

The Standard Model (and flavour (anomalies))

Andrea Romanino, SISSA, romanino@sissa.it

Particle physics in three lines

The SM describes the
Strong, Electromagnetic, and Weak interactions
of the known elementary particles

$$\mathcal{L}_{\text{SM}}^{\text{ren}} = \bar{\Psi}_i i \gamma^\mu D_\mu \Psi_i - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \quad \text{gauge} \quad \sim \text{\textperthousand}$$
$$+ \lambda_{ij} \Psi_i \Psi_j H + \text{h.c.} \quad \text{flavor} \quad \sim \%$$
$$+ |D_\mu H|^2 - V(H) \quad \text{symmetry breaking} \quad \sim 10\%$$

+ gravity: all known phenomena (with few notable exceptions)

Outline

- Preliminaries
- The SM
 - Gauge sector
 - Higgs sector
 - Flavour sector
- in a new physics perspective

Preliminaries (I)

the SM as a relativistic renormalisable gauge QFT

Relativistic Quantum Field Theory (QFT)

- $\hbar = c = 1$
- Degrees of freedom: **fields**. $\mathbf{S} = \mathbf{0}$ (scalar) $\mathbf{S} = \mathbf{1/2}$ (spinor) $\mathbf{S} = \mathbf{1}$ (vector)
- Dynamics: $S = \int d^4x \mathcal{L}(x)$ $[\mathcal{L}] = 4$ $\mathcal{L}(x) = \sum_i c_i \mathcal{O}_i(x)$ Lorentz Scalar
- $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$
 - $\mathcal{L}_{\text{free}}$ 2-linear in the fields
 - $\mathcal{L}_{\text{int}}$ ≥ 3 -linear in the fields
- Linear terms (scalars) can be reabsorbed by shifting fields (SSB)
- Constant term not physical without gravity

Renormalisability and relevance

- $\mathcal{L}(x) = \sum_i c_i \mathcal{O}_i(x)$ dimension **D** of **O(x)**:

- **D = 0** constant terms, not physical without gravity
- **D = 1** linear (scalar) terms, reabsorbed by shifting fields (SSB)
- **D = 2,3** “relevant” operators **[c] = M², M**
- **D = 4** “marginal” operators **[c] = g**
- **D ≥ 5** “irrelevant” operators **[c] = 1/M^{D-4}**

renormalisable
theory (SM)
finite # of c.t.

- Irrelevant operators: effect suppressed by **(E/M)^{D-4}** at $E \ll M$
 - any theory looks renormalizable at sufficiently low scales

Gauge theories

- Specific form of QFT
 - gauge transformations $\leftrightarrow$ equivalent field configurations
 - necessary to consistently describe vector fields
 - extremely successful (QED & photons, QCD & gluons)
- Defined by
 - Gauge (compact) Lie group G
 - A massless vector field to each generator of G : $t_A \leftrightarrow V^A_\mu$
 - Action on “matter” fields (commuting with Lorentz)
 - Globally invariant $\mathcal{L}_0(\phi, \partial\phi)$

$$\mathcal{L} = \mathcal{L}_0(\phi, D_\mu \phi) - \frac{1}{4} v_{\mu\nu}^A v_A^{\mu\nu}$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig_A v_\mu^A T_A \qquad t_A \rightarrow T_A$$

$$v_{\mu\nu}^A = \partial_\mu v_\nu^A - \partial_\nu v_\mu^A - gf_{BC}^A v_\mu^B v_\nu^C \qquad [t_A, t_B] = if_{AB}^C t_C$$

$$\bar{\psi} i \gamma^\mu D_\mu \psi = \bar{\psi} i \gamma^\mu \partial_\mu \psi - g_A \bar{\psi}_i \gamma^\mu T_A^{ij} \psi_j v_\mu^A$$

Preliminaries (II)

digression on L and R spinors

QED and QCD written in terms of Dirac spinors

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$$

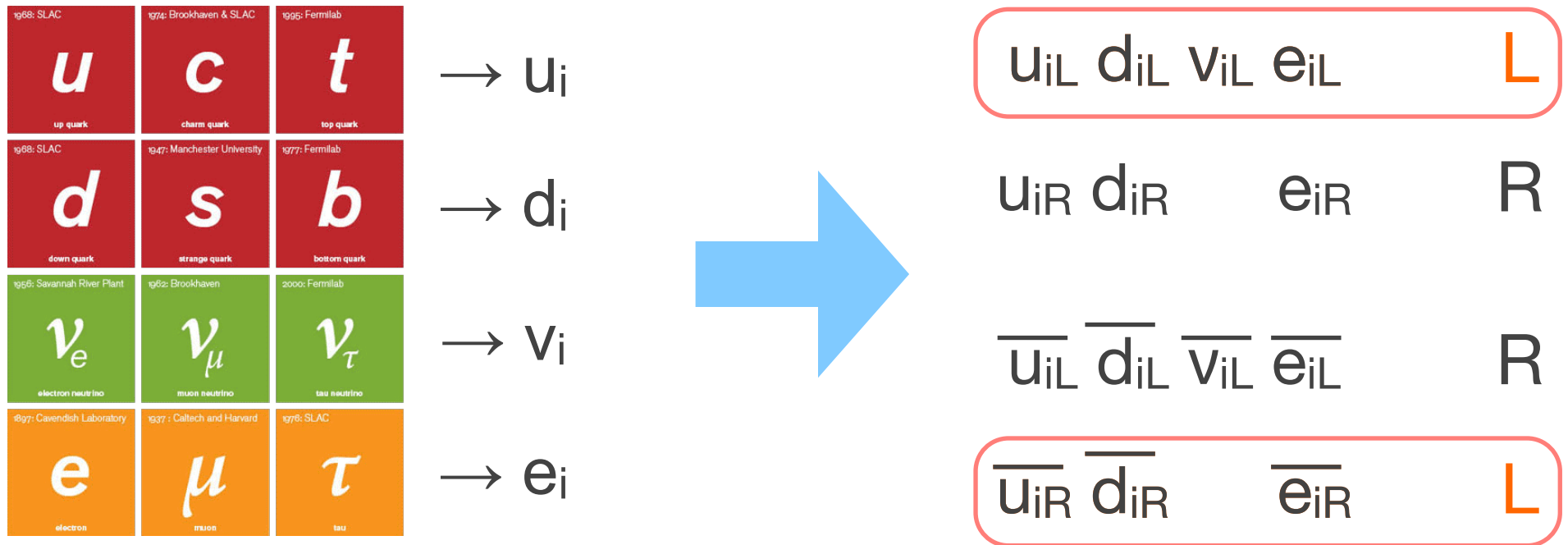
A Dirac spinor is not “elementary”

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \rightarrow \left(\begin{array}{c|c} L^{\dagger-1} & \\ \hline & L \end{array} \right) \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \quad (\det L = 1)$$

$$\Psi_R(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \\ 0 \\ 0 \end{pmatrix} \quad \Psi_L(x) = \begin{pmatrix} 0 \\ 0 \\ \Psi_3(x) \\ \Psi_4(x) \end{pmatrix} \quad \Psi = \Psi_L + \Psi_R$$

inequivalent, conjugated representations of $SL(2, \mathbb{C})$

Left- and Right-handed



Most general gauge transformation can mix all L

The gauge sector of the SM lagrangian

$$\bar{\Psi}_i i \gamma^\mu D_\mu \Psi_i - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \quad \text{gauge}$$

$$\mathcal{L}_{\text{SM}}^{\text{ren}} = +\lambda_{ij} \Psi_i \Psi_j H + \text{h.c.} \quad \text{flavor}$$
$$+ |D_\mu H|^2 - V(H) \quad \text{symmetry breaking}$$

Gauge sector

$$G_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$$

		SU(3)	SU(2)	U(1)	
$Q_i = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$ $L_i = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$	L_i	1	2	-1/2	
	$\bar{e}_{iR}$	1	1	1	L-handed spinors
	Q_i	3	2	1/6	i = 1,2,3
	$\bar{u}_{iR}$	3*	1	-2/3	
	$\bar{d}_{iR}$	3*	1	1/3	

Y

$$\mathcal{L}_{\text{SM}}^{\text{gauge}} = \sum_{\psi=Q_i, u_{iR}, d_{iR}, L_i, e_{iR}} \bar{\psi} i \gamma^\mu D_\mu \psi - \frac{1}{4} \sum_{v=G_A, W_a, B} v^{\mu\nu} v_{\mu\nu}$$

“Blackboard”

QED + QCD + effective weak interactions + gauge principle
= Standard Model gauge theory

“Blackboard”

Gauge sector

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
$\bar{e}_{iR}$	1	1	1
Q_i	3	2	1/6
$\bar{u}_{iR}$	3^*	1	-2/3
$\bar{d}_{iR}$	3^*	1	1/3

Y

$$\mathcal{L}_{\text{SM}}^{\text{gauge}} = \sum_{\psi=Q_i, u_{iR}, d_{iR}, L_i, e_{iR}} \bar{\psi} i \gamma^\mu D_\mu \psi - \frac{1}{4} \sum_{v=G_A, W_a, B} v^{\mu\nu} v_{\mu\nu}$$

The Standard Model II

Andrea Romanino, SISSA, romanino@sissa.it

Puzzles (gauge sector)

- Why 3 copies of the same representation of G_{SM} ?
- Why G_{SM} ?
- Why those quantum numbers? And in particular
 - why Y / Q is quantised?
 - why anomalies cancel (see below)?

Notable features (gauge sector)

- chirality
- anomaly cancellation
- electroweak precision tests

Gauge sector

$$G_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$$

$$Q_i = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$$

$$L_i = \begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$$

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
$\bar{e}_{iR}$	1	1	1
Q_i	3	2	1/6
$\bar{u}_{iR}$	3^*	1	-2/3
$\bar{d}_{iR}$	3^*	1	1/3

L-handed
spinors

$i = 1, 2, 3$

Y

A nice property

- The fermion content is “chiral”: no L and R with same G_{SM} quantum numbers
 - Equivalently: r irrep on L $\Rightarrow r^*$ is not
 - Equivalently: no explicit (G_{SM} symmetric) fermion mass term is allowed
- SM fermion masses $\propto \langle H \rangle$
- Extra heavy fermions ($M \gg \langle H \rangle$) should be “vectorlike”
- A puzzle, a blessing, or what expected?

Anomalies

- A symmetry of the lagrangian is not necessarily preserved by quantum corrections. If not, the symmetry is **anomalous**
- Global symmetry: OK + interesting physics
 - axial U(1) in QCD
 - scale invariance
- In the case of gauge transformations: INCONSISTENCY
- A gauge symmetry survives quantum corrections iff $\text{Tr} (T_a \{T_b T_c\}) = 0$ where T_a are the generators and the trace is on L-handed fermions
- QED and QCD automatically non-anomalous

Anomaly cancellations in the SM

- Anomaly cancellation

- Is $T_{ijk} \equiv \text{Tr} (\tau_i \{ \tau_j, \tau_k \}) = 0$? $\tau_i = T_A, T_a, Y$

$$SU(3)^3 \quad \text{vectorlike}$$

$$SU(3)^2 \times SU(2) \quad \text{Tr}(\sigma_a) = 0$$

$$SU(3)^2 \times U(1) \quad 2Y_q + Y_{\overline{u_R}} + Y_{\overline{d_R}} = 0$$

$$SU(3) \times (\text{not } SU(3))^2 \quad \text{Tr}(\lambda_A) = 0$$

...

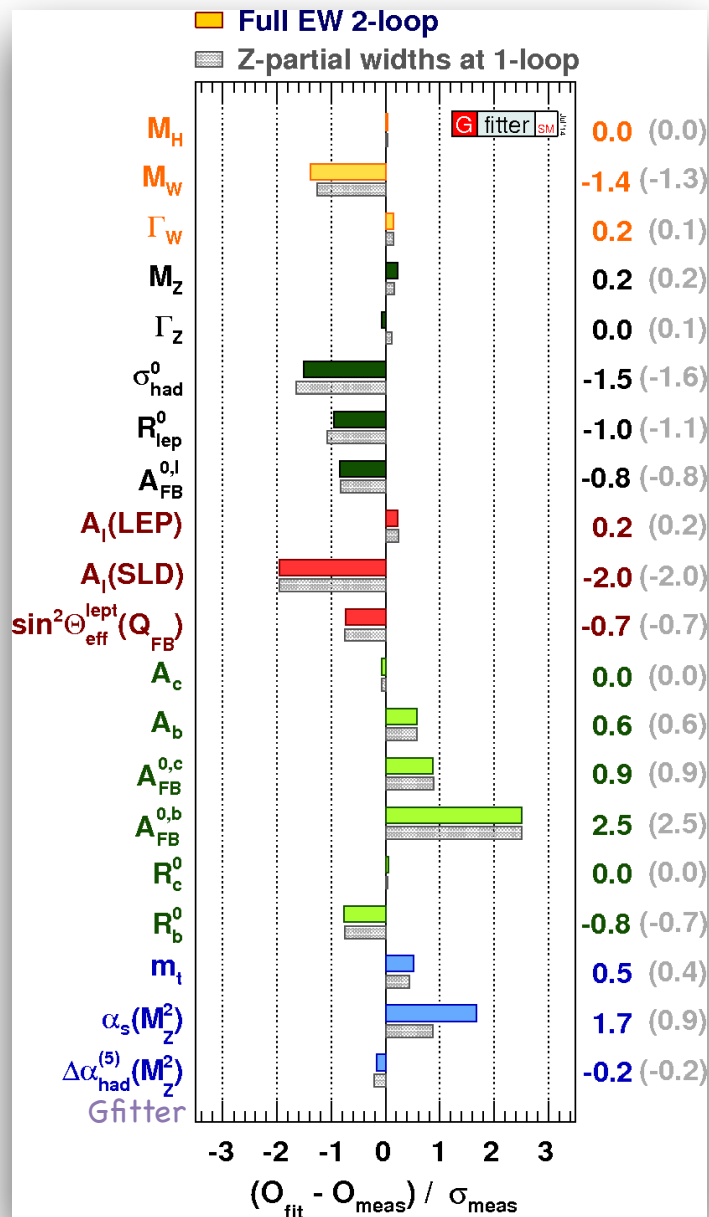
$$SU(2)^2 \times U(1) \quad Y_l + 3Y_q$$

$$U(1)^3 \quad 2Y_l^3 + 6Y_q^3 + 3Y_{\overline{u_R}}^3 + 3Y_{\overline{d_R}}^3 + Y_{\overline{e_R}}^3 = 0$$

$$\text{grav. anomaly} \quad 2Y_l + 6Y_q + 3Y_{\overline{u_R}} + 3Y_{\overline{d_R}} + Y_{\overline{e_R}} = 0$$

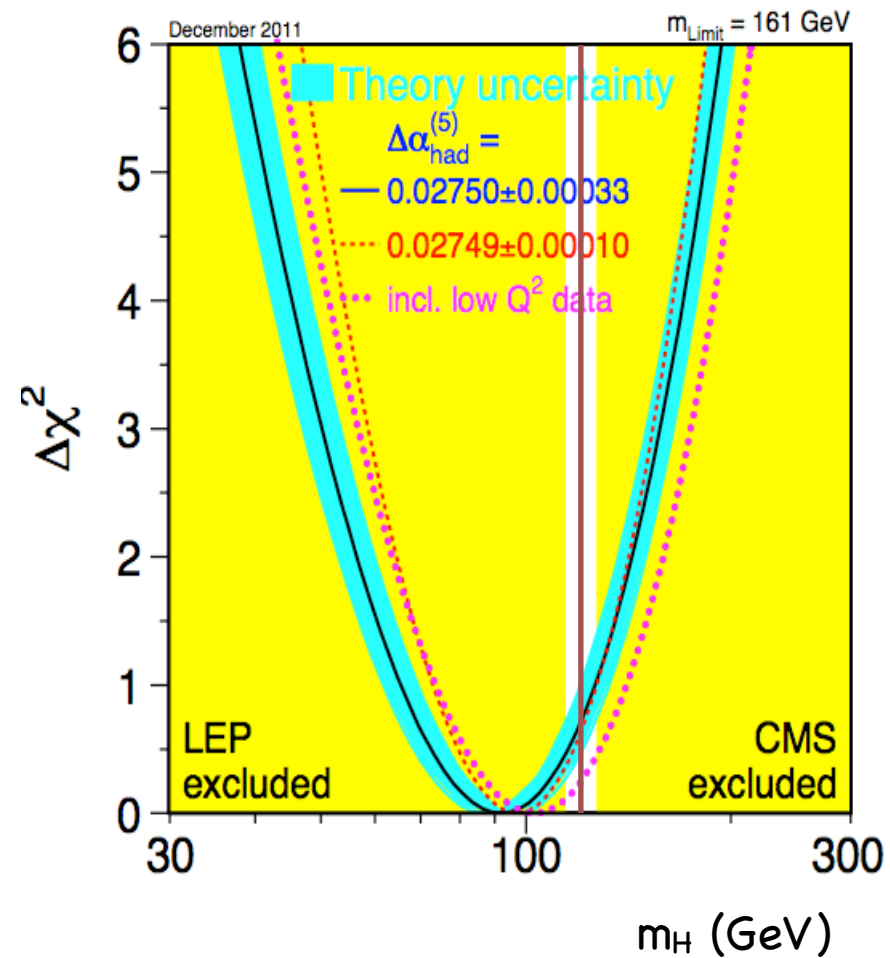
(nice, but why??)

Electroweak precision tests (EWPT)



- Accuracy up to the ‰ level → sensitivity to 1-loop corrections, which involve
 - g, g', v
 - $m_t, \alpha_s(M_Z), \Delta\alpha_{had}(M_Z)$
 - m_h
- and bring together
 - the gauge sector: $g^2/(4\pi)^2, g'^2/(4\pi)^2$
 - the flavour sector: $\lambda^2/(4\pi)^2$
 - the EW-breaking sector: $g^2/(4\pi)^2 \log(m_h/M_W)$
- The agreement works because of the relatively low value of m_h

Experimental determination of Higgs mass: direct vs indirect



The flavour sector and spontaneous SB

$$\mathcal{L}_{\text{SM}}^{\text{ren}} = \bar{\Psi}_i i \gamma^\mu D_\mu \Psi_i - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \quad \text{gauge}$$

$+ \lambda_{ij} \Psi_i \Psi_j H + \text{h.c.} \quad \text{flavor}$ $+ |D_\mu H|^2 - V(H) \quad \text{symmetry breaking}$

Fermion and gauge boson masses

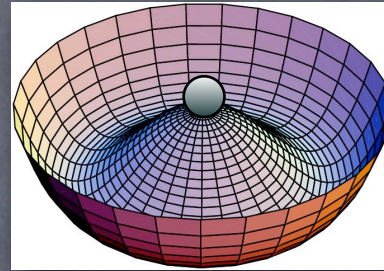
- Gauge invariance forbids (gauge invariant) vector masses $M_W?$
- G_{SM} forbids (gauge invariant) fermion masses $m_e?$
- $SU(2)_L$ predicts ν_L e_L to have same masses and couplings ν ints?

Spontaneous Symmetry Breaking (SSB)

- Characterization of spontaneous symmetry breaking
 - The lagrangian is invariant
 - The currents associated with the symmetry are conserved
 - The vacuum of the theory is not invariant
 - The spectrum is not invariant
- Features
 - Allows a consistent breaking of gauge symmetries (symmetry is not totally lost); in particular, allows a consistent quantization of massive vectors
 - SSB of both global (e.g. approximate chiral symmetry of QCD) and gauge (electroweak) symmetries does take place in nature

- The spontaneous breaking of a symmetry arises when the ground state of the system is degenerate because of a symmetry

- Example: classical mechanics:



- Example: quantum mechanics: a rotationally invariant system of coupled spins (ferromagnet)
 - The spins are aligned in the ground state
 - They spontaneously choose a direction, thus breaking rotational invariance
- In the limit of ∞ degrees of freedom (QFT), superpositions of degenerate vacua are not allowed: the symmetry is indeed broken

SSB in QFT

- SSB $\Leftrightarrow \langle \phi \rangle \equiv \langle \Omega | \phi(x) | \Omega \rangle$ not invariant under the symmetry ($\neq 0$)
("vev" $\equiv$ non-vanishing vacuum expectation value)

(if Ω is invariant, $\langle \phi \rangle$ is invariant)

- Poincaré invariance:
 - Only (4D) scalars can get a vev
 - $\langle \phi(x) \rangle$ does not depend on x
- $\langle \phi \rangle$ minimizes the effective potential

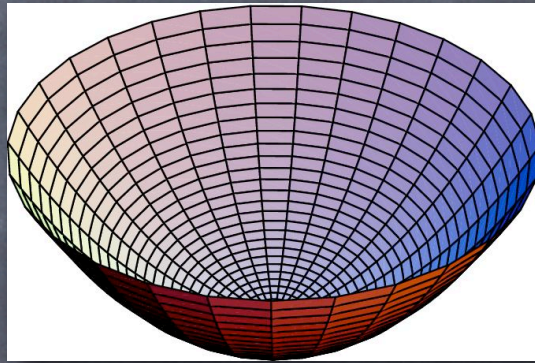
$$V_{\text{eff}} = V_{\text{tree}} + \text{quantum corrections (1PI diagrams)}$$

SSB of global symmetry: complex scalar field

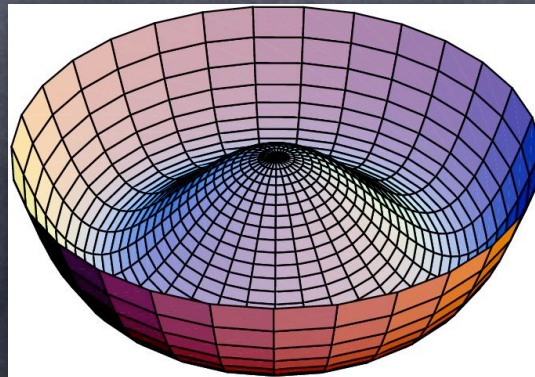
• $\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi^\dagger \phi), \quad V(\phi^\dagger \phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$
is invariant under $\phi \rightarrow e^{i\alpha} \phi$ (global U(1))

• $\lambda > 0$ (V bounded from below)

• $\mu^2 > 0$



• $\mu^2 < 0$



$$\langle \phi \rangle = v e^{i\theta} \quad v^2 = \frac{|\mu^2|}{2\lambda} \neq 0$$

The system chooses an arbitrary value of θ , thus breaking U(1)

Assume $\theta = 0$ in the minimum
(no loss of generality: $\tilde{\varphi}(x) = \varphi(x) e^{-i\theta}$)

SSB of gauge transformations (Higgs mechanism)

- Gauge bosons can get a longitudinal component and a mass
- Correspondingly, the Goldstone bosons associated to the global symmetry become unphysical. The degree of freedom associated to those bosons is eaten up by the gauge bosons
- massive vectors $\leftrightarrow$ broken generators $\leftrightarrow$ unphysical Goldstones

Example: the complex scalar field again

- Promote the global U(1) to a gauge U(1)

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) + \text{g.f.}, \quad D_\mu = \partial_\mu + igA_\mu$$

- Break U(1) spontaneously: $\mu^2 < 0$, $\langle\phi\rangle = v$; a vector mass term is generated:

$$(D_\mu\phi)^\dagger(D^\mu\phi) = (\partial_\mu\phi')^\dagger(\partial^\mu\phi') + \frac{1}{2}M^2 A_\mu A^\mu + \sqrt{2}gv A^\mu \partial_\mu G + \text{interactions}$$
$$M^2 = 2g^2v^2$$

- Gauge transformation:

$$\begin{cases} \phi(x) = \left(v + \frac{h(x)}{\sqrt{2}}\right) e^{ig(x)} \\ A_\mu(x) \end{cases} \text{ is equivalent to } \begin{cases} \phi(x) = v + \frac{h(x)}{\sqrt{2}} \\ A_\mu(x) - \frac{1}{g}\partial_\mu g(x) \end{cases} \quad \text{Unitarity gauge}$$

The Goldstone boson disappears, A_μ gets a longitudinal component

“Blackboard”