Complex scalar field ϕ

$U(1) = G$

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad D_\mu = \partial_\mu + i g A_\mu$$

$$V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \lambda > 0 \quad \mu^2 < 0$$

$$\langle \phi \rangle = v \qquad v^2 = -\frac{\mu^2}{2\lambda} > 0$$

$$\phi = \langle \phi \rangle + \phi' + v + \frac{h + iG}{\sqrt{2}} \qquad m_h^2 = \# \lambda v^2$$

$$(D_\mu \phi)^\dagger (D^\mu \phi) = \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} (\partial_\mu G)^2 + \frac{1}{2} M_A^2 A_\mu A^\mu + \sqrt{2} g v A^\mu \partial_\mu G$$

$$M_A^2 = 2 g^2 v^2$$

+ interactions

G is "eaten" $\phi(x) = r(x)^{\,i\theta(x)}$

$$\phi(x) \to e^{ig\alpha(x)}\,\phi(x) \qquad g\alpha(x) = \frac{G(x)}{\sqrt{2}\,v}$$

$$\begin{cases} \phi(x) = \left(v + \dfrac{h(x)}{\sqrt{2}}\right) e^{i\frac{G(x)}{\sqrt{2}v}} \\ A_\mu(x) \end{cases} \to \begin{cases} \phi'(x) = v + \dfrac{h(x)}{\sqrt{2}} \\ A'_\mu(x) = A_\mu(x) - \dfrac{\partial_\mu G(x)}{\sqrt{2}\,v\,g} \end{cases}$$

$\exists$ gauge choice $\phi(x) = v + \frac{h(x)}{\sqrt{2}}$ A_μ long. comp.

"Unitary" gauge $G = 0$

R_ξ gauge $\partial_\mu A^\mu = \sqrt{2} g v \xi \underline{G}$ ξ free parameter

$\mathcal{L}$: add $\mathcal{L}_{g.f.} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu - \sqrt{2} g v \xi G\right)^2$

$$= -\frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \xi g^2 v^2 G^2 + \sqrt{2} g v \, \partial_\mu A^\mu G$$

$\uparrow$ propagator of A^μ $\mu_G^2 = \xi \mu_A^2$

gauge propagator $i\left[-g^{\mu\nu}+\frac{(1-\xi)\,k^\mu k^\nu}{k^2-\xi M^2}\right]\frac{1}{k^2-M^2}$

Unitary gauge $\xi \to \infty$

't Hooft $\xi = 1$

In general!

$$\{t_A\} = \{\underbrace{t_1 \ldots t_m}_{\text{unbroken}},\ \underbrace{\hat{t}_{m+1} \ldots \hat{t}_N}_{\text{broken}}\}$$

broken $\longrightarrow$ $\hat{t}_A\langle\phi\rangle = 0$

unbroken: $e^{-i\varepsilon t_A}\langle\phi\rangle = \langle\phi\rangle \qquad t_A\langle\phi\rangle = 0$

$$\bar{\psi}\,(i\gamma^\mu\partial_\mu - \not{m})\,\psi$$

$$\psi_L \to e^{-igQ_L\alpha}\,\psi_L \qquad Q_L \neq Q_R$$

$$\psi_R \to e^{-igQ_R\alpha}\,\psi_R \qquad m\bar{\psi}\psi = m\overline{\psi_R}\,\psi_L + h.c.$$

$$m = 0$$

$$Q_R = Q_L + 1$$

$$\lambda\,\phi\,\overline{\psi_R}\,\psi_L + h.c. \qquad \phi = \langle\phi\rangle + \phi'$$

$$\hookrightarrow \langle\phi\rangle \qquad m\overline{\psi_R}\,\psi_L \qquad m = \lambda v$$

SM: "Higgs field" H

$$m_e \bar{e} e = m_e \overline{e_R} e_L + h.c. \qquad e_L \subseteq L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\underline{\lambda \, \overline{e_R} L_r} H^*_r + h.c. \text{ gauge invariant} \Rightarrow$$

H $\begin{cases} SU(3)_c \text{ singlet} \\ SU(2)_L \text{ doublet} \\ Y = \frac{1}{2} \end{cases}$

$L \to UL$

$H \to UH$

$LH^* \to LH^*$

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad v > 0 \qquad H = \langle H \rangle + \underline{H'}$$

$\overline{\quad \cdot \cdot \quad}$ $= m_e \overline{e_R} e_L + h.c. + \ldots$ $m_e = \lambda v$

(1 family)

$Y = -\frac{1}{2}$

d $\overline{d_R} d_L \subseteq \overline{d_R} Q H^*$

$\overline{u_R} u_L \subseteq \overline{u_R} Q \varepsilon H$ $\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$Q \to UQ$

$H \to UH$

$Y = -\frac{1}{2}$ $Y = +\frac{1}{2}$

$QH^* \to QH^*$ QH is not inv. $Q\varepsilon H \to Q\varepsilon H$