The Standard Model and its flavour anomalies V

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Standard parameterisation

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}s^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}s^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$\sin\theta_{12}$	$\sin\theta_{23}$	$\sin\theta_{13}$	δ/π
0.225 ± 0.001	0.0420 ± 0.0006	0.0037 ± 0.0001	0.37 ± 0.01

[UTfit]

Standard parameterisation

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Standard → Wolfestein parameterisation

$$V_{\rm CKM} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\bar\rho - i\bar\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$$\lambda \approx 0.22$$

$$A, \bar\rho, \bar\eta = \mathcal{O}\left(1\right)$$

“Blackboard”

Tests of SM flavour predictions and anomalies

One source : $-\frac{g}{\sqrt{2}} V_{ij} \overline{u_{iL}} \gamma^\mu d_{jL} W^+_\mu + h.c.$

V is unitary

Tests of unitarity :

- $\sum_k |V_{ik}|^2 = \sum_k |V_{kj}|^2 = 1$

- $\sum_k V_{ik} V^*_{jk} = \sum_k V_{ki} V^*_{kj} = 0 \quad i \neq j$

$$\sum_h |V_{ih}|^2 = \sum_k |V_{kj}|^2 = 1$$

Measure $|V_{ij}|$ e.g. through $u_i \leftrightarrow d_j$ tree level decays

$|V_{ud}|$ $\quad d \to u e^- \bar{\nu}$ $\quad n \to p e \nu$ $\quad 0.97370 \pm 0.00014$

$u \to d e^+ \bar{\nu}$ $\quad \pi^+ \to \pi^0 e^+ \nu$

$0^+ \to 0^+$ nuclear transitions

$|V_{us}|$ $\quad s \to u \ell^- \nu$ $\quad K_L^0 \to \pi^- e^+ \nu$ $\quad 0.2245 \pm 0.0008$

$\ell = e, \mu$ $\quad K^+ \to \pi^0 e^+ \nu$

$$\sum_h |V_{ih}|^2 = \sum_k |V_{kj}|^2 = 1$$

Analogously:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

$$|V_{cd}| = 0.221 \pm 0.004$$

$$|V_{cs}| = 0.987 \pm 0.011$$

$$|V_{cb}| = (0.410 \pm 0.014) \cdot 10^{-2}$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.079 \pm 0.006$$

Moreover:

$$|V_{ts}| = (0.388 \pm 0.011) \cdot 10^{-3}$$

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.205 \pm 0.006$$

$$|V_{tb}| = 1.013 \pm 0.030$$

tensions in V_{cb} and V_{ub}
(inclusive vs exclusive)

$$\sum_h |V_{ih}|^2 = \sum_k |V_{kj}|^2 = 1$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022$$

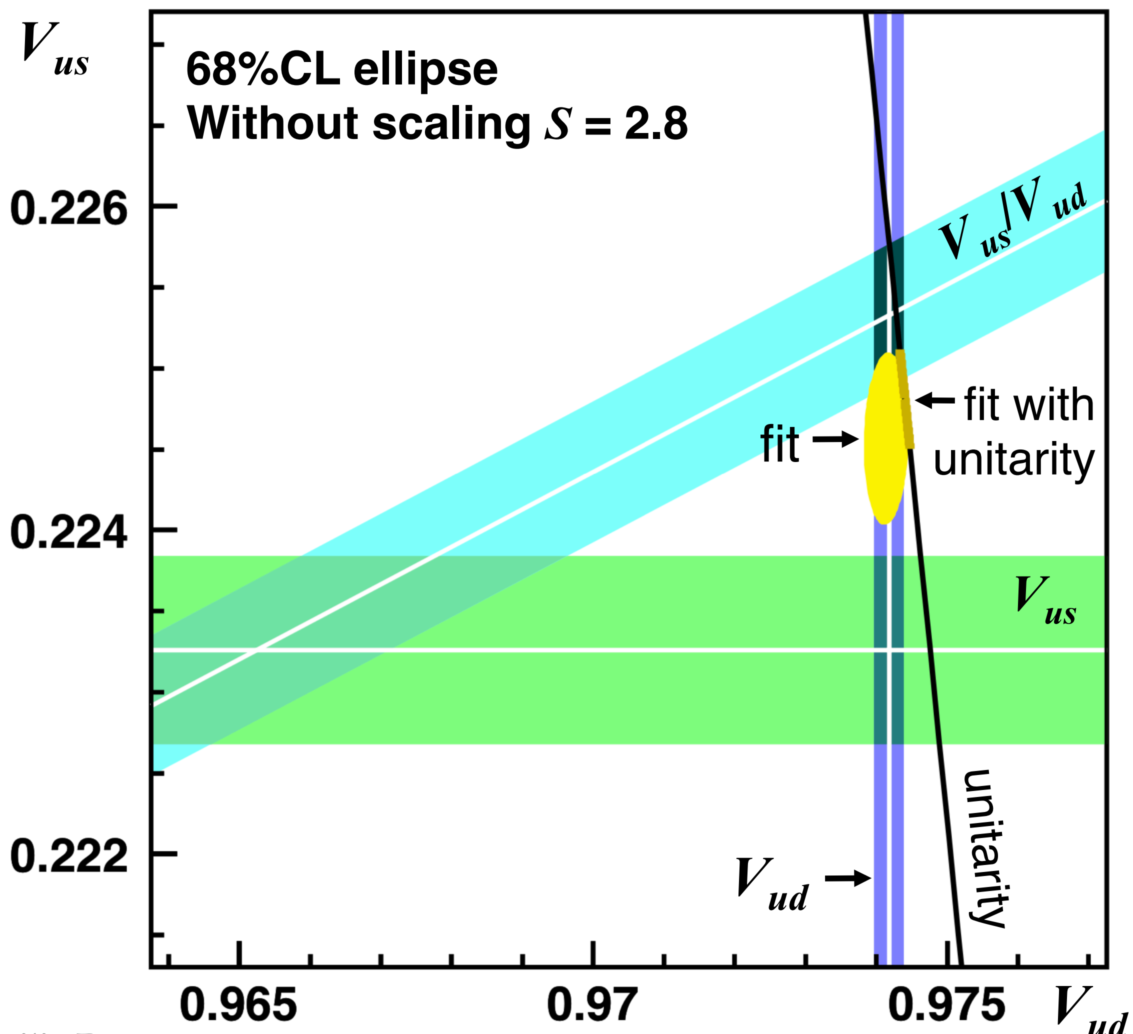
$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9970 \pm 0.0018$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.026 \pm 0.022$$

$$\sum_{\substack{i=u,c \\ j=d,s,b}} |V_{ij}|^2 = 2.002 \pm 0.027$$

$$\left(\text{from } BR(W \to e\nu) \propto \frac{1}{3 + 3\sum |V_{ij}|^2}\right)$$

Cabibbo tension



Emilie Passemar

$$\underline{V_{ui} V_{uj}^* = 0}$$

$ij = sd$ $\quad V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^* = 0$

$\quad\quad O(\lambda) \quad O(\lambda^5) \quad O(\lambda^5)$

$ij = bs$ $\quad V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0$

$\quad\quad O(\lambda^4) \quad O(\lambda^2) \quad O(\lambda^2)$

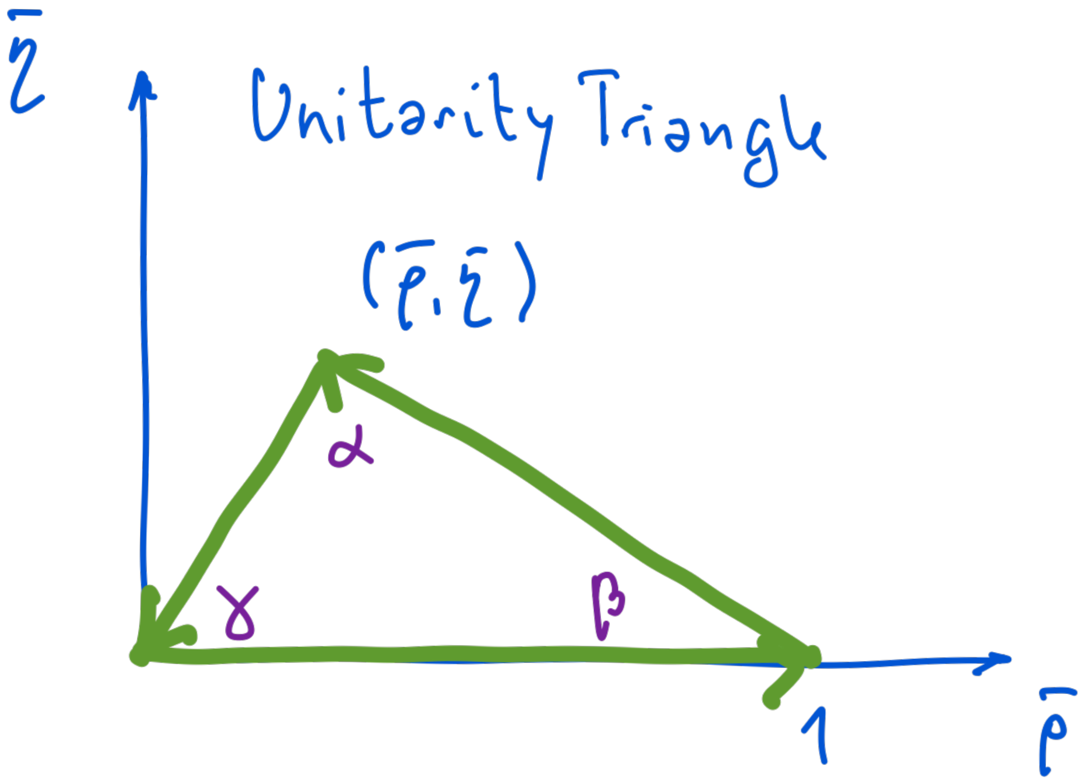
$ij = bd$ $\quad V_{ub} V_{ud}^* + V_{cb} V_{cd}^* + V_{tb} V_{td}^* = 0$ $\quad / V_{cb} V_{cd}^*$

$\quad\quad O(\lambda^3) \quad O(\lambda^3) \quad O(\lambda^3)$

$$V_{ub}V_{ud}^* = 0$$

$$\frac{V_{ub}V_{ud}^*}{V_{cb}V_{cd}^*} + 1 + \frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*} = 0$$

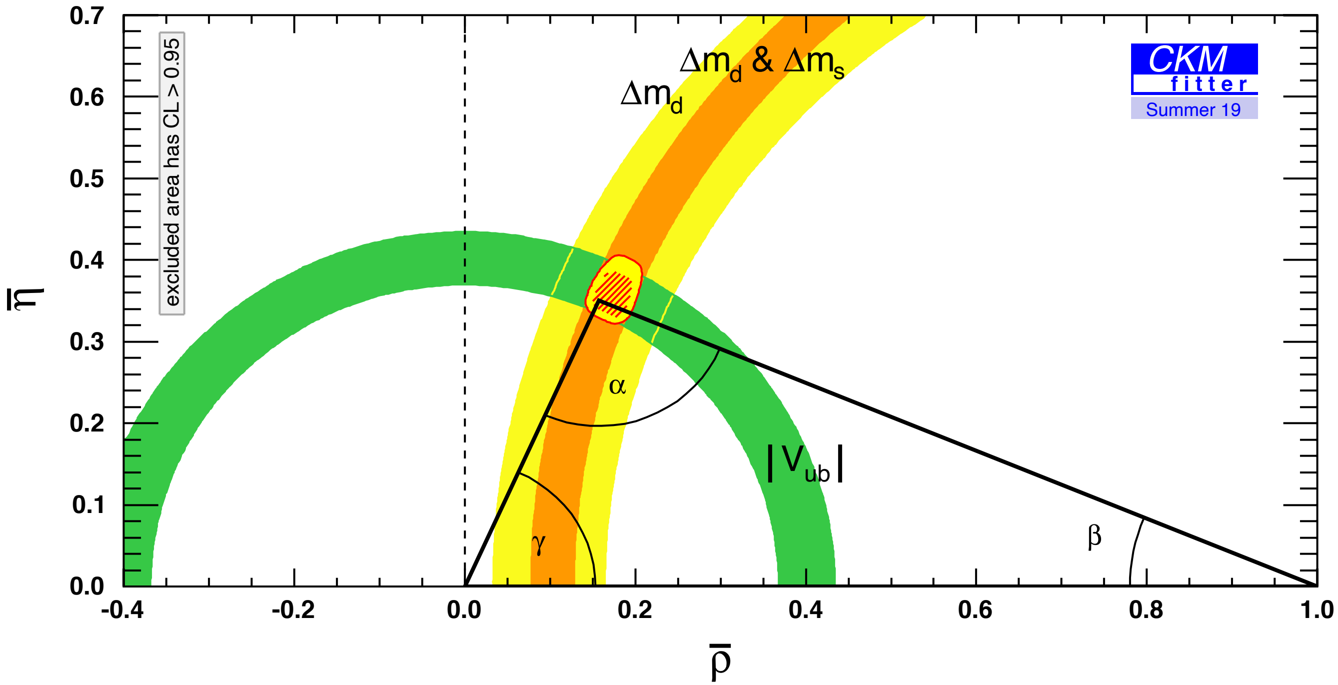
$$-(\bar{\rho} + i\bar{\eta}) \qquad \bar{\rho} + i\bar{\eta} - 1$$



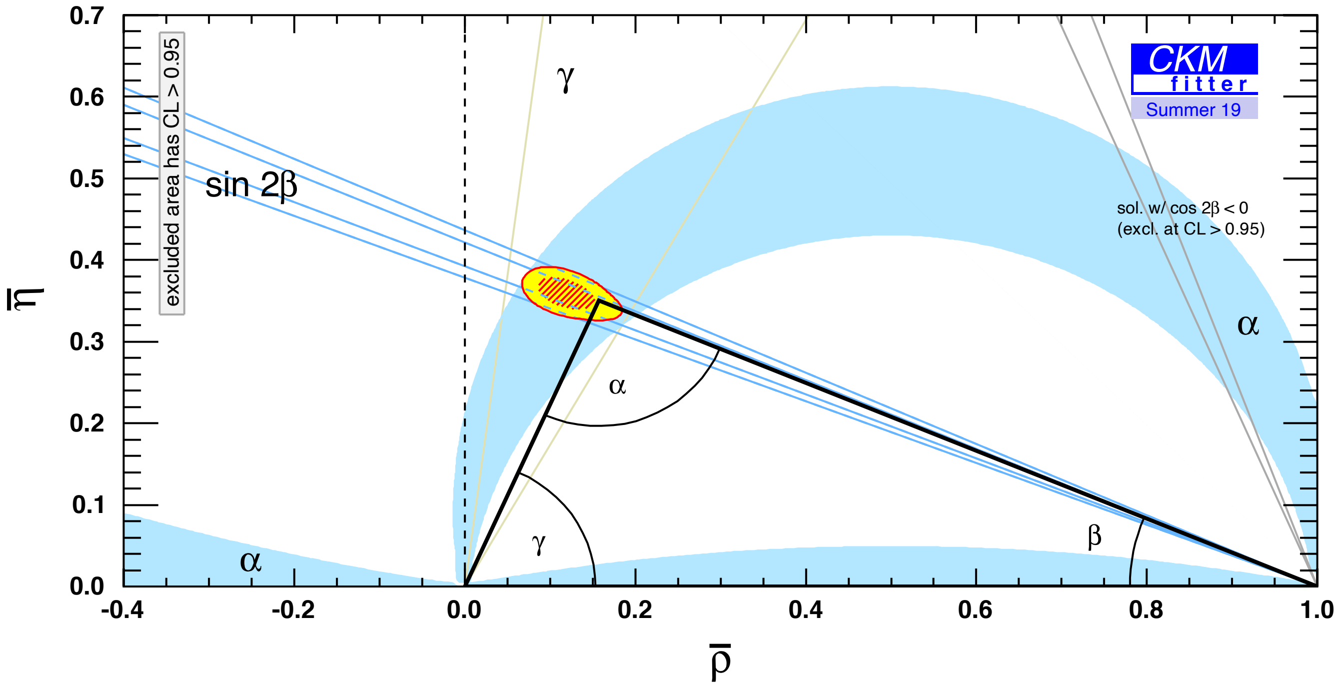
$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$



excluded area has CL > 0.95
Δm_d & Δm_s
Δm_d
|V_ub|
α
β
γ
CKM fitter
Summer 19
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
η̄
ρ̄



β from time-dependent asymmetries in $B \to J/\psi K_S$ + ...

α " " " " $B \to \pi\pi$ + ...

γ " " " " $B \to DK$ + ...

Time-dependent CP-asymmetries in B decays

$M^0-\bar{M}^0$ system $\qquad M = K \sim \bar{s}d$, $B_d \sim \bar{b}d$, $B_s \sim \bar{b}s$

Evolution governed by $H = M - i\Gamma/2$

$\langle M^0|H|\bar{M}^0\rangle \neq 0$: $M^0 \leftrightarrow \bar{M}^0$ allowed (loop)

$M_L, M_H = p M^0 \pm q \bar{M}^0$ mass eigenstates (mass $m_{L,H}$ width $\Gamma_{L,H}$)

$M = B$ $\qquad$ can approximate $\Gamma_L = \Gamma_H \equiv \Gamma$, $|q/p| = 1$. Then

$$\frac{\Gamma(B^0(t)\to f) - \Gamma(\bar{B}^0(t)\to f)}{\Gamma(\quad) + \Gamma(\quad)} = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)$$

$$S_f = \frac{2\,\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2} \qquad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \qquad \lambda_f = \frac{q}{p}\frac{\langle f|H|B^0\rangle}{\langle f|H|\bar{B}^0\rangle}$$

f CP eigenstate

↳ dependence on angles

Example: the "gold-plated" $B \to J/\psi K_S$ decay (and β)

$b \to c\bar{c}s$

amplitude: $V_{cb}^* V_{cs} T + V_{u_i b}^* V_{u_i s} P_i =$

$$" + \underbrace{V_{cb}^* V_{cs} (P_c - P_t)}_{\text{same phase}} + \underbrace{V_{ub}^* V_{us} (P_u - P_t)}_{\lambda^2 \times \text{loop}}$$

suppressed: CLEAN

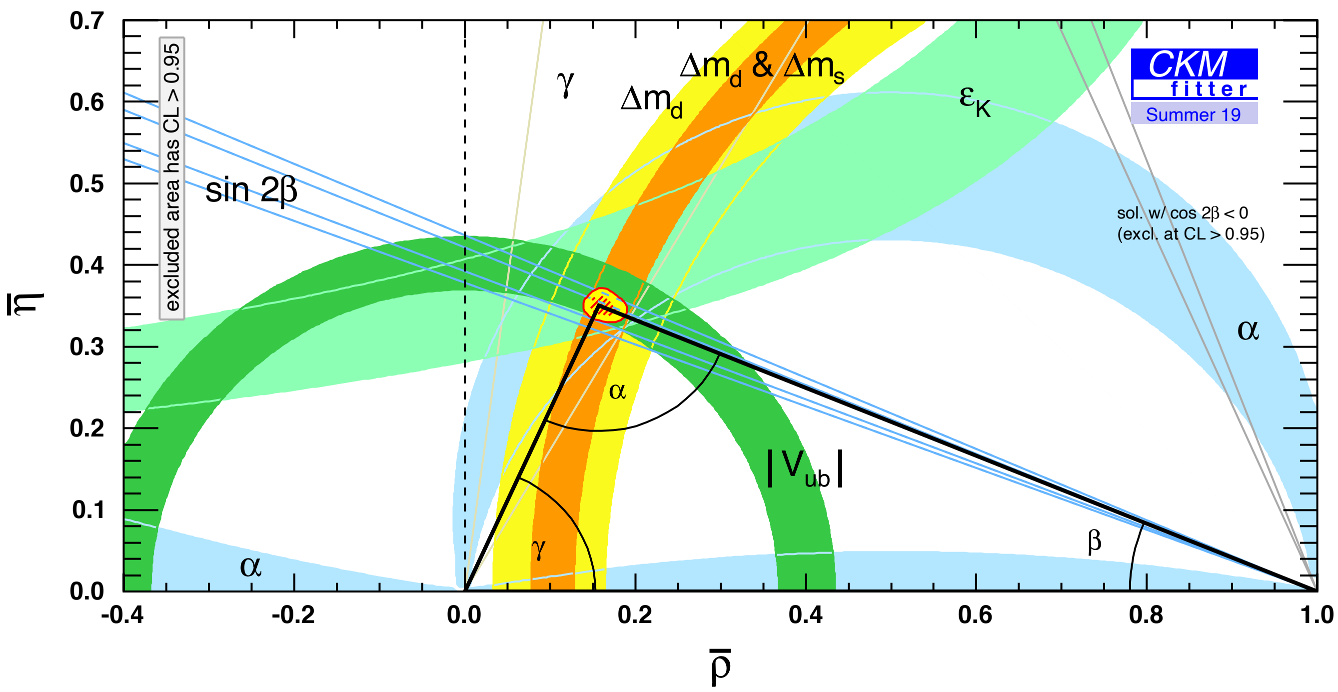
the tree-level phase combines with others to give

$\lambda_{f=J/\psi K_S} = -e^{-2i\beta}$ $\qquad S_{\psi K_S} = \sin 2\beta$ $\qquad C_{\psi K_S} = 0$

$\bar{b} \to \bar{q}q\bar{q}'$	$B^0 \to f$	$B_s \to f$	CKM dependence of A_f	Suppression	
$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$\psi\phi$	$(V_{cb}^* V_{cs})T + (V_{ub}^* V_{us})P^u$	loop $\times\ \lambda^2$	
$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$\phi\phi$	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})P^u$	λ^2	
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})T$	λ^2/loop	β
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	ψK_S	$(V_{cb}^* V_{cd})T + (V_{tb}^* V_{td})P^t$	loop	
$\bar{b} \to \bar{s}s\bar{d}$	$\phi\pi$	ϕK_S	$(V_{tb}^* V_{td})P^t + (V_{cb}^* V_{cd})P^c$	$\lesssim 1$	
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\pi^0 K_S$	$(V_{ub}^* V_{ud})T + (V_{tb}^* V_{td})P^t$	loop	α

$b \to c\bar{u}s$ vs $b \to \bar{c}us$ $\quad\quad$ $B^\pm \to D^0 K^\pm$ vs $B^\pm \to \bar{D}^0 K^\pm$ $\quad$ γ

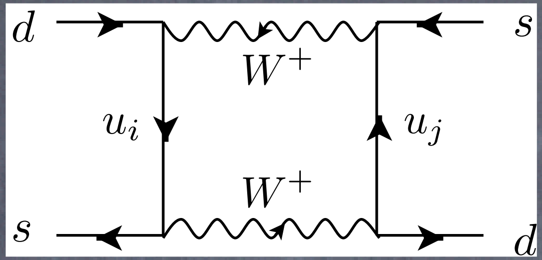
$$\alpha + \beta + \gamma = (180.6 \pm 7.2)^\circ$$



Test of unitarity
and of new physics

Anomalously small loop-induced FCNC

- Expect:



$$\sim \quad \frac{1}{M_W^2} \times \frac{g^4}{(4\pi)^2} \times \epsilon$$

K^0 - $\overline{K}^0$ oscillations

- Instead: 10^6 smaller $\quad \epsilon \sim 10^{-6}$

- $\epsilon = (V^\dagger_{su_i} V_{u_i d})(V^\dagger_{su_j} V_{u_j d}) f\left(\frac{m^2_{u_i}}{M_W^2}, \frac{m^2_{u_j}}{M_W^2}\right)$

  i = 3: f = O(1), $|V_{td}V_{ts}| \ll 1$

  i = 1,2: $|V_{id}V_{is}| = O(1)$, f $\ll$ 1

Symmetry origin of suppression: U(2)^5

- In an appropriate basis

$$\lambda = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix} + \text{small} \quad (U,\, D,\, E)$$

- Approximately U(2)^5 symmetric
- ε = 0 in the symmetric limit

Flavour tensions / anomalies

“Clean” observables?

- Isospin relations
  - Koto anomaly: $K_L \to \pi^0 \nu \bar{\nu}$, $K^\pm \to \pi^\pm \nu \bar{\nu}$
  - hadronic matrix elements related to $K^+ \to \pi^0 e \nu$ by isospin

- Lepton flavour universality (LFU)
  - R_K, R_{K*}, R_D, R_{D*} anomalies
  - uncertainties partially cancel in ratios of observables

KOTO

- Koto
- $K_L \to \pi^0 \nu \bar{\nu}$
- 4 events, 0.05 expected
- BR$(K_L \to \pi^0 \nu \bar{\nu})_{\rm KOTO} = 21^{+20}_{-11} \cdot 10^{-10}$
- BR$(K_L \to \pi^0 \nu \bar{\nu})_{\rm SM} \approx 0.34 \cdot 10^{-10}$
- $3.8\,\sigma$ tension

- NA62
- $K^+ \to \pi^+ \nu \bar{\nu}$
- 2 events, 1.5 expected
- BR$(K^+ \to \pi^+ \nu \bar{\nu})_{\rm NA62} < 1.85 \cdot 10^{-10}$
- BR$(K^+ \to \pi^+ \nu \bar{\nu})_{\rm SM} \approx 0.84 \cdot 10^{-10}$

Grossman-Nir bound

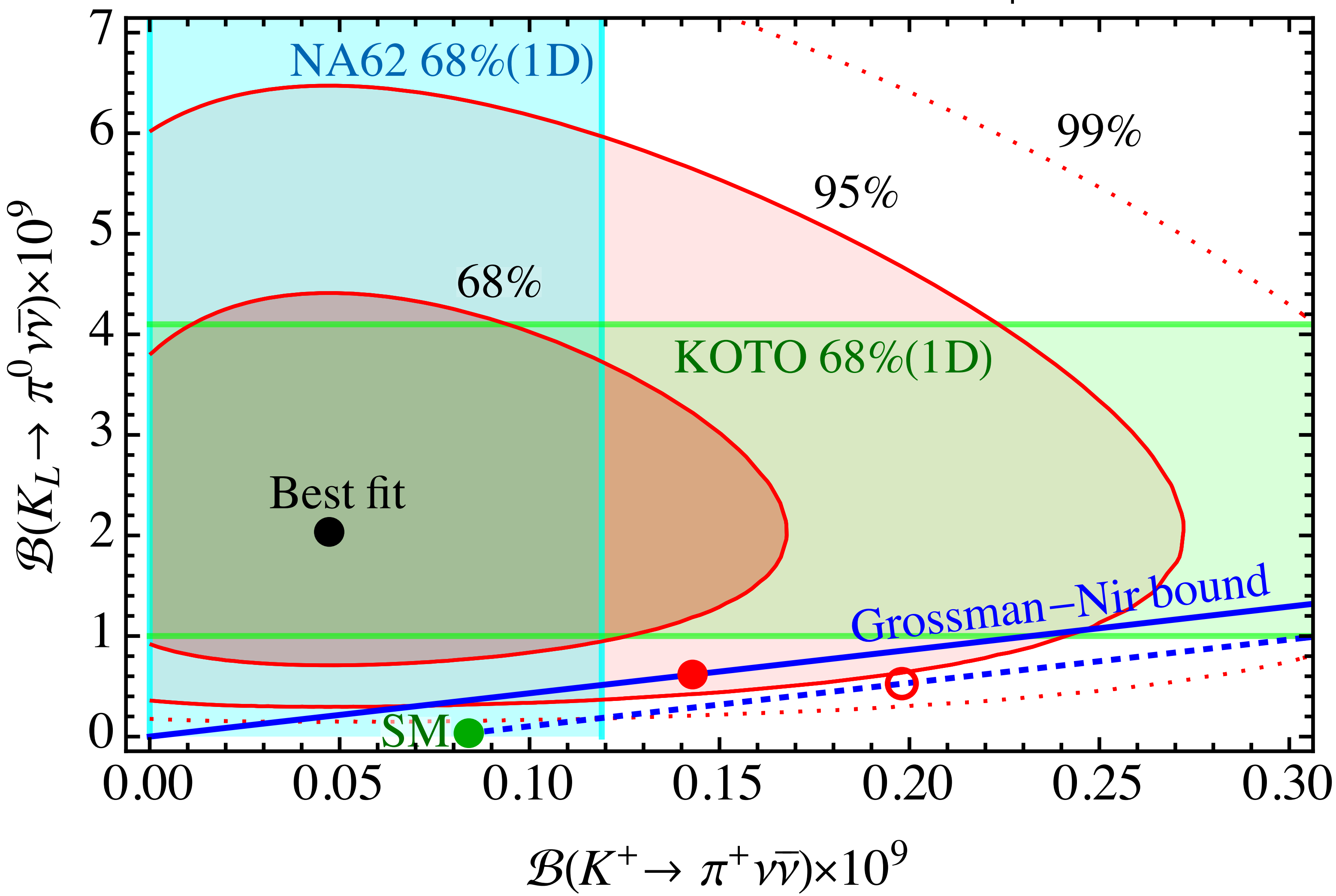
BR$(K_L \to \pi^0 \nu \bar{\nu}) \lesssim 4.3\,BR(K^+ \to \pi^+ \nu \bar{\nu}) \lesssim 8.1\; 10^{-10}$

2.1 tension with BSM

(asumes $s \to dX$ origin for both)

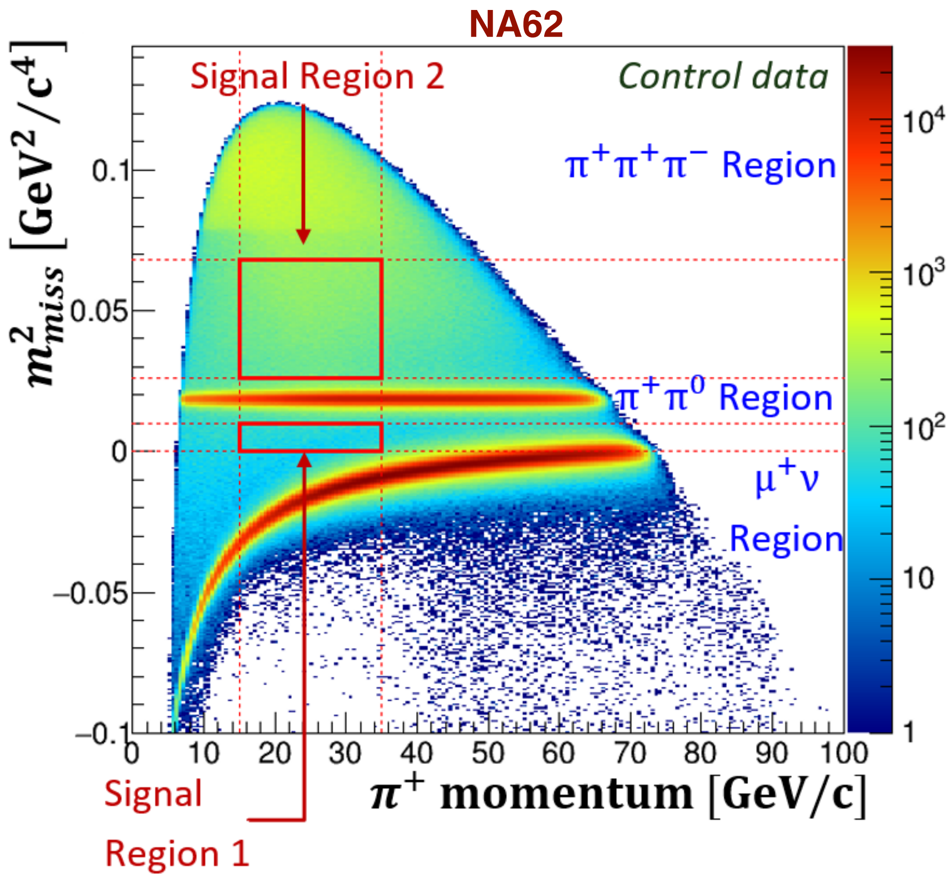
Grossman-Nir bound

Kitahara Okui Perez Soreq Tobioka 1909.11111



Pheno-gym: how to violate the GN bound

- $K \to \pi X$ with $M_X \sim m_\pi$
  - blind to NA62
    not to KOTO
- $K \to \pi X,\ X \to$ SM
  - NA62 X decay in detector, rejects
    KOTO X decay outside detector
- $X \to \gamma\gamma$
  - NA62 looks for π^+, rejects
    KOTO sees $\gamma\gamma$ from off axis mimicking π^0 + missing p_T



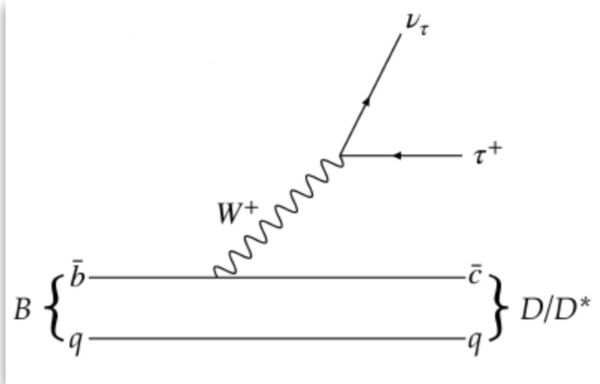
Lepton Flavour Universality (LFU)

- $\text{BR}(B \to A e^+ e^-) = \text{BR}(B \to A \mu^+ \mu^-) = \text{BR}(B \to A \tau^+ \tau^-)$
    - up to $U(3)_l \times U(3)_{eR}$ breaking effects
    - small when $m^2 \ll q^2$ and anyway calculable

- $\dfrac{\text{BR}(B \to A l_1^+ l_1^-)}{\text{BR}(B \to A l_2^+ l_2^-)}$ reduces hadronic uncertainties

LFU in CC B-decays

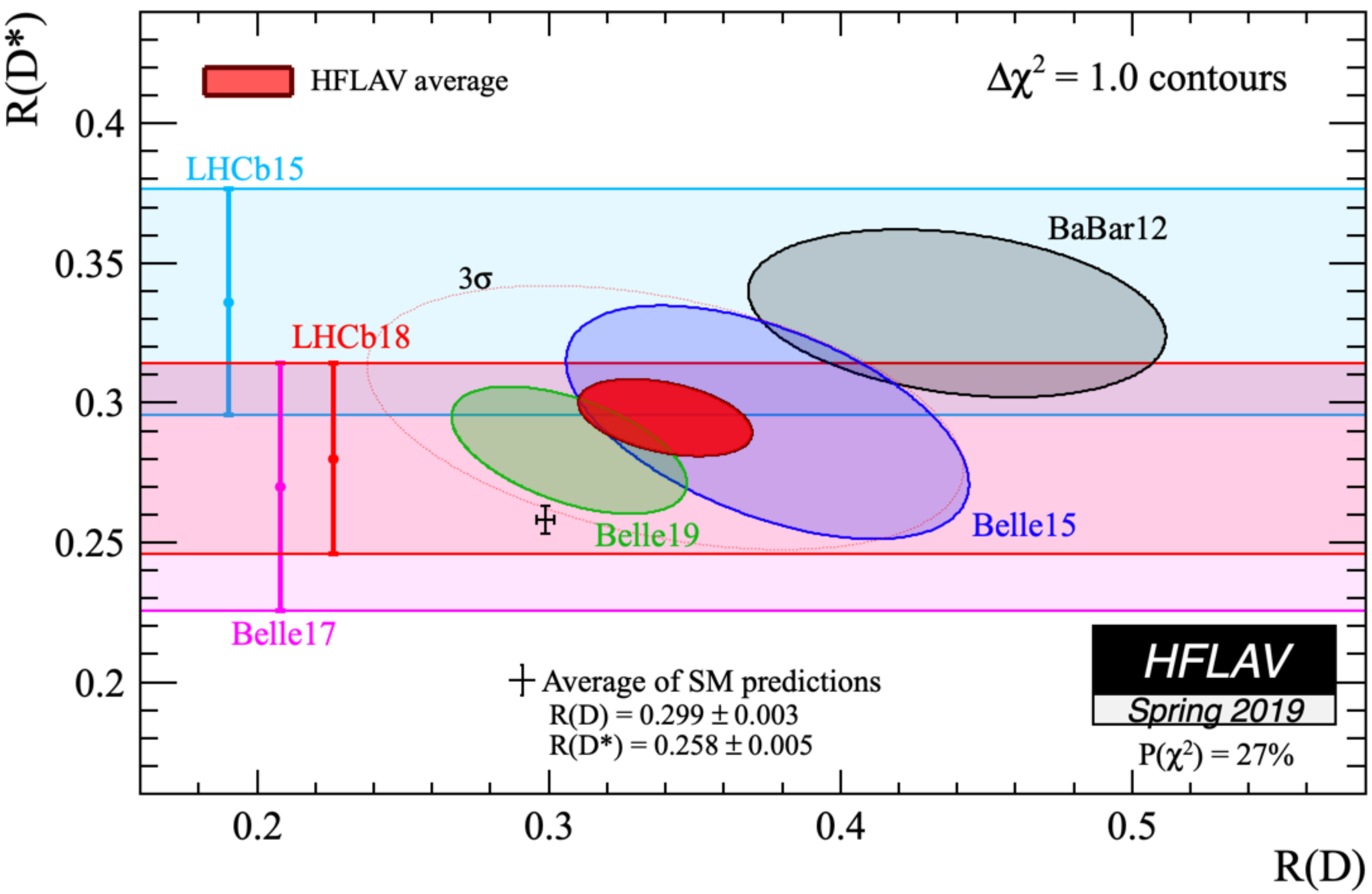
- $b \to c\tau\bar{\nu}$
    - tree-level in the SM



- $$\frac{\text{BR}(B^0 \to D^* e^+ \bar{\nu})}{\text{BR}(B^0 \to D^* \mu^+ \bar{\nu})} = 1.01 \pm 0.03$$

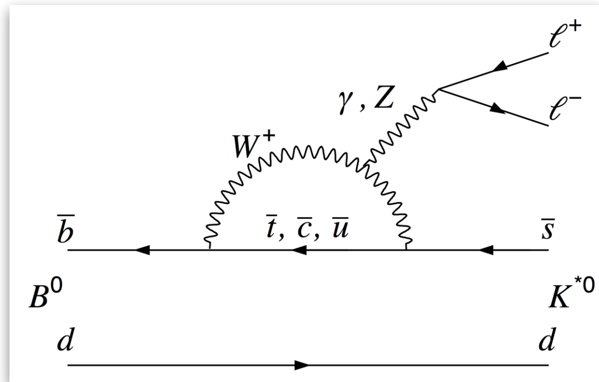
- $$\frac{\text{BR}(B^0 \to D^{(*)} \tau^+ \bar{\nu})}{\text{BR}(B^0 \to D^{(*)} l^+ \bar{\nu})} \equiv R_{D^{(*)}} \neq (R_{D^{(*)}})_{\text{SM}}$$

LFU in CC B-decays



LFU in NC B-decays

- $b \to s\mu^+\mu^-$
  - 1-loop in the SM

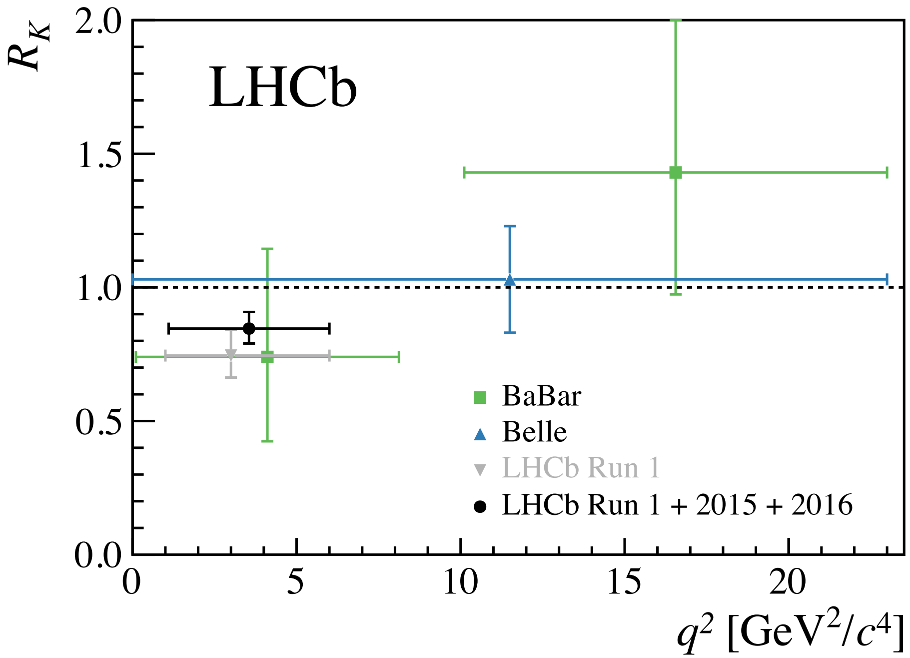


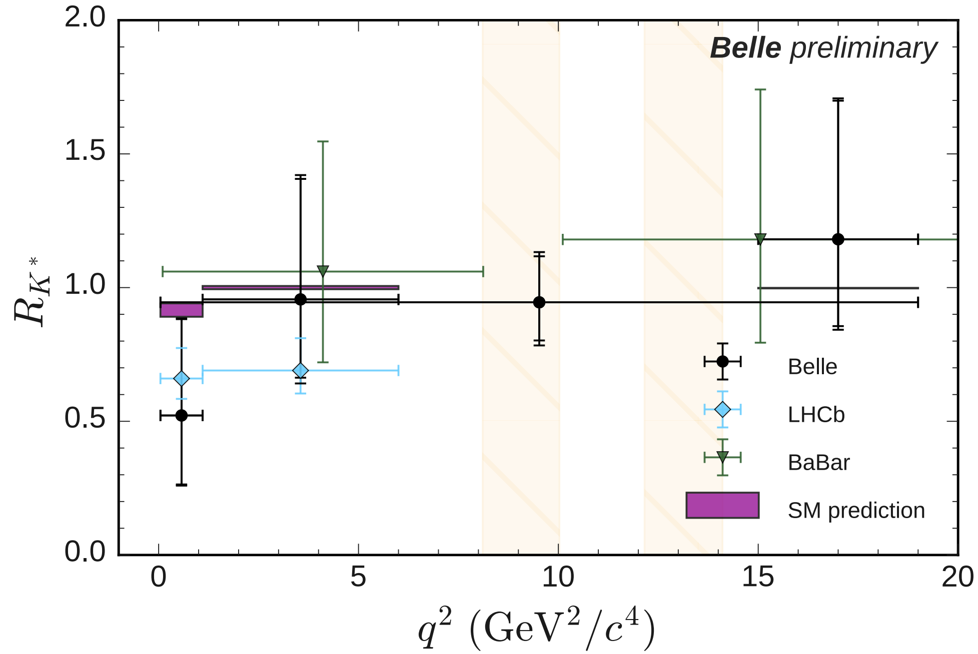
- $\frac{\mathrm{BR}(B \to K^{(*)}\mu^+\mu^-)}{\mathrm{BR}(B \to K^{(*)}e^+e^-)} \equiv R_{K^{(*)}} \neq (R_{K^{(*)}})_{\mathrm{SM}}$

- Clean: $R_K^{[1,6]}$, $R_{K*}^{[0.045,1.1]}$, $R_{K*}^{[1.1,6]}$: each about 2.5 σ tension (q^2/GeV^2)
  also: $B_s \to \mu^+\mu^-$: 2 σ tension

- Further deviations, and constraints, in observables with hadronic uncertainties

LFU in NC B-decays





New Physics parameterisation in WET

- WET = SU(3)$_c$ x U(1)$_{em}$ invariant EFT below EW scale
- 10(μ) + 10(e) independent operators (10 = 4S + 4V + 2T)
- 6(μ) + 6(e) from tree-level match with D=6 SMEFT (6 = 4V + 2S)
- Global fits prefer μ, vector, Q$_L$ (with some flexibility)

$$C_L^{bs}\,(\overline{s_L}\gamma^\mu b_L)(\overline{\mu_L}\gamma_\mu\mu_L) + C_R^{bs}\,(\overline{s_L}\gamma^\mu b_L)(\overline{\mu_R}\gamma_\mu\mu_R)$$

needed
sufficient

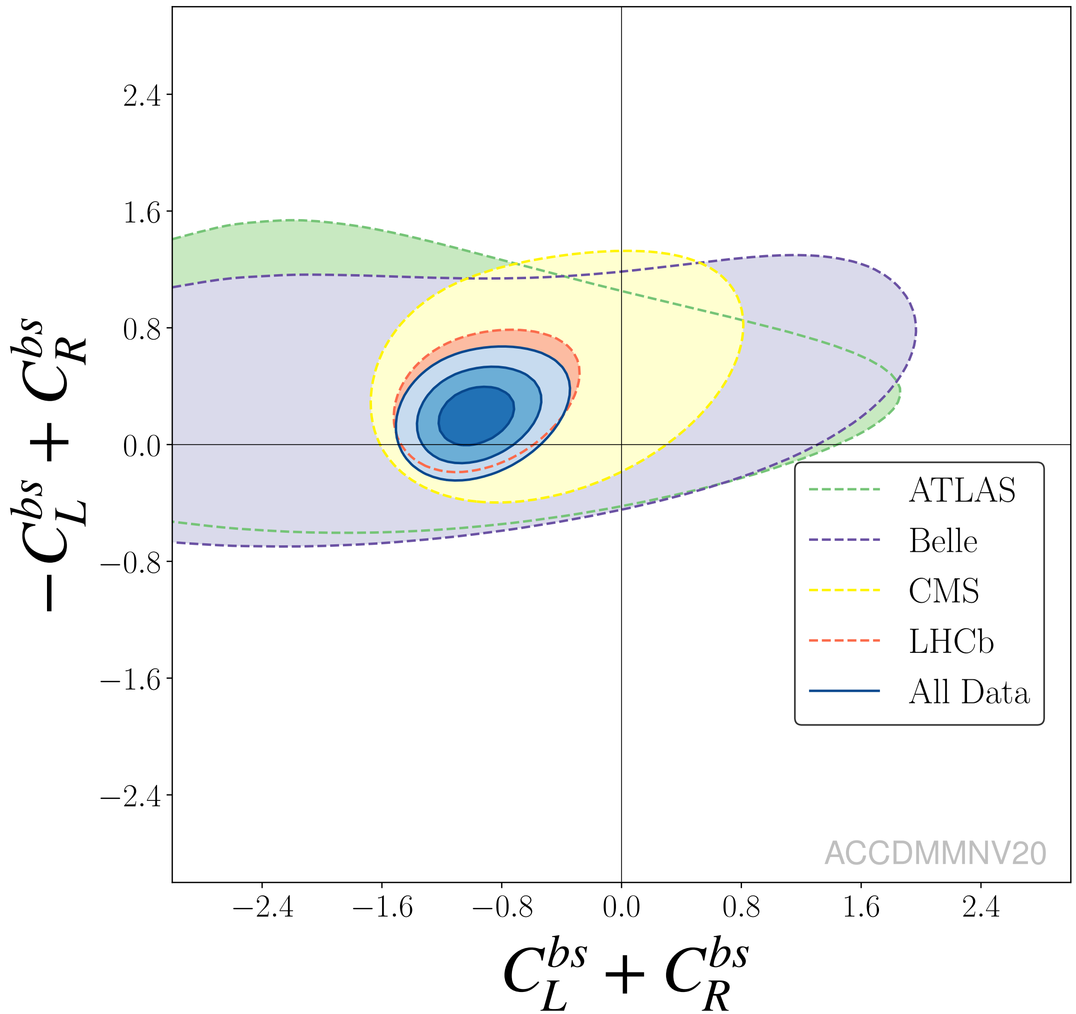
optional

- (Subdominant contributions from d$_R$ and e possibly welcome)

New Physics parameterisation in WET

$$C_L^{bs}\,(\overline{s_L}\gamma^\mu b_L)(\overline{\mu_L}\gamma_\mu\mu_L) + C_R^{bs}\,(\overline{s_L}\gamma^\mu b_L)(\overline{\mu_R}\gamma_\mu\mu_R)$$

Pull-SM
$\chi^2_{\rm SM} - \chi^2_{min}$
6.2 σ



Algueró, Capdevila, Crivellin, Descotes-Genon, Masjuan, Matias, Novoa-Brunet, Virto
Addendum 06/2020 of arXiv:1903.09578

New Physics parameterisation in SMEFT

- SMEFT = G_{SM} invariant EFT below NP scale

- $\mathcal{O}_{ij}^{L} = (\overline{d_{iL}}\gamma^{\mu} d_{jL})(\overline{\mu_L}\gamma_{\mu}\mu_L),\ \mathcal{O}_{ij}^{R} = (\overline{d_{iL}}\gamma^{\mu} d_{jL})(\overline{\mu_R}\gamma_{\mu}\mu_R)$ from

$$\begin{aligned}
\mathcal{O}_{ij}^{S} &= (\overline{Q_i}\gamma^{\mu} Q_j)(\overline{L_2}\gamma_{\mu} L_2) &\leftrightarrow\quad C_{ij}^{S}\\
\mathcal{O}_{ij}^{T} &= (\overline{Q_i}\gamma^{\mu}\sigma_a Q_j)(\overline{L_2}\gamma_{\mu}\sigma_a L_2) &\leftrightarrow\quad C_{ij}^{T}\\
\mathcal{O}_{ij}^{R} &= (\overline{Q_i}\gamma^{\mu} Q_j)(\overline{\mu_R}\gamma_{\mu}\mu_R) &\leftrightarrow\quad C_{ij}^{R}
\end{aligned}
\qquad Q_i = \begin{pmatrix} V_{ij}^{\dagger} u_j \\ d_i \end{pmatrix}$$

- Redundancy

$$\mathcal{O}_{ij}^{+} = \frac{\mathcal{O}_{ij}^{S} + \mathcal{O}_{ij}^{T}}{2} = (\overline{Q_i}\gamma^{\mu} L_2)(\overline{L_2}\gamma_{\mu} Q_j) \quad\leftrightarrow\quad C_{ij}^{+} = C_{ij}^{S} + C_{ij}^{T} = C_{ij}^{L}$$

$$\mathcal{O}_{ij}^{-} = \frac{\mathcal{O}_{ij}^{S} - \mathcal{O}_{ij}^{T}}{2} = 2(\overline{Q_i^c} L_2)(\overline{L_2} Q_j^c) \quad\leftrightarrow\quad C_{ij}^{-} = C_{ij}^{S} - C_{ij}^{T}$$

A coherent picture?

- $b \to c\tau\bar{\nu}$ $b \to s\mu^+\mu^-$ $b \to se^+e^-$
- $3_q \to 2_q\ 3_l\ 3_l$ $3_q \to 2_q\ 2_l\ 2_l$ $3_q \to 2_q\ 1_l\ 1_l$
- $(C_\tau)^{-1/2}$ » $(C_\mu)^{-1/2}$ » $(C_e)^{-1/2}$
- λ_τ » λ_μ » λ_e

Why Beyond?

Some reasons to go beyond the SM

- Experimental "problems" of the SM
  - Gravity
  - Dark matter
  - Baryon asymmetry
  - Neutrino masses
- Experimental "hints" of physics beyond the SM
  - Quantum number unification
- Theoretical puzzles of the SM
  - $\langle H \rangle \ll M_{Pl}$
  - Family replication
  - Small Yukawa couplings, pattern of masses and mixings
  - Gauge group, no anomaly, charge quantization, quantum numbers
- Theoretical problems of the SM
  - Higgs mass naturalness problem
  - Cosmological constant problem
  - Strong CP problem
  - Landau poles

Experimental “problems” of the SM

- Gravity
- Dark matter
- Baryon asymmetry
- Neutrino masses

Experimental "hints" of physics beyond the SM

- Quantum number unification

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
e^c_i	1	1	1
Q_i	3	2	1/6
u^c_i	3^*	1	-2/3
d^c_i	3^*	1	1/3
			Y

SO(10)
16

p-decay bounds: $M \gg m_H$

an accident?

Theoretical puzzles of the SM

- $\langle H \rangle \ll M_{Pl}$
- Family replication
- Small Yukawa couplings, masses and mixings

Theoretical problems of the SM

- Landau poles
- Strong CP problem $\theta\, G_{\mu\nu}\tilde{G}^{\mu\nu}$ $D = 4$
- Naturalness problem $\alpha\, Q^2_{\max} H^\dagger H$ $D = 2$
- Cosmological constant problem $\beta\, Q^4_{\max}\sqrt{g}$ $D = 0$