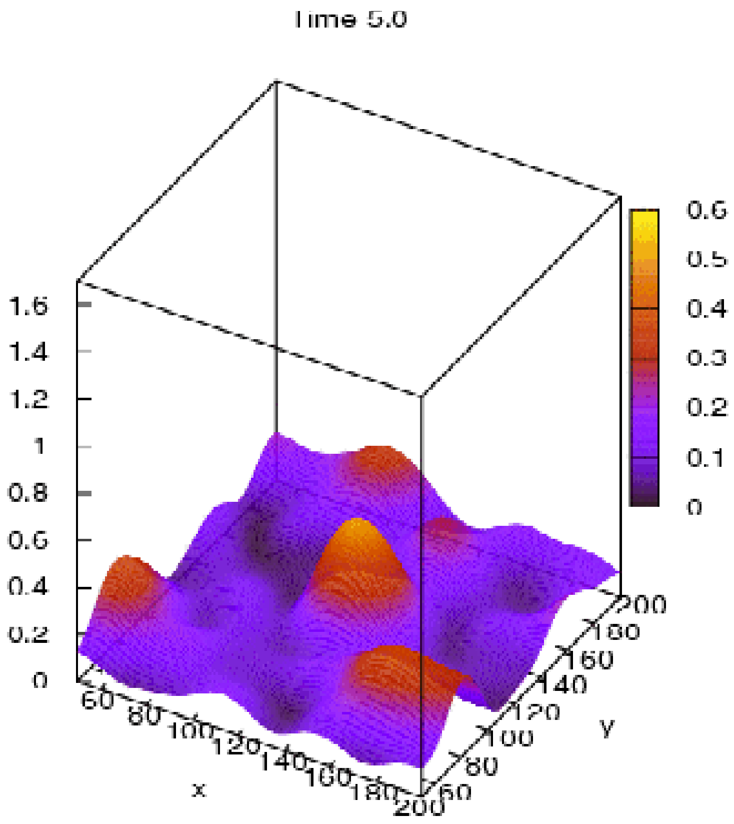

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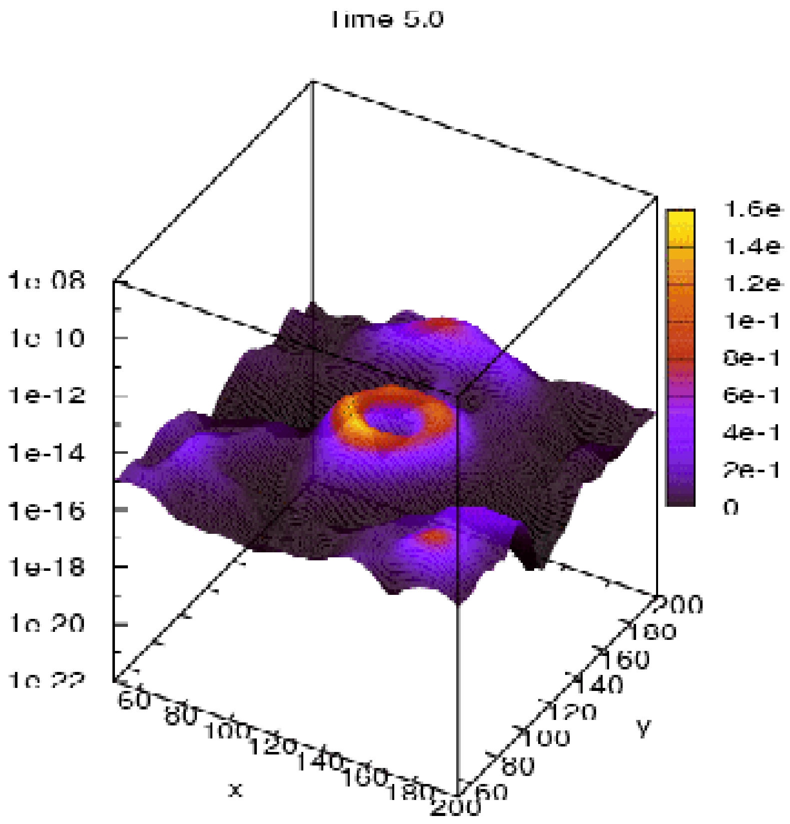

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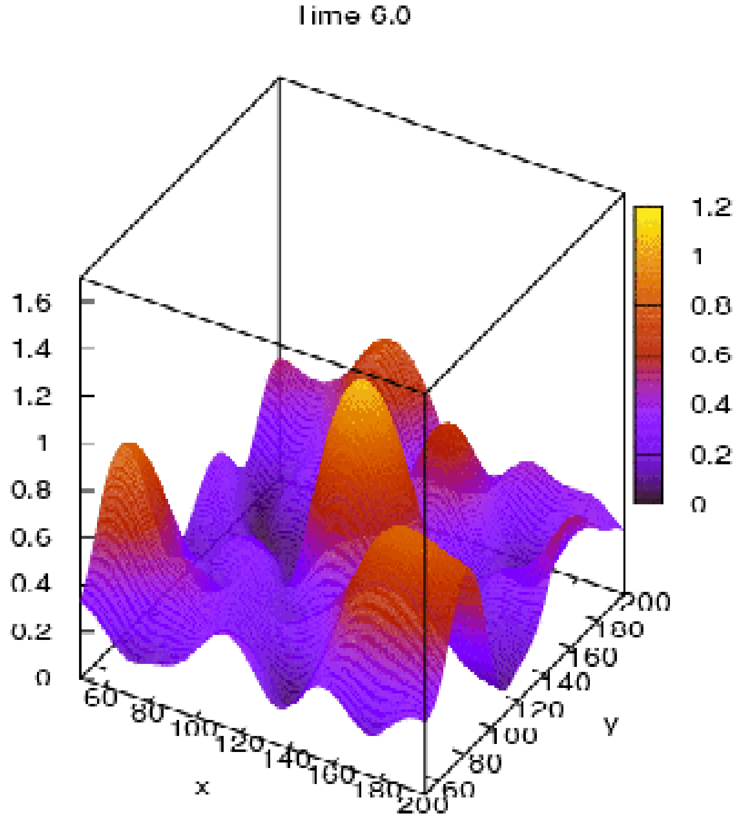

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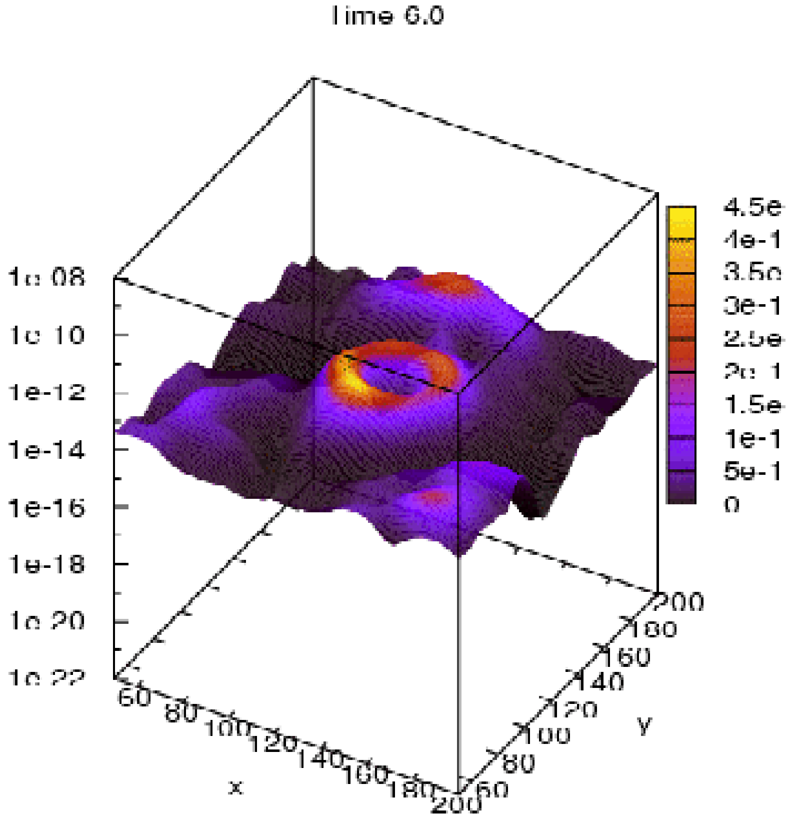

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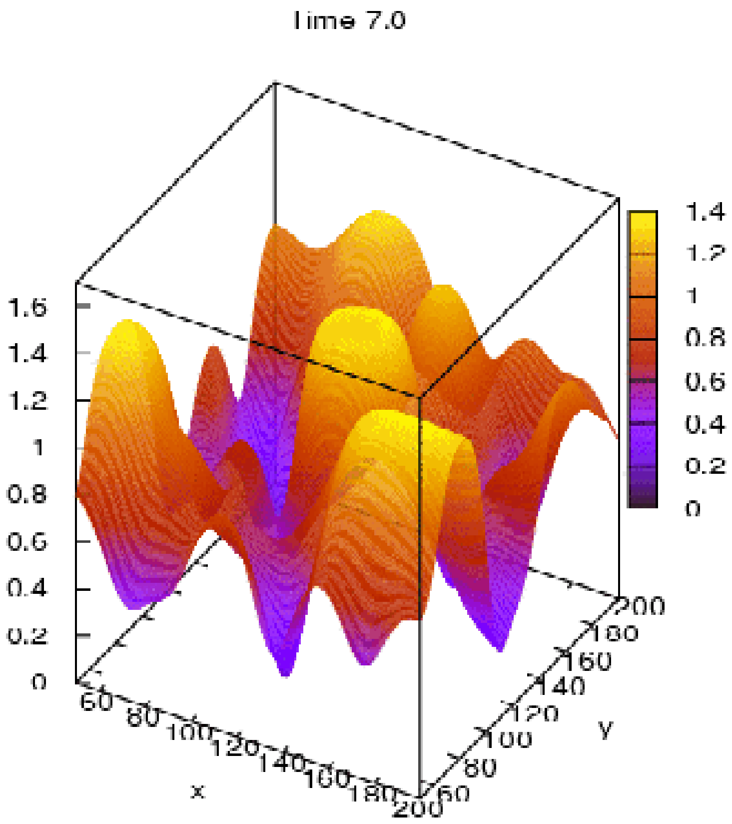

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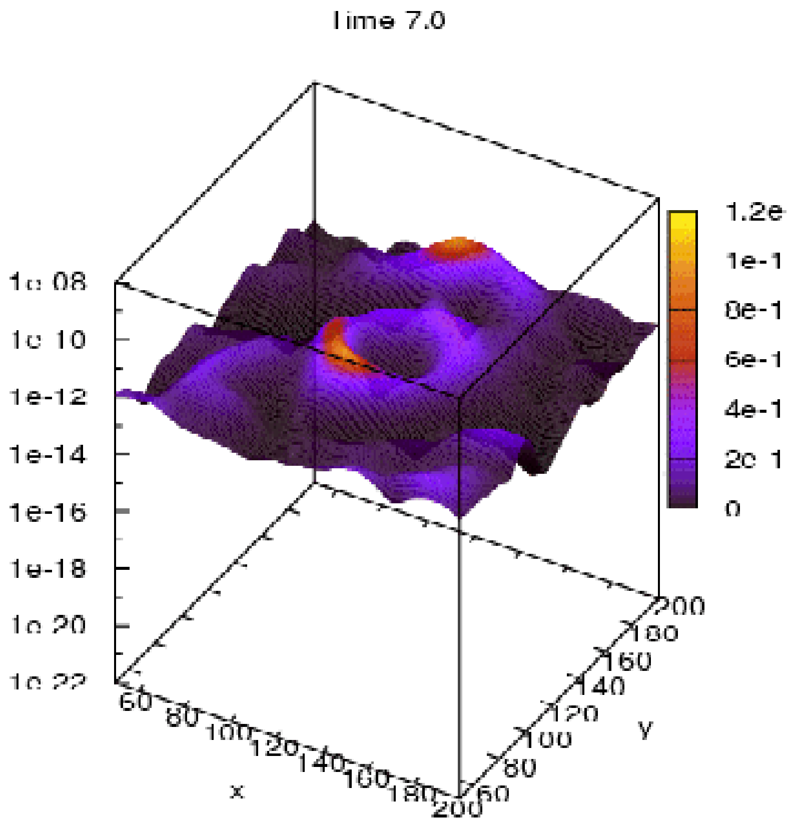

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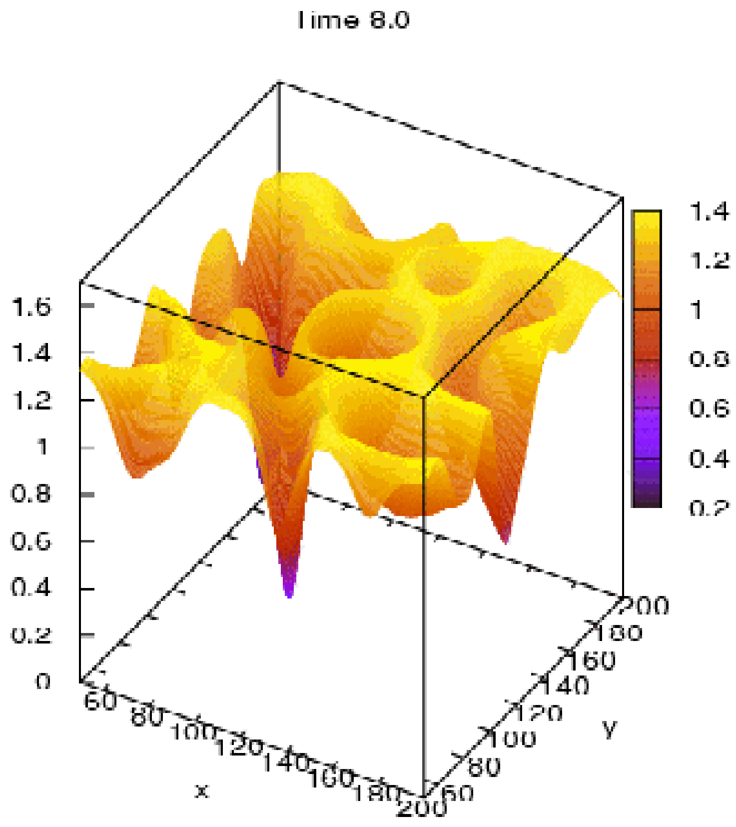

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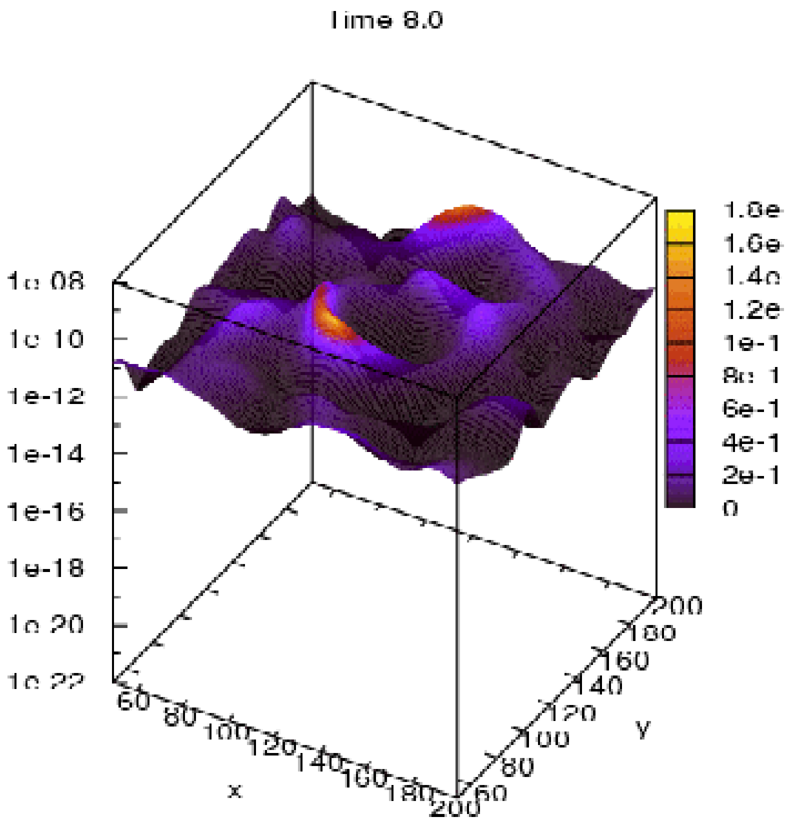

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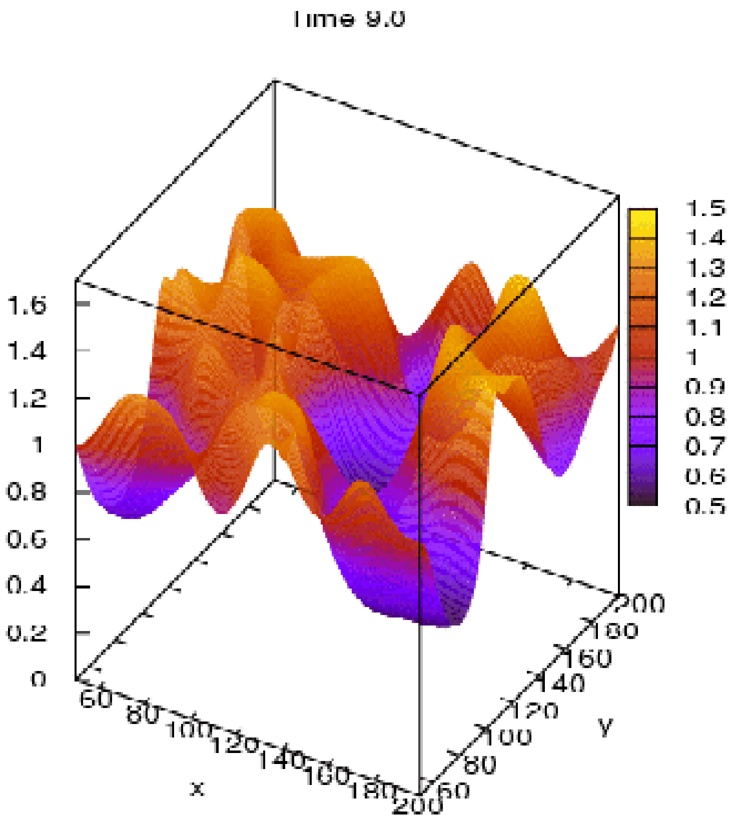

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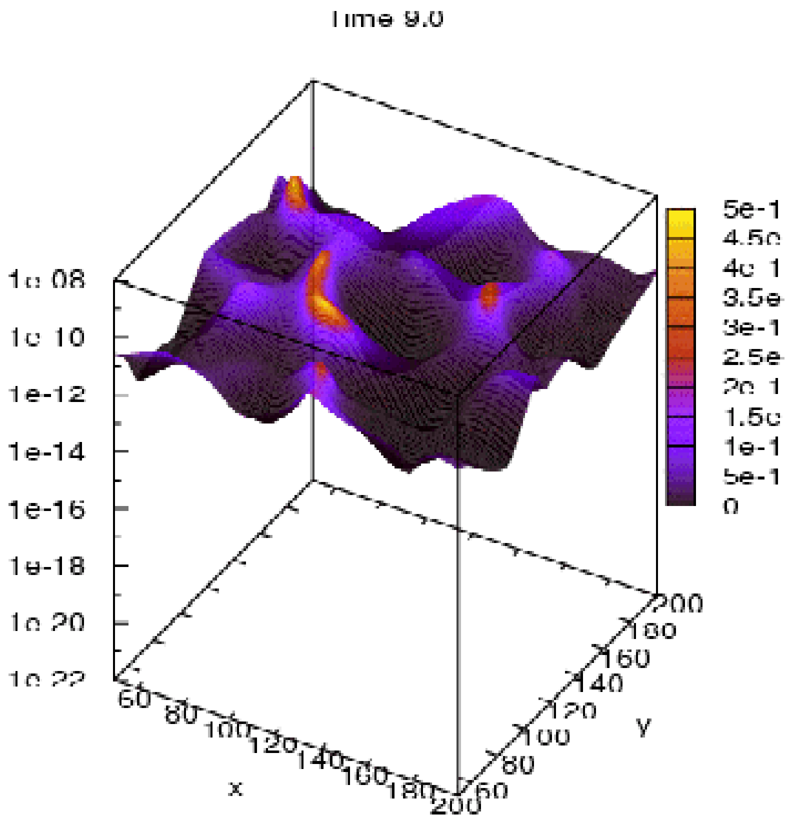

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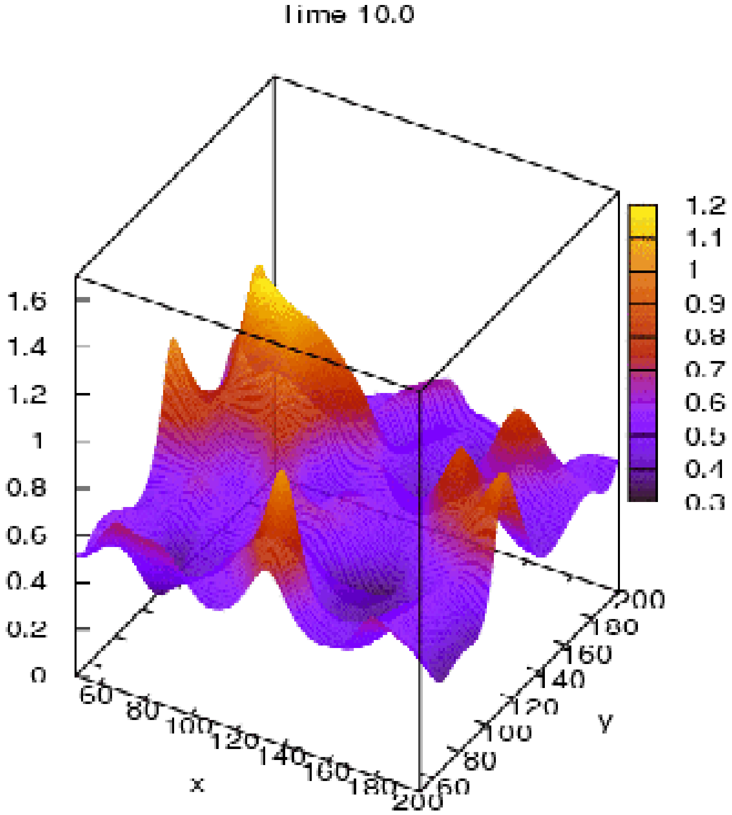

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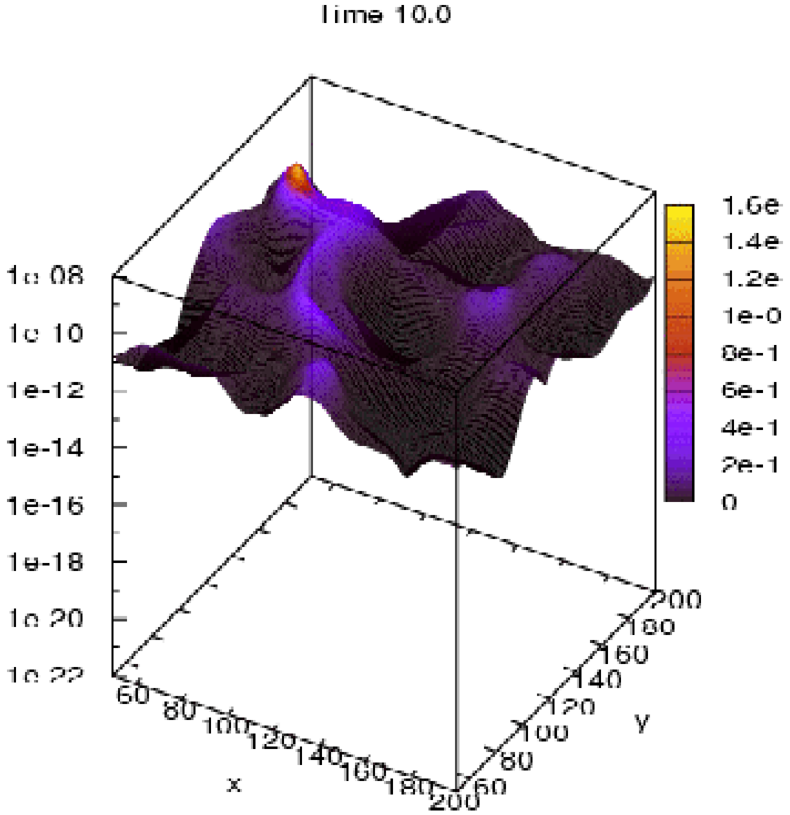

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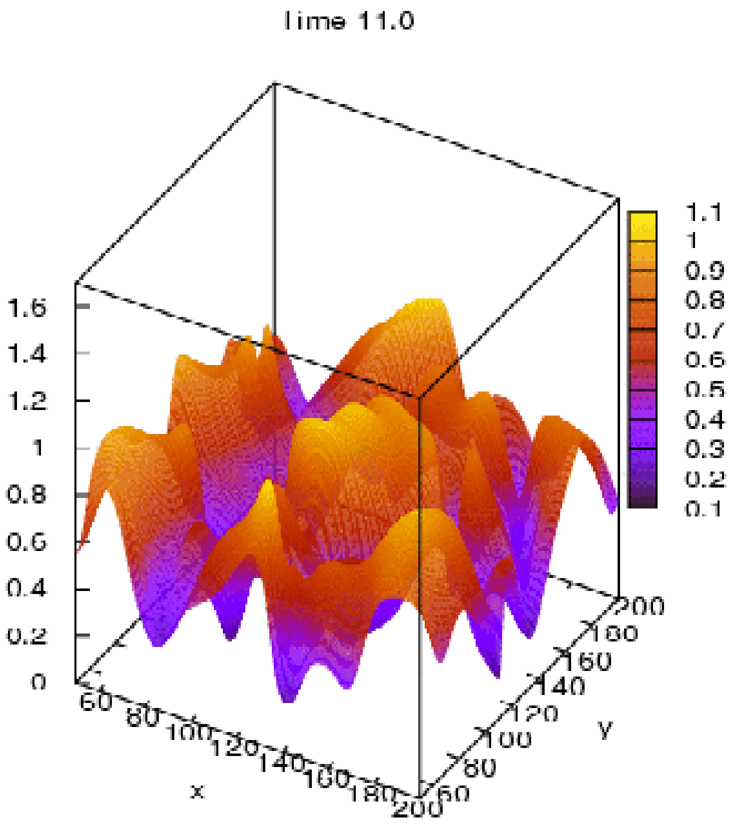

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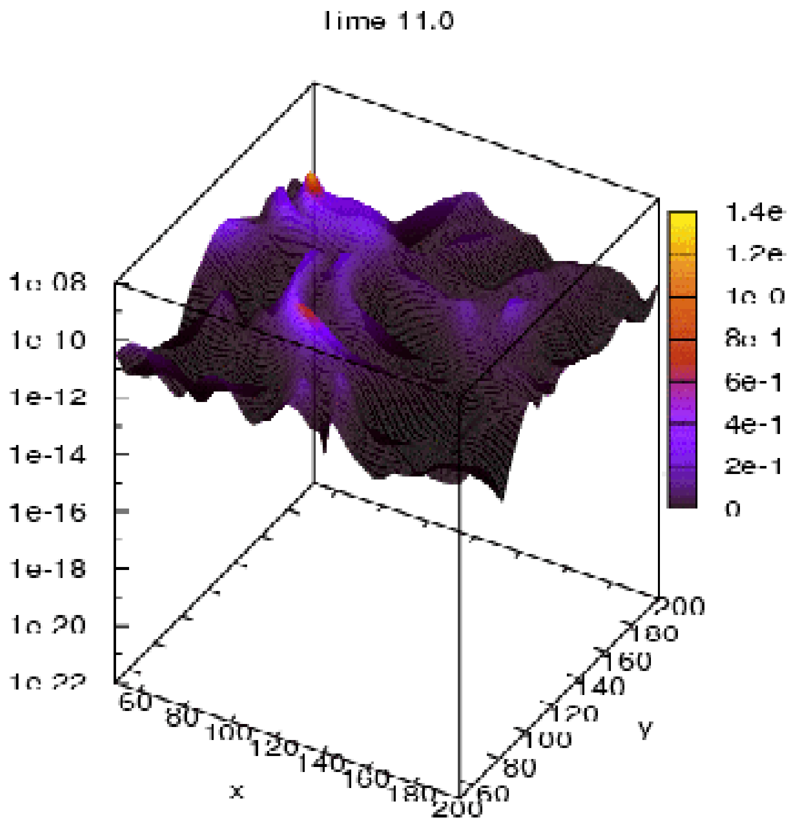

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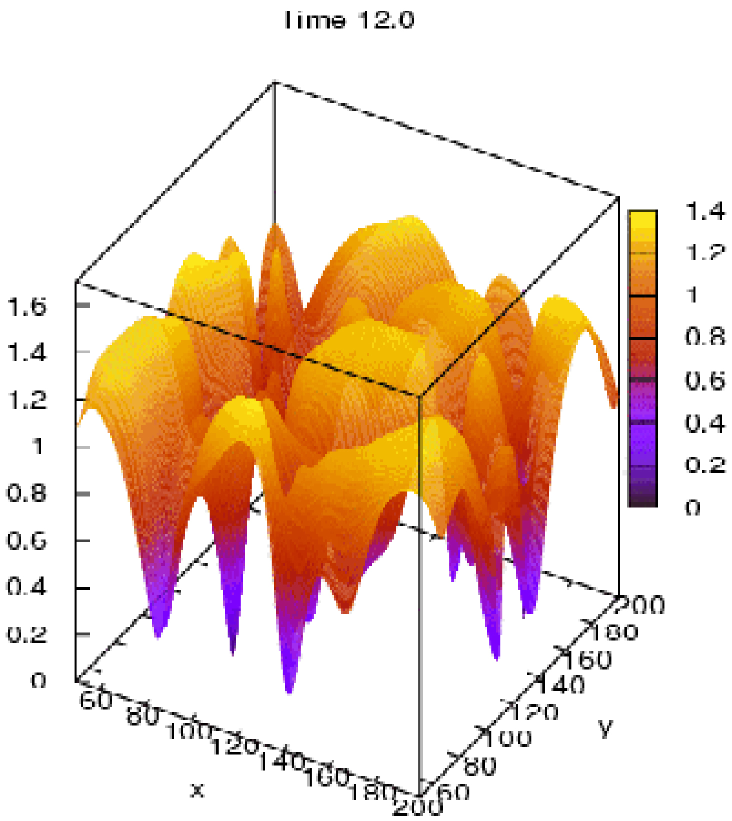

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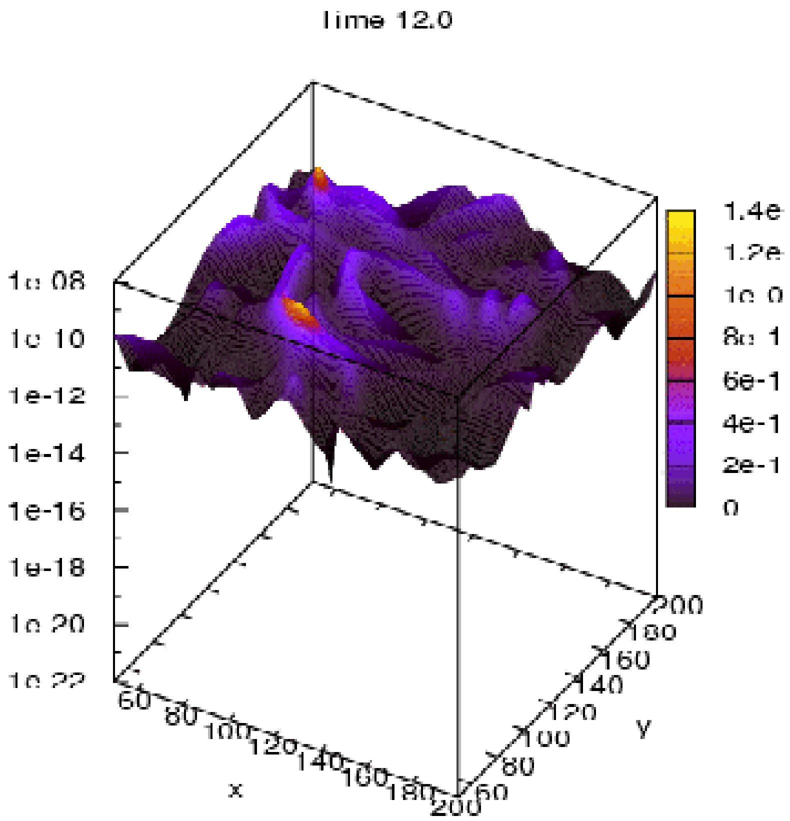

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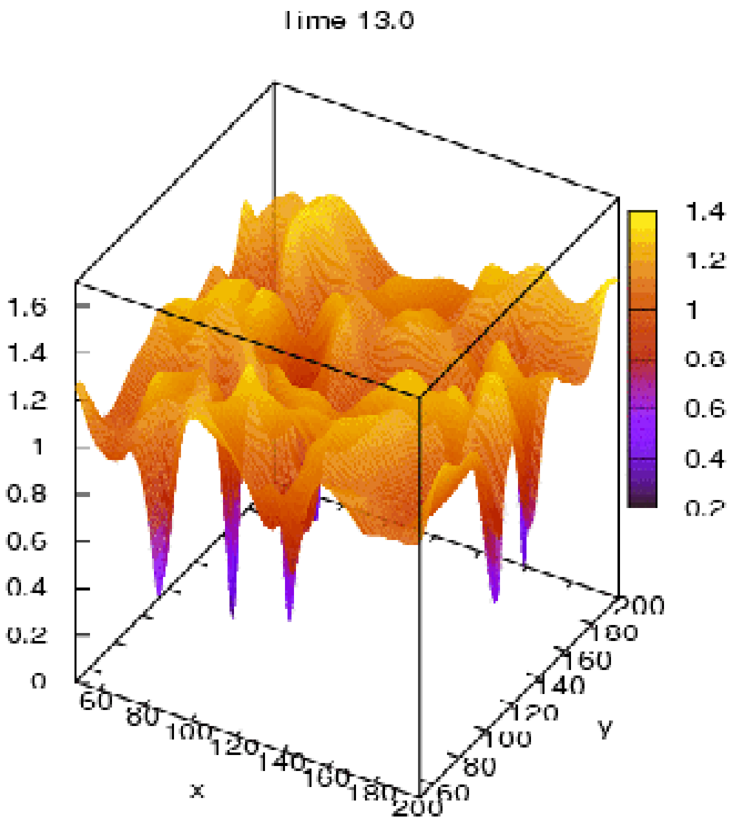

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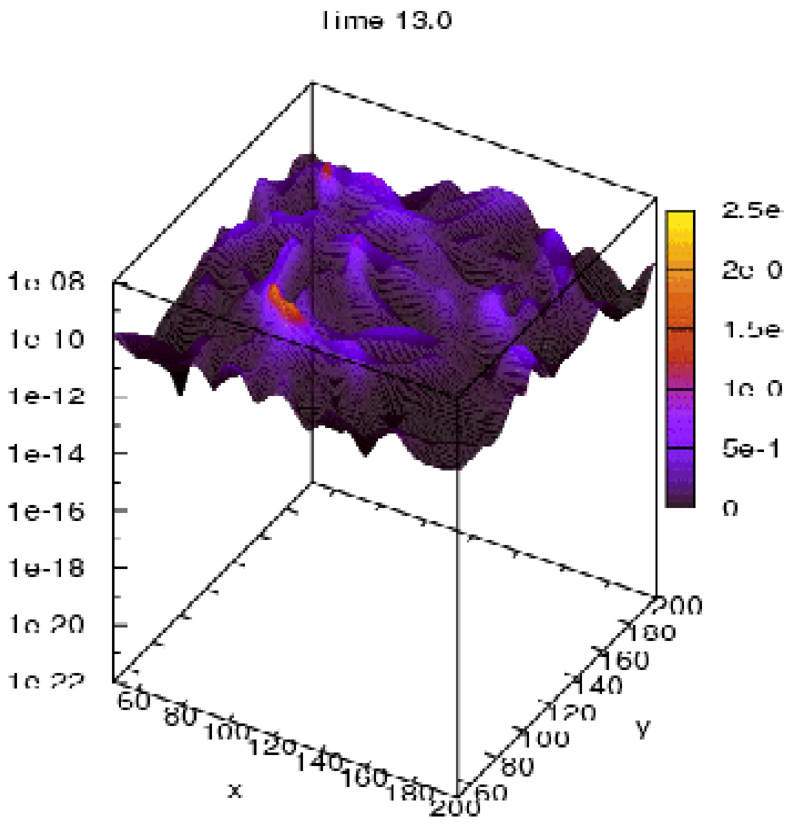

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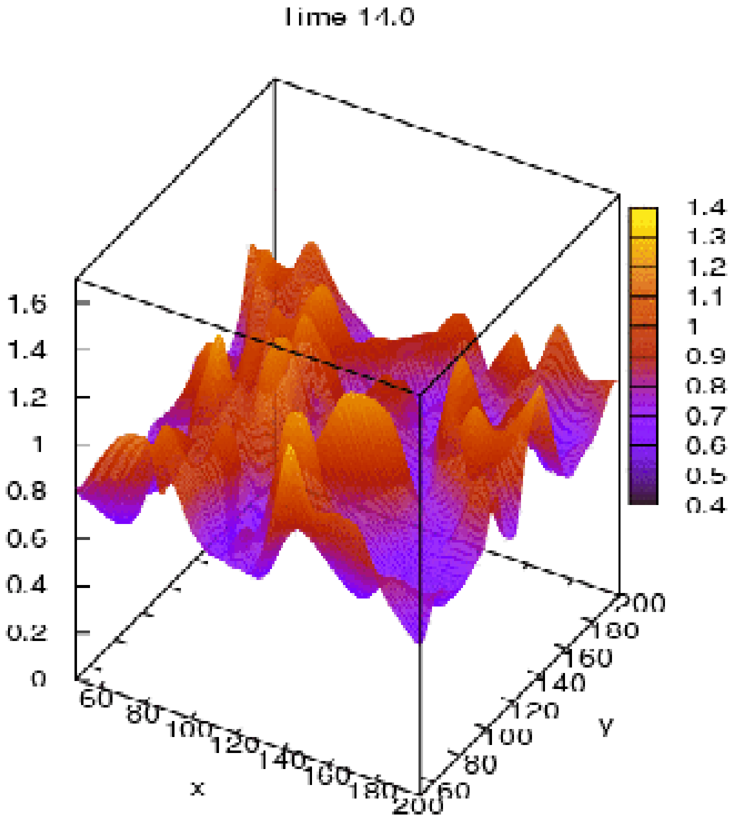

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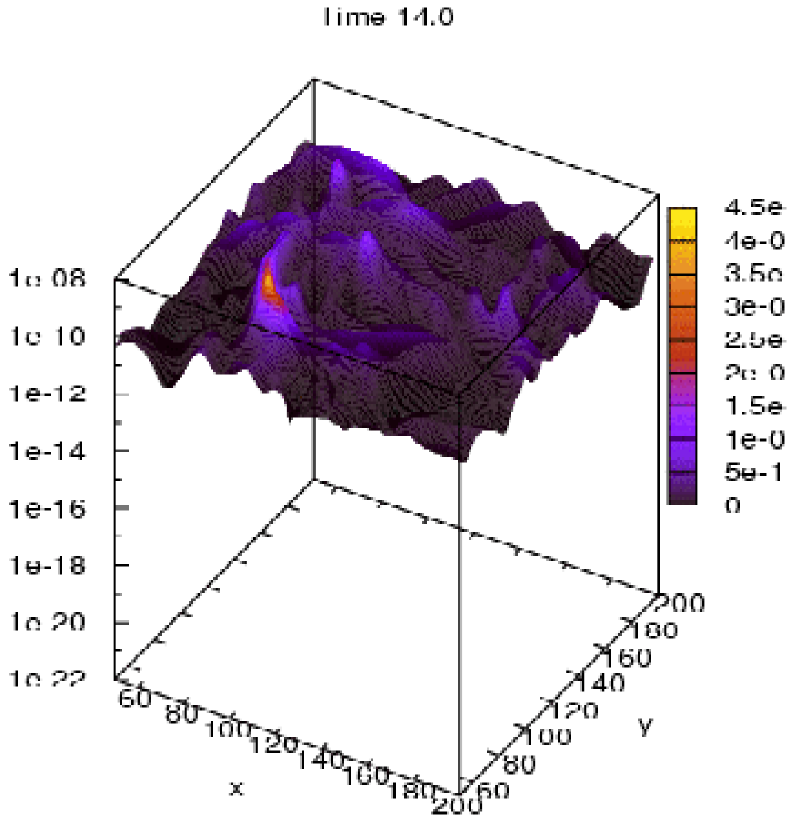

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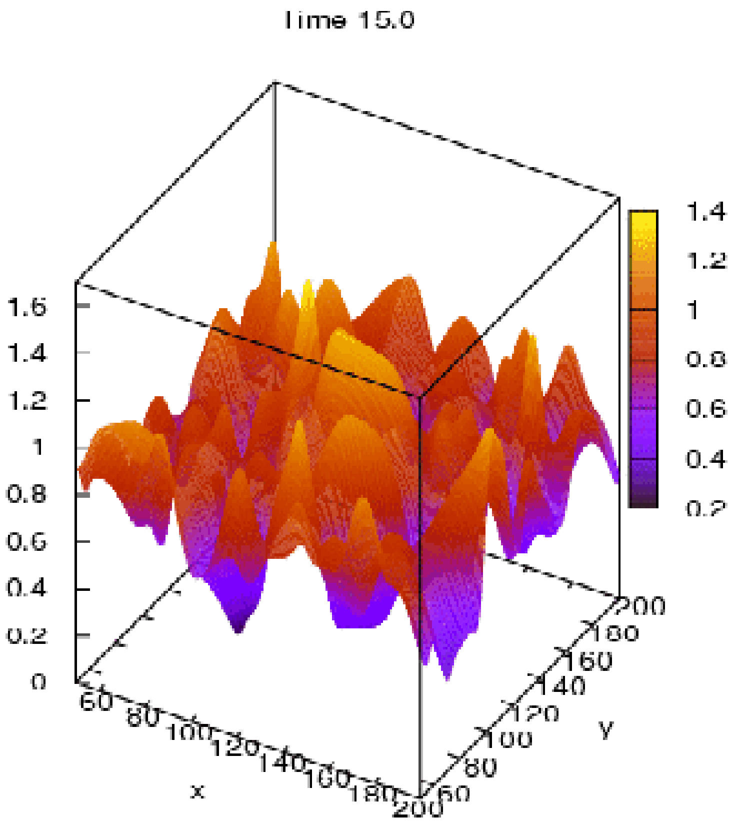

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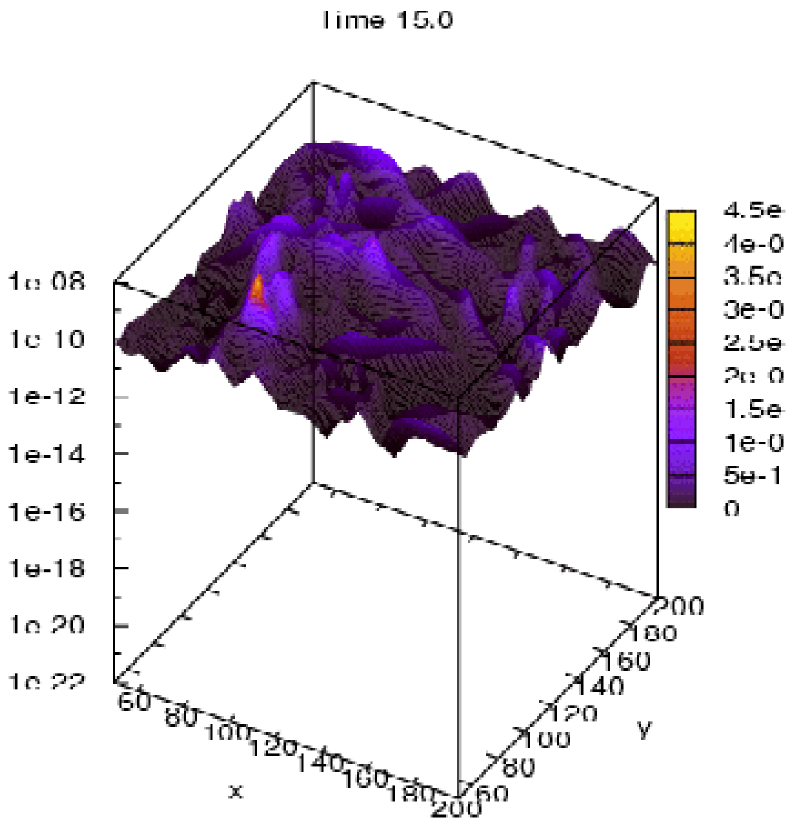

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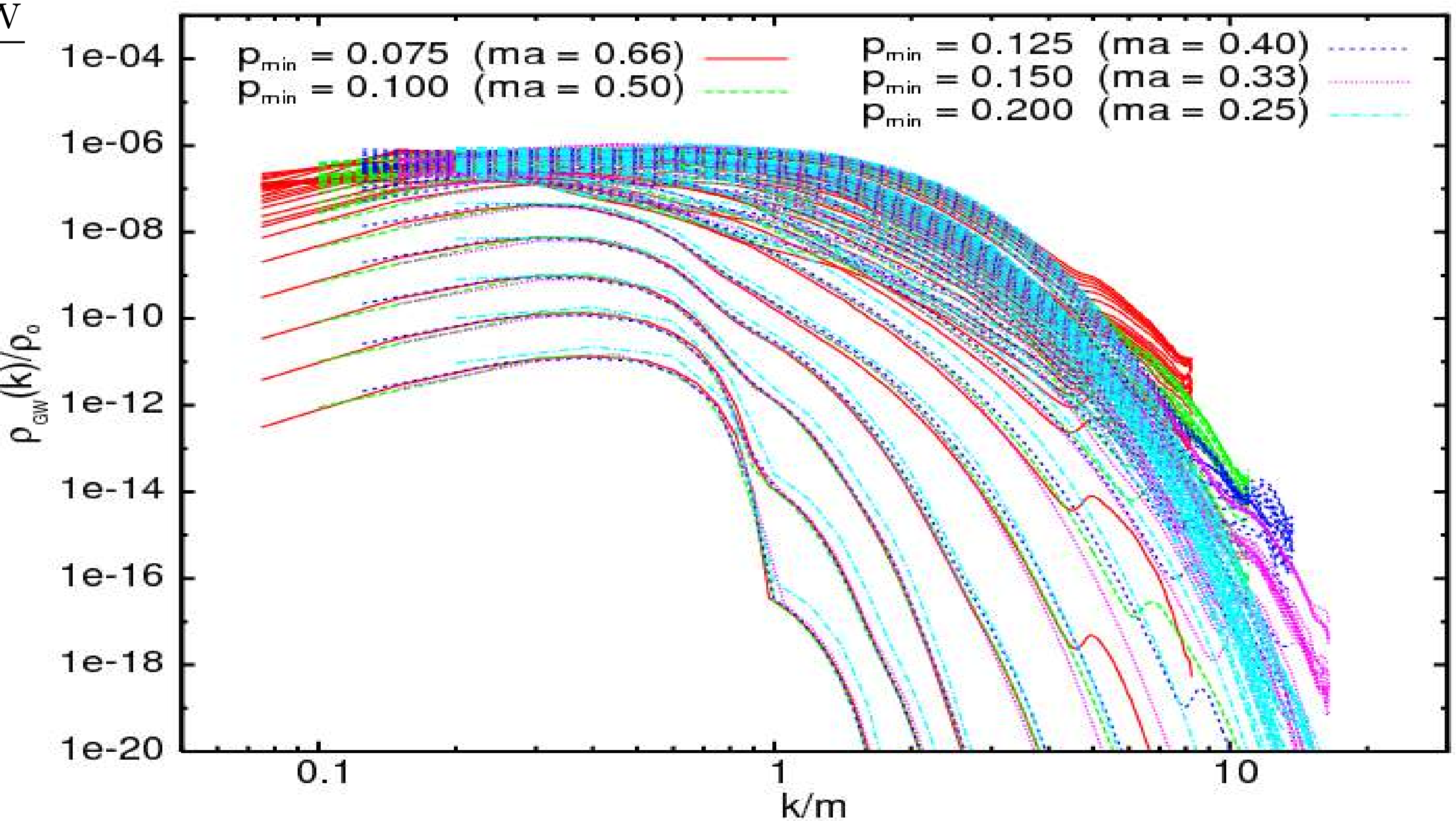
3 stages: **Exp. Instabilities** → **Non-linearities** → **Relaxation**

$$\frac{1}{\rho_{\rm tot}}\frac{d\log\rho_{\rm GW}}{d\log k}$$

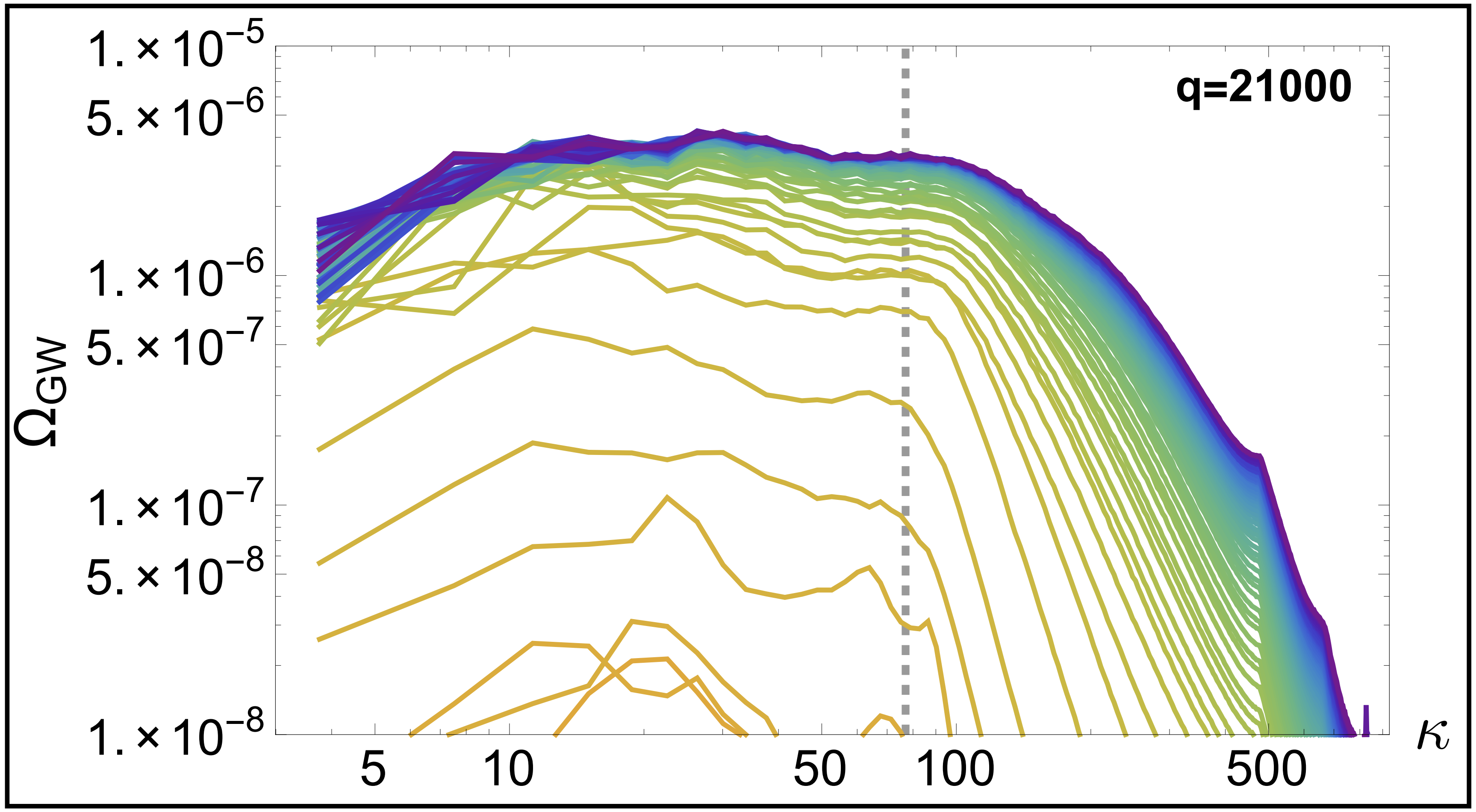


INFLATIONARY PREHEATING

Parameter Dependence (Peak amplitude)

INFLATIONARY PREHEATING

Parameter Dependence (Peak amplitude**)**



(DGF, Torrentí 2017)

INFLATIONARY PREHEATING

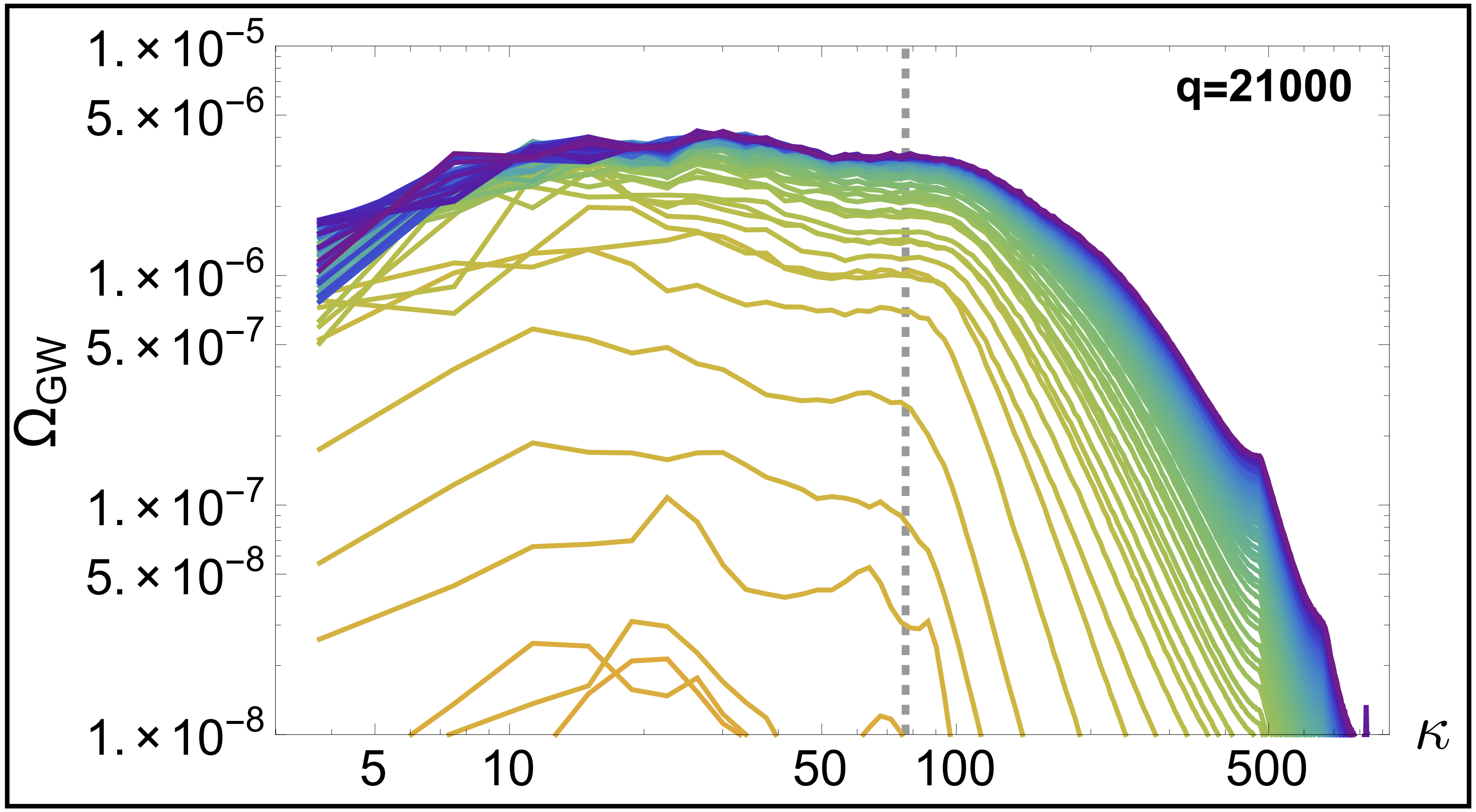
Parameter Dependence (Peak amplitude**)**

$$\boxed{\omega^2 \equiv V''(\Phi_I)}$$

Chaotic Models: $$\Omega^{(o)}_{\rm GW} \sim A^2 \frac{\omega^6}{\rho m_p^2}\, q^{-1/2}$$

$$\boxed{q \equiv \frac{g^2\Phi_{\rm i}^2}{\omega^2}}$$

Resonance Param.



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INFLATIONARY PREHEATING

Parameter Dependence (Peak amplitude**)**

Chaotic Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \quad f_o \sim 10^8 - 10^9 \ {\rm Hz}$

Large amplitude ! **… at high Frequency !**

INFLATIONARY PREHEATING

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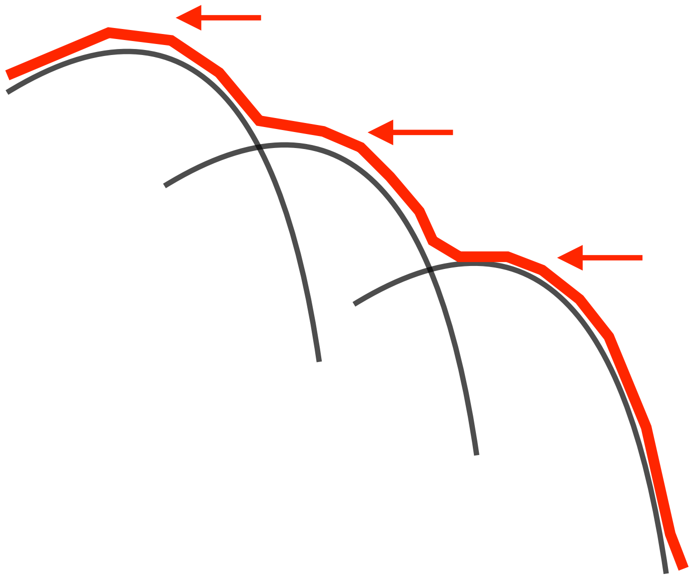
$\Omega_{\rm GW} \propto q^{-1/2} \longrightarrow$ **Spectroscopy of particle couplings ?**



different couplings
... different peaks ?

INFLATIONARY PREHEATING

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Large amplitude ! **... at high Frequency !**

Very unfortunate... no detectors there !

INFLATIONARY PREHEATING

Parameter Dependence (Peak amplitude**)**

Hybrid Models: $\Omega_{\rm GW}^{(o)} \propto \left(\frac{v}{m_p}\right)^2 \times f(\lambda, g^2)$, $f_o \sim \lambda^{1/4} \times 10^9$ Hz

$$\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \quad \begin{cases} f_o \sim 10^8 - 10^9 \text{ Hz} & \boxed{\lambda \sim 0.1} \\ f_o \sim 10^2 \text{ Hz} & \boxed{\lambda \sim 10^{-28}} \end{cases}$$

(natural)

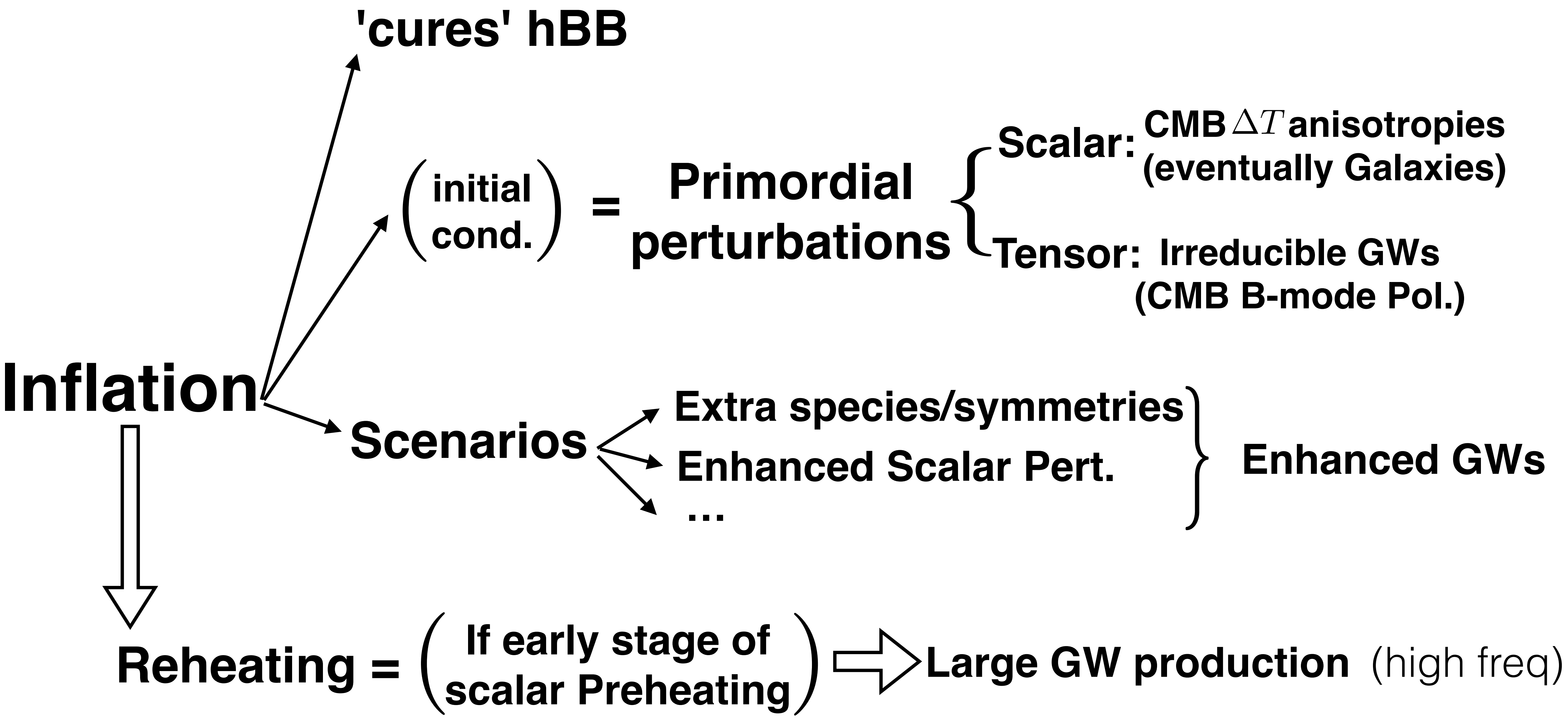
(fine-tuning)

Large amplitude !

(for $v \simeq 10^{16}$ GeV)

realistically speaking ...

EARLY UNIVERSE



'cures' hBB

(initial cond.) = Primordial perturbations

Scalar: CMB ΔT anisotropies (eventually Galaxies)

Tensor: Irreducible GWs (CMB B-mode Pol.)

Inflation

Scenarios

Extra species/symmetries

Enhanced Scalar Pert.

...

Enhanced GWs

Reheating = (If early stage of scalar Preheating) ⇨ Large GW production (high freq)

Gravitational Waves as a probe of the early Universe

OUTLINE

1) GWs definition ✓

Early Universe

2) GWs from Inflation

3) GWs from Preheating ✓

4) GWs from Phase Transitions

5) GWs from Cosmic Defects

BACK SLIDES

PROPAGATION OF TENSORS

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

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quantum fields

Polarizations: +, x

$$\boxed{t = \eta = \tau}$$

conformal time

Bad notation

Irreducible GW background from Inflation

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$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\begin{aligned} \rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \\ &\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \end{aligned}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\begin{aligned}
\rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_V \\
&\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \\
&= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\; \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k}',t) \\
&\qquad\qquad \times \frac{1}{V} \int_V d\mathbf{x}\; e^{-i\mathbf{x}(\mathbf{k}-\mathbf{k}')}\,,
\end{aligned}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\begin{aligned}
\rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \\
&\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \\
&= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \cancel{\frac{d\mathbf{k}'}{(2\pi)^3}}\; \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k}',t) \\
&\qquad \times \frac{1}{V} \boxed{(2\pi)^3 \delta^{(3)}(\mathbf{k}-\mathbf{k}')}
\end{aligned}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\begin{aligned}
\boxed{\rho_{\rm GW}(t)} &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_V \\
&\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \\
&= \boxed{\frac{1}{32\pi G a^2(t) V} \int \frac{d\mathbf{k}}{(2\pi)^3}\; \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k},t)}
\end{aligned}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r}^{+}\right)\; e_{ij}^{r}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle_V$$

→ Volume/Time Average

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\, e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\rm QM}$$

→ ensemble average

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\ e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\begin{aligned}
\rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\text{QM}} \longrightarrow \text{ensemble average} \\
&= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\ e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} \left\langle \dot{h}_{ij}\,(\mathbf{k},t)\,\dot{h}^*_{ij}\,(\mathbf{k}',t) \right\rangle
\end{aligned}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\rm QM} \longrightarrow \text{ensemble average}$$

$$= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\; e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} \left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}^*_{ij}\,(\mathbf{k}',t) \right\rangle$$

$$\boxed{\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}^*_{ij}\,(\mathbf{k}',t) \right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot{h}}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times}\int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\,\hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\,\hat{a}^+_{\mathbf{k}r}\right)\ e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{(4\pi)^3 G a^2(t)}\int \frac{dk}{k}\, k^3\, \mathcal{P}_{\dot h}(k,t) \ = \int \frac{d\rho_{\rm GW}}{d\log k}\, d\log k$$

$$\boxed{\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\,\dot{h}^*_{ij}\,(\mathbf{k}',t)\right\rangle \equiv (2\pi)^3\,\mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{(4\pi)^3 G a^2(t)} \int \frac{dk}{k}\, k^3\, \mathcal{P}_{\dot h}(k,t) \;=\; \int \frac{d\rho_{\rm GW}}{d\log k}\, d\log k$$

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

$$\boxed{\left\langle \dot h_{ij}(\mathbf{k},t)\, \dot h^*_{ij}(\mathbf{k}',t) \right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Irreducible GW background from Inflation

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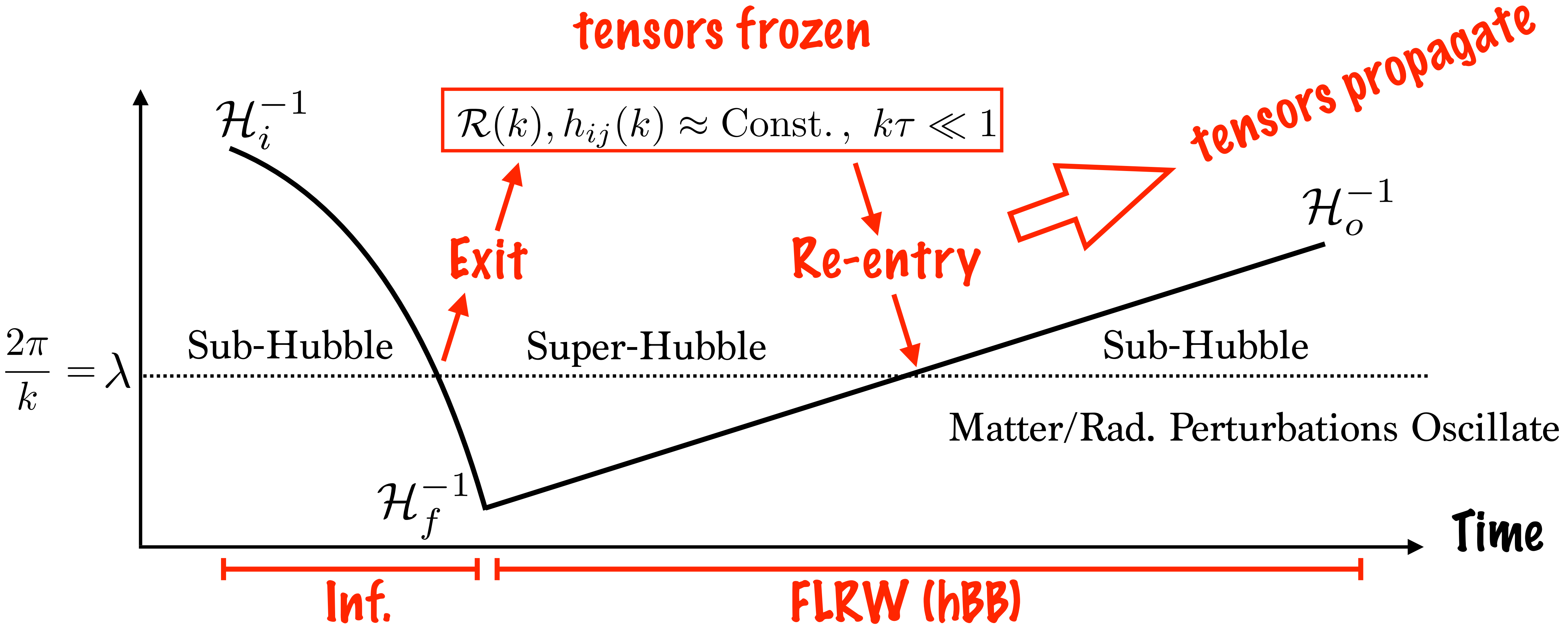

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot{h}}(k,t)}$$

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Horizon Re-entry ⇒ **tensors propagate**

Rad Dom: $h_r(\mathbf{k},\eta) = \frac{A_r(\mathbf{k})}{a(\eta)}\,e^{ik\eta} + \frac{B_r(\mathbf{k})}{a(\eta)}\,e^{-ik\eta}$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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Horizon Re-entry ⇒ **tensors propagate**

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$$\boxed{\begin{array}{c} \text{@ Horizon :} \begin{cases} h = h_* \\ \dot h_* = 0 \end{cases} \\[2ex] A = B = \frac{1}{2} a_* h_* \end{array}}$$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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$$\boxed{\begin{array}{c} \text{@ Horizon :} \left\{ \begin{array}{l} h = h_* \\ \dot h_* = 0 \end{array} \right. \\ A = B = \frac{1}{2} a_* h_* \end{array}}$$

$$\left\langle \dot h \dot h \right\rangle = k^2 \langle h h \rangle = \left(\frac{a_*}{a}\right)^2 \frac{k^2}{2} \langle |h_*|^2 \rangle = \left(\frac{a_o}{a}\right)^2 \frac{k^2}{2(1+z_*)^2} \frac{2\pi^2}{k^3} \Delta^2_{h_*}$$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

$$\boxed{\left\langle \dot h_{ij}\,(\mathbf{k},t)\, \dot h^*_{ij}\,(\mathbf{k}',t) \right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Horizon Re-entry ⇒ tensors propagate

Rad Dom: $h_r(\mathbf{k},\eta) = \dfrac{A_r(\mathbf{k})}{a(\eta)}\, e^{ik\eta} + \dfrac{B_r(\mathbf{k})}{a(\eta)}\, e^{-ik\eta}$

$$\boxed{\begin{array}{c} @\ \text{Horizon} : \begin{cases} h = h_* \\ \dot h_* = 0 \end{cases} \\ A = B = \dfrac{1}{2} a_* h_* \end{array}}$$

$$\left\langle \dot h \dot h \right\rangle = k^2 \langle hh \rangle = \left(\frac{a_*}{a}\right)^2 \frac{k^2}{2} \langle |h_*|^2 \rangle = \left(\frac{a_o}{a}\right)^2 \frac{k^2}{2(1+z_*)^2}\, \frac{2\pi^2}{k^3} \Delta^2_{h_*}$$

Redshift

Inflationary Tensor Spectrum !

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

$$\boxed{\left\langle \dot h_{ij}\,(\mathbf{k},t)\,\dot h^*_{ij}\,(\mathbf{k}',t)\right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

$$\boxed{\mathcal{P}_{\dot h} = \left(\frac{a_o}{a}\right)^2 \frac{k^2}{2(1+z_*)^2}\,\frac{2\pi^2}{k^3}\Delta^2_{h_*}} \Rightarrow \boxed{\frac{d\log\rho_{\rm GW}}{d\log k} = \frac{1}{8}\,\frac{a_o^2}{a^4}\,\frac{m_p^2 k^2}{(1+z_*)^2}\Delta^2_{h_*}}$$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

$$\boxed{\left\langle \dot h_{ij}\,(\mathbf{k},t)\, \dot h^*_{ij}\,(\mathbf{k}',t)\right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

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$$(1+z_*)^{-2}_{\rm RD} = \Omega^{(o)}_{\rm Rad}\frac{a_o^2H_o^2}{k^2} \Rightarrow \frac{d\log\rho_{\rm GW}}{d\log k} = \frac{\Omega^{(o)}_{\rm Rad}}{24}\left(\frac{a_o}{a}\right)^4 3m_p^2H_o^2\Delta^2_{h_*}$$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

$$\boxed{\left\langle \dot h_{ij}\left(\mathbf{k},t\right) \dot h^*_{ij}\left(\mathbf{k}',t\right)\right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

$$\boxed{\mathcal{P}_{\dot h} = \left(\frac{a_o}{a}\right)^2 \frac{k^2}{2(1+z_*)^2}\frac{2\pi^2}{k^3}\Delta^2_{h_*}} \Rightarrow \boxed{\frac{d\log\rho_{\rm GW}}{d\log k} = \frac{1}{8}\frac{a_o^2}{a^4}\frac{m_p^2 k^2}{(1+z_*)^2}\Delta^2_{h_*}}$$

$$(1+z_*)^{-2}_{\rm RD} = \Omega^{(o)}_{\rm Rad}\frac{a_o^2 H_o^2}{k^2} \quad\Rightarrow\quad \frac{d\log\rho_{\rm GW}}{d\log k} = \frac{\Omega^{(o)}_{\rm Rad}}{24}\left(\frac{a_o}{a}\right)^4 3m_p^2 H_o^2 \Delta^2_{h_*}$$

$$\boxed{\Omega^{(o)}_{\rm GW} \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega^{(o)}_{\rm Rad}}{24}\Delta^2_{h_*}}$$

Irreducible GW background from Inflation

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

$$\boxed{\left\langle \dot h_{ij}\left(\mathbf{k},t\right) \dot h^*_{ij}\left(\mathbf{k}',t\right)\right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

$$\boxed{\mathcal{P}_{\dot h} = \left(\frac{a_o}{a}\right)^2 \frac{k^2}{2(1+z_*)^2}\frac{2\pi^2}{k^3}\Delta^2_{h_*}} \Longrightarrow \boxed{\frac{d\log\rho_{\rm GW}}{d\log k} = \frac{1}{8}\frac{a_o^2}{a^4}\frac{m_p^2 k^2}{(1+z_*)^2}\Delta^2_{h_*}}$$

$$\boxed{\Omega^{(o)}_{\rm GW}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega^{(o)}_{\rm Rad}}{24}\Delta^2_{h_*}(k)} \qquad (k = 2\pi f)$$

GW normalized energy density spectrum (today)

Inflationary tensor spectrum

Irreducible GW background from Inflation

$$\boxed{\Omega_{\rm GW}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega_{\rm Rad}^{(o)}}{24}\Delta^2_{h_*}(k)} \qquad (k = 2\pi f)$$

GW normalized energy density spectrum (today)

Transfer Funct

$$T(k)\begin{cases}\propto k^0(\mathrm{RD})\\ \propto k^{-2}(\mathrm{MD})\end{cases}$$

Inflationary tensor spectrum

Irreducible GW background from Inflation

$$\boxed{\Omega_{\rm GW}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega_{\rm Rad}^{(o)}}{24}\Delta_{h*}^2(k)} \qquad (k = 2\pi f)$$

GW normalized energy density spectrum (today)

Transfer Funct

$$T(k)\begin{cases}\propto k^0(\text{RD})\\ \propto k^{-2}(\text{MD})\end{cases}$$

Inflationary tensor spectrum

Inflationary Hubble Rate

$$\boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

Small red-tilt, i.e. (almost-) scale-invariant

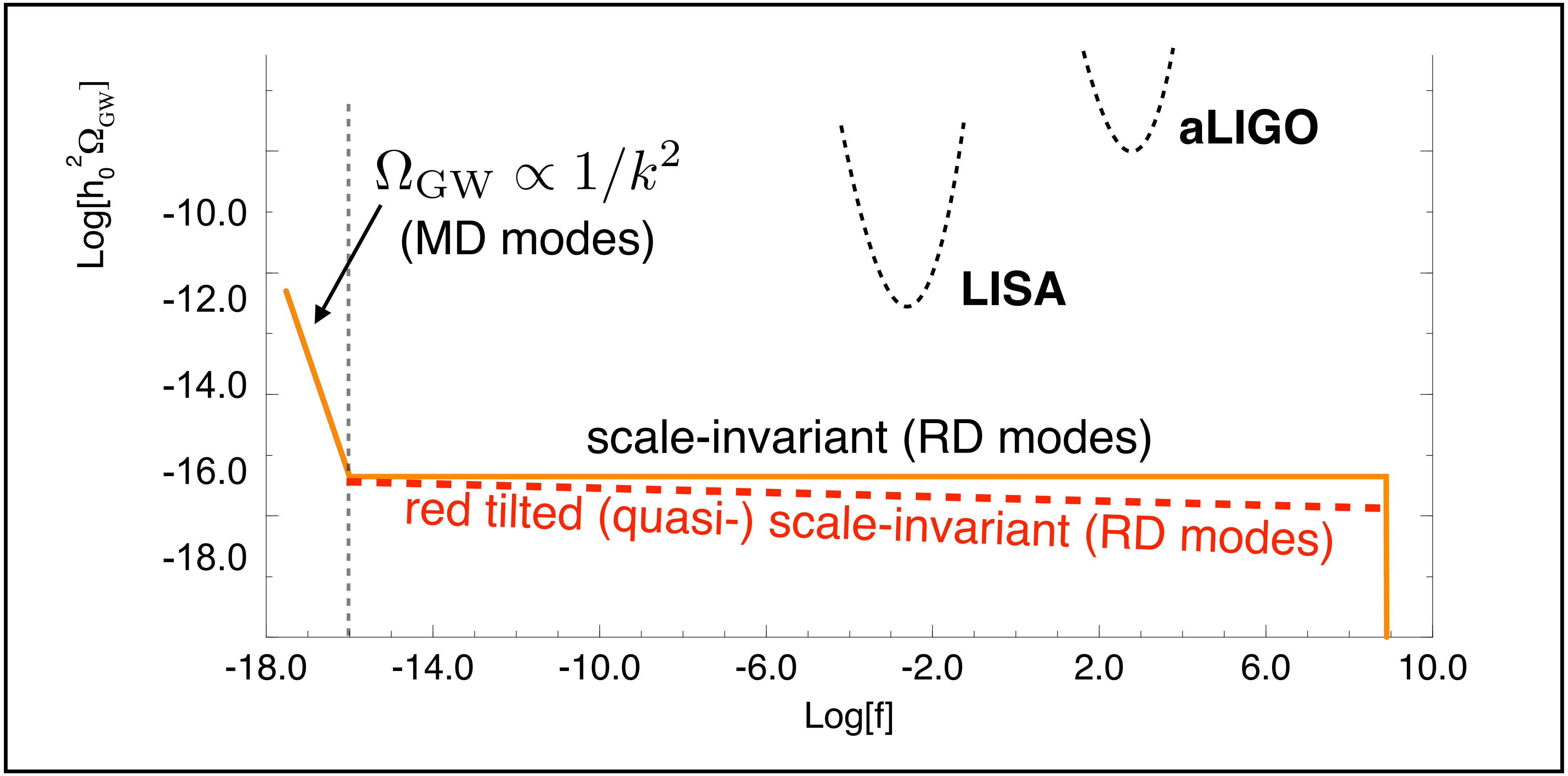
Irreducible GW background from Inflation

$$\boxed{\Omega^{(o)}_{\rm GW}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \underbrace{\frac{\Omega^{(o)}_{\rm Rad}}{24}}\Delta^2_{h_*}(k)}$$

$$\boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$$\boxed{n_t \equiv -2\epsilon}$$

Transfer Funct.: $T(k) \propto k^0 (\rm RD)$



STIFF EQUATION of STATE
& INFLATIONARY GW HF-TAIL

Irreducible GW background from Inflation

$$\boxed{\Omega^{(o)}_{\rm GW}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \underbrace{\frac{\Omega^{(o)}_{\rm Rad}}{24}}\Delta^2_{h_*}(k)}$$

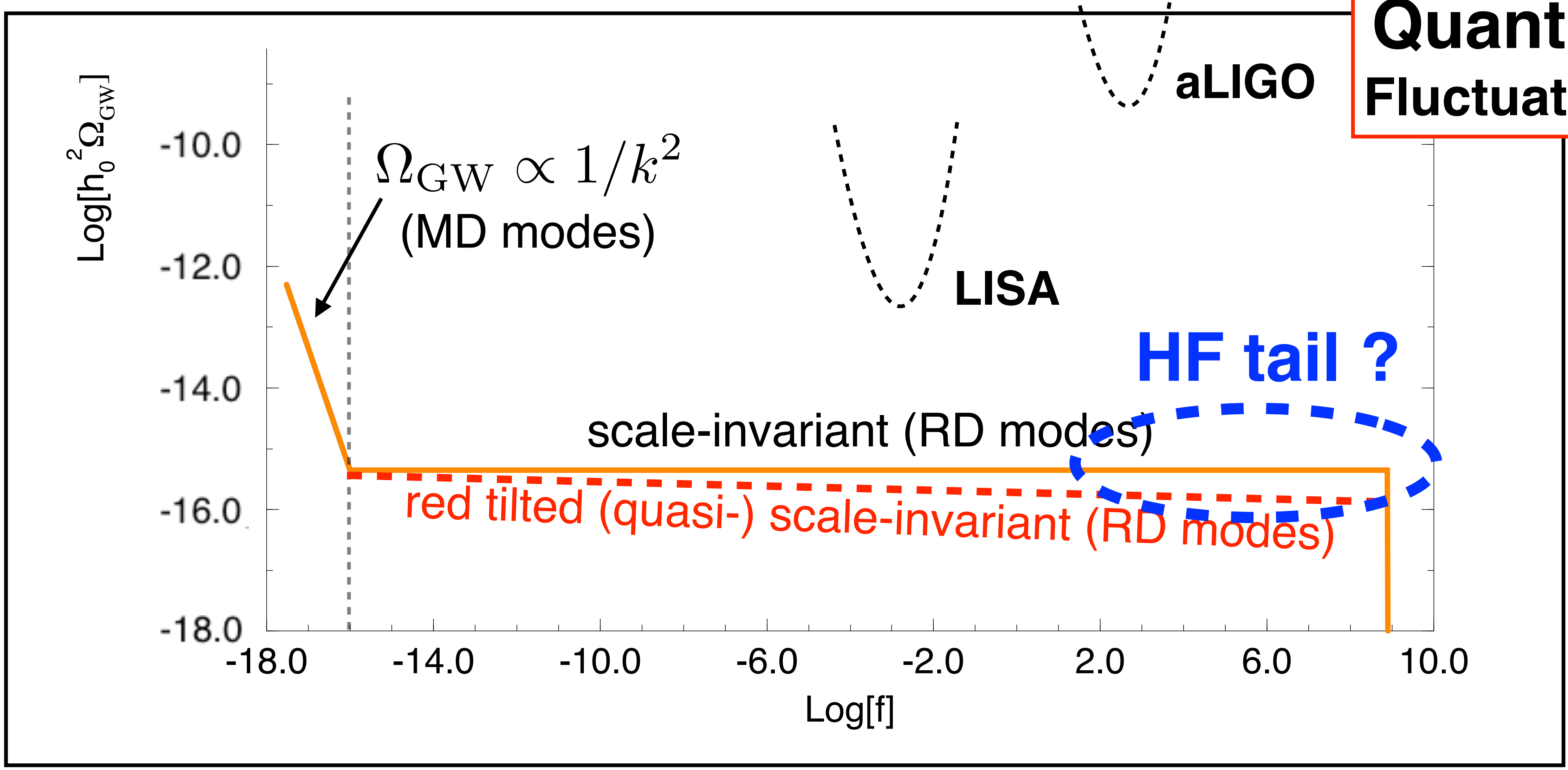
Transfer Funct.: $T(k) \propto k^0 (\mathrm{RD})$

$$\boxed{\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$\boxed{n_t \equiv -2\epsilon}$

energy scale

Quantum Fluctuations



STIFF EQ of STATE

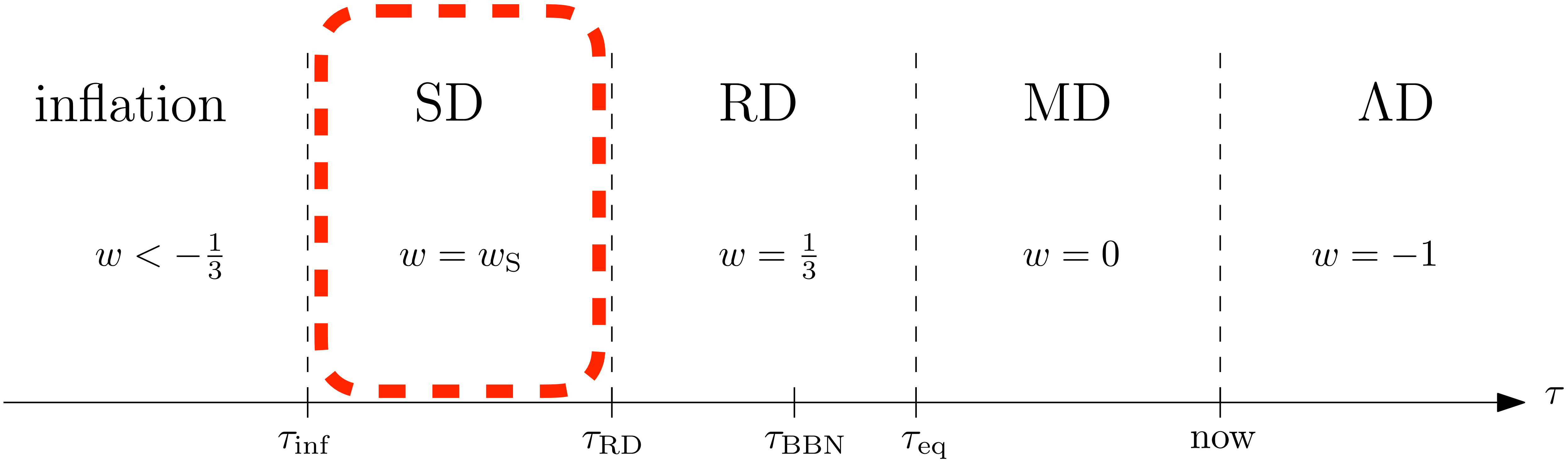
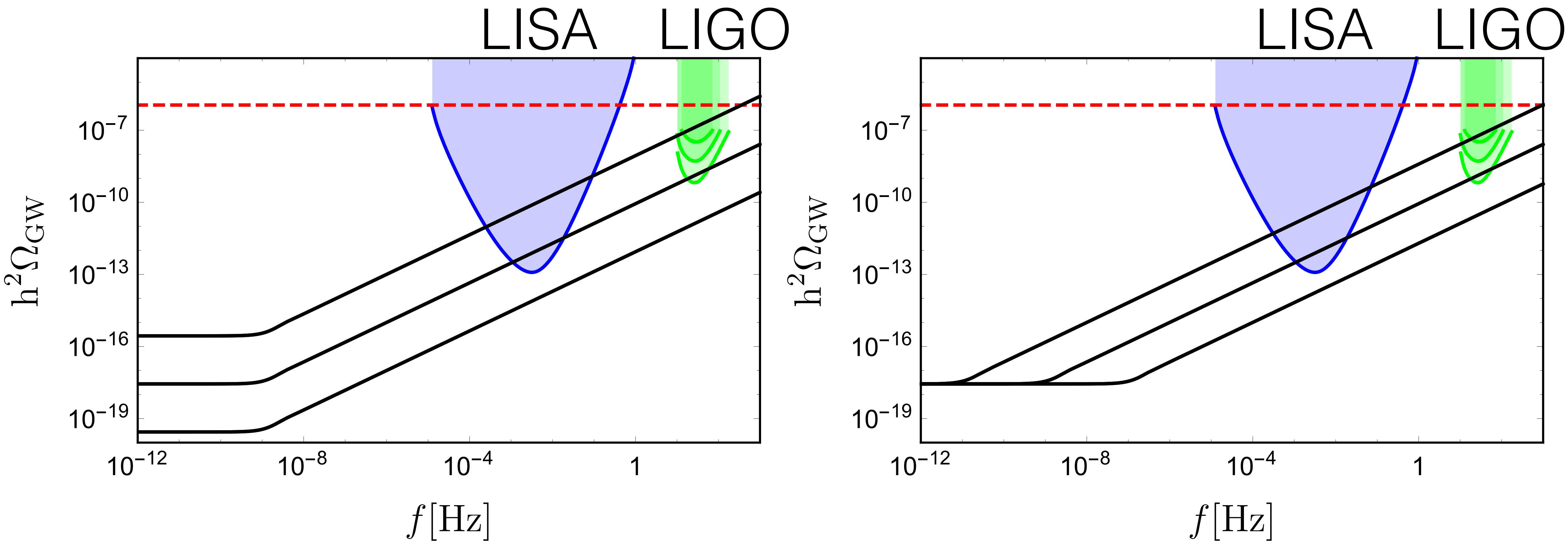


Figure 1. ΛCDM+inflation expansion history with a stiff epoch.

$$SD: \; 1/3 < w_S \lesssim 1$$

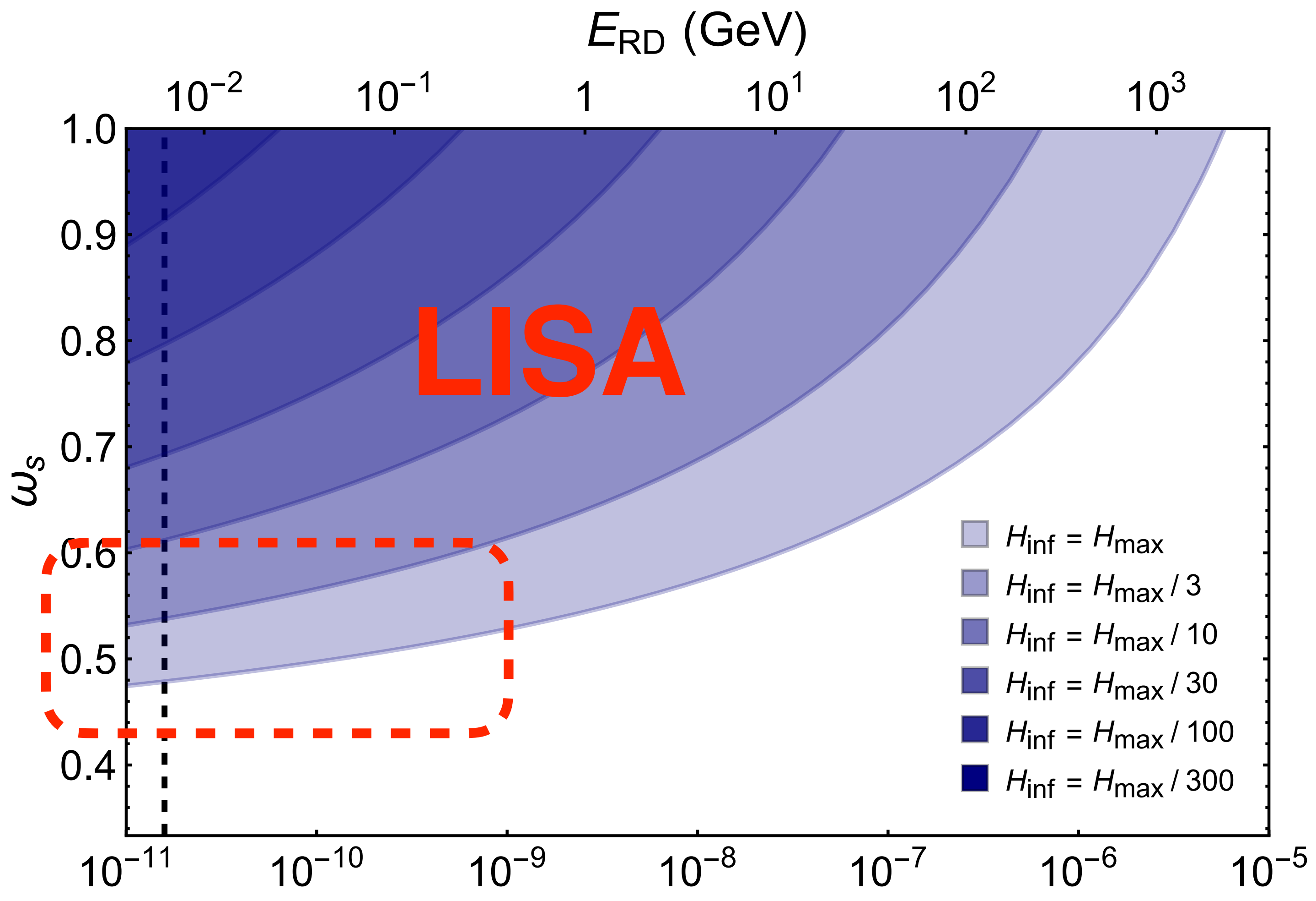
STIFF EQ of STATE

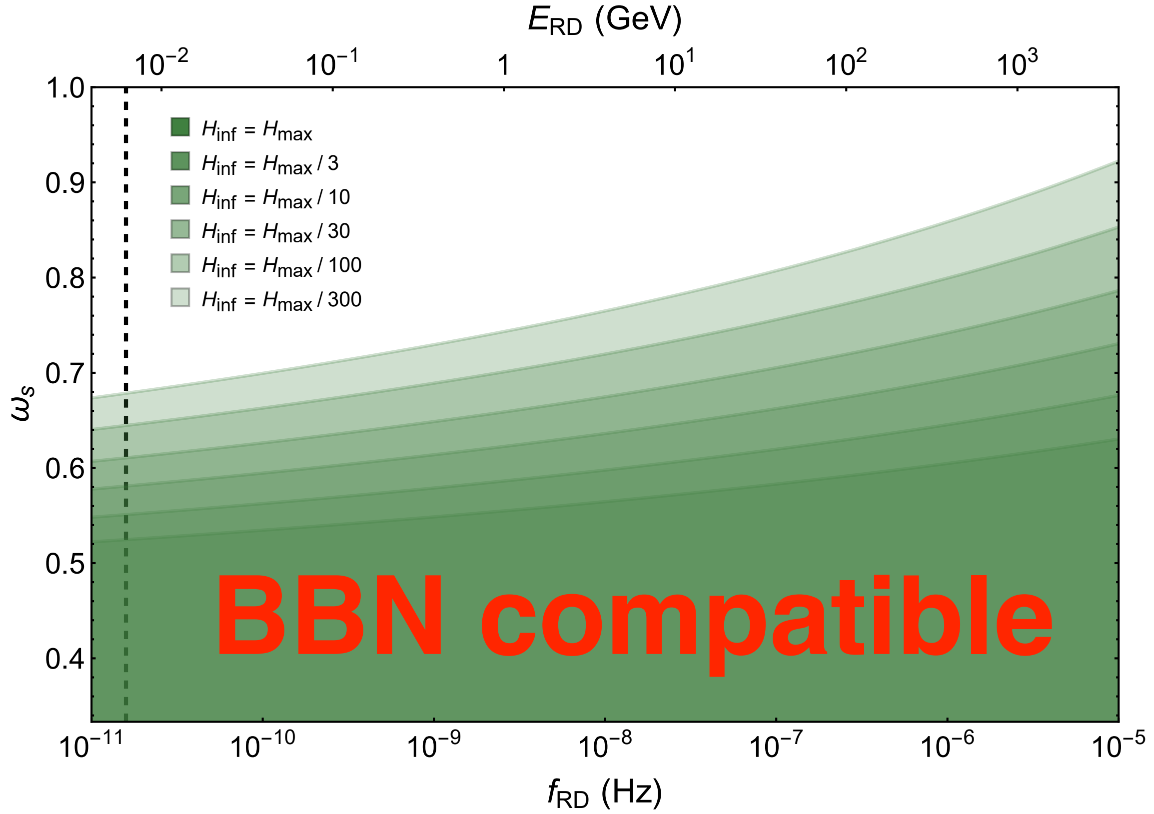


$$\Omega_{\rm GW}(f) \propto H_{\rm inf}^2 \left(\frac{f}{f_{\rm RD}}\right)^{\frac{2(w-1/3)}{(w+1/3)}}$$

Not Scale Invariant !

STIFF EQ of STATE

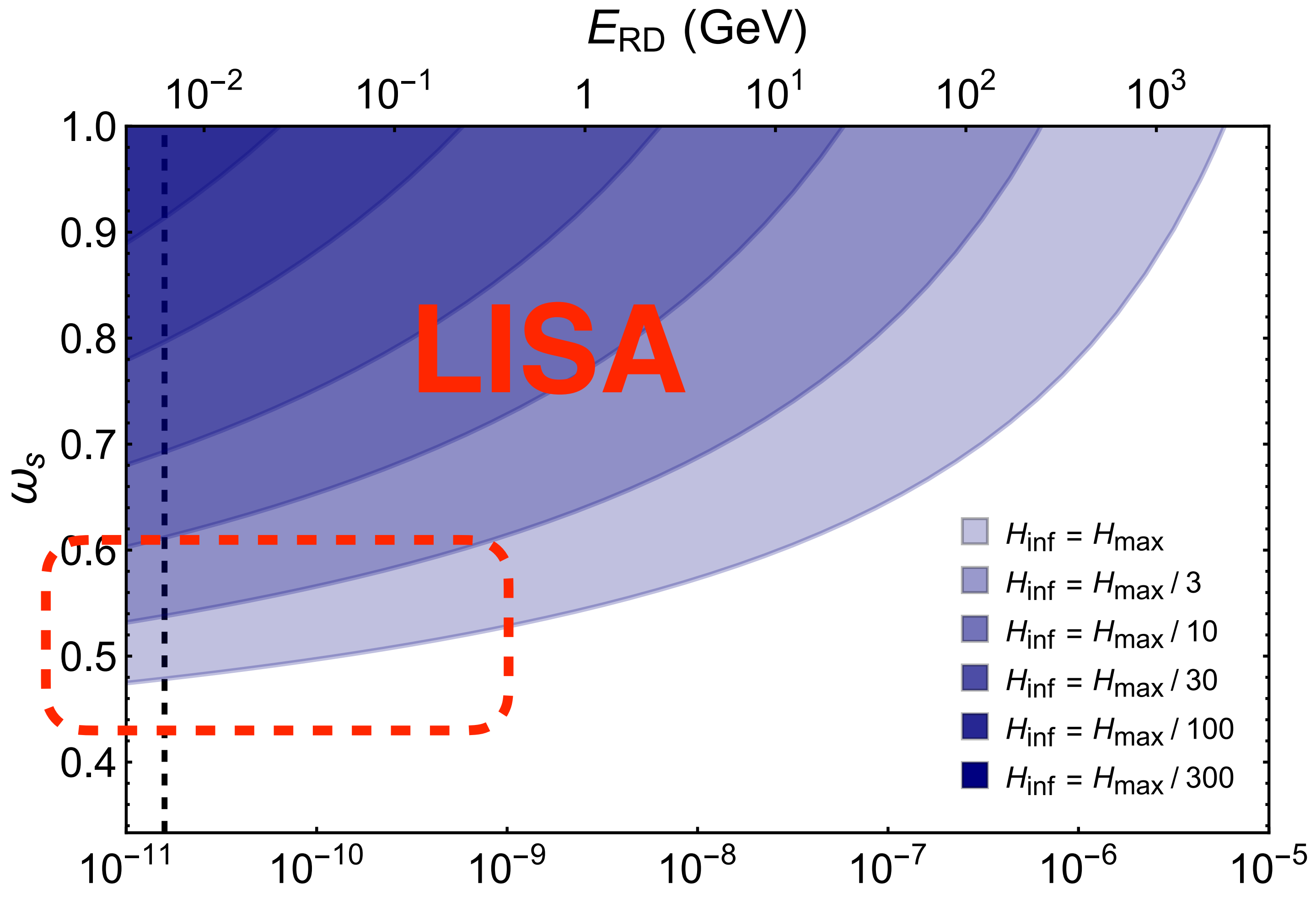




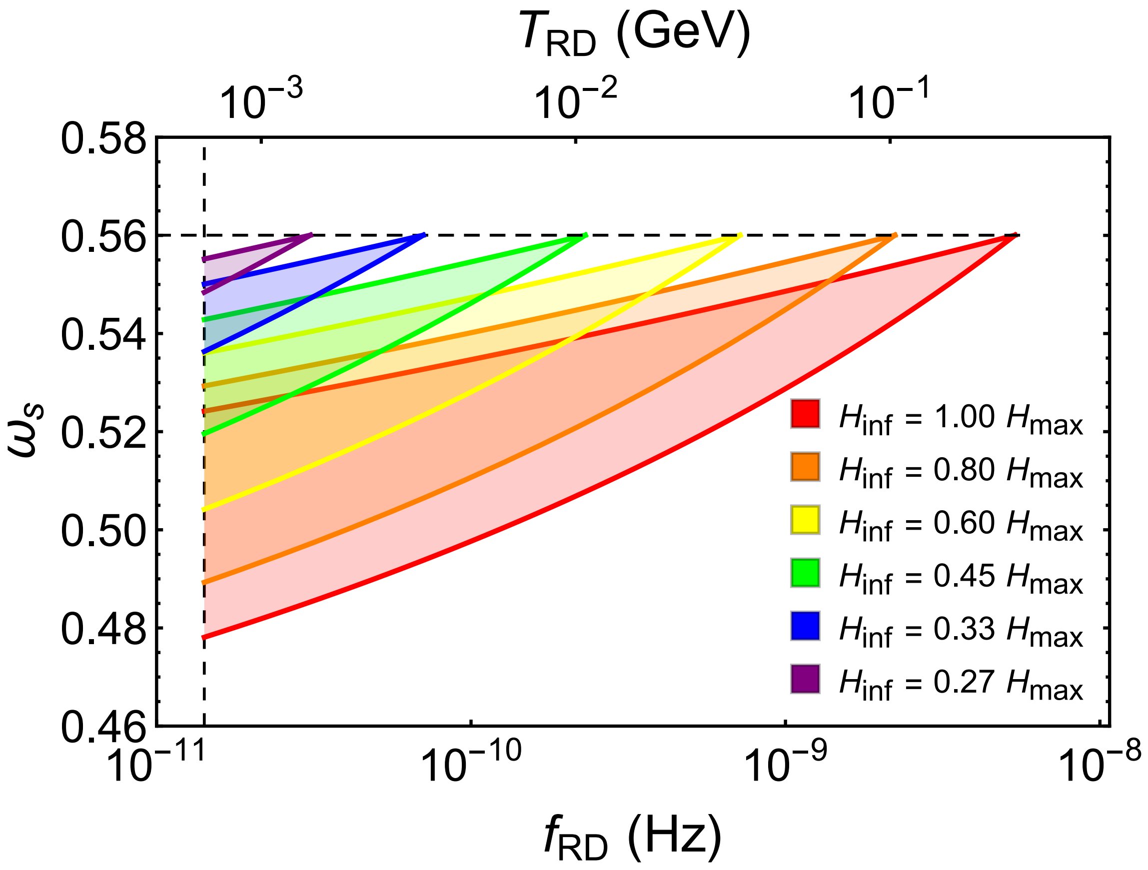
$$\Omega_{\rm GW}(f) \propto H_{\rm inf}^2 \left(\frac{f}{f_{\rm RD}}\right)^{\frac{2(w-1/3)}{(w+1/3)}}$$

DGF & Tanin
(preliminar)

STIFF EQ of STATE



after BBN cut



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CMB B-Modes
from INFLATION

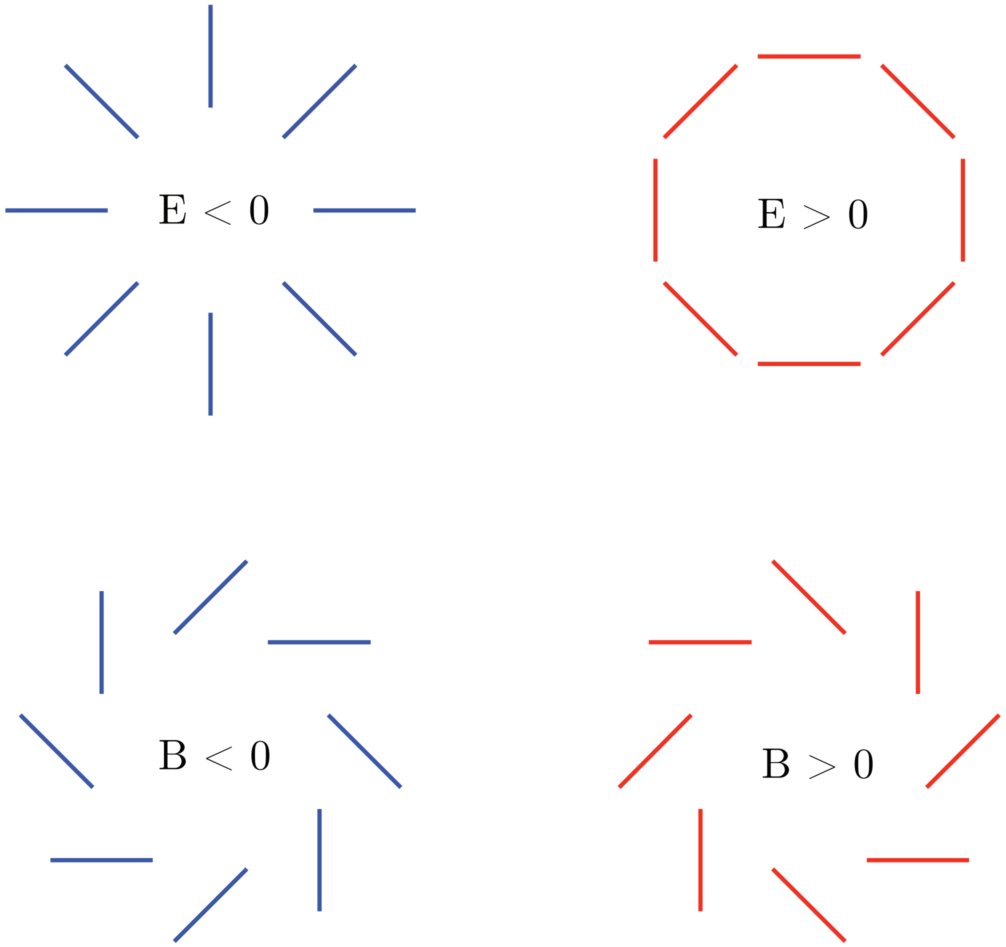
Inflation: Observables

$$\boxed{\langle \mathcal{E}^2 \rangle\,,\ \ \langle \mathcal{B}^2 \rangle \ \ \rightarrow \ \langle |e_{lm}|^2 \rangle \equiv C_l^E,\quad \langle |b_{lm}|^2 \rangle \equiv C_l^B}$$

Polarization Angular Power Spectrum

Depends on Scalar (also tensor) Perturbations

Depends only on Tensor Perturbations !



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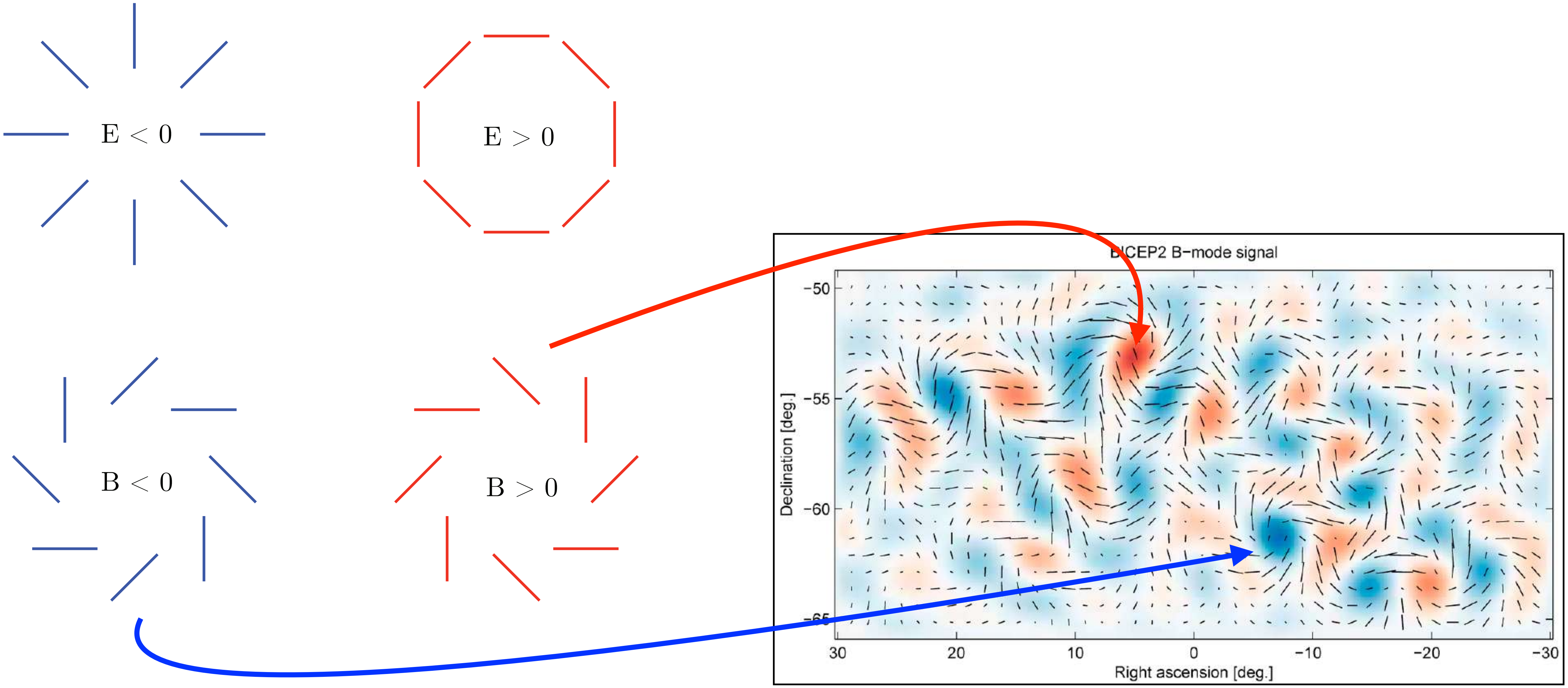

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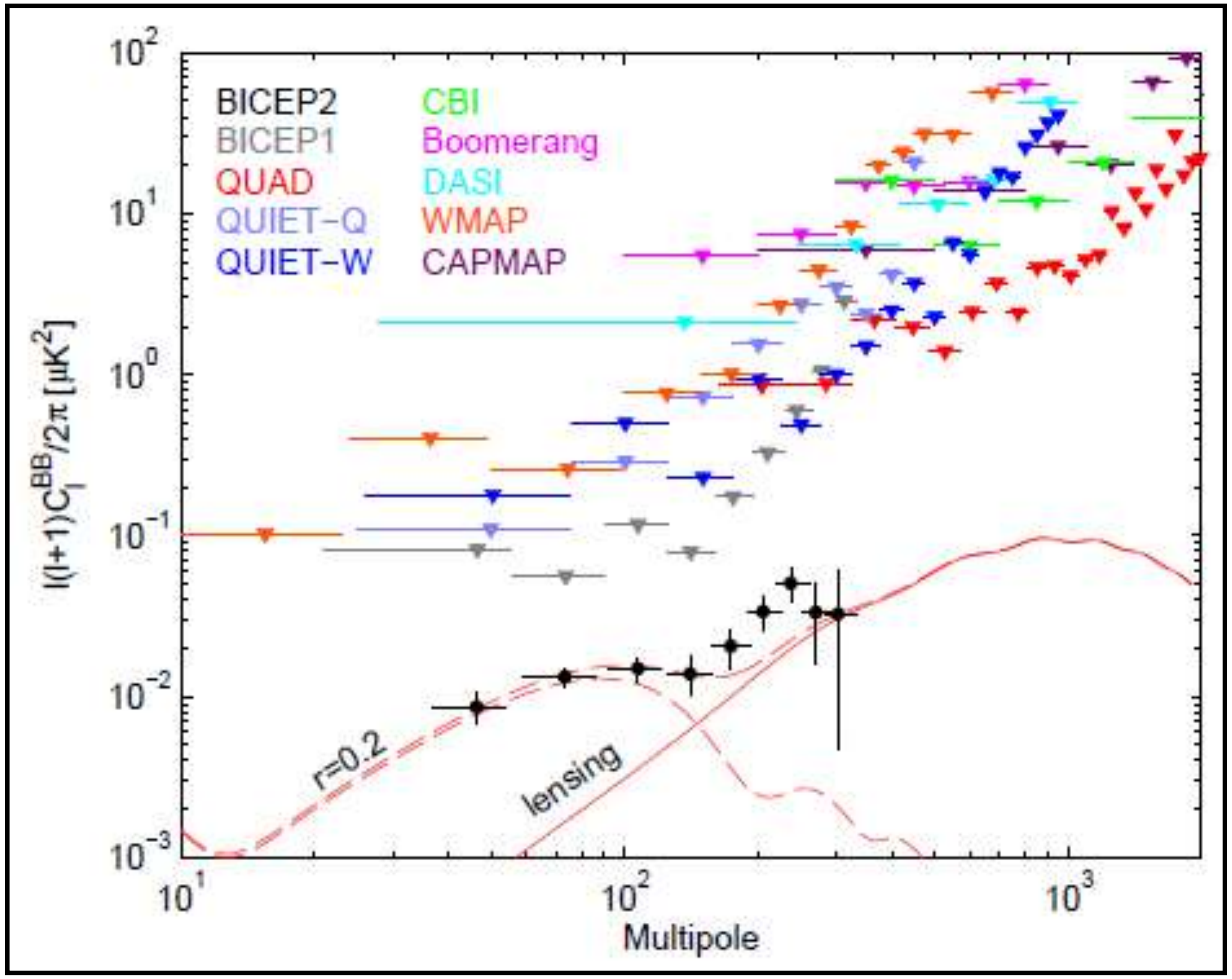

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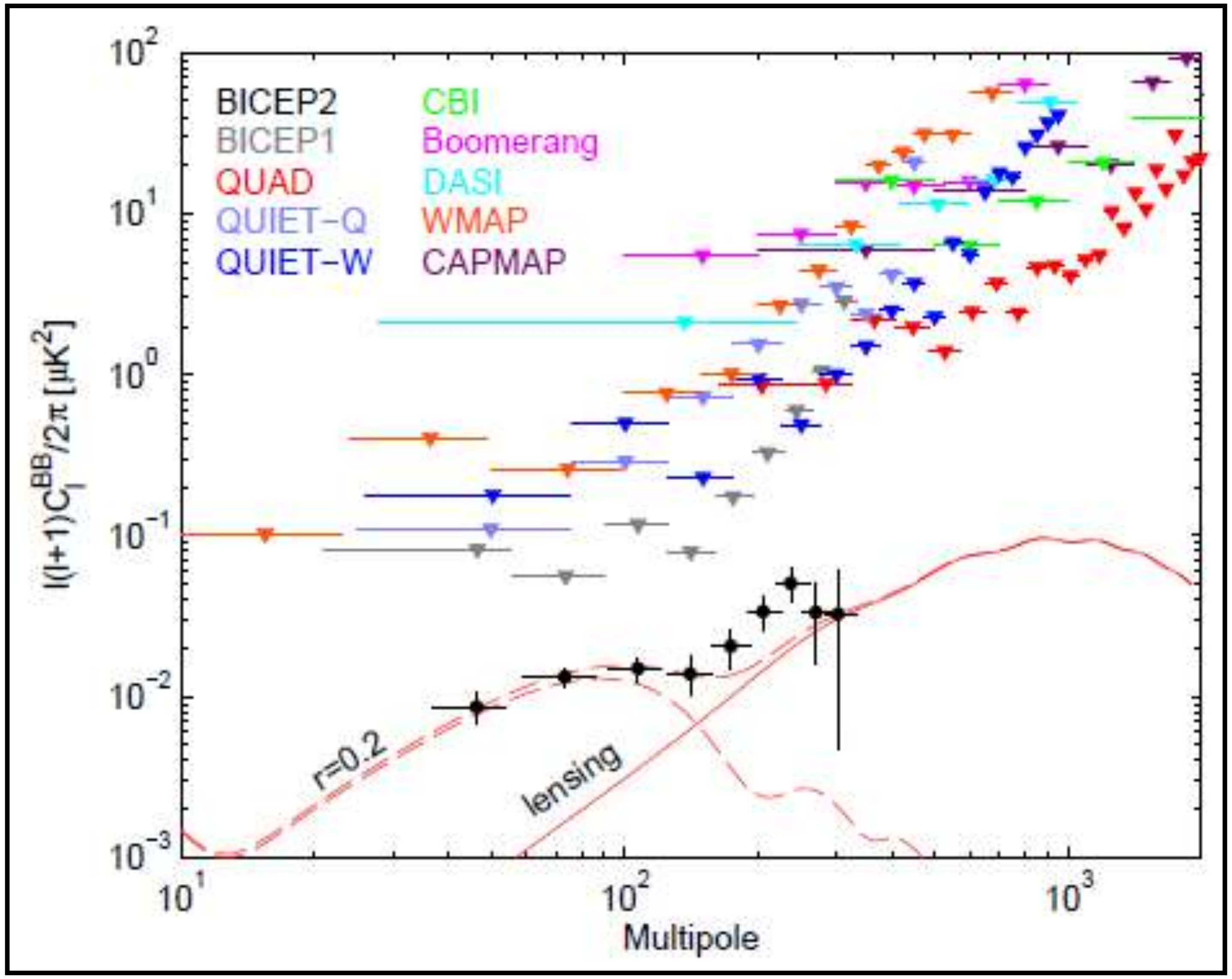

Dashed Line Theoretical Expectation from Inflation

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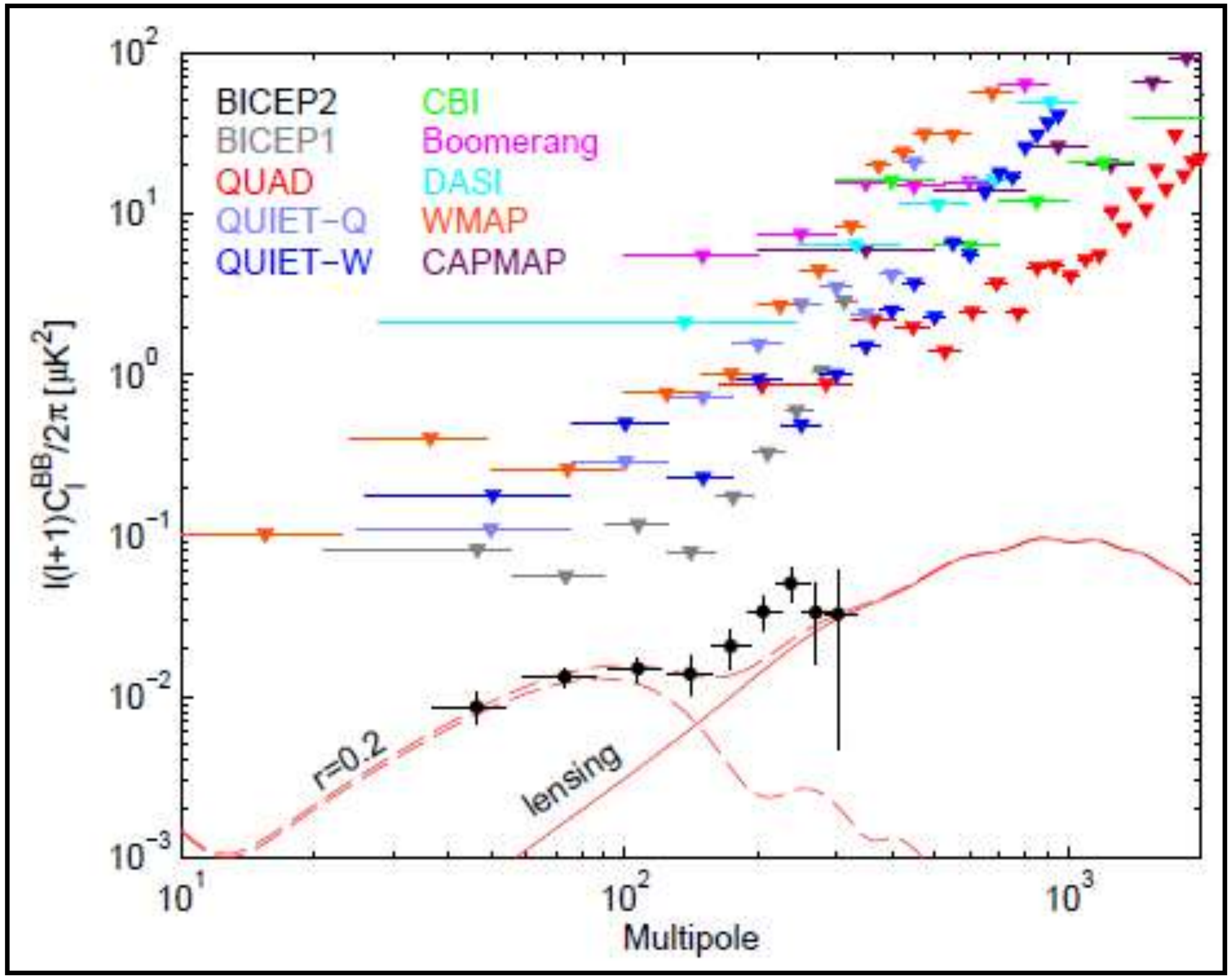

Dashed Line Theoretical Expectation from Inflation

BICEP-2 thought "we found B-modes due to Inflationary tensors"

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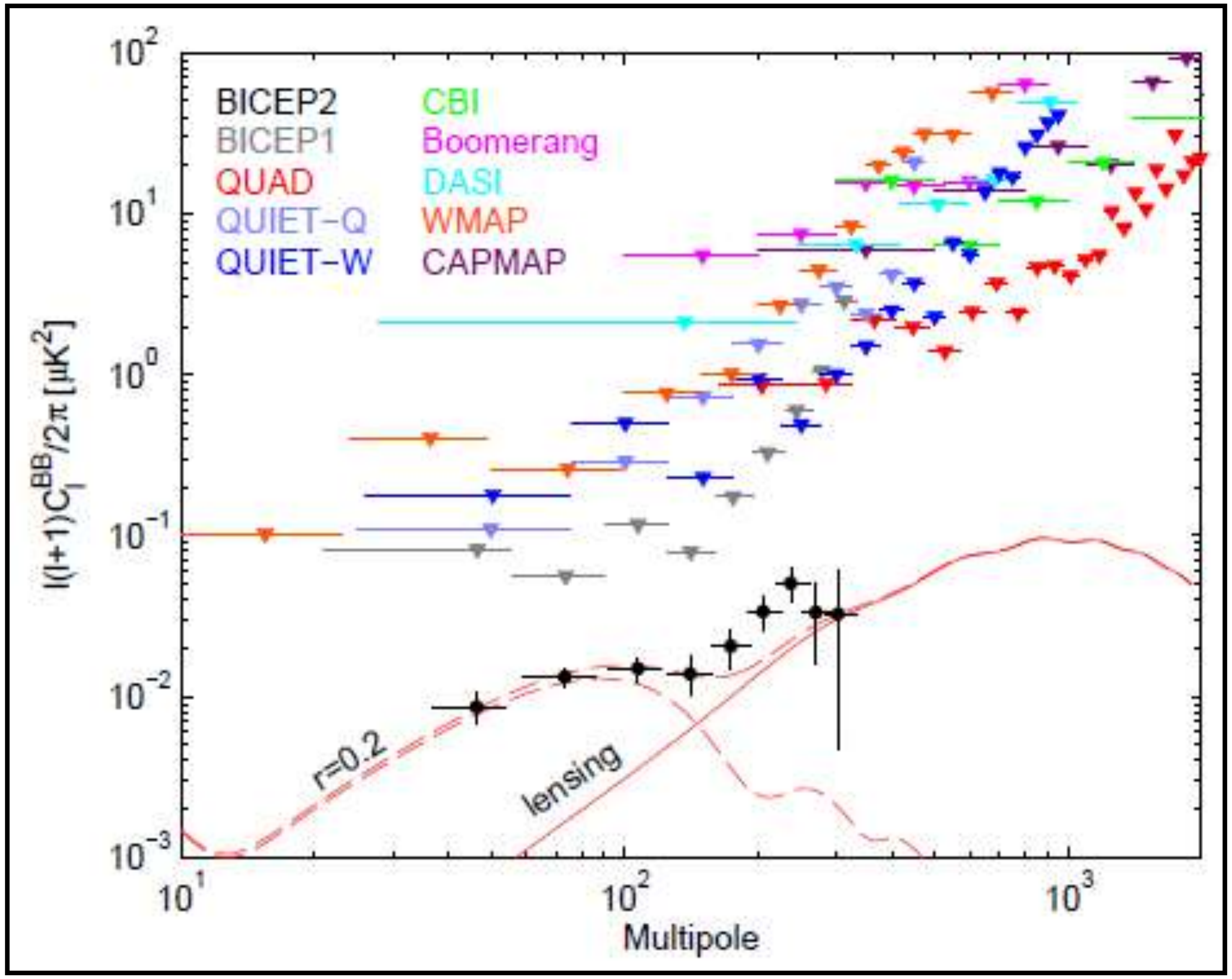

Dashed Line Theoretical Expectation from Inflation

but ...
signal (even real) was only due
[at least mostly dominated by]
... dust contamination

Inflation: Observables

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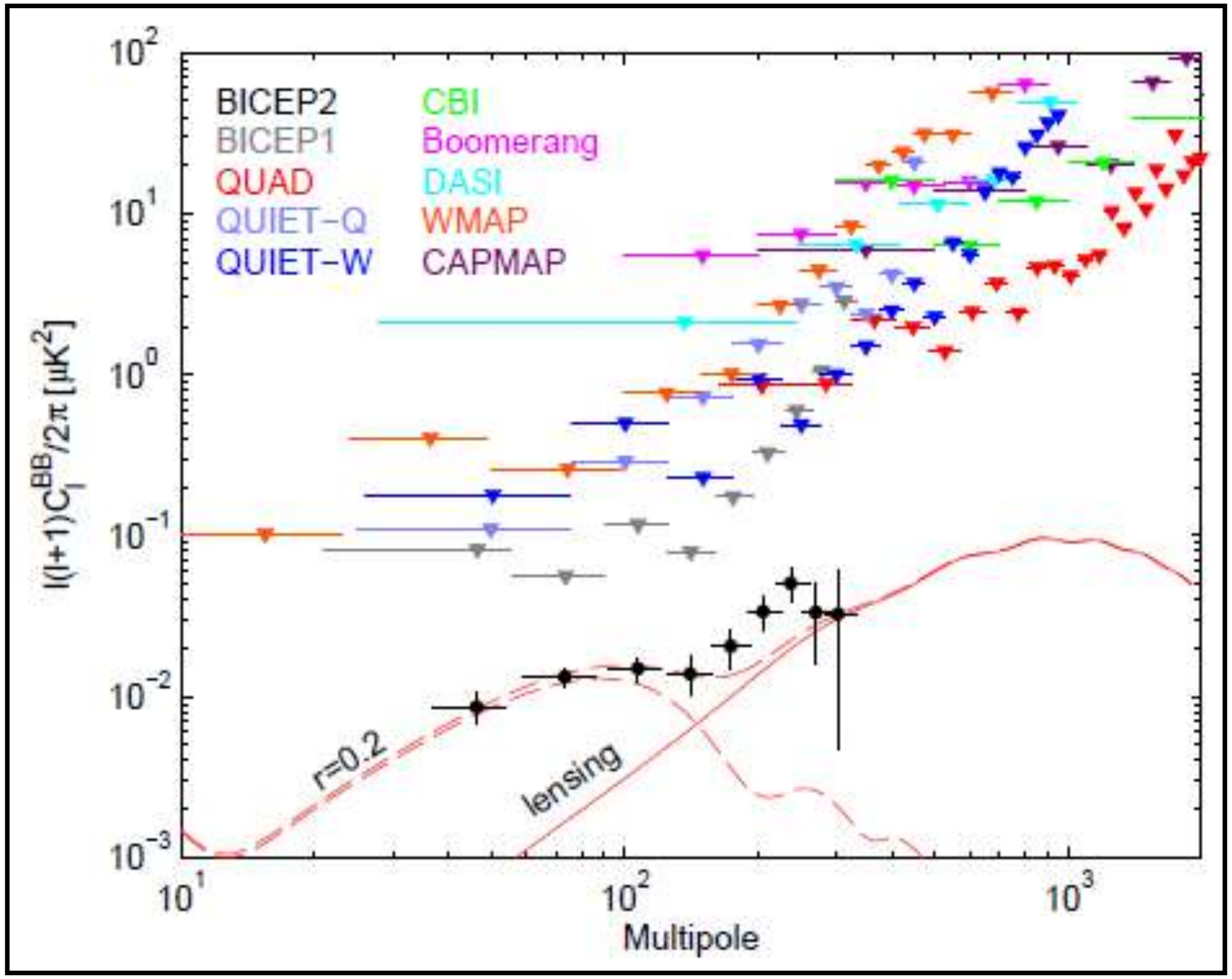

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Power Spectrum

Depends on Scalar
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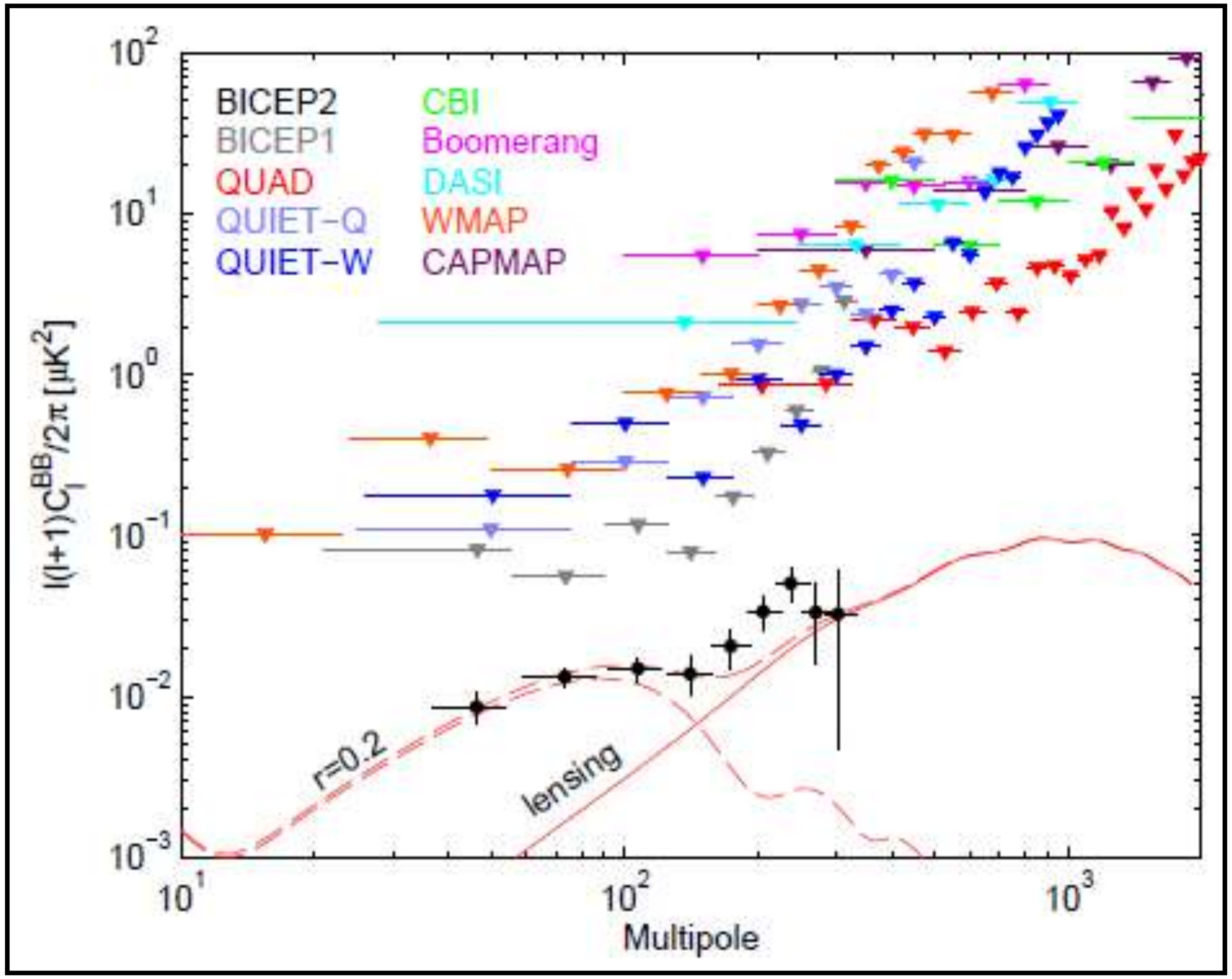

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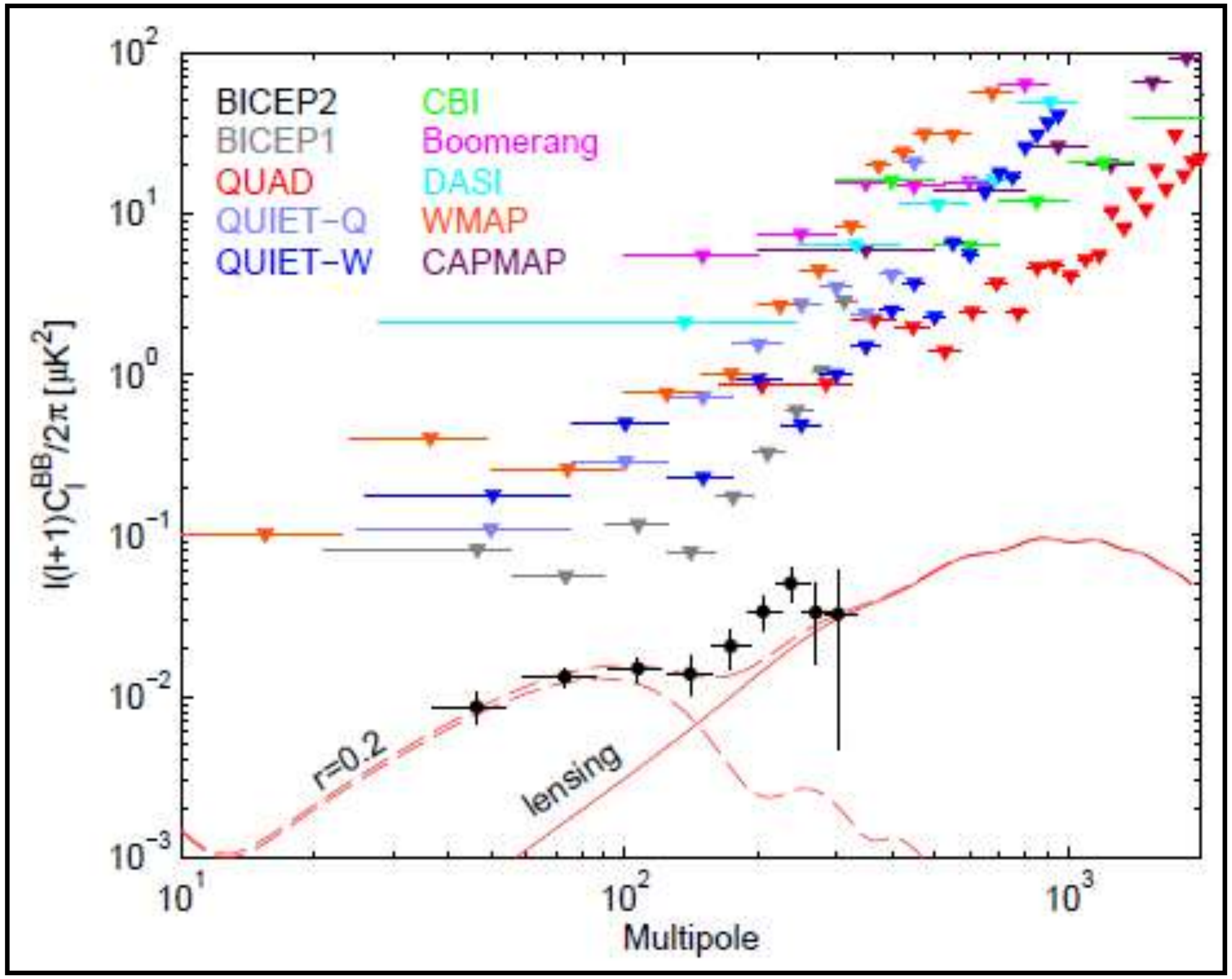

Dashed Line Theoretical Expectation from Inflation

$$r \equiv \Delta_t^2 / \Delta_s^2 < 0.07 \ (2\sigma)$$

Planck/Keck

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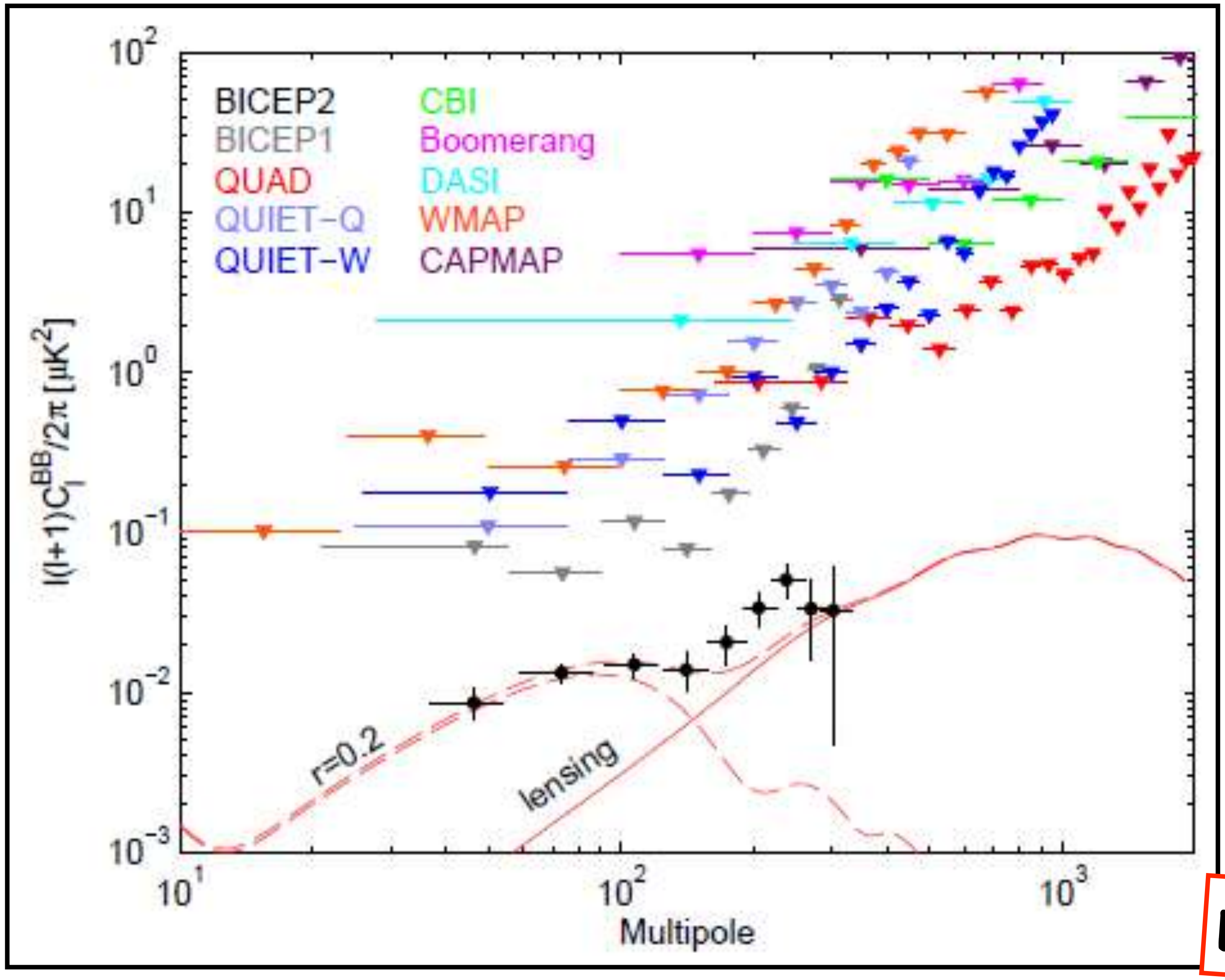

Dashed Line Theoretical Expectation from Inflation

Planck/Keck

$$r \equiv \Delta_t^2/\Delta_s^2 < 0.07\ (2\sigma)$$

$$r \sim 10^{-2} - 10^{-3} \Rightarrow \boxed{E_* \lesssim 5 \cdot 10^{15}\mathrm{GeV}\ (!)}$$

next generation

Inflation: Observables

$$\langle \mathcal{E}^2 \rangle , \ \langle \mathcal{B}^2 \rangle \ \rightarrow \ \langle |e_{lm}|^2 \rangle \equiv C_l^E , \quad \langle |b_{lm}|^2 \rangle \equiv C_l^B$$

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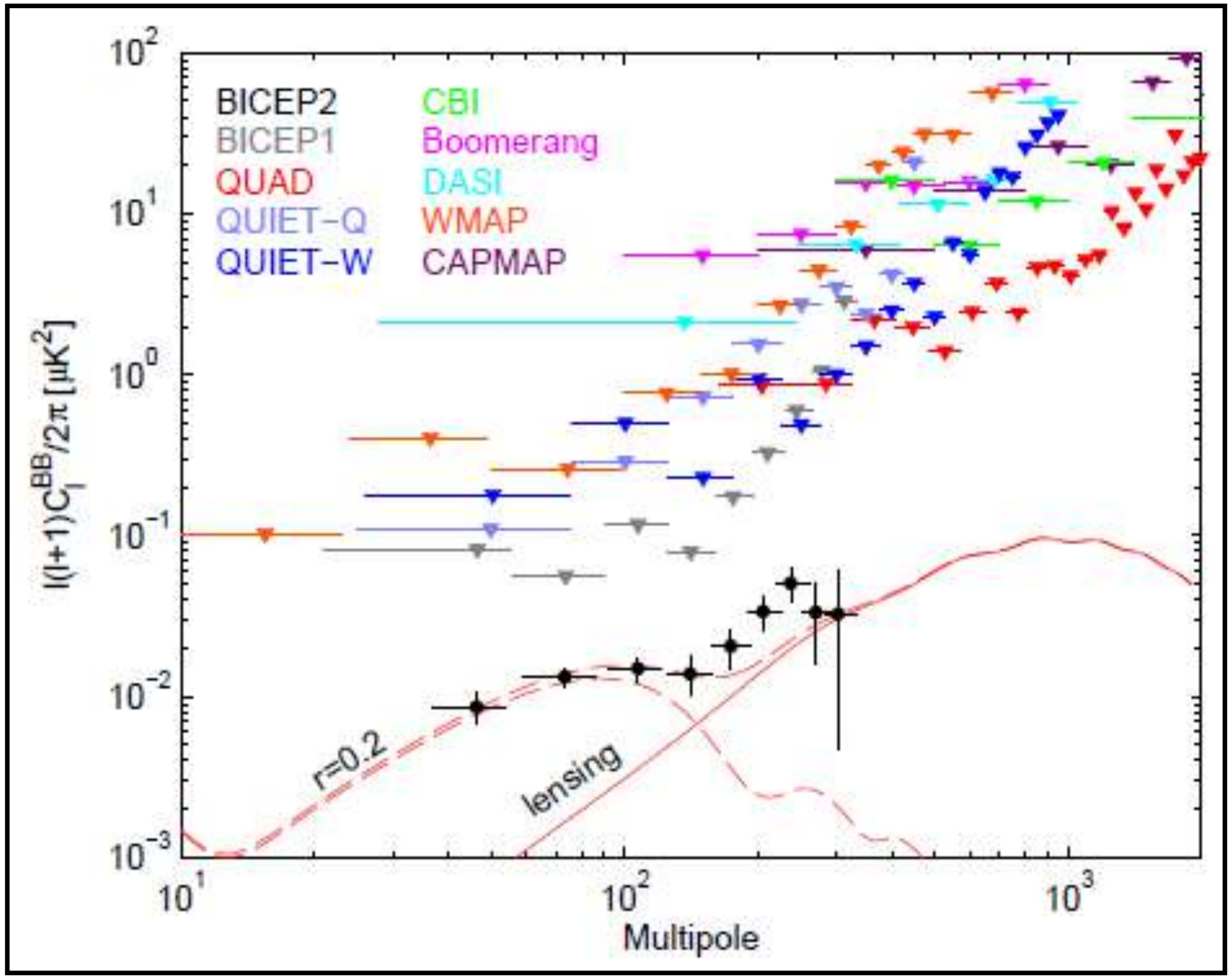

Search of B-modes @ CMB might be only change to detect Inflationary Tensors !

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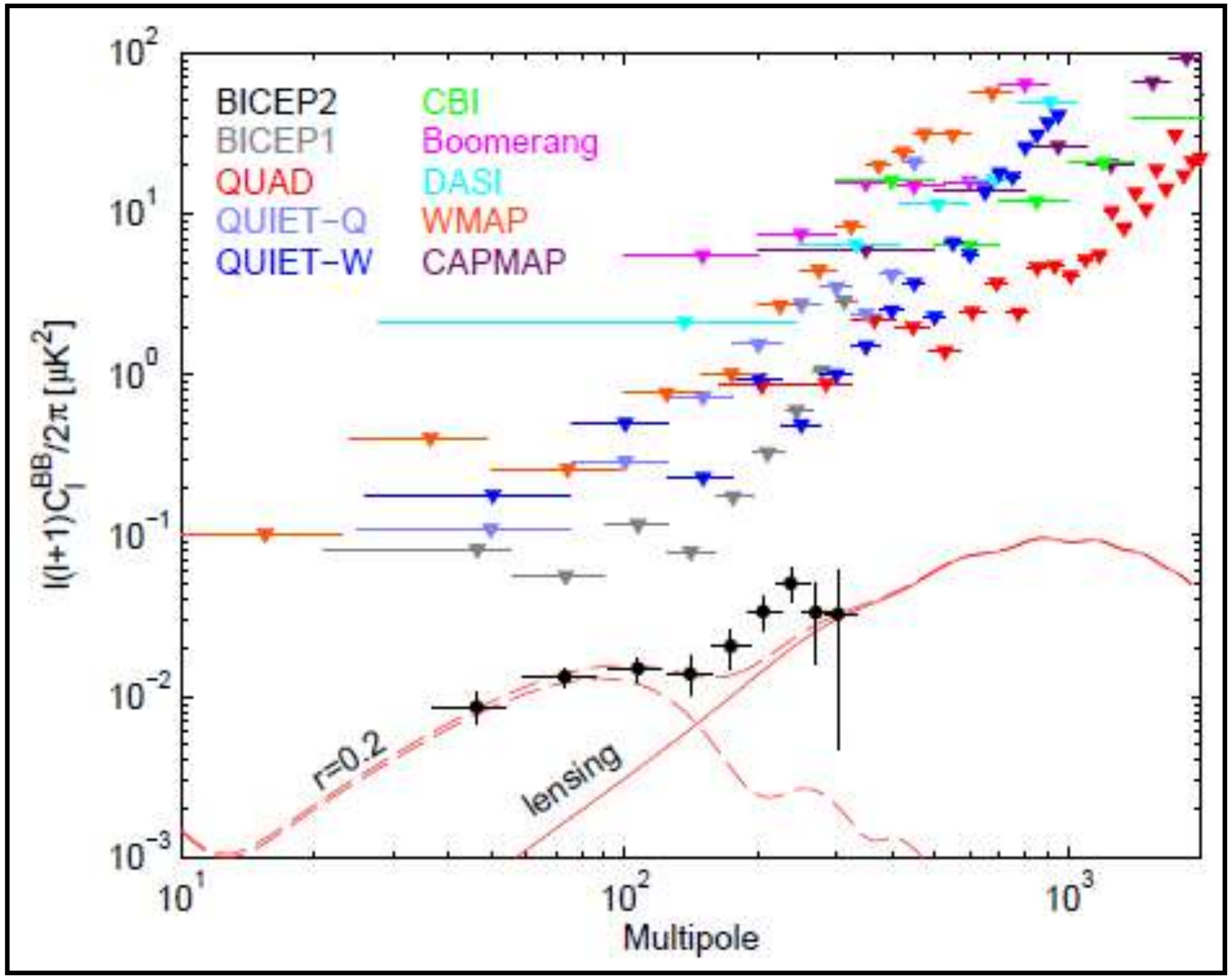

Sear

Holy Grial of Inflation, great effort put forward by CMB community

sors !