Classical Solutions and Background Independence in OSFT

Carlo Maccaferri
Torino University and INFN

Based on 1909.11675, JHEP 01 (2020) 021
w/ Ted Erler
Relevance of classical solutions in SFT

• SFT equation of motion can give new handles on exact (B)CFT’s which are not easily accessible from the standard CFT approach: the “honeycomb” c=2 BCFT (Kudrna-Schnabl-Vosmera), RR backgrounds in RNS (Cho-Collier-Yin), etc…

• Some backgrounds just don’t have a direct (B)CFT description (e.g. the tachyon condensate) and to study their physics one needs field theory-like tools.

• In QFT’s based on a path integral, classical solutions are the saddle points of the action and they account for non-perturbative contributions to amplitudes. If we want to have a non-perturbative understanding of string theory from SFT it is necessary to understand how string theory backgrounds appear as classical solutions.
Background independence in string field theory

- We can construct a string field theory on any given exact string background \((B)CFT_0\), with a dynamical field variable \(\phi^{(0)}\) and an action \(S[\phi^{(0)}]\)

- Classical solutions \(\phi^{(0)} = \Psi^{(0)}\), represent other consistent backgrounds \(B(CFT)^*_*\)

- Expand \(\phi^{(0)} = \Psi^{(0)} + \phi^{(0)}\): dynamics of fluctuations around the solution

\[
S \left[ \Psi^{(0)} + \phi^{(0)} \right] = S \left[ \Psi^{(0)} \right] + S^* \left[ \phi^{(0)} \right]
\]

- Background independence: the expanded action and the action directly formulated around \(B(CFT)^*_*\) should be related by field redefinition

\[
\phi^{(0)} = f \left( \phi^{(*)} \right)
\]

\[
S^* \left[ \phi^{(0)} \right] = S \left[ \phi^{(*)} \right]
\]

- A Jacobian is also generated from the field redefinition in the path integral measure (quantum effect)

\[
\mathcal{D} \phi^{(0)} = \mathcal{D} \phi^{(0)} = \mathcal{D} \phi^{(*)} \left| \frac{\mathcal{D} \phi^{(0)}}{\mathcal{D} \phi^{(*)}} \right|
\]
From now on let us focus on (classical) Witten bosonic Open String Field Theory

\[ S[\phi] = -\frac{1}{g_s} \text{Tr} \left[ \frac{1}{2} \phi Q \phi + \frac{1}{3} \phi^3 \right] \]

\[ \phi \in H_{\text{BCFT}_0} \]
- Solution to the equation of motion $Q\Psi + \Psi^2 = 0$ (Erler, C.M. 2014-2019)

\[
\Psi = \Psi_0 - \sum \Psi^*_v \sum
\]

- Equation of motion

\[
Q_{tv}\Sigma = Q\Sigma + \Psi^0_{tv}\Sigma - \sum \Psi^*_v \sum = 0
\]

\[
Q_{tv}\overline{\Sigma} = Q\overline{\Sigma} + \Psi^*_v \overline{\sum} - \overline{\sum} \Psi^0_{tv} = 0
\]
• In our construction we have

\[ \Sigma \Sigma = 1 \in H_{BCFT_*} \]
\[ \Sigma \Sigma = P \in H_{BCFT_0} \]

• The emerging star algebra projector \( P^2 = P \) has an interesting role in the proof of background independence, as we will see.

• Other important properties which descend from the previous ones

\[ Q_{\Psi} \left( \Sigma \phi^{(*)} \Sigma \right) = \Sigma \left( Q_B \phi^{(*)} \right) \Sigma \]
\[ \text{Tr}_0 [\Sigma \phi^{(*)} \Sigma] = \text{Tr}_* [\phi^{(*)}] \]
Observables

- **Action** (it computes the energy for static solutions)

\[
E = -\frac{1}{6} \text{Tr}[\Psi^3] = -\frac{1}{6} \text{Tr}[(\Psi_{tv})^3] + \frac{1}{6} \text{Tr}[\Sigma(\Psi_{tv}^*)^3 \Sigma] \tag{EOM}
\]

\[
= -\frac{1}{6} \text{Tr}[(\Psi_{tv})^3] + \frac{1}{6} \text{Tr}[(\Psi_{tv}^*)^3] = \frac{1}{2\pi^2} (-g_0 + g_*)
\]

- **Ellwood Invariants** (Boundary State of the new background)

\[
\langle I|V(i, -i)|\Psi \rangle \equiv \text{Tr}_V[\Psi] = \text{Tr}_V[\Psi_{tv}] - \text{Tr}_V[\Sigma \Psi_{tv}^* \Sigma]
\]

\[
= \text{Tr}_V[\Psi_{tv}] - \text{Tr}_V[\Psi_{tv}^*]
\]

\[
= \frac{1}{2\pi i} \left\langle c\bar{c}V^{(m)}(0)c(1) \right\rangle_{\text{disk}}^{\text{BCFT}_0} - \frac{1}{2\pi i} \left\langle c\bar{c}V^{(m)}(0)c(1) \right\rangle_{\text{disk}}^{\text{BCFT}_*}
\]
Background Independence

- To study the physics around the solution we shift

\[ \phi^{(0)} = \Psi^{(0)} + \phi^{(0)}_* \]

- Fluctuations \( \phi^{(0)}_* \) can be decomposed according to \( P \) and \( \bar{P} = 1 - P \)

\[ \phi^{(0)}_* = (P + \bar{P})\phi^{(0)}_*(P + \bar{P}) = \begin{pmatrix} \phi^{(0)}_{PP} & \phi^{(0)}_{P\bar{P}} \\ \phi^{(0)}_{\bar{P}P} & \phi^{(0)}_{\bar{P}\bar{P}} \end{pmatrix} \]

Matrix with \textbf{constrained} entries

- One-to-one linear field redefinition

\[
\begin{pmatrix}
\phi^{(0)}_{PP} & \phi^{(0)}_{P\bar{P}} \\
\phi^{(0)}_{\bar{P}P} & \phi^{(0)}_{\bar{P}\bar{P}}
\end{pmatrix} = \begin{pmatrix} \Sigma & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi^{(*)}_{11} & \bar{x}_{1\bar{P}} \\ \chi_{\bar{P}1} & t_{\bar{P}\bar{P}}^{(0)} \end{pmatrix} = \begin{pmatrix} \Sigma & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi^{(0)}_{PP} & \phi^{(0)}_{P\bar{P}} \\ \phi^{(0)}_{\bar{P}P} & \phi^{(0)}_{\bar{P}\bar{P}} \end{pmatrix} \]

\[ \phi^{(*)}_{11} \in H_{BCFT_0} \]

Expected new variables

\( \chi_{\bar{P}1} \in H_{0^*} \)

Un-expected extra variables

\( \bar{x}_{1\bar{P}} \in H_{*0} \)

(constrained)
• The action can then be rewritten as (see also Kishimoto, Masuda, Takahashi)

\[ S[\Psi + \phi^{(0)}] = \frac{1}{2\pi^2 g_s} (g^{(0)} - g^{(*)}) - \frac{1}{g_s} \text{Tr}_* \left[ \frac{1}{2} \phi^{(*)} Q\phi^{(*)} + \frac{1}{3} (\phi^{(*)})^3 \right] \]

\[ -\frac{1}{g_s} \text{Tr}_0 \left[ \frac{1}{2} tQ_{tv} t + \frac{1}{3} t^3 + \chi Q_{tv,*} \bar{\chi} + t\chi\bar{\chi} + \chi\phi^{(*)} \bar{\chi} \right] \]

Extra constrained fluctuations at the TV

\[ P\chi = \bar{\chi}P = Pt = tP = 0 \]

• However the only perturbative solution around the TV is \( \chi = \bar{\chi} = t = 0 \)

• In the perturbative expansion around the saddle, the TV sector can be integrated out, setting it to zero. The action of the physical fluctuation \( \phi^{(*)} \) DOES NOT change.

• This is very different from the integration out of the massive fields (\( Q_{tv} \) has no cohomology, “nothing is left”)

• Amplitudes in the background of the solution are mapped to amplitudes in the new BCFT background.
**Background independence:**

*shift + field redefinition + integration out of trivial fields*

(expected)  (something genuinely new!)

- The constrained TV sector would be absent if $\Sigma \overline{\Sigma} = 1$. Can this be possible?

- In this case the cohomologies of the two backgrounds would be strictly isomorphic but this is something we don’t expect in general (except for special exactly marginal deformations)

- So, physically, $\Sigma \overline{\Sigma} \neq 1$ is there to allow to connect genuinely different backgrounds.

- Gauge equivalence in $H_{BCFT_0}$ implies gauge equivalence in $H_{BCFT^*}$ and viceversa. The extra pure gauge TV sector accounts for that.
Explicit realization: the flags

- The intertwining fields can be explicitly constructed from a TV solution and bcc operators ("twist fields")

\[
\Psi_{tv} = \sqrt{F(K)}c \frac{BK}{1 - F^2(K)} c\sqrt{F(K)}
\]

\[
Bc + cB = 1 \\
QB = K \\
c^2 = B^2 = 0 \\
Qc = cKc
\]

\[
\Sigma = Q_{tv}\left(\sqrt{\frac{1 - F^2}{K}}B|\sigma\right)\sqrt{\frac{1 - F^2}{K}}
\]

\[
(\bar{\sigma} | | \sigma) = 1 \in H_{BCFT*}
\]

\[
\bar{\Sigma} = Q_{tv}\left(\sqrt{\frac{1 - F^2}{K}}(\bar{\sigma}|B\right)\sqrt{\frac{1 - F^2}{K}}
\]

\[
(\bar{\sigma} | B | \sigma) = B \in H_{BCFT*}
\]

\[
\bar{\Sigma}\Sigma = 1
\]
The Flags as surface states with bcc insertion

\[ | \sigma \rangle = \bordermatrix{ & L & R \\ & \text{BCFT}_0 & \text{BCFT}_* \\ \circ \sigma(0) & f & \circ \sigma(0) } \]

\[ (\bar{\sigma} | = \bordermatrix{ & L & R \\ & \text{BCFT}_* & \text{BCFT}_0 \\ \bar{\circ} \sigma(0) & \bar{f} & \bar{\circ} \sigma(0) } \]

It is possible to map the surfaces obtained by gluing flags and wedge states to the UHP: Schwarz-Christoffel map
Multiplying in the order $(\bar{\sigma} \mid \star \mid \sigma)$: degenerating surface

$$\epsilon \rightarrow 0$$

$$\begin{array}{c}
\begin{tikzpicture}
\draw[blue, very thick] (-2,0) -- (2,0);
\draw[red, very thick] (-2,0) -- (-2,2);
\draw[red, very thick] (2,0) -- (2,2);
\filldraw[blue] (-2,0) circle (2pt);
\filldraw[blue] (2,0) circle (2pt);
\end{tikzpicture}
\end{array}$$

$$= \begin{array}{c}
\begin{tikzpicture}
\draw[blue, very thick] (-2,0) -- (2,0);
\filldraw[blue] (-2,0) circle (2pt);
\filldraw[blue] (2,0) circle (2pt);
\end{tikzpicture}
\end{array}$$

$$= \frac{1}{g^*} \langle I \circ \bar{\sigma}(0) \sigma(0) \rangle \times 1^{(*)}$$

(normalization)

$$(\bar{\sigma} \mid \mid \sigma) = 1$$
• Multiplying in the order $|\sigma\rangle \star (\bar{\sigma}|$ : **new kind of surface state**

It looks like the identity string field towards the midpoint but it has a non degenerate boundary and it is “left/right” factorized towards the endpoints!

$|\sigma\rangle(\bar{\sigma}| = \text{new projector}$
MULTIBRANES

\[ \Psi = \Psi_{tv} - \sum_{i} \Sigma_i \Psi_{tv} \Sigma_i \]

- Easily realized by orthogonal flags (generation of Chan-Paton’s factors)

\[ \Sigma_i \Sigma_j = \delta_{ij} \]

- Obtained by choosing bcc operators as

\[ \delta_{ij} = \langle I \circ \bar{\sigma}_i(0) \sigma_j(0) \rangle \]

- **Universal multibranes**: take \((\sigma_i, \sigma_j)\) in the matter Verma module of the identity and diagonalize the Gram matrix.
Comments on non-perturbative amplitudes

- The **multi-brane solution** can be used to account for multi D-instantons contributions to closed string’s scattering in two-dimensional string theory (Sen, Baltazar-Rodriguez-Yin) using **Ellwood invariants** and **open string propagators** (Witten vertex covers all bosonic moduli space). Open-closed amplitudes. Integration on moduli space from open string massless states (D-instanton moduli)

\[
< V_1 \cdots V_n >^{(1D-Instanton)} \xrightarrow{2-brane} < V_1 \cdots V_n >^{(2D-Instantons)} \xrightarrow{3-brane} \text{ etc}
\]

- The **0-instanton** sector should be the tachyon vacuum. Scattering of Ellwood invariants at the tachyon vacuum should give purely closed string amplitudes (Gaiotto-Rastelli-Sen-Zwiebach, et al)

\[
< V_1 \cdots V_n >^{(0D-Instanton)} \xleftrightarrow{TV} < V_1 \cdots V_n >^{(1D-Instantons)}
\]

- Since bosonic two-dimensional string theory makes sense at the quantum level, this is a concrete arena to test the solution.
EXPLICIT REALIZATION OF THE FLAGS

\[
\text{Tr} \left[ \Omega^{\beta/2} (\bar{\sigma} | \Omega^\alpha | \sigma) \Omega^{\beta/2} \right] = \\
\frac{2\ell}{\pi} \left( \frac{p(1 + s^2)}{s^2 - p^2} \tan^{-1} u + \tanh^{-1} \frac{u}{p} \right)
\]
FOCK SPACE COEFFICIENTS

• The solution has a Fock space expansion

\[ \Psi = \sum_i \psi_i \ c \phi^i(0) \ |0\rangle + (\ldots) \]

\[ \psi_i = K(h_i, h_\sigma) \ C_{i\sigma\bar{\sigma}} \]

• \(K\) is a universal function which depends on the choice of tachyon vacuum and on the details of the Schwarz-Christoffel map

• \(C_{i\sigma\bar{\sigma}}\) is the basic three-point function \(\langle I \circ \bar{\sigma}(0) \ \hat{\phi}_i(1) \ \sigma(0) \rangle\)

• We have analysed the \(gh=0\) toy model (\(gh=1\) needs 7-dimensional integral on an implicitly defined region, the toy model “only” 3 )

\[ \Gamma_* = 1 - \sqrt{1 + K \ |\sigma\rangle \frac{1}{1 + K} (\bar{\sigma}) \sqrt{1 + K} } \]
EXAMPLE: the Cosh Rolling Tachyon

• One of the advantage of this solution is the possibility to describe time dependent background (just as any other background)

• Sen’s Rolling tachyon BCFT: exactly marginal deformation of Neumann bc

\[ e^{\lambda \int_{a}^{b} ds \cosh(X^0)(s)} = \sigma_{\lambda}(b) \bar{\sigma}_{\lambda}(a) \]

• Periodic moduli space \( \lambda \in [0,1) \),

• \( \lambda = 0 \): Neumann b.c. for \( X^0 \) (perturbative vacuum)

• \( \lambda = \frac{1}{2} \): multiple Dirichlet b.c. at imaginary values \( X^0 = 2\pi i(n + 1/2) \)

• The boundary state vanishes at \( \lambda = 1/2 \) for real time (but non-trivial support at imaginary time) : Is it the Tachyon Vacuum?
OSFT solution for rolling tachyon

- The tachyon profile of $\Gamma_*$ is given as

$$T^{toy}(x^0, \lambda) = \sum_{n \in \mathbb{Z}} T_n^{toy}(\lambda) e^{nx^0} = \langle \phi_n^{toy}(\lambda) | \Gamma_* \rangle$$

$$|\phi_n^{toy}(\lambda)\rangle = -\frac{1}{2 \langle 0|0 \rangle_{\text{matter}}} c \partial c \partial^2 c e^{-n \chi^0(0)} |0\rangle.$$ 

- The needed input is the three-point function which we computed

$$\langle I \circ (\phi_n^{toy}(\lambda)(0)) \sigma_\lambda(1) \overline{\sigma}_\lambda(0) \rangle_{\text{UHP}} = (-1)^n 4^{-n^2} \frac{\mathcal{P}_n(\lambda)}{\mathcal{P}_n(\frac{1}{2})}, \quad (n \geq 0)$$

$$\mathcal{P}_n(\lambda) = \lambda^n \prod_{j=1}^{n-1} (j^2 - \lambda^2)^{n-j}$$
• The obtained tachyon profile displays the well-know oscillations at late times

\[ \lambda = 0.01 \quad \lambda = 1/2 \]

• It doesn’t look that \( \lambda = 1/2 \) corresponds to the tachyon vacuum. Moreover the marginal current would be a physical state at the tachyon vacuum, which is not expected to happen

• In fact at \( \lambda = 1/2 \) a new branch of moduli space opens up, allowing to translate the (imaginary) D-branes in imaginary time (Gaiotto-Itzhaki-Rastelli). The TV should correspond to pushing these D-branes at imaginary infinity.
• To give a new viewpoint on this problem, we can follow the new imaginary branch with our solution (multiple imaginary lumps at increasing separation)

• We constructed solutions for increasing value of imaginary separation

\[ x^0 = 2\pi i \left( n + \frac{1}{2} \right) \quad \xrightarrow{\text{imaginary branch}} \quad x^0 = ia \left( n + \frac{1}{2} \right), \quad n \in \mathbb{Z} \]

• Now indeed the tachyon profile sits at the tachyon vacuum for larger and larger time

• Notice that these different backgrounds have all vanishing boundary state (in real time).
Conclusions

• For the first time we have a non perturbative realisation of background independence in bosonic open string field theory.

• Can we use this solution to account for non-perturbative contributions when the bosonic string makes sense at the quantum level?

• Can we better understand time-dependent backgrounds with vanishing boundary state in OSFT? (closed string radiation expressed in open string variables)

• It would be very desirable to have a similar solution for open-superstring field theory. Field theory understanding of RR charge? Instanton contributions etc… As of now we have it for cubic superstring field theory at picture zero (see Noris talk).

• Ideally we would like to be able to connect closed string backgrounds in a similar way.