Maximally supersymmetric RG flows in 4D and Integrability

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Upstream the RG?

In this talk, I will revisit a famous irrelevant deformation of $\mathcal{N} = 4$ SYM,

$$\mathcal{L}_{\text{SYM}} + h\,\mathcal{O}_8 + \ldots$$

$\mathcal{O}_8$ is the leading single-trace *irrelevant* operator $(E = 8)$ that preserves all 16 Q's in $\mathbb{R}^4$.

A maximally SUSY RG flow ending in planar $\mathcal{N} = 4$ SYM generically takes this form in the IR.

Recent remarkable progress in flowing up the RG in $d = 2$:
$T\bar{T}$ deformation appears to be well-defined. The UV theory is *not* a conventional local QFT.

$\mathcal{O}_8$ is in some ways a $d = 4$, $\mathcal{N} = 4$ version of $T\bar{T}$. Analogous story in $\mathcal{N} = 4$ SYM?

To preserve SUSY, need to add top component of a multiplet.

Leading irrelevant deformation is $Q^4\tilde{Q}^4\mathbf{105}$, top component of the 1/2 BPS multiplet in the four-index symmetric traceless irrep of $SO(6)_R$. Unique F-term that preserves full R-symmetry.

Single-trace version:

$$\begin{aligned}\mathcal{O}_8^{\rm ST} &= Q^4\tilde{Q}^4\,{\rm Tr}\,\Phi^{(I}\Phi^J\Phi^K\Phi^{L)}\\ &= {\rm Tr}\Big[F^4-\frac{1}{4}(F^2)^2+4\big(F_{mp}F^{np}-\tfrac{1}{4}F_{pq}F^{pq}\delta^n_m\big)D^m\Phi_I D_n\Phi_I\\ &\qquad -(D_m\Phi_I)(D^m\Phi_I)(D_n\Phi_J)(D^n\Phi_J)+2(D_m\Phi_I)(D^m\Phi_J)(D_n\Phi_I)(D^n\Phi_J)+\ldots\Big]\end{aligned}$$

Double-trace version:

$$Q^4\tilde{Q}^4\,{\rm Tr}\,\phi^{(I}\phi^J\,{\rm Tr}\,\phi^K\phi^{L)}=T_{mn}T_{mn}+\mathcal{O}_\tau\mathcal{O}_{\bar\tau}+\ldots$$

A bit like $T\bar{T}$ indeed! But it clearly does *not* have the same "semi-topological" nature.
We will attempt to define the deformation in conformal perturbation theory, not in terms of an evolution equation at finite h.

Most intriguing example of such a flow is the full EFT on N D3 branes, $h \sim (\alpha')^2$.

Long-standing dream to generalize the AdS/CFT duality:
Full D3 brane geometry (asymptotic to flat space) $\leftrightarrow$ full D3 brane effective action?

Intriligator speculated that closed string theory on

$$ds^2 = H^{-1/2} dx^m dx_m + H^{1/2} dx_I dx_I \,, \quad H(r) = \tilde{h} + \frac{R^4}{r^4} \,, \quad R^4 = 4\pi g_s N (\alpha')^4$$

is dual to to

$$\mathcal{L}_{\text{SYM}} + \tilde{h} R^4 \mathcal{O}_8 \,.$$

This proposal passes a leading order check. LR van Raamsdonk

An obvious difficulty of this idea is making sense of the field theory side. Full open SFT?
Can we now make progress?

On $S^3 \times \mathbb{R}$

One novelty of our approach is that we will study the deformation on $S^3 \times \mathbb{R}$.

For $h = 0$, map $\mathbb{R}^4 \to S^3 \times \mathbb{R}$ by Weyl transformation. Full $\mathfrak{psu}(2,2|4)$ superalgebra is of course preserved.

For $h \neq 0$, we can preserve the subgroup of "rigid" (i.e. non-conformal) superisometries

$$\mathfrak{psu}(2|2) \times \mathfrak{psu}(2|2) \ltimes \mathbb{R}^2$$

The preserved bosonic symmetries comprise the isometries

$$SO(4) \times \mathbb{R}_\tau \cong SU(2)_\alpha \times SU(2)_{\dot\alpha} \times \mathbb{R}_\tau$$

and the R-symmetries

$$SU(2)_a \times SU(2)_{\dot a} \times U(1)_J \subset SU(4)_R$$

Superalgebra is *two* copies of $\mathfrak{su}(2|2)$, with *common* central extension $H - J$, where H is the generator of τ translations.

To preserve SUSY on $S^3 \times \mathbb{R}$, the irrelevant deformation $h\mathcal{O}_8$ must be supplemented with curvature corrections $O(1/\ell^k)$, where ℓ is the radius of S^3.

A priori a hard problem, leading to an *infinite* expansion.

Fortunately, we found an elegant off-shell formalism (following Berkovits, Evans, Pestun). We can linearly realize 8 of the 16 supersymmetries, the ones with say $J = 1/2$.

Seven auxiliary fields, split as 3+4: $K^{\hat{\mu}}$ spatial vector on S^3 and K^i vector of $SO(4)_R$, preserving the full isometries of $S^3 \times \mathbb{R}$ and the full $SO(4)_R \times U(1)_J$ R-symmetry,

The remaining 8 supersymmetries are realized on-shell, provided we turn on an imaginary $U(1)_J$ background connection along (Euclidean) time direction τ

$$V = \frac{i}{\ell} d\tau \,, \quad D_\tau \equiv \partial_\tau + \frac{J}{\ell}$$

It is convenient to rename the six scalar fields as

$$Z\,, \qquad \bar{Z}\,, \qquad \phi_{a\dot{a}}\,,$$

with $U(1)_J$ assignments $J(Z) = +1$, $J(\bar{Z}) = -1$, $J(\phi_{a\dot{a}}) = 0$.

It is immediate to set up a superspace formalism, defining the superfield

$$\bar{Z}(\theta_{a\alpha}, \tilde{\theta}^{\dot{a}\dot{\alpha}}) = \bar{Z} - 2i\epsilon^{ab}\epsilon^{\alpha\beta}\Psi_{-a\alpha}\,\theta_{b\beta} - 2i\epsilon_{\dot{a}\dot{b}}\epsilon_{\dot{\alpha}\dot{\beta}}\Psi_{-}^{\dot{a}\dot{\alpha}}\,\tilde{\theta}^{\dot{b}\dot{\beta}} + \dots$$

We now have the full classical action

$$S(g_{\rm YM}, h) = S_{\rm SYM} + h\int_{S^3\times\mathbb{R}}\sqrt{g}\,d^4x\int d^4\theta d^4\tilde{\theta}\,{\rm Tr}\,\bar{Z}(\theta,\tilde{\theta})^4 + {\rm h.c.}$$

Integrating out the auxiliary fields generate an infinite power expansion in h.

To leading order in h, classical Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SYM} + h\left[\mathcal{O}_8 + \frac{\mathcal{O}_7}{\ell} + \ldots \frac{\mathcal{O}_4}{\ell^4}\right] + O(h^2)$$

where $\mathcal{O}_i$ are components of the $\mathbf{105}$ supermultiplet. For example,

$$\mathcal{O}_4 = \mathrm{STr}(3Z^2\bar{Z}^2 - 6Z\bar{Z}\phi_j\phi_j + \phi_i\phi_i\phi_j\phi_j)$$

is the $SO(4)_R \times U(1)_J$ singlet piece of the superprimary.

At the quantum level, must of add counterterms and fine tune.
The procedure is intrinsically ambiguous, barring additional input.

Spectral problem and spin chains

State/operator map is lost, but it makes perfect sense to ask how the energy spectrum of *states*. Usual spin-chain picture.

The spectrum of the planar theory is calculated by a deformation $H(g^2, h)$ of the spin-chain Hamiltonian $H(g^2)$ of $\mathcal{N} = 4$ SYM.

Much of the familiar story goes through.
Our deformation is hermitian and preserve "parity" (in the spin chain sense).

For $h \neq 0$, magnons are *not* Goldstones, but their dispersion relation and their S-matrix is still constrainted by Beisert's triply centrally extended $\mathfrak{su}(2|2)$.

$$\{\mathcal{S}^\alpha{}_a, \mathcal{Q}^b{}_\beta\} = \delta^a_b \mathcal{L}^\alpha{}_\beta + \delta^\alpha_\beta \mathcal{R}^b{}_a + \frac{1}{2}\delta^a_b \delta^\alpha_\beta \left(H - J\right) \,.$$

$$\{\mathcal{Q}^a_\alpha\,, \mathcal{Q}^b_\beta\} = \epsilon^{ab}\epsilon_{\alpha\beta}\,\mathcal{P}\,, \quad \{\mathcal{S}^\alpha_a\,, \mathcal{S}^b_\beta\} = \epsilon_{ab}\epsilon^{\alpha\beta}\,\mathcal{K}\,,$$

By the *same* argument, magnon dispersion relation

$$E(p) - J = \frac{1}{2}\sqrt{1 + 16\,\alpha(g^2, h/\ell^4)\sin^2\left(\frac{p}{2}\right)}$$

The $\mathfrak{su}(2|2)$ symmetry also completely fixes the matrix structure of $2 \to 2$ scattering.

The only freedom in the $2 \to 2$ S-matrix is thus in the dressing phase.

Of course, still non-trivial to ask whether $n \to n$ scattering factorizes.
To try and answer this question we turn to explicit calculations.

The $SU(2|2) \ltimes U(1)$ sector

Impractical to do perturbation theory using the complicated action we derived.
But symmetry-based methods are very powerful.

We restrict to the subsector with elementary fields

$$\phi_a \equiv \phi_{a\mathrm{i}}\,, \qquad \psi_\alpha \equiv \psi_{\alpha\mathrm{i}} \quad , \qquad Z\,.$$

where $a = 1, 2$ and $\alpha = \pm$. This subsector is analogous to the $SU(2|3)$ sector of $\mathcal{N} = 4$ SYM (with $Z \to \phi_3$), but we have the *smaller* symmetry $SU(2|2) \ltimes U(1)$.

We have a double expansion in g^2 and h, but from the abstract symmetry viewpoint they appear on the same footing. A term $O(g^{2i}h^j)$ corresponds to $2i + j$ "loop" order.

Following Beisert, we use symmetry to constrain the action of the generators of $SU(2|2) \ltimes U(1)$ acting on spin-chain states.
As usual, the symbols

$$\left\{ {A_1 \ldots A_n \atop B_1 \ldots B_m} \right\}$$

represent the tensor structures. At a given "loop" order k the generators should take the form

$$J_k \sim \left\{ {A_1 \ldots A_n \atop B_1 \ldots B_m} \right\}, \text{ with } n + m = k + 2$$

By imposing closure of the algebra, hermiticity and parity we find that at one-loop

$$\begin{aligned} H_2 \quad &= d_1 (\left\{ {ab \atop ab} \right\} + \left\{ {a\beta \atop a\beta} \right\} + \left\{ {\alpha b \atop \alpha b} \right\} + \left\{ {\alpha\beta \atop \alpha\beta} \right\} - \left\{ {ab \atop ba} \right\} - \left\{ {a\beta \atop \beta a} \right\} - \left\{ {\alpha b \atop b\alpha} \right\} + \left\{ {\alpha\beta \atop \beta\alpha} \right\} \\ &+ \left\{ {Z\beta \atop Z\beta} \right\} + \left\{ {\beta Z \atop \beta Z} \right\} + \left\{ {Zb \atop Zb} \right\} + \left\{ {bZ \atop bZ} \right\} - \left\{ {Z\beta \atop \beta Z} \right\} - \left\{ {Zb \atop bZ} \right\} - \left\{ {bZ \atop Zb} \right\} - \left\{ {\beta Z \atop Z\beta} \right\}) \end{aligned}$$

There is only one overall constant, and this is in fact the familiar one-loop result for the $SU(2|3)$ chain. Automatic symmetry enhancement to this lowest order.

At two loops, calculation more involved but still feasible.

Closure of the $SU(2|2) \ltimes U(1)$ algebra, hermiticity and dispersion relation fix H_4 uniquely, up to similarity and redefinition of the coupling.

The deformation makes no difference even at two loops!
To this order, again automatic symmetry enhancement to $SU(2|3)$.

Clearly, no dynamical test of integrability yet.

We can however ask more structural questions.
Does there *exist* an integrable long-range spin-chain which is *different* from the $\mathcal{N} = 4$ case?

Beisert, Fievét, de Leeuw, Loebbert: general analysis of integrable long range XXZ chains.
They found a large class of models encoded in the Bethe ansatz

$$\exp(ip(u_k)L) = \exp(i\phi L) \prod_{j \neq k} \exp(-2i\theta(u_k, u_j)) \frac{\sinh \hbar(u_k - u_j + i)}{\sinh \hbar(u_k - u_j - i)}$$

$$\theta(u_k, u_j) = \sum_{s>r=2}^{\infty} \beta_{r,s}(q_r(u_k)q_s(u_j) - q_s(u_k)q_r(u_j)) + \sum_{r=2}^{\infty} \eta_r(q_r(u_k) - q_r(u_j))$$

We should repeat the analysis for the $SU(2|2) \times U(1)$ chain!

Note that the XXZ chain can be viewed as a closed of subsector of the $SU(2|2) \ltimes U(1)$ chain.
Since the only departure from $\mathcal{N} = 4$ can be in the dressing phase, we must take $\hbar = \phi \equiv 0$.

Still, ample freedom is left. Crossing equation gives extra constraints, but *a priori* no obstacle.

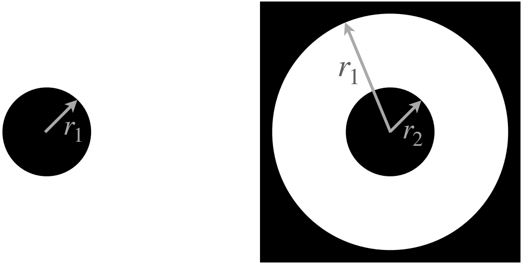
Holographic interpretation?

A natural setting are the "bubbling" geometries of Lin Lunin Maldacena (LLM).
Generally, $SU(2|2) \times SU(2|2)$ symmetry and one can impose additional $U(1)_J$.

LLM geometries describe the backreaction of "additional" D3 branes (giant gravitons) in global $AdS_5 \times S^5$. The extreme UV asymptotics are always $AdS_5 \times S^5$.

An LLM geometry is fully specified by prescribing ± 1 boundary conditions on a two-dimensional plane: bicoloring of the plane.

Dually, it corresponds to considering $\mathcal{N} = 4$ in a non-trivial half-BPS *state*, with $E \sim N^2$.
In a sense, an $S^3 \times \mathbb{R}$ version of a Coulomb branch flow.



(a) (b)

(a) is the LLM picture for $AdS_5 \times S^5$

(b) is the simplest non-trivial LLM geometry with $U(1)_J$ isometry. We can identify

$$\frac{h}{\ell^4} \sim \frac{r_2^2}{r_1^2}$$

Chervonyi and Lunin: the classical sigma model for (b) is not integrable.

Outlook

- $SU(2|2)$ integrable long-range chain?
  Which RG flow would it describe on field theory side?
- Interesting to explore relation with LLM, regardless of integrability.
- Other geometries: $\mathbb{R}^4$, S^4.
- Double-trace version.