The Standard Model III

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Notable features (flavour sector)

- $U(3)^5$ and accidental symmetries
- no neutrino masses
- (anomalous suppression of loop FCNC)

The flavour sector

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \psi_i^\dagger i\sigma^\mu D_\mu \psi_i - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} & \text{gauge} \\ +\lambda_{ij}\psi_i\psi_j H + \text{h.c.} & \text{flavor} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \end{array}$$

	1	2	3
l	l_1	l_2	l_3
e^c	$(e^c)_1$	$(e^c)_2$	$(e^c)_3$
q	q_1	q_2	q_3
u^c	$(u^c)_1$	$(u^c)_2$	$(u^c)_3$
d^c	$(d^c)_1$	$(d^c)_2$	$(d^c)_3$

The flavour sector allows to tell the three families: gauge interactions do not

$U(3)^5 \times U(1)_H$

The gauge lagrangian cannot tell families ↔ is $U(3)^5$ invariant:

$$U(3)^5: \begin{array}{l} L_i \to U^L_{ij} L_j \\ e^c_i \to U^{e^c}_{ij} e^c_j \\ Q_i \to U^Q_{ij} Q_j \\ u^c_i \to U^{u^c}_{ij} u^c_j \\ d^c_i \to U^{d^c}_{ij} d^c_j \end{array} \Rightarrow \mathcal{L}^{\text{gauge}}_{\text{SM}} \to \mathcal{L}^{\text{gauge}}_{\text{SM}}$$

also $U(1): \; H \to e^{i\alpha} H \Rightarrow \mathcal{L}^{\text{EWSB}}_{\text{SM}} \to \mathcal{L}^{\text{EWSB}}_{\text{SM}}$

$U(3)^5 \times U(1)_H$

The flavour (Yukawa) lagrangian is is not $U(3)^5$ invariant (unless $\lambda_{ij}=0$)

$$
U(3)^5: \quad
\begin{aligned}
l_i &\to U^l_{ij} l_j \\
e^c_i &\to U^{e^c}_{ij} e^c_j \\
q_i &\to U^q_{ij} q_j \\
u^c_i &\to U^{u^c}_{ij} u^c_j \\
d^c_i &\to U^{d^c}_{ij} d^c_j
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\lambda_E &\to U^T_{e^c} \lambda_E U_L \\
\lambda_D &\to U^T_{d^c} \lambda_D U_Q \\
\lambda_U &\to U^T_{u^c} \lambda_U U_Q
\end{aligned}
\quad \mathcal{L}^{\text{flavour}}_{\text{SM}} \not\to \mathcal{L}^{\text{flavour}}_{\text{SM}}
$$

$$
\mathcal{L}^{\text{flavor}}_{\text{SM}} = \lambda^E_{ij} e^c_i l_j H^\dagger + \lambda^D_{ij} d^c_i q_j H^\dagger + \lambda^U_{ij} u^c_i q_j H + \text{h.c.}
$$

Accidental symmetries (ren lagrangian)

- The flavour lagrangian breaks $U(3)^5 \times U(1)_H$ to $U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_B \times U(1)_Y$

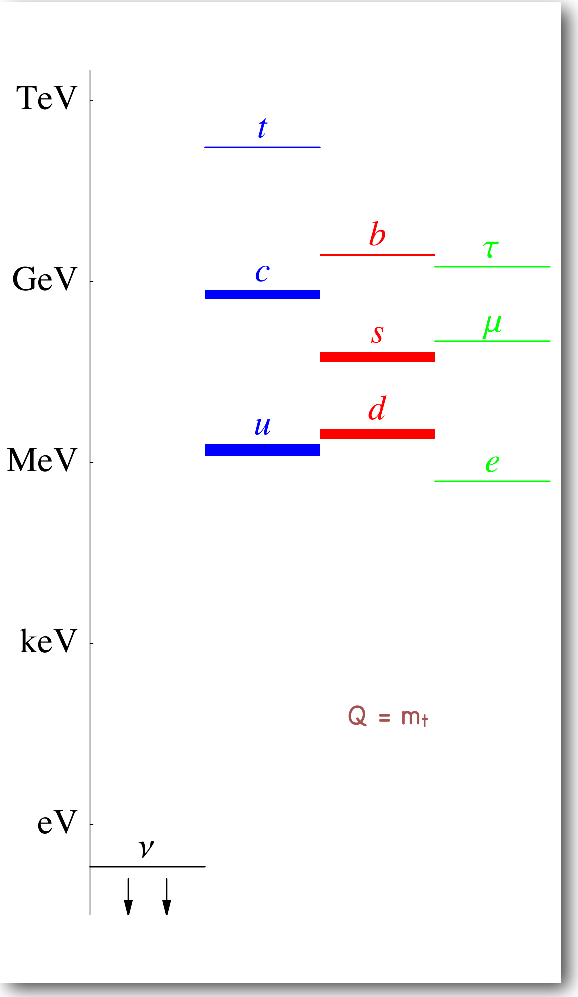
$$\lambda^E_{ij} e^c_i L_j H^\dagger \to \lambda_{e_i} e^{c\prime}_i L'_i H^\dagger$$
$$\lambda^D_{ij} d^c_i Q_j H^\dagger \to \lambda_{d_i} d^{c\prime}_i Q'_i H^\dagger$$
$$\lambda^U_{ij} u^c_i Q_j H \to \lambda_{u_i} V_{ij} u^{c\prime}_i Q'_i H$$

- L_e L_μ L_τ individual lepton numbers (also $L = L_e + L_\mu + L_\tau$ total)
  B Baryon number

- Welcome that they arise as accidental symmetries
  - prediction of the SM, not by hand
  - « 1 predicted, not = 0
  - allows SM extensions in which they are broken (see-saw, GUT) (L_i necessarily broken by neutrino masses, possibly L)
  - broken by non perturbative effects

No neutrino masses

- Within the SM
    - $m_\nu = 0$ 👍 (U(1)$_{em}$: $m_\nu \neq 0$, $L \neq 0$)
    - $m_\nu = 0 \leftrightarrow L$ accidentally conserved 👍
- $m_\nu / \langle H \rangle < 10^{-12}$
- Plausibly unrelated to $m_e / \langle H \rangle \approx 0.3 \times 10^{-5}$
    - family independent
    - (compelling reason for m_ν to be small)



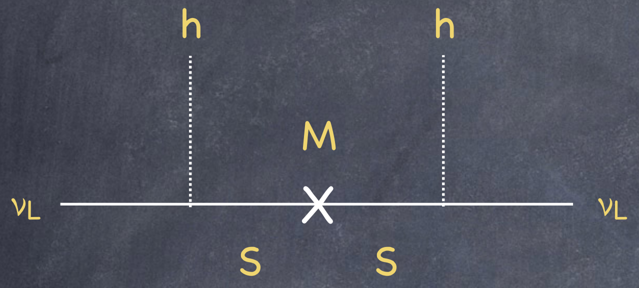
- Beyond the SM, standard framework
    - if the origin of $m_\nu \neq 0$ lies above the EW scale

$$\mathcal{L}_{\rm SM}^{\rm eff} = \mathcal{L}_{\rm SM}^{\rm ren} + \frac{c_{ij}}{2\Lambda}(l_i h)(l_j h) + \text{h.c.} + \ldots$$

$$m_{u,d,e} = \lambda_{u,d,e}\, v \qquad m_\nu = c\, v \times \frac{v}{\Lambda}$$

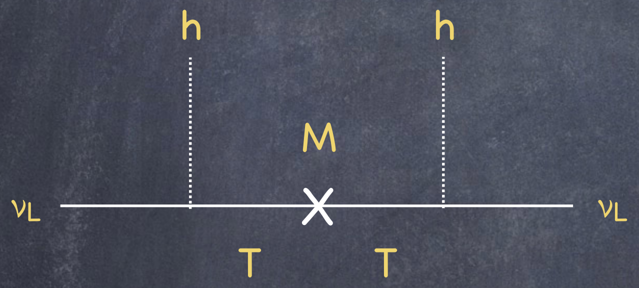
$$\Lambda \sim 0.5 \cdot 10^{15}\,\text{GeV}\, c \left(\frac{0.05\,\text{eV}}{m_\nu}\right)$$

Possible tree level origins of LHLH



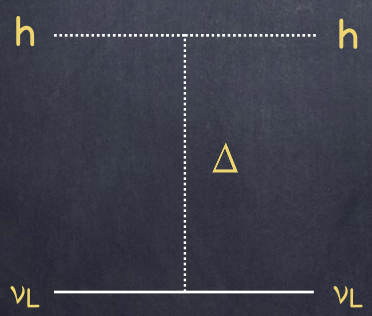
See-saw type I
S: SM singlet

at least 2



See-saw type III
T: $SU(2)_L$ triplet, $Y = 0$

at least 2



See-saw type II
T: $SU(2)_L$ triplet, $Y = 1$

at least 1

Analysis of the SM lagrangian: SSB

The Higgs sector

- Most general gauge invariant ren. lagrangian for H:

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) - V(H^\dagger H)$$
$$V(H^\dagger H) = \mu^2 H^\dagger H + \lambda_H (H^\dagger H)^2$$

- $\lambda_H > 0$
- $\mu^2 < 0 \Rightarrow \langle H \rangle \neq 0 \Rightarrow$ electroweak symmetry breaking
- ($\mu^2 > 0 \Rightarrow$ still electroweak symmetry breaking, but at $\Lambda \approx m_\pi$)

QED unbroken

- Fix the Higgs quantum numbers from fermion masses. Then the electric charge is automatically conserved

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \; v > 0, \; v^2 = \frac{|\mu^2|}{2\lambda_H} \approx (174\,\mathrm{GeV})^2 \qquad m_H^2 = 4\,\lambda_H(v^2)\,v^2$$

$$T = aY + b_a T_a, \; a, b_a \text{ real}, \; T_a = \frac{\sigma}{2}, \; Y = \frac{\mathbf{1}}{2}$$

$$0 = T\,\langle H \rangle = \frac{v}{2}\begin{pmatrix} b_1 - ib_2 \\ a - b_3 \end{pmatrix} \Rightarrow T \propto Q$$

- 3 broken generators ↔ 3 massive vectors ↔ 3 unphysical Goldstone bosons ↔ 1 real physical Higgs particle

“Blackboard”

SM spectrum: $s = 1$

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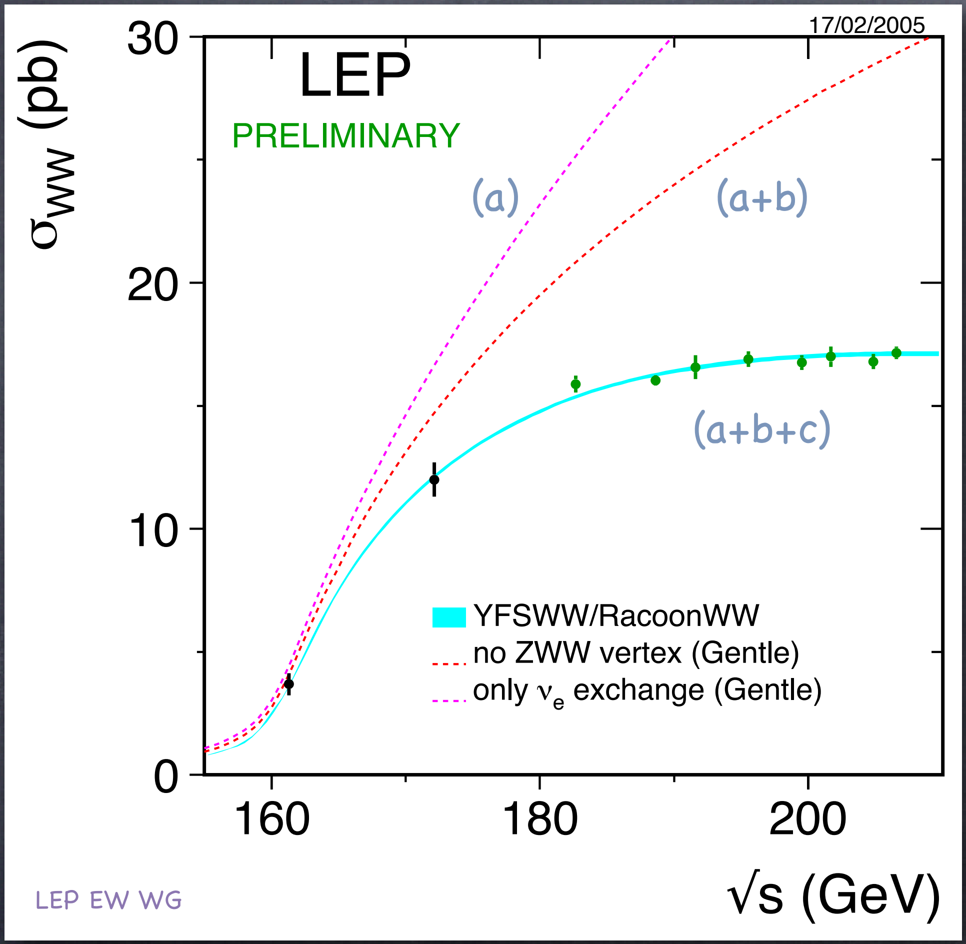
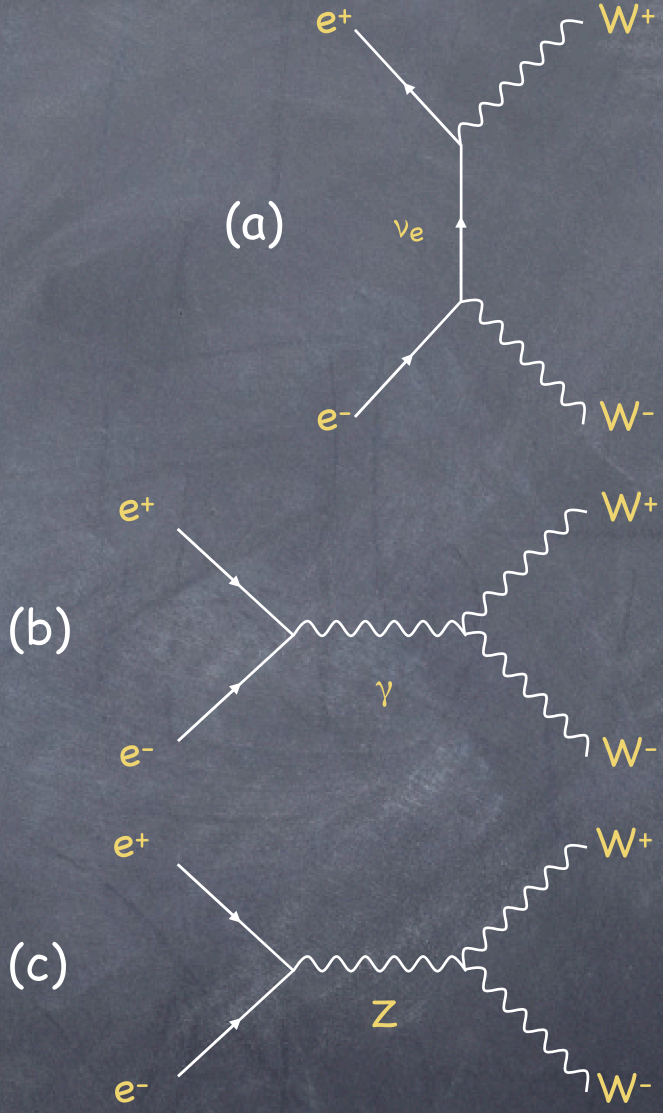

17/02/2005
LEP
PRELIMINARY
σWW (pb)
30
20
10
0
160
180
200
√s (GeV)
(a)
(a+b)
(a+b+c)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only νe exchange (Gentle)
LEP EW WG


(a)
e+
e-
νe
W+
W-
(b)
e+
e-
γ
W+
W-
(c)
e+
e-
Z
W+
W-

SM spectrum: $s = 0$

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The Standard Model (and its flavour anomalies) IV

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II Joint ICTP-Trieste/ICTP-SAIFR School on Particle Physics

Notable features (Higgs sector)

- electric charge conserved
- custodial symmetry
- perturbative extrapolation

QED unbroken

- Higgs quantum numbers from fermion masses
- Just what needed to break (or not) $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$
    - e.g. $SU(2)_L$ doublet with different Y would break to $U(1)_{Q'}$
    - e.g. $SU(2)_L$ triplet with $Y \neq 0, 1, -1$ would break to nothing
$Y = 0$ would break to $U(1)_Y \times U(1)_{T3}$
    - e.g. two $SU(2)_L$ doublets with correct Y could break the whole $SU(2)_L \times U(1)_Y$

Custodial symmetry

- $\rho \equiv \dfrac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \text{ (tree level)}$
  - Not guaranteed by gauge invariance or breaking pattern
  - Peculiar of EW breaking by a doublet
    (dominant contribution from triplets ruled out)
  - Exact in the limit $g' = 0$, $\lambda_U = \lambda_D$

$\rho \approx 1 \leftrightarrow$ (approximate) custodial SU(2)

- $\rho = 1$ if in the $g' = 0$ limit $W^{1,2,3}$ have equal mass

- I.e. if a $SO(3) \approx SU(2)$ symmetry rotates the real fields $W^{1,2,3}$

- The custodial symmetry in the Higgs sector: the Higgs lagrangian is accidentally SO(4) symmetric, as

$$|H|^2 = h_{1R}^2 + h_{1I}^2 + h_{2R}^2 + h_{2I}^2$$

- SO(4) is spontaneously broken to SO(3) by $\langle h_{2R} \rangle \neq 0$

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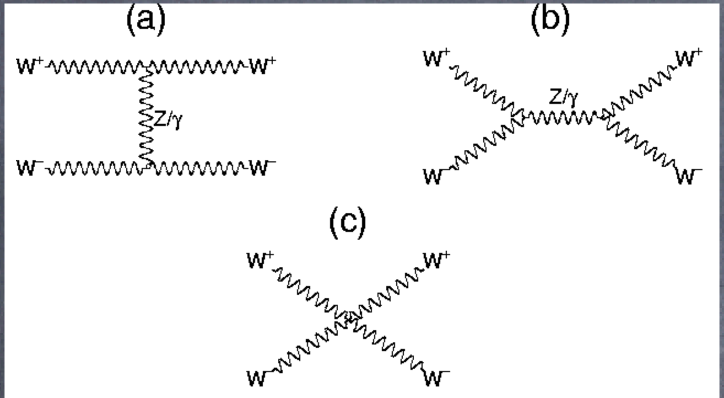
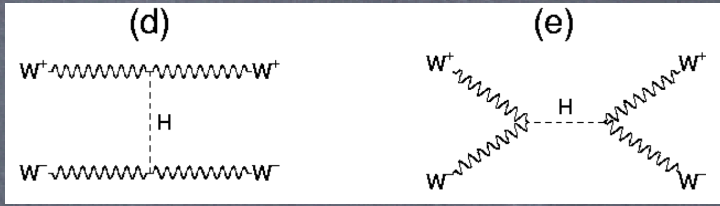
Light Higgs + SM couplings = perturbativity

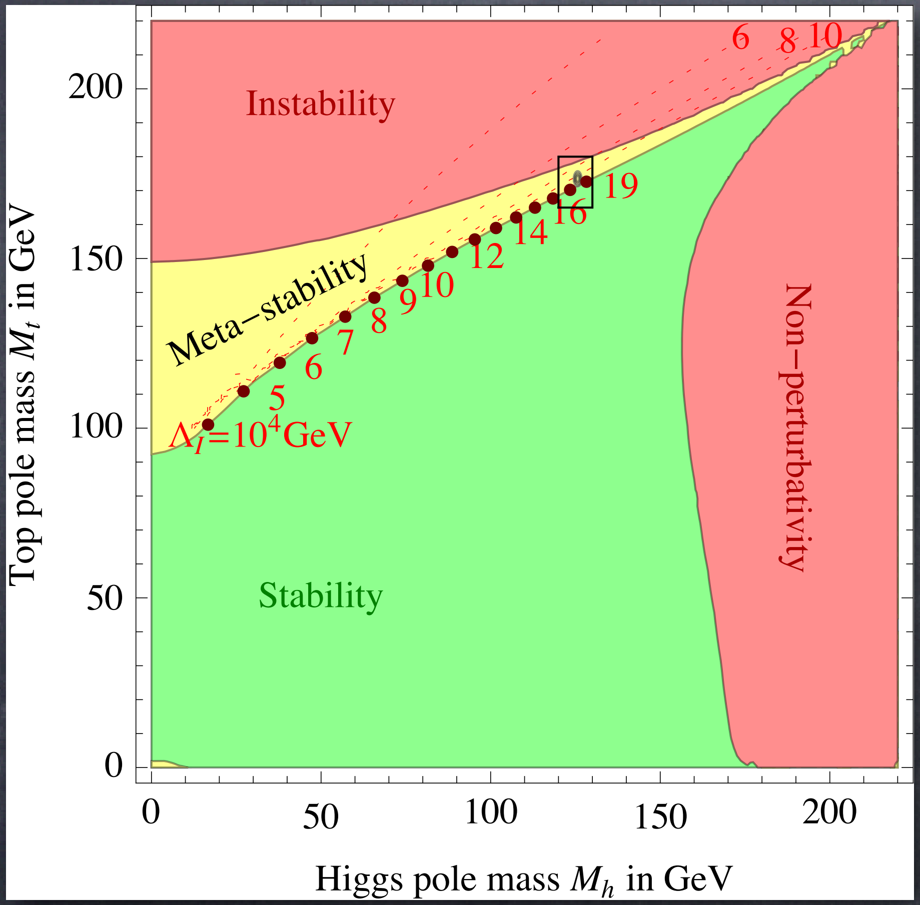
- $A(W_L W_L \to W_L W_L) = \Sigma_l\, a_l\, A_l$, a_l = partial wave amplitude
- Unitarity bound: $|a_0| \leq 1$
- Tree level, no Higgs: $a_0 \sim \frac{s}{16\pi v^2}$, $s = (p_1+p_2)^2$, $v \approx 174$ GeV

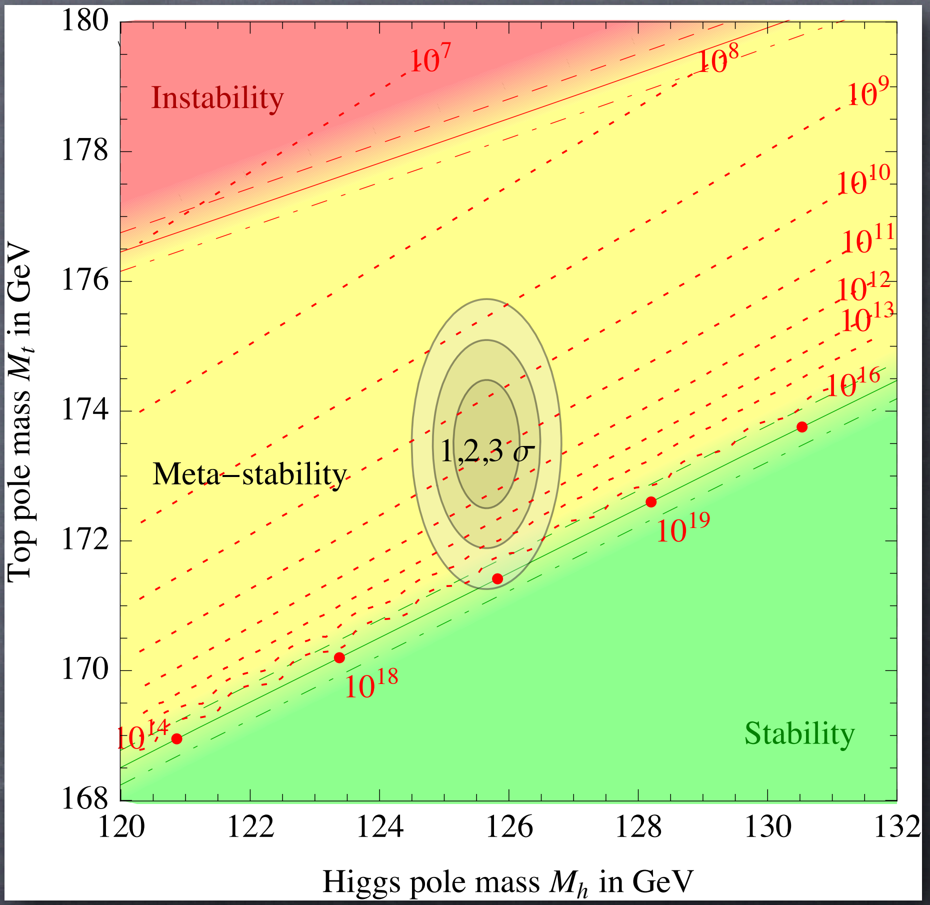




- Unitarity bound saturated at $s \approx (1.2\ \text{TeV})^2$
- Bad behaviour of a_0 due to the longitudinal part of the W propagator $\sim p_\mu p_\nu/(M_W)^2$, cancelled by Higgs exchange

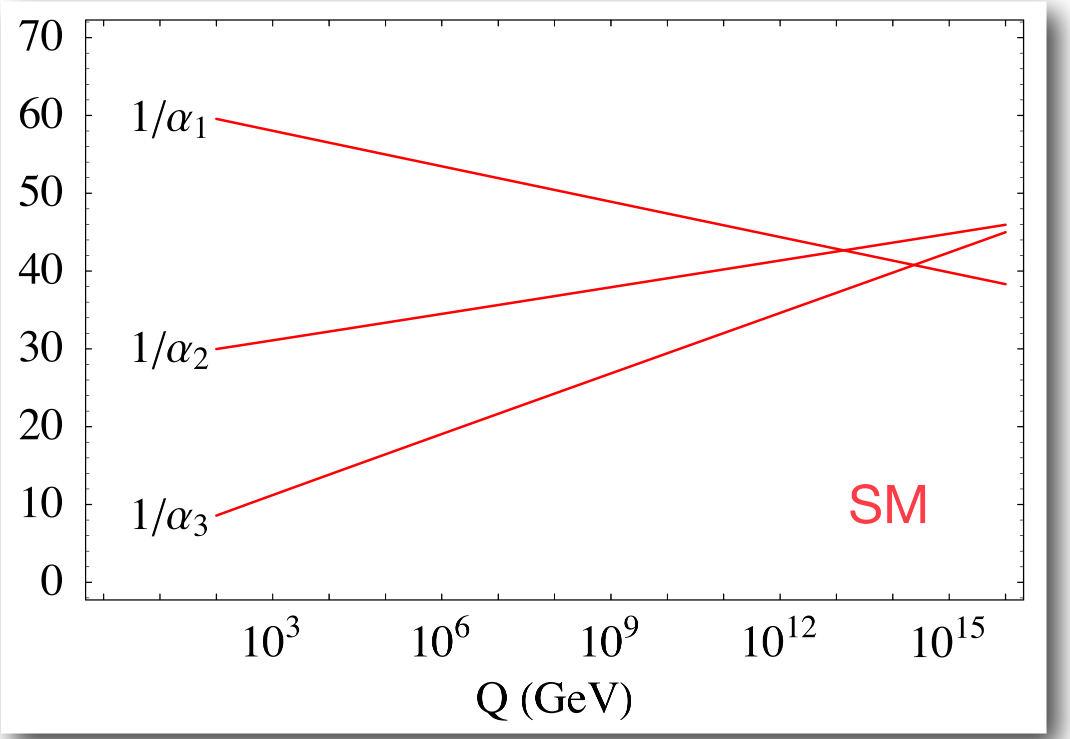
(perturbative extrapolation possible up to M_{Pl})





Buttazzo et al

(perturbative extrapolation possible up to M_{Pl})



SM spectrum: $s = 1/2$

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