G_2 3 NG bosons

φ 1 real scalar singlet under $SU(3)_c \times U(1)_{em}$

CCWZ

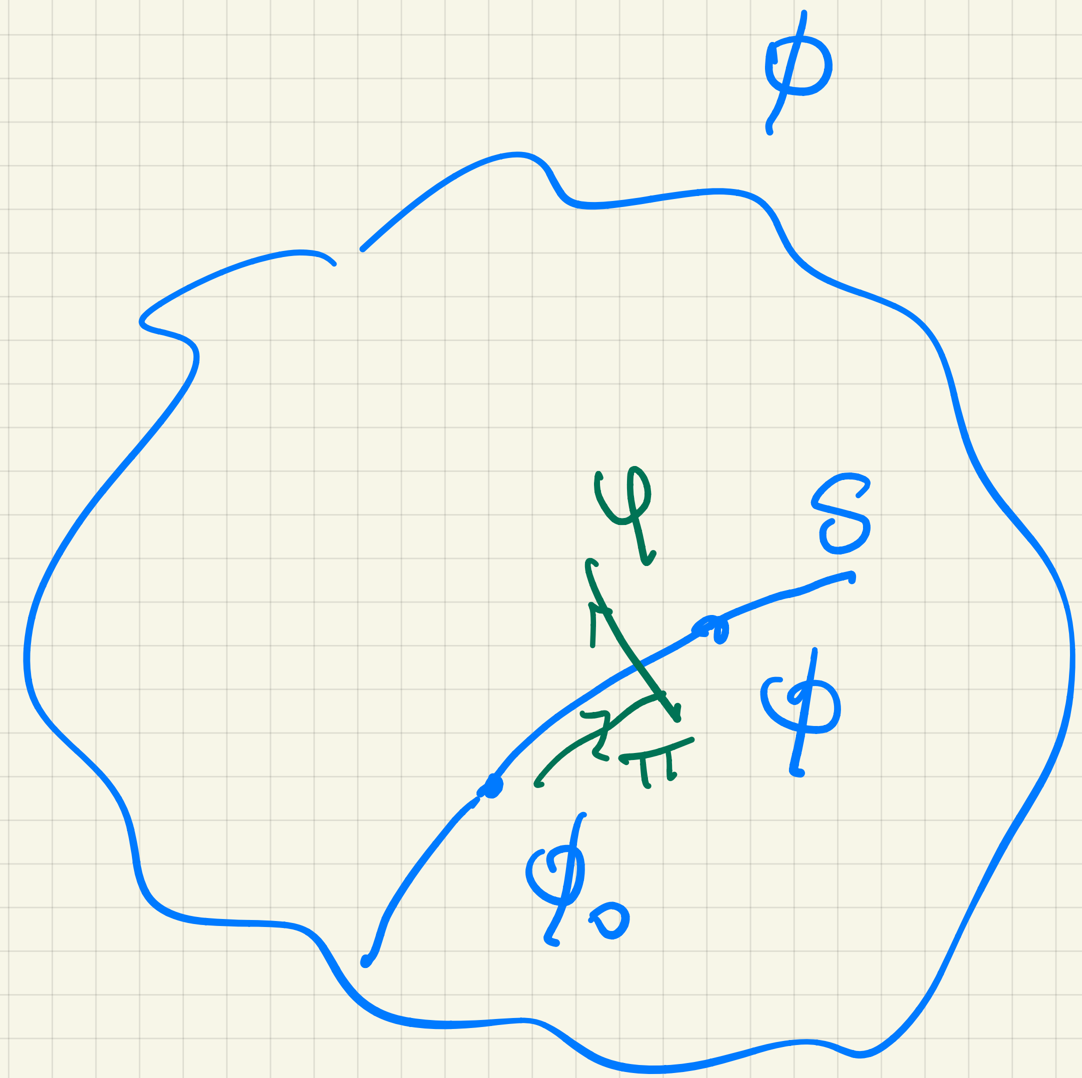
ϕ scalars

$g \in G: \quad \phi \to g\phi$

ϕ_0 ground state

$H = \{h \in G : h\phi_0 = \phi_0\}$ unbroken

$H \neq G$



Along γ: $\phi = \exp(\pi_a \hat{t}_a)\phi_0$ $\hat{t}_a$ broken generators

transf. of π under G

$\phi = \exp(\pi_a \hat{t}_a)$ $g\phi = \exp(\pi'_a \hat{t}_a)\phi_0$ $g: \pi \to \pi'$

$$\boxed{g \exp(\pi_a \hat{t}_a) = \exp(\pi'_a \hat{t}_a) h} \qquad h = h(g, \pi) \in H$$

$$g(\phi) = g \exp(\pi_a \hat{t}_a)\phi_0 = \exp(\pi'_a \hat{t}_a) h \phi_0$$
$$= \exp(\pi'_a \hat{t}_a)\phi_0$$

CCWZ:

$\exists\ \varphi$ "standard coordinates"

$\phi = (\pi_a, \varphi)$

$\phi_0 \hookrightarrow \varphi = 0$

φ transform linearly under H $\quad \varphi \to D(h)\varphi \quad D(h)$ linear repr. of H

$$\left\{\begin{array}{l} \pi \to \pi' \\ \varphi \to D(h(g,\pi))\varphi \end{array}\right\} g \in G$$

$G = SU(2)_L \times U(1)_Y$

$H = U(1)_{em}$

$g \in G \qquad g = u_L e^{i\alpha y} \qquad u_L \in SU(2)_L$

$h \in H \qquad h = e^{i\alpha q}$

q unbroken

t^L_a gen of $SU(2)_L$ broken

$\Sigma = e^{i G_a t^L_a} \qquad g\Sigma = \Sigma' h \qquad h(g, \Sigma)$

$$g\Sigma = u_L e^{i\alpha y} \underset{\underset{SU(2)_L}{|}}{\Sigma} = u_L \Sigma e^{i\alpha(q-t_3)} =$$

$$= \underbrace{u_L \Sigma e^{-i\alpha t_3^L}}_{SU(2)_L} \underbrace{e^{i\alpha q}}_{U(1)_{em}} = \Sigma' h$$

$$\boxed{\begin{aligned} &\Sigma \to \Sigma' = u_L \Sigma e^{-i\alpha t_3^L} \\ &h = e^{i\alpha q} \end{aligned}}$$

$$\mathcal{L}_\Sigma = \frac{v^2}{2} Tr\left[(D_\mu\Sigma)^\dagger (D^\mu \Sigma)\right] + a_T \frac{v^2}{4} Tr\left(\Sigma^\dagger D_\mu \Sigma \sigma_3\right)^2$$

$$D_\mu \Sigma = \partial_\mu \Sigma + i g W_\mu^a \frac{\sigma_a}{2}\Sigma - i g' \Sigma \frac{\sigma_3}{2} B_\mu$$

$$W_\mu = W_\mu^a \frac{\sigma_a}{2} \qquad \tilde{W}_\mu = \Sigma^{-1} W_\mu \Sigma - \frac{i}{g} \Sigma^{-1} \partial_\mu \Sigma$$

$SU(2)_L$ gauge transf. by Σ on W_μ

$$\mathcal{L}_\Sigma = M^2_{W,0}\left(1+\frac{a_T}{2}\right)\tilde{W}^+_\mu \tilde{W}^{\mu -} + \frac{M^2_{Z,0}}{2}\left(1-\frac{a_T}{2}\right)\tilde{Z}_\mu \tilde{Z}^\mu$$

$$M^2_{W,0} = \frac{g^2 v^2}{2} \qquad \tilde{W}^\pm_\mu = \ldots$$

$$M^2_{Z,0} = \frac{g^2+g'^2}{2} v^2 \qquad \tilde{Z}_\mu = \ldots$$

Unitary gauge $\Sigma = \mathbb{1}$ $\quad \mathcal{L}_\Sigma = M_W^2 W^+_\mu W^{\mu -} + \frac{M_Z^2}{2} Z_\mu Z^\mu$

$$M_W^2 = M_{W,0}^2 \left(1 + \frac{\alpha T}{2}\right) \qquad \rho = \frac{M_W^2}{c_W^2 M_Z^2} = \frac{1+\alpha T/2}{1-\alpha T/2}$$

$$M_Z^2 = M_{Z,0}^2 \left(1 - \frac{\alpha T}{2}\right)$$

$$\rho_{exp} = 1 + O\left(\frac{1}{200}\right) \qquad |\alpha T| \lesssim \frac{1}{200}$$

$\longleftrightarrow \quad \mathcal{L}(\Sigma)$ is approximately $SU(2)_L \times SU(2)_R$ symm.

$$\Sigma \to U_L \Sigma U_R^\dagger \qquad U_L \in SU(2)_L \qquad U_R \in SU(2)_R$$

$\Sigma = 1 \qquad SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$ diagonal

(custodial $SU(2)$)

$$\varphi \to \mathcal{D}\left(h(g,\Sigma)\right)\varphi$$

φ singlet of $SU(3)_c \times U(1)_{em}$.

$$\varphi \to \varphi \qquad \mathcal{D} = \mathbb{1}$$

Singlet under the full $SU(2)_L \times U(1)_Y$

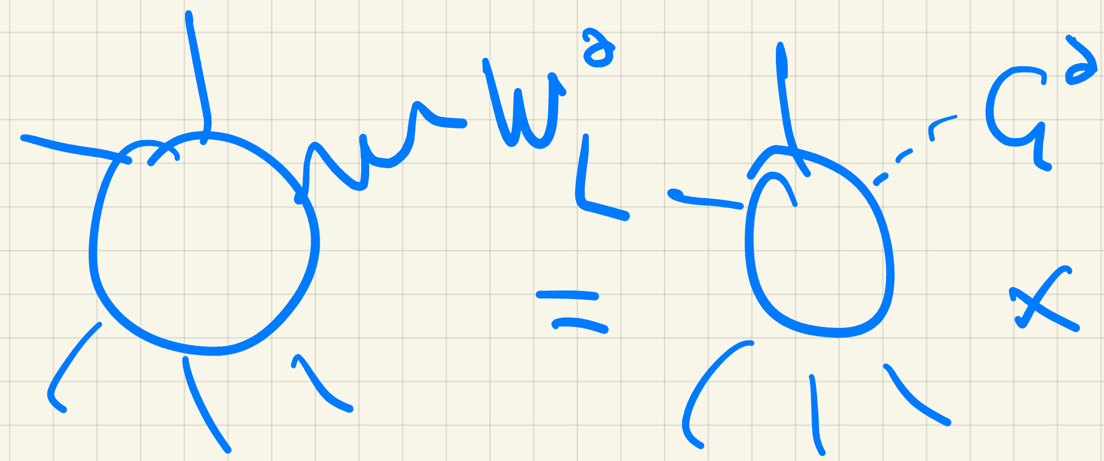
$$\mathcal{L} = \frac{v^2}{2} \mathrm{Tr}\left[(D_\mu \Sigma)^\dagger (D^\mu \Sigma)\right]\left(1 + \sqrt{2}\, a \frac{\varphi}{v} + b \frac{\varphi^2}{2v^2} + \dots\right)$$

$$+ \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$$

$a_T = 0$

$W_L W_L \to W_L W_L$

Equivalence theorem:



$\times\left(1+\mathcal{O}\left(\frac{v^2}{E^2}\right)\right)$

$$A(G^+G^- \to G^+G^-)_{\text{tree}} = \frac{s+t}{2v^2}(1-a^2) + \mathcal{O}\left(\frac{v^2}{E^2}\right)$$

"unitarisation" for $a = 1$

$$A(G^+G^- \to \varphi\varphi)_{\text{tree}} = \frac{s}{2v^2}(b-a^2) + \mathcal{O}(\ldots)$$

"unit." for $b = a^2 = 1$

$a \approx 1$

SM: $a = b = c = 1$ (no higher orders in φ^n)

$H = \Sigma \begin{pmatrix} 0 \\ v + \frac{\varphi}{\sqrt{2}} \end{pmatrix}$ recover $\mathcal{L}_{SM}$

$V_{SM} \rightarrow$ $V\left(\sqrt{|H|^2 - v^2}\right)$

$$\delta = \frac{1}{2} \qquad H = \langle H \rangle + H'$$

$$-\mathcal{L}_{SM}^{flavour} = \lambda^{E}_{ij}\, \overline{e_{R}}_{i} L_{j} H^{*} + \lambda^{D}_{ij}\, \overline{d_{R}}_{i} Q_{j} H^{*} + \lambda^{U}_{ij}\, \overline{u_{R}}_{i} Q_{j} \varepsilon H + h.c.$$

$$= \cdots \langle H \rangle^{*} \cdots \langle H \rangle^{*} \cdots \langle H \rangle \cdots$$

$$+ \qquad H'^{*} \qquad H'^{*} \qquad H'$$

$$\mathcal{L}^{\text{free}}_{\text{fermion}} = \overline{u_{iL}}\, i\gamma^\mu \partial_\mu u_{iL} + \overline{d_{iL}}\, i\gamma^\mu \partial_\mu d_{iL} + \overline{e_{iL}} \ldots$$

$$\overline{u_{iR}}\, i\gamma^\mu \partial_\mu u_{iR} + \overline{d_{iR}}\, i\gamma^\mu \partial_\mu d_{iR} + \ldots$$

$$- m^U_{ij}\, \overline{u_{iR}}\, u_{jL} - m^D_{ij}\, \overline{d_{iR}}\, d_{jL} - m^E_{ij}\, \overline{e_{iR}}\, e_{jL} + h.c.$$

$$u'_{iR} = U^{u_R}_{ij}\, u_{jR}$$

$$u'_{iL} = U^{u_L}_{ij}\, d_{jL}$$

$$d'_{iR} = U^{d_R}_{ij}\, d_{jR}$$

$$d'_{iL} = U^{d_L}_{ij}\, d_{iL}$$

1) kin. term preserved

2) mass term $\longrightarrow$

$$- m_{u_i}\, \overline{u'_{iR}}\, u'_{iL} - m_{d_i}\, \overline{d'_{iR}}\, d'_{iL} + h.c.$$

$$- m_{e_i}\, \overline{e_{iR}}'\, e'_{iL}$$

1) U unitary

2) $m_u = U_{uR}^\dagger m_u^{diag} U_{uL}$ $\qquad m_D = U_{dR}^\dagger m_D^{diag} U_{dL}$

$\qquad \underset{\text{diagonal} \geq 0}{}$ $\qquad\qquad m_E = U_{eR}^\dagger m_E^{diag} U_{eL}$

diagonal ≥ 0

Th. m generic $n \times n$ complex matrix

$\exists$ U_R U_L $n \times n$ unitary s.t. $m = U_R^\dagger m_{diag} U_L$

≥ 0