The Standard Model (and its flavour anomalies)

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Particle physics in three lines

The SM describes the
Strong, Electromagnetic, and Weak interactions
of the known elementary particles

	$\bar{\Psi}_i i\gamma^\mu D_\mu \Psi_i - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$	gauge	~ ‰
$\mathcal{L}^{\text{ren}}_{\text{SM}} =$	$+\lambda_{ij} \Psi_i \Psi_j H + \text{h.c.}$	flavor	~ %
	$+\lvert D_\mu H\rvert^2 - V(H)$	symmetry breaking	~ 10%

+ gravity: all known phenomena (with few notable exceptions)

Outline

- Preliminaries
- The SM
  - Gauge sector
  - Higgs sector
  - Flavour sector
- in a new physics perspective

the SM as a relativistic renormalisable gauge QFT

Preliminaries (I)

Relativistic Quantum Field Theory (QFT)

- $\hbar = c = 1$
- Degrees of freedom: fields. **S = 0** (scalar) **S = 1/2** (spinor) **S = 1** (vector)
- Dynamics: $S = \int d^4x\, \mathcal{L}(x)$ $[\mathcal{L}] = 4$ $\mathcal{L}(x) = \sum_i c_i \mathcal{O}_i(x)$ Lorentz Scalar
- $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$
  - $\mathcal{L}_{\text{free}}$ 2-linear in the fields
  - $\mathcal{L}_{\text{int}}$ ≥ 3-linear in the fields
- Linear terms (scalars) can be reabsorbed by shifting fields (SSB)
- Constant term not physical without gravity

Renormalisability and relevance

- $\mathcal{L}(x) = \sum_i c_i \mathcal{O}_i(x)$ dimension **D** of **O(x)**:
  - D = 0   constant terms, not physical without gravity
  - D = 1   linear (scalar) terms, reabsorbed by shifting fields (SSB)
  - **D = 2,3**   "relevant" operators   **[c] = M^2, M**
  - **D = 4**   "marginal" operators   **[c] = g**
  - **D ≥ 5**   "irrelevant" operators   **[c] = $1/M^{D-4}$**

renormalisable
theory (SM)
finite # of c.t.

- Irrelevant operators: effect suppressed by **$(E/M)^{D-4}$** at E « M
  - any theory looks renormalizable at sufficiently low scales

Gauge theories

- Specific form of QFT
  - gauge transformations ↔ equivalent field configurations
  - necessary to consistently describe vector fields
  - extremely successful (QED & photons, QCD & gluons)
- Defined by
  - Gauge (compact) Lie group G
  - A massless vector field to each generator of G: $t_A \leftrightarrow v^A_\mu$
  - Action on ``matter" fields (commuting with Lorentz)
  - Globally invariant $\mathcal{L}_0(\phi, \partial\phi)$

$$\mathcal{L} = \mathcal{L}_0(\phi, D_\mu \phi) - \frac{1}{4} v^A_{\mu\nu} \, v^{\mu\nu}_A$$

$$\partial_\mu \to D_\mu = \partial_\mu + i g_A v^A_\mu T_A \qquad t_A \to T_A$$

$$v^A_{\mu\nu} = \partial_\mu v^A_\nu - \partial_\nu v^A_\mu - g f^A_{BC} v^B_\mu v^C_\nu \qquad [t_A, t_B] = i f^C_{AB} t_C$$

$$\overline{\psi} i \gamma^\mu D_\mu \psi = \overline{\psi} i \gamma^\mu \partial_\mu \psi - \boxed{g_A \overline{\psi}_i \gamma^\mu T^{ij}_A \psi_j v^A_\mu}$$

digression on L and R spinors

Preliminaries (II)

QED and QCD written in terms of Dirac spinors

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \overline{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi$$

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$$

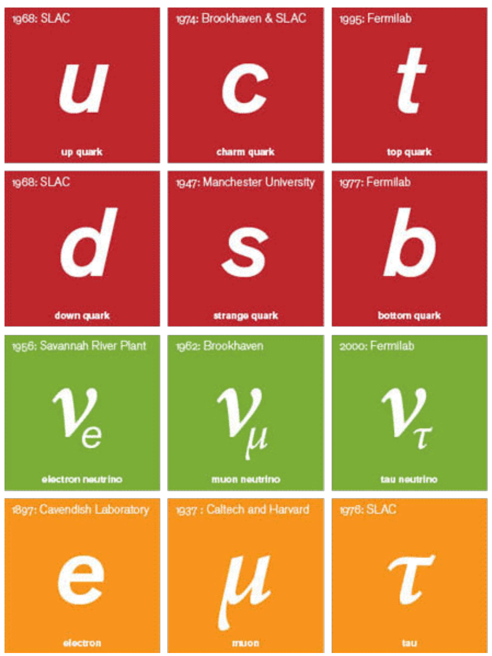
A Dirac spinor is not "elementary"

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \to \left(\begin{array}{c|c} L^{\dagger -1} & \\ \hline & L \end{array}\right) \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \qquad (\det L = 1)$$

$$\Psi_R(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ 0 \\ 0 \end{pmatrix} \qquad \Psi_L(x) = \begin{pmatrix} 0 \\ 0 \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \qquad \Psi = \Psi_L + \Psi_R$$

inequivalent, conjugated representations of SL(2,C)

Left- and Right-handed



$\rightarrow u_i$

$\rightarrow d_i$

$\rightarrow \nu_i$

$\rightarrow e_i$

$u_{iL}\ d_{iL}\ \nu_{iL}\ e_{iL}$ L

$u_{iR}\ d_{iR}\ e_{iR}$ R

$\overline{u_{iL}}\ \overline{d_{iL}}\ \overline{\nu_{iL}}\ \overline{e_{iL}}$ R

$\overline{u_{iR}}\ \overline{d_{iR}}\ \overline{e_{iR}}$ L

Most general gauge transformation can mix all L

The gauge sector of the SM lagrangian

$$\mathcal{L}_{\rm SM}^{\rm ren} = \boxed{\bar{\Psi}_i i\gamma^\mu D_\mu \Psi_i - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \qquad \text{gauge}}$$

$$+\lambda_{ij} \Psi_i \Psi_j H + \text{h.c.} \qquad \text{flavor}$$

$$+|D_\mu H|^2 - V(H) \qquad \text{symmetry breaking}$$

Gauge sector

$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$$Q_i = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \qquad L_i = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$$

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
$\bar{e}_{iR}$	1	1	1
Q_i	3	2	1/6
$\bar{u}_{iR}$	3^*	1	-2/3
$\bar{d}_{iR}$	3^*	1	1/3

Y

L-handed spinors

i = 1,2,3

$$\mathcal{L}_{\rm SM}^{\rm gauge} = \sum_{\psi = Q_i, u_{iR}, d_{iR}, L_i, e_{iR}} \bar{\psi} i \gamma^\mu D_\mu \psi - \frac{1}{4} \sum_{v = G_A, W_a, B} v^{\mu\nu} \, v_{\mu\nu}$$

“Blackboard”

QED + QCD + effective weak interactions + gauge principle

= Standard Model gauge theory

“Blackboard”

Gauge sector

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
$\overline{e}_{iR}$	1	1	1
Q_i	3	2	1/6
$\overline{u}_{iR}$	3^*	1	-2/3
$\overline{d}_{iR}$	3^*	1	1/3

Y

$$\mathcal{L}_{\rm SM}^{\rm gauge} = \sum_{\psi=Q_i,u_{iR},d_{iR},L_i,e_{iR}} \overline{\psi} i\gamma^\mu D_\mu \psi - \frac{1}{4} \sum_{v=G_A,W_a,B} v^{\mu\nu}\, v_{\mu\nu}$$

The Standard Model II

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Puzzles (gauge sector)

- Why 3 copies of the same representation of G_{SM}?
- Why G_{SM}?
- Why those quantum numbers? And in particular
  - why Y / Q is quantised?
  - why anomalies cancel (see below)?

Notable features (gauge sector)

- chirality
- anomaly cancellation
- electroweak precision tests

Gauge sector

$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$$Q_i = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$$

$$L_i = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$$

	SU(3)	SU(2)	U(1)
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$\bar{d}_{iR}$	3^*	1	1/3
			Y

L-handed spinors

i = 1,2,3

A nice property

- The fermion content is “chiral”: no L and R with same G_{SM} quantum numbers
    - Equivalently: r irrep on L $\Rightarrow$ r* is not
    - Equivalently: no explicit (G_{SM} symmetric) fermion mass term is allowed
- SM fermion masses $\propto \langle H \rangle$
- Extra heavy fermions ($M \gg \langle H \rangle$) should be “vectorlike”
- A puzzle, a blessing, or what expected?

Anomalies

- A symmetry of the lagrangian is not necessarily preserved by quantum corrections. If not, the symmetry is **anomalous**
- Global symmetry: OK + interesting physics
  - axial U(1) in QCD
  - scale invariance
- In the case of gauge transformations: INCONSISTENCY
- A gauge symmetry survives quantum corrections iff $\mathrm{Tr}\,(T_a \{T_b\, T_c\}) = 0$ where T_a are the generators and the trace is on L-handed fermions
- QED and QCD automatically non-anomalous

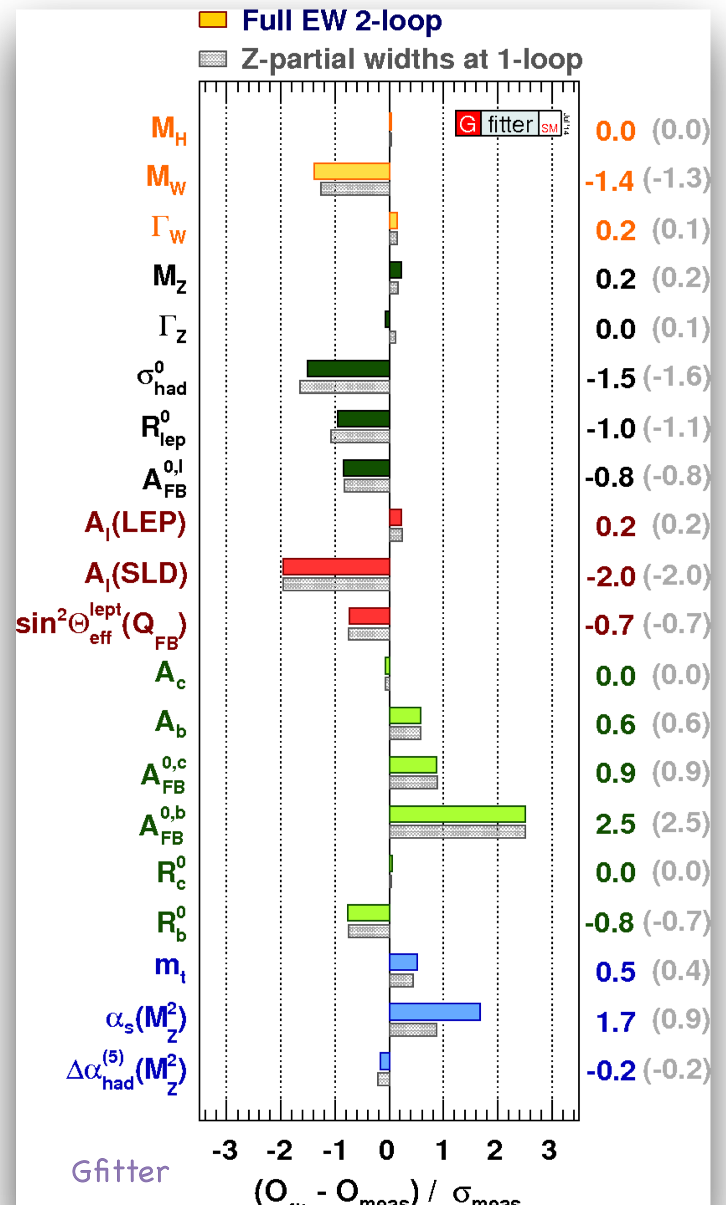
Anomaly cancellations in the SM

- Anomaly cancellation
- Is $T_{ijk} \equiv \mathrm{Tr}\,(\tau_i \{\tau_j, \tau_k\}) = 0$? $\tau_i = T_A, T_a, Y$

$SU(3)^3$	vectorlike
$SU(3)^2 \times SU(2)$	$\mathrm{Tr}(\sigma_a) = 0$
$SU(3)^2 \times U(1)$	$2\,Y_q + Y_{\overline{u_R}} + Y_{\overline{d_R}} = 0$
$SU(3) \times (\text{not } SU(3))^2$	$\mathrm{Tr}(\lambda_A) = 0$
$\ldots$	
$SU(2)^2 \times U(1)$	$Y_l + 3Y_q$
$U(1)^3$	$2Y_l^3 + 6Y_q^3 + 3Y_{\overline{u_R}}^3 + 3Y_{\overline{d_R}}^3 + Y_{\overline{e_R}}^3 = 0$
grav. anomaly	$2Y_l + 6Y_q + 3Y_{\overline{u_R}} + 3Y_{\overline{d_R}} + Y_{\overline{e_R}} = 0$

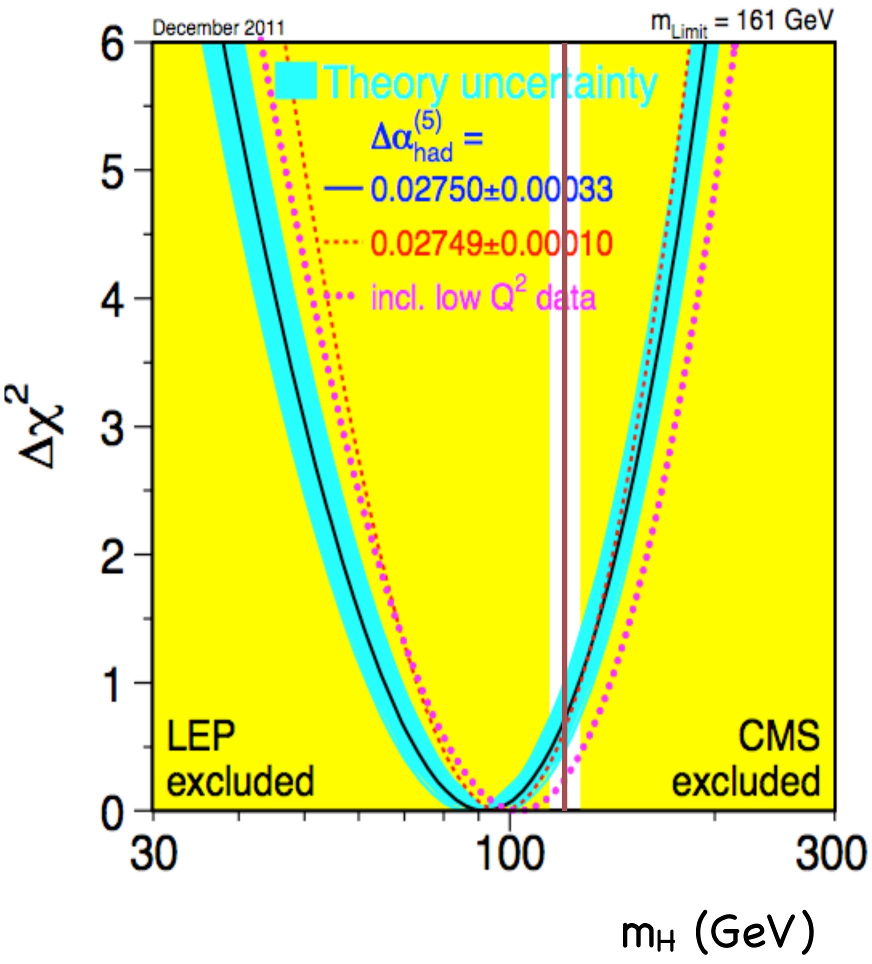
(nice, but why??)

Electroweak precision tests (EWPT)



- Accuracy up to the ‰ level → sensitivity to 1-loop corrections, which involve
  - g, g', v
  - m_t , $\alpha_s(MZ)$, $\Delta\alpha_{had}(MZ)$
  - m_h
- and bring together
  - the gauge sector: $g^2/(4\pi)^2$, $g'^2/(4\pi)^2$
  - the flavour sector: $\lambda^2/(4\pi)^2$
  - the EW-breaking sector: $g^2/(4\pi)^2 \log(m_h/M_W)$
- The agreement works because of the relatively low value of m_h

Experimental determination of Higgs mass: direct vs indirect



The flavour sector and spontaneous SB

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar{\Psi}_i i\gamma^\mu D_\mu \Psi_i - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} & \text{gauge} \\ +\lambda_{ij} \Psi_i \Psi_j H + \text{h.c.} & \text{flavor} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \end{array}$$

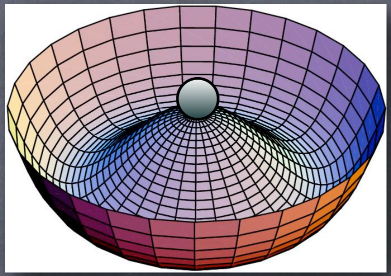
Fermion and gauge boson masses

- Gauge invariance forbids (gauge invariant) vector masses M_W?
- G_{SM} forbids (gauge invariant) fermion masses m_e?
- $SU(2)_L$ predicts ν_L e_L to have same masses and couplings ν ints?

Spontaneous Symmetry Breaking (SSB)

- Characterization of spontaneous symmetry breaking
  - The lagrangian is invariant
  - The currents associated with the symmetry are conserved
  - The vacuum of the theory is not invariant
  - The spectrum is not invariant
- Features
  - Allows a consistent breaking of gauge symmetries (symmetry is not totally lost); in particular, allows a consistent quantization of massive vectors
  - SSB of both global (e.g. approximate chiral symmetry of QCD) and gauge (electroweak) symmetries does take place in nature

- The spontaneous breaking of a symmetry arises when the ground state of the system is degenerate because of a symmetry
- Example: classical mechanics:



- Example: quantum mechanics: a lattice of spins coupled in a rotationally invariant way (ferromagnet)
  - The spins are aligned in the ground state
  - They spontaneously choose a direction, thus breaking rotational invariance
- In the limit of ∞ degrees of freedom (QFT), superpositions of degenerate vacua are not allowed: the symmetry is indeed broken

SSB in QFT

- SSB ⇔ $\langle\phi\rangle \equiv \langle\Omega|\phi(x)|\Omega\rangle$ not invariant under the symmetry (≠ 0)

  (“vev” ≡ non-vanishing vacuum expectation value)

  (if Ω is invariant, $<\varphi>$ is invariant)

- Poincaré invariance:
  - Only (4D) scalars can get a vev
  - $<\varphi(x)>$ does not depend on x

- $<\varphi>$ minimizes the effective potential

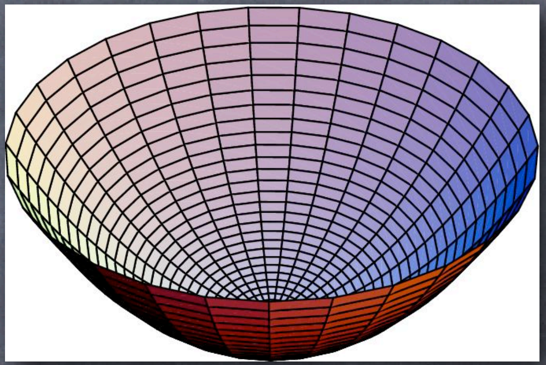
  $V_{eff} = V_{tree}$ + quantum corrections (1PI diagrams)

SSB of global symmetry: complex scalar field

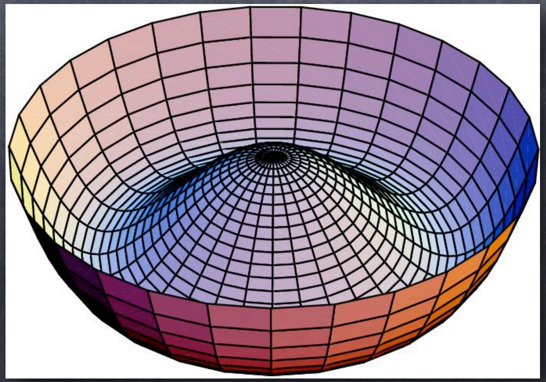
- $\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi^\dagger \phi), \quad V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$
  is invariant under $\phi \to e^{i\alpha}\phi$ (global U(1))

- $\lambda > 0$ (V bounded from below)

- $\mu^2 > 0$



- $\mu^2 < 0$



$$\langle \phi \rangle = v e^{i\theta} \quad v^2 = \frac{|\mu^2|}{2\lambda} \neq 0$$

The system chooses an arbitrary value of θ, thus breaking U(1)
Assume $\theta = 0$ in the minimum
(no loss of generality: $\tilde{\varphi}(x) = \varphi(x) e^{-i\theta}$)

SSB of gauge transformations (Higgs mechanism)

- Gauge bosons can get a longitudinal component and a mass
- Correspondingly, the Goldstone bosons associated to the global symmetry become unphysical. The degree of freedom associated to those bosons is eaten up by the gauge bosons
- massive vectors $\leftrightarrow$ broken generators $\leftrightarrow$ unphysical Goldstones

Example: the complex scalar field again

- Promote the global U(1) to a gauge U(1)

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) + \text{g.f.}, \quad D_\mu = \partial_\mu + igA_\mu$$

- Break U(1) spontaneously: $\mu^2 < 0,\ \langle\phi\rangle = v$; a vector mass term is generated:

$$(D_\mu\phi)^\dagger(D^\mu\phi) = (\partial_\mu\phi')^\dagger(\partial^\mu\phi') + \frac{1}{2}M^2 A_\mu A^\mu + \sqrt{2}gvA^\mu\partial_\mu G + \text{interactions}$$

$$M^2 = 2\,g^2v^2$$

- Gauge transformation:

$$\begin{cases} \phi(x) = \left(v + \dfrac{h(x)}{\sqrt{2}}\right)e^{ig(x)} \\ A_\mu(x) \end{cases} \text{is equivalent to} \begin{cases} \phi(x) = v + \dfrac{h(x)}{\sqrt{2}} \\ A_\mu(x) - \dfrac{1}{g}\partial_\mu g(x) \end{cases}$$

Unitarity gauge

The Goldstone boson disappears, A_μ gets a longitudinal component

“Blackboard”