$$\cdots = \mathcal{L}_{kin} - g_s G^A_\mu j^\mu_{SA} - \frac{g}{\sqrt{2}} W^+_\mu j^\mu_+ - \frac{g}{\sqrt{2}} W^-_\mu j^\mu_- - eA_\mu j_{em} - \frac{g}{c_w} Z_\mu j^\mu_n$$

$$j^\mu_{em} = Q_u \bar{u}_{iL} \gamma^\mu u_{iL} + Q_d \bar{d}_{iL} \gamma^\mu d_{iL} + \cdots - Q_u \bar{u}_{Ri} \gamma^\mu u_{Ri} \cdots$$

$$= Q_u \bar{u}^i_{iL} \gamma^\mu u^i_L \qquad \text{(same form)}$$

$$j^\mu_n = \sum_{\substack{f = u_i\, d_i\, \nu_i\, e_i \\ x = L, R}} \bar{f}_x \gamma^\mu (T_3 - s^2_w Q) f_x = \cdots \text{(same form)}$$

$$j^\mu_+ = \bar{Q}_i \gamma^\mu T_+ Q_i + \bar{l}_i \gamma^\mu T_+ l_i =$$

$$= \overline{u}_{iL} \gamma^\mu d_{iL} + \overline{\nu}_{iL} \gamma^\mu e_{iL}$$

$$u'_{iL} = U^{u_L}_{ij} u_{jL}$$

$$= V_{ij} \overline{u}'_{iL} \gamma^\mu d'_{jL} + \overline{\nu}'_{iL} \gamma^\mu e'_{iL}$$

$$d'_{iL} = U^{d_L}_{ij} d_{jL}$$

$$\boxed{V = U_{u_L} U^\dagger_{u_d}}$$

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$$e'_{iL} = U^{e_L}_{ij} e_{jL}$$

neutrino flavor eigenstate $\leftarrow$ $\nu'_{iL} = U^{e_L}_{ij} \nu_{jL}$

(if $m_\nu \neq 0$ PMNS U)

$$H = \begin{pmatrix} iG^+ \\ v + \frac{\varphi - iG^0}{\sqrt{2}} \end{pmatrix} \quad H' = \begin{pmatrix} iG^+ \\ \frac{\varphi - iG^0}{\sqrt{2}} \end{pmatrix} \quad \varphi \quad G^0 \quad G^+ \quad G^-$$

$$\ldots = \frac{\lambda_{e_i}}{\sqrt{2}} \overline{e'_{R_i}} e'_{L_i} \varphi + \frac{\lambda_{d_i}}{\sqrt{2}} \overline{d'_{R_i}} d'_{L_i} \varphi + \frac{\lambda_{u_i}}{\sqrt{2}} \overline{u'_{R_i}} u'_{L_i} + \text{h.c.}$$

$$\lambda_U = U^+_{u_R} \lambda_U^{\text{diag}} U_{u_L} \qquad m = v\lambda$$

$$\lambda_{e_i} = \frac{m_{e_i}}{v}$$

G^0 : $\varphi \to iG^0$

$G^\pm$:

$$\lambda_{d_i} V^\dagger_{ij}\, \overline{d'_{R_i}}\, u'_{L_j}\, i G^- + \lambda_{u_i} V_{ij}\, \overline{u'_{R_i}}\, d_{L_j}\, i G^+ + h.c.,$$

DROP " ' " from now on

$$V = \begin{pmatrix} e^{i\tau_1} & & 0 \\ & e^{i\tau_2} & \\ 0 & & e^{i\tau_3} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\sigma_1} & & 0 \\ & e^{i\sigma_2} & \\ 0 & & e^{i\sigma_3} \end{pmatrix} \quad e^{i\sigma_1} = 1$$

$$g = 3 + 4 + 2$$

$$\frac{g}{\sqrt{2}} V_{ij}\, \overline{u_{iL}}\, \gamma^\mu d_{jL}\, W^+_\mu = \frac{g}{\sqrt{2}} V^{CKM}_{ij}\, \overline{u'_{iL}}\, \gamma^\mu d'_{jL}\, W^+_\mu$$

$u'_{iL} = e^{i\tau_i} u_{iL} \qquad d'_{iL} = e^{i\sigma_i} d_{iL}$

P and CP

$(a, \Lambda) \qquad x^\mu \to \Lambda^\mu{}_\nu x^\nu + a^\mu \qquad P : x \to x_p = (x^0, -\vec{x})$

$P(a, \Lambda) P^{-1} = (a_p, \Lambda^{T-1}) \qquad \Lambda \to L \qquad L \to L^{T-1}$

$Q \to Q \qquad$ P $\qquad \psi_L \to \psi_R \ \overline{\psi_L}$

$Q \to -Q \qquad$ CP $\qquad \psi_L \to \overline{\psi_L}$

$\exists$ CP under which gauge int's are invariant

$\mathcal{L}_{QCD+QED} \subseteq \mathcal{L}_{SM}$ invariant under CP

$$\psi(x) \rightarrow \underline{e^{i\theta_\psi}} \, C \, \psi^*(x_P) \qquad C \hat{=} \begin{pmatrix} \varepsilon & 0 \\ 0 & -\varepsilon \end{pmatrix}$$

$$\psi = u_L, u_R, d_L, d_R, e_L, e_R \qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$V^\mu(x) \rightarrow -V_\mu^*(x_P)$$

$$\frac{g}{\sqrt{2}} V_{ij} \overline{u_{iL}} \gamma^\mu d_{jL} W_\mu^+ + \frac{g}{\sqrt{2}} V_{ij}^\dagger \overline{d_{iL}} \gamma^\mu u_{jL} W_\mu^- \longrightarrow$$

$$\frac{g}{\sqrt{2}} V_{ij}^\dagger \overline{u_{jL}} \gamma^\mu d_{iL} W_\mu^+ \, e^{i(\theta_{u_i} - \theta_{d_j})} + \ldots =$$

CP conserved $\Leftrightarrow$ $\underbrace{e^{-i\theta_{u_i}} V_{ij}\, e^{i\theta_{d_j}}}_{\text{III}\; V'_{ij}}$ real

1, 2 families yes

3 no if $\sin\delta \neq 0$

$$\Im m\left(V'_{us} V'^{*}_{cs} V'_{cb} V^{*\prime}_{us}\right) \quad \text{Jarlskog invariant}$$

$$= \Im m\left(V_{us} V^{*}_{cs} V_{cb} V^{*}_{ub}\right)$$

$$= \underbrace{c_{12}\, c_{23}\, c^2_{13}\, s_{12}\, s_{23}\, s_{13}}_{\neq 0} \sin\delta$$