Type II SUGRA from the spinning world line

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Motivation

① classical SFT as a theory of background fields

② point particle vs. ambitwistor strings

based on 1807.07989
2004.06129
w/ R. Bonezzi and A. Meyer

① BV vs. string theory:

BV — SFT — S.T.

$$(\mathcal{F}, \Delta, S) \rightsquigarrow \Omega \xrightarrow[\text{crit. point}]{\text{exp'n around}} \{\Omega^{(k)}\} \xrightarrow{\text{Feynman}} \{S^{(k)}\}$$

fields, BV-op, action

homol. v.f.

this talk

smart CFT repr.

② ambitwistor string: an S-matrix theory for the world line

(L. Mason + D. Skinner '14)

sums up diagrams

n.b.: BRST quantisation of ambitwistor worldsheet => Type II SUGRA

(T. Adamo, E. Casali, D. Skinner '15)

Result 1: as a theory of background fields the spinning world line is almost as rigid as string theory.

Result 2: there is a version of operator-state correspondance for the world line.

The model:

"gravitini"

$$S = \int d\tau \left[p_\mu \dot{x}^\mu + i\,\bar{\theta}^i_\mu \dot{\theta}^\mu_i - \frac{e}{2}\,p^2 - i\,\chi_i\,\bar{\theta}^{\mu i} p_\mu - i\bar{\chi}^i\,\theta^\mu_i p_\mu\right] \,,$$

world line spinors, $i = 1, 2$

- $N = 4$ world line SUSY : $\overset{(-)}{\theta}{}^\mu_i \sim \overset{(-)}{\psi}_{\pm 1/2}$ of type II w.s.

History: $N=0$: no constraints on background.

$N=2$: $Q_{BRST}^2 = 0 \Rightarrow \begin{cases} \text{Y-M e.m.} \\ \text{any metric} \end{cases}$ (P. Dai, Y. Huang, W. Siegel '08)

$N=4$: $\Rightarrow$ e.m. for $(m=0)$ NS-NS fields of type II

Recall: $\Omega = \Omega^{(0)}_{x_0} + \underbrace{\Omega^{(1)}_{x_0}}_{Q_{BRST}:\, T_{x_0}\mathcal{F} \to T_{x_0}\tilde{\mathcal{F}}} + \Omega^{(2)}_{x_0} + \cdots$; $\Omega^2 = \underbrace{\Omega^{(0)} \circ \Omega^{(2)}}_{\substack{=0 \\ \text{e.m.}}} + \underbrace{\Omega^{(1)} \circ \Omega^{(1)}}_{= Q_{BRST}^2} + \cdots$

Thus $Q_{BRST}^2 = 0 \Rightarrow$ e.m. for background fields.

world line supercharges

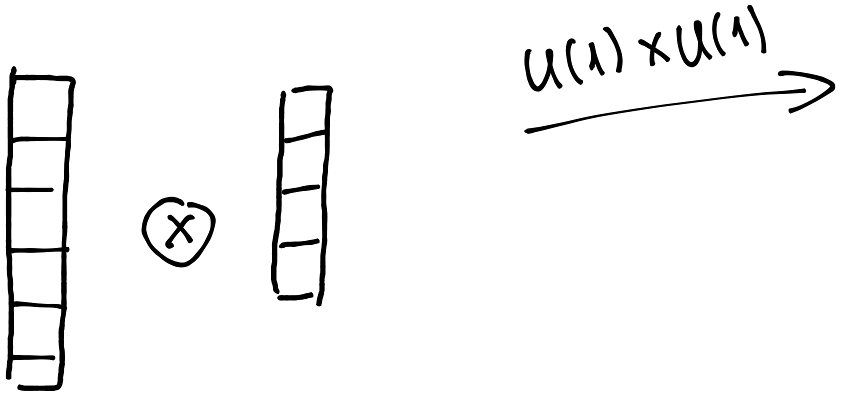
Here, $Q := c\,\Box + \gamma_i\,\bar{q}^i + \bar{\gamma}^i\,q_i + \bar{\gamma}^i\gamma_i\,b$, is a linear op. on

$\mathcal{H} = L^2(\mathbb{R}^d) \otimes \mathrm{Cliff}^{(n)} \otimes \mathrm{Weyl}^{(N)} \Rightarrow \Phi = \mathrm{Pol}(\theta_i, \gamma_i, \beta_i, c)$
w/ coeff in $L^2(\mathbb{R}^d)$

Dirac:

$$\Phi(x,\theta_i) = \sum_{m,n=0}^{d} \phi_{\mu_1\ldots\mu_m|\,\nu_1\ldots\nu_n}(x)\,\theta_1^{\mu_1}\ldots\theta_1^{\mu_m}\,\theta_2^{\nu_1}\ldots\theta_2^{\nu_n} \sim \bigoplus_{m,n} m\left\{\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline\end{array}\right. \otimes\, n\left\{\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline\end{array}\right.$$

- $\mathcal{H}$ can be reduced by gauging subgroups of $SO(4)_R$:



$\square \otimes \square$ (level match + level trunc.)

$\big\downarrow$ Yang

$\begin{array}{|c|c|}\hline \; & \; \\ \hline\end{array}$ (w.s. parity, $\cancel{B_{\mu\nu}}$)

$\big\downarrow$ Tr

$\begin{array}{|c|c|}\hline \; & \; \\ \hline\end{array}$ − tr (no interpr. on w.s., $\cancel{\Phi}$)

BRST:

$$\begin{aligned}\Psi(x,\theta_i\,|c,\gamma_i,\beta_i) = {} & h_{\mu\nu}(x)\,\theta_1^\mu\theta_2^\nu + \tfrac{1}{2}\,h(x)\,(\gamma_1\beta_2-\gamma_2\beta_1) - \tfrac{i}{2}\,v_\mu(x)\,(\theta_1^\mu\beta_2-\theta_2^\mu\beta_1)c \\ & - \tfrac{i}{2}\,\xi_\mu(x)\,(\theta_1^\mu\beta_2-\theta_2^\mu\beta_1) \\ & + h^*_{\mu\nu}(x)\,\theta_1^\mu\theta_2^\nu c + \tfrac{1}{2}\,h^*(x)\,(\gamma_1\beta_2-\gamma_2\beta_1)c - \tfrac{i}{2}\,v^*_\mu(x)\,(\theta_1^\mu\gamma_2-\theta_2^\mu\gamma_1) \\ & - \tfrac{i}{2}\,\xi^*_\mu(x)\,(\theta_1^\mu\gamma_2-\theta_2^\mu\gamma_1)c\,,\end{aligned}$$

SO(4): maximal gauging

Background fields ($\square \otimes \square$, minimal gauging)

$$q_i = \theta^\mu_i \left(\partial_\mu + \omega_{\mu ab}\, \theta^a \cdot \bar\theta^b + H_{\mu\lambda\varsigma}\, \theta^\lambda \cdot \tilde{\bar\theta}^\varsigma + \partial_\mu \Phi \right)$$

$$\square = D^\mu D_\mu + \underbrace{R_{\mu\nu\lambda\varsigma}\, \theta^\mu \cdot \bar\theta^\nu \theta^\lambda \cdot \bar\theta^\varsigma + (\nabla \cdot H)_{\lambda\varsigma}\, \theta^\lambda \cdot \tilde{\bar\theta}^\varsigma - \nabla_\mu \partial_\nu \Phi\, \theta^\nu \cdot \bar\theta^\mu - \Lambda R}_{\text{non-min. couplings}} \Big)$$

- in contrast to the string (but not the ambitwistor string) this construction is backgr'd ind.
- $B_{\mu\nu}$ does not couple to a string (cf. swampland)
- $\Lambda R \;\not\swarrow\; B_{\mu\nu}$

$$Q^2 \Big|_{\mathcal{H}_{red}} = \left. \gamma_i \bar\gamma^j \{\bar q^i, q_j\} + \gamma \cdot \bar\gamma\, \square + c\, \underbrace{[\square, \gamma \cdot \bar q + \bar\gamma\, q]}_{\overset{!}{=}\, 0} \right|_{\mathcal{H}_{red}} \overset{!}{=} 0$$

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - H_{\mu\lambda\sigma} H_\nu{}^{\lambda\sigma} = 0\,, \quad \nabla^\lambda H_{\lambda\mu\nu} - 2\, H_{\mu\nu\lambda} \nabla^\lambda \Phi = 0\,. \qquad \text{for } \Lambda = 0$$

$$R_{\mu\nu} - \Lambda g_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi = 0\,, \quad \nabla^2\Phi - 2\,\nabla^\mu\Phi\,\nabla_\mu\Phi + 2\Lambda\Phi = 0\,, \qquad \text{for } \Lambda \neq 0$$

$\therefore$ $\exists$ another way to couple Φ:

$$Q = Q_0 + 2c\left(\mathcal{G}\,\bar\theta^{1\mu}\bar\theta^{2\nu} + \theta_1^\mu\theta_2^\nu\, Tr\right)\nabla_\mu\nabla_\nu\Phi - 2i\left(\mathcal{G}\,\bar\gamma^{[1}\bar\theta^{2]\mu} + \gamma_{[1}\theta^\mu_{2]}\, Tr\right)\nabla_\mu\Phi$$

$$\theta_1\cdot\theta_2 - \beta_1\gamma_2 + \beta_2\gamma_1. \qquad \bar\theta^1\cdot\bar\theta^2 - \bar\beta^1\bar\gamma^2 + \bar\beta^2\bar\gamma^1,$$

vertex op of Kataoka and Sato '90

Then, $Q^2 = 0 \Rightarrow \Box\Phi = 0$

Operator-state: (also: Dai, Huang, Siegel)

Let $Q = Q_0 + \delta Q$; $Q_0^2 = 0 \Rightarrow [Q_0, \delta Q] = 0$

Then, $\delta Q |\tilde{o}\rangle$ is a physical state if $\exists\ |\tilde{o}\rangle$ of gh# -1
$Q_0 |\tilde{o}\rangle = 0$

Here, $|\tilde{o}\rangle = |\xi\rangle := \xi_\mu \left(\theta_1^\mu \beta_2 - \theta_2^\mu \beta_1\right) |0\rangle$; Diffeo ghost.

$$\delta Q = V(\delta g_{\mu\nu}, \delta B_{\mu\nu}, \delta\Phi) = c \underbrace{W_I}_{\text{pict. } \phi} (\cdots) + \underbrace{W_{II}}_{\text{pict. } -1} (\cdots)$$

e.g. for $Q_0\big|_{g_{\mu\nu} = \eta_{\mu\nu}}$ and $\delta B_{\mu\nu} = \delta\tilde{\Phi} = 0$, $\delta g_{\mu\nu} = h_{\mu\nu}$

$W_{\rm II} |\xi\rangle$: $\varepsilon_{\mu\nu} \theta_1^\mu \theta_2^\nu e^{ik\cdot x} |0\rangle = |h\rangle$

graviton

 $= \left\langle h^{(3)} \middle| V^{(2)} \middle| h^{(1)} \right\rangle = \left\langle \xi^{(3)} \middle| T\{V^{(3)} V^{(2)} V^{(1)}\} \middle| \xi^{(1)} \right\rangle$

Discussion:

- spinning world line is quite stringy 😊
- higher spin ($N > 4$) ?
- connection to ambitwistor ?
- vertex operators in AdS (dS)
- pure spinor ?
- Generalised geometry / doubled field theory ?
- Completness: Did we identify all background fields?

Outlook: (w/ A. Meyer + M. Grigoriev, in progress)

Let $\{ \dot{X}^\mu, p_\mu, \theta^\mu, \bar{\theta}^\mu, c, b, \beta, \bar{\beta}, \gamma, \bar{\gamma} \}$ be generators of an associative Lie super algebra $\mathcal{A}$ with a filtration given by dimension: $\mathcal{A}_0 \subset \mathcal{A}_1 \subset \mathcal{A}_2 \subset \cdots$

and $Q_{BRST} \in \mathcal{A}_1$ with $[Q_0, Q_0] = 0$ interpreted as e.m. for background fields.

$\rightsquigarrow$ determine $\mathrm{coh}([Q_0, \cdot])$ for each filtration

$$\text{e.g. } (N=2): \quad \mathrm{coh}([Q_0, \cdot])\Big|_{\mathcal{A}_0} = \begin{cases} \text{Yang-Mills (on-shell)} \\ \text{dilaton (off-shell)} \end{cases}$$

- makes no reference to an underlying world-line.