Scattering in chiral strong backgrounds

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based on work w/ L. Mason & A. Sharma

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Motivation

Tree-level gauge theory/gravity in Minkowski space:

We know everything

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But what about gauge theory/gravity/string theory in **non-trivial/strong** background fields?

Much less known – even in simplest strong backgrounds

- Tree-level 4-points (AdS, plane waves) [D'Hoker et al., Raju, TA-Casali-Mason-Nekovar]
- Unclear how 'novel' methods (unitarity, double copy, scattering equations) extend to strong backgrounds

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Today: Can we make all-multiplicity statements on (any) strong backgrounds?

Yes!

Class of chiral, asymp. flat strong backgrounds in 4d gauge theory and gravity

Radiative backgrounds

Radiative field is completely characterized by free data at $\mathscr{I}^{\pm}$

Asymp. flat gauge/gravitational fields:

$$A = \mathcal{A}^0(u,z,\bar{z})\,\mathrm{d}z + \bar{\mathcal{A}}^0(u,z,\bar{z})\,\mathrm{d}\bar{z} + O(r^{-1})$$

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\mathbb{M}} - 2\frac{m_B(u,z,\bar{z})}{r}\,\mathrm{d}u^2 + \frac{\sigma^0(u,z,\bar{z})}{r}\,\mathrm{d}z^2 + \frac{\bar{\sigma}^0(u,z,\bar{z})}{r}\,\mathrm{d}\bar{z}^2 + O(r^{-2})$$

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Free data are spin-weighted functions on $\mathscr{I}^{+}$:

$$\mathcal{A}^0(u,z,\bar{z}) \leftrightarrow s = 1\,, \qquad \sigma^0(u,z,\bar{z}) \leftrightarrow s = 2$$

SD Radiative fields

Lorentzian-real fields $\mathcal{A}^0$, $\bar{\mathcal{A}}^0$ and σ^0, $\bar{\sigma}^0$ related by complex conjugation

Complexify $\Rightarrow$ $\mathcal{A}^0$, $\tilde{\mathcal{A}}^0$ and σ^0, $\tilde{\sigma}^0$ *independent*

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Idea:

- Exploit integrability of SD sector to compute tree-level S-matrix

Twistor theory

Twistor space: $Z^A = (\mu^{\dot{\alpha}}, \lambda_\alpha)$ homog. coords. on $\mathbb{CP}^3$

$$\mathbb{PT} = \mathbb{CP}^3 \setminus \{\lambda_\alpha = 0\}$$

$x \in \mathbb{C}^4$ given by $X \cong \mathbb{CP}^1 \subset \mathbb{PT}$ via $\mu^{\dot{\alpha}} = x^{\alpha\dot{\alpha}}\lambda_\alpha$

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Many applications in flat background:

- Massless free fields $\leftrightarrow$ cohomology on $\mathbb{PT}$ [Penrose, Sparling, Eastwood-Penrose-Wells]
- Representation for on-shell scattering kinematics [Hodges]
- Full tree-level S-matrix of $\mathcal{N} = 4$ SYM [Witten, Berkovits, Roiban-Spradlin-Volovich]
- Full tree-level S-matrix of $\mathcal{N} = 8$ SUGRA [Cachazo-Skinner]

Trivializing SD sector

Theorem [Ward 1977]

There is a 1:1 correspondence between:

- SD SU(N) Yang-Mills fields on $\mathbb{C}^4$, and
- rank N holomorphic vector bundles $E \to \mathbb{PT}$ trivial on every $X \subset \mathbb{PT}$ (+ technical conditions)

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Theorem [Penrose 1976]

There is a 1:1 correspondence between:

- SD 4-manifolds $\mathscr{M}$, and
- $\mathbb{P}\mathscr{T}$ a complex deformation of $\mathbb{PT}$ with rational curve $X \subset \mathbb{P}\mathscr{T}$ with normal bundle $N_X \cong \mathcal{O}(1) \oplus \mathcal{O}(1)$

Upshot

SD sector encoded by integrable $\mathbb{C}$-structures on twistor space

- Gauge theory: $E \to \mathbb{PT}$ partial connection $\bar{D} = \bar{\partial} + \mathsf{A}$, $\mathsf{A} \in \Omega^{0,1}(\mathrm{End} E)$, $\bar{D}^2 = 0$
- Gravity: $T_{\mathbb{P}\mathscr{T}}$ complex structure $\bar{\nabla} = \bar{\partial} + V$, $V \in \Omega^{0,1}(T_{\mathbb{P}\mathscr{T}})$, $\bar{\nabla}^2 = 0$

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For SD radiative fields [Sparling, Newman, Eastwood-Tod]

- $\mathsf{A}(Z) = \partial_u \mathcal{A}^0(\mu^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}}, \lambda, \bar{\lambda})\, \mathrm{D}\bar{\lambda}$
- $V(Z) = \sigma^0(\mu^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}}, \lambda, \bar{\lambda})\, \mathrm{D}\bar{\lambda}\, \bar{\lambda}^{\dot{\alpha}} \frac{\partial}{\partial \mu^{\dot{\alpha}}}$

Gauge theory

Restrict to SD rad. gauge fields valued in Cartan

$E|_X$ trivialized by holomorphic frame

$$\bar{\partial}|_X \mathsf{H}(x,\lambda) = -\mathsf{A}|_X\, \mathsf{H}(x,\lambda)\,,$$
$$\mathsf{A}|_X = \bar{\partial} g(x,\lambda)\,, \qquad \text{as } H^{0,1}(\mathbb{P}^1, \mathcal{O}) = \emptyset\,,$$
$$\Rightarrow \quad \mathsf{H}(x,\lambda) = \exp\left[-g(x,\lambda)\right]$$

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Recover space-time gauge field from

$$\partial_{\alpha\dot{\alpha}} g(x,\lambda) = A_{\alpha\dot{\alpha}}(x) + \lambda_\alpha\, g_{\dot{\alpha}}(x,\lambda)$$

Gluon perturbations

Gluon perturbations in background encoded by cohomology:
$+$ helicity gluon $\leftrightarrow a \in H^{0,1}_{\bar{D}}(\mathbb{PT}, \mathcal{O} \otimes \mathrm{End} E)$
$-$ helicity gluon $\leftrightarrow b \in H^{0,1}_{\bar{D}}(\mathbb{PT}, \mathcal{O}(-4) \otimes \mathrm{End} E)$

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E.g. $+$ helicity with asymp. momentum $k_{\alpha\dot{\alpha}} = \kappa_\alpha \tilde{\kappa}_{\dot{\alpha}}$

$$a(Z) = \mathsf{T} \frac{\langle \xi\, \lambda \rangle}{\langle \xi\, \kappa \rangle} \bar{\partial} \left(\frac{1}{\langle \lambda\, \kappa \rangle} \right) \mathrm{e}^{\mathrm{i} \frac{\langle \xi\, \kappa \rangle}{\langle \xi\, \lambda \rangle} [\mu\, \tilde{\kappa}]}$$

Leads to

$$a^{(+)}_{\alpha\dot{\alpha}}(x) = \mathsf{T} \frac{\xi_\alpha (\tilde{\kappa}_{\dot{\alpha}} + e\, g_{\dot{\alpha}}(x, \kappa))}{\langle \xi\, \kappa \rangle} \exp\left[\mathrm{i}\, k \cdot x + e\, g(x, \kappa)\right]$$

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Lesson from flat background: [Nair, Witten, Berkovits, Roiban-Spradlin-Volovich]

- Tree-level YM amplitudes live on rational holomorphic curves in $\mathbb{PT}$
- N^{k}MHV degree related to degree of curve $d = k + 1$

$$\mu^{\dot{\alpha}}(\sigma) = u^{\dot{\alpha}}_{\mathbf{a}(d)}\, \sigma^{\mathbf{a}(d)}\,, \qquad \lambda_{\alpha}(\sigma) = \lambda_{\alpha\,\mathbf{a}(d)}\, \sigma^{\mathbf{a}(d)}$$

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Colour-ordered N^{d-1}MHV amplitude of $\mathcal{N} = 4$ SYM:

$$\int \frac{\mathrm{d}^{2d+2}\lambda}{\mathrm{vol}\,\mathrm{GL}(2,\mathbb{C})}\,\delta^{2|4(d+1)}\left(\sum_{i=1}^{n} s_i\,\tilde\kappa_i\,\sigma_i^{\mathbf{a}(d)}\right) \times \prod_{j=1}^{n}\frac{\mathrm{d}s_j\,\mathrm{D}\sigma_j}{s_j\,(j\,j+1)}\,\bar\delta^2(\kappa_j - s_j\,\lambda(\sigma_j))$$

Central conjecture

Twistor string theory coupled to background field A (trivial extension to $\mathcal{N} = 4$ SYM)

Worldsheet action:

$$S = \frac{1}{2\pi} \int_\Sigma Y_I \bar{\partial} Z^I + \mathrm{tr}(j\, \mathsf{A}) + S_{\mathfrak{g}} + \mathrm{ghosts}\,,$$

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Conjecture

N^kMHV amplitude on SD rad. background given by worldsheet correlator at degree $d = k + 1$

Amplitude formula

Let $Z^I(\sigma) = U^I_{\mathbf{a}(d)}\,\sigma^{\mathbf{a}(d)}$, and

$$\mathsf{A}(Z(\sigma)) = \bar{\partial}\, g(U, \sigma)$$

Result for N^kMHV color-ordered tree amplitude:

$$\int \frac{\mathrm{d}^{4|4(d+1)}U}{\mathrm{vol}\,\mathrm{GL}(2,\mathbb{C})} \prod_{i=1}^{n} \frac{\mathrm{d}s_i\,\mathrm{D}\sigma_i}{s_j\,(i\;i+1)}\,\bar{\delta}^2(\kappa_i - s_i\,\lambda(\sigma_i))$$
$$\times \exp\left([\mu(\sigma_i)\,\tilde{\kappa}_i] + e_i\, g(U, \sigma_i)\right)$$

Properties

Formula passes many checks

- Explicit 3- and 4-point checks
- Flat & perturbative limits
- Proved for MHV ($d = 1$)

Important differences from RSVW

- No momentum conservation
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Similar – more complicated – story for gravity!

All-multiplicity formulae on some strong backgrounds!

$\Rightarrow$ interesting probe of non-linearities in QFT

Any use/interest for string theory?

- SD *plane waves* $\subset$ SD radiative backgrounds
- Vacuum plane waves are string backgrounds [Amati-Klimcik, Horowitz-Steif]
- So these formulae provide data for the $\alpha' \to 0$ limit of strings in plane wave backgrounds

Further directions

Many interesting open questions:

- Prove N^{k}MHV formulae for $k > 1$ (analyticity constraints [Ilderton-MacLeod] ?)
- Other chiral backgrounds (with sources, not asymptotically flat)
- Non-chiral backgrounds, scattering equations, ambitwistor strings

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