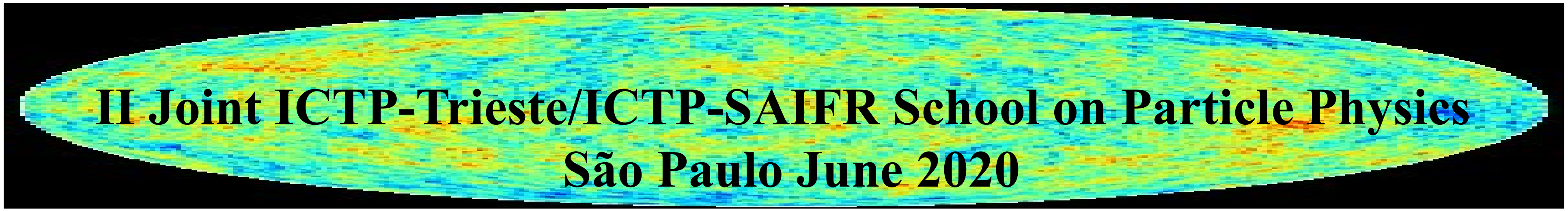
Cosmology and Particle Physics

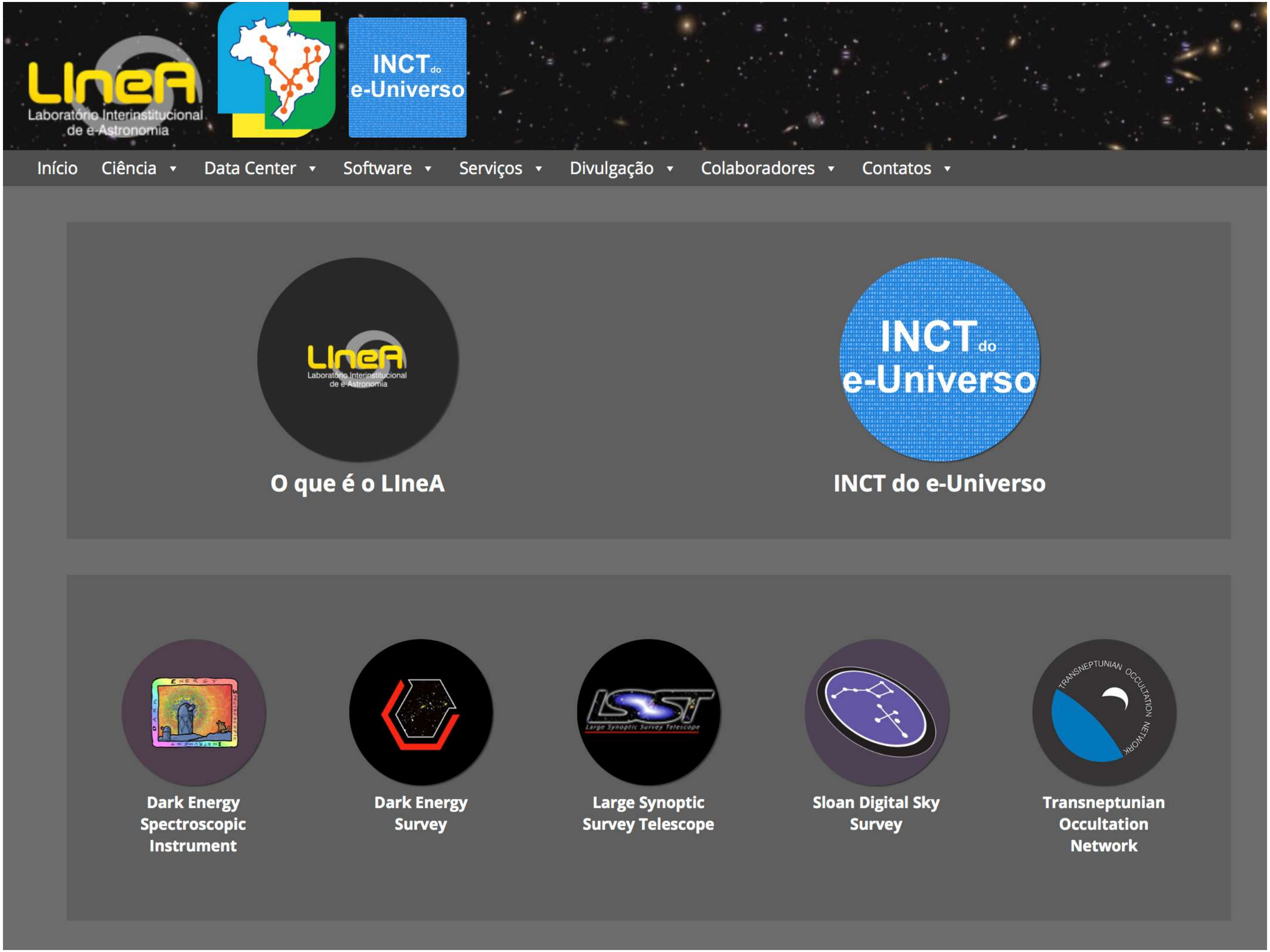
Rogerio Rosenfeld
IFT-UNESP & ICTP-SAIFR & LIneA

Part I: The average Universe

Part II: Origins

Part III: The perturbed Universe




LIneA
Laboratório Interinstitucional de e-Astronomia
INCT do e-Universo
Início
Ciência
Data Center
Software
Serviços
Divulgação
Colaboradores
Contatos
O que é o LIneA
INCT do e-Universo
Dark Energy Spectroscopic Instrument
Dark Energy Survey
Large Synoptic Survey Telescope
Sloan Digital Sky Survey
Transneptunian Occultation Network

ent lectures:
umann’s lectures:
//www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf
1807.03098
ne’s lectures:
7.08749
uer and Plehn:
5.01987

ssic books:
Weinberg: Gravitation and Cosmology (1973); Cosmology (2008)
Kolb and M. Turner: The Early Universe (1994)
eebles: Principles of Physical Cosmology (1993)
Dodelson and Fabian Schmidt – Modern Cosmology (2020)

Plan:

I.0 – Introduction and motivation

I.1 – Brief review of GR

I.2 – Dynamics of the Universe

I.3 – Thermal history of the Universe

"Our whole universe was in a hot dense state
Then nearly fourteen billion years ago expansion started"

I.0- Introduction

Why should a particle physicist learn cosmology?

- main evidences from BSM comes from cosmology: dark matter, dark energy, inflation;

- particle physics affect cosmology: eg origin of matter-anti-matter asymmetry, Higgs as inflaton, neutrinos and the formation of structures, phase transitions;

- cosmology affects particle physics: eg evolution of the Universe may be responsible for electroweak symmetry breaking (relaxion idea).

• early Universe is a testbed for SM and BSM: stability or metastability of SM vacuum, new physics tests from CMB, inflation, matter-antimatter asymmetry, primordial non-gaussianity, primordial gravitational waves, stochatic gravitational waves from phase transitions (GUT, etc) “cosmological collider physics”, modified gravity, ...

• gravity (geometry) may play an important role in particle physics: eg models with warped extra dimensions

• new particles from geometry: KK excitations, radion, etc

• models with extra dimensions can change the evolution of the Universe (and hence be tested).

Standard Model of Particle Physics works fine but it is unsatisfactory (neutrino masses, dark matter, hierarchy problem, etc). Beyond SM!

Standard Model of Cosmology (ΛCDM) works fine but it is unsatisfatory (value and nature of Λ). Beyond ΛCDM!

Models abound! We have to see what Nature has chosen...

Cosmology has recently become a data driven science. Era of precision cosmology!

$t_U = (13.799 \pm 0.021) \times 10^9$ years [used to be $10^{9\pm1}$ years]

Many experiment are taking a huge amount of data that are being analyzed in order to find out which model best describes the universe.

Cosmology became a respectable Science due in great part to Jim Peebles.

The Nobel Prize in Physics 2019

© Nobel Media. Photo: A. Mahmoud

James Peebles

Prize share: 1/2

© Nobel Media. Photo: A. Mahmoud

Michel Mayor

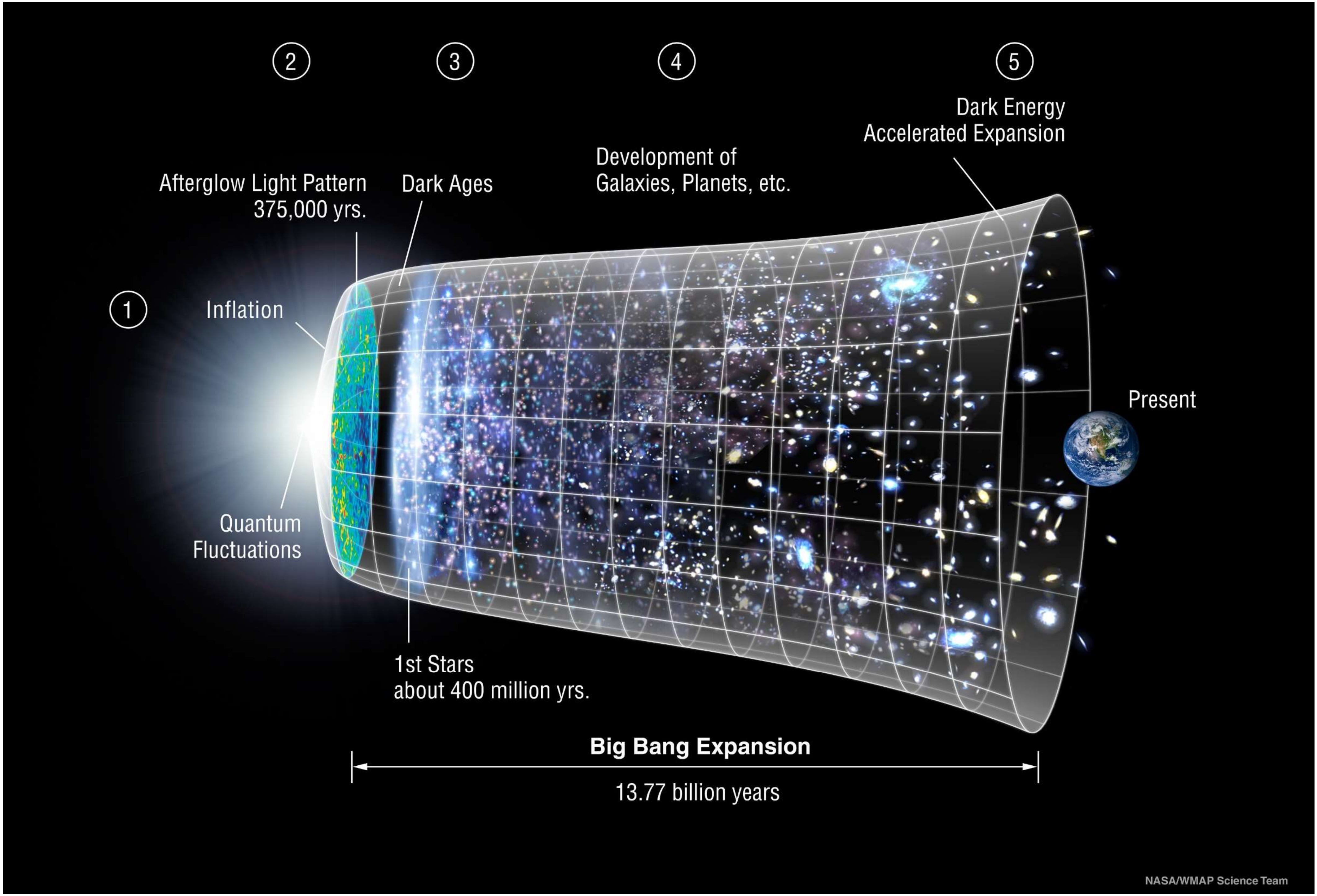
Prize share: 1/4

© Nobel Media. Photo: A. Mahmoud

Didier Queloz

Prize share: 1/4

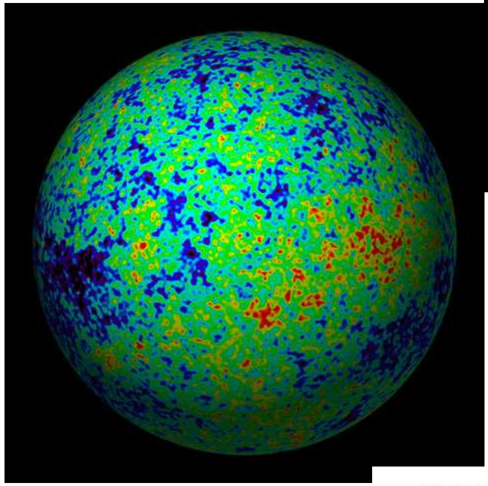
The Nobel Prize in Physics 2019 was awarded "for contributions to our understanding of the evolution of the universe and Earth's place in the cosmos" with one half to James Peebles "for theoretical discoveries in physical cosmology", the other half jointly to Michel Mayor and Didier Queloz "for the discovery of an exoplanet orbiting a solar-type star."

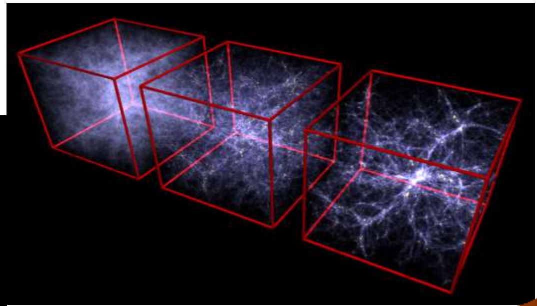

1
2
3
4
5
Dark Energy
Accelerated Expansion
Development of
Galaxies, Planets, etc.
Afterglow Light Pattern
375,000 yrs.
Dark Ages
Inflation
Quantum
Fluctuations
Present
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Team

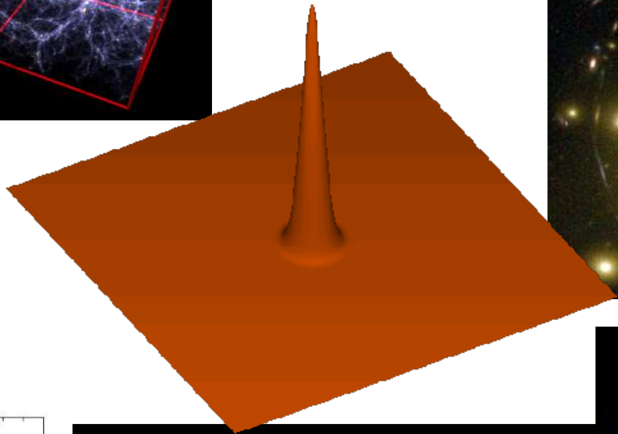
Cosmological probes

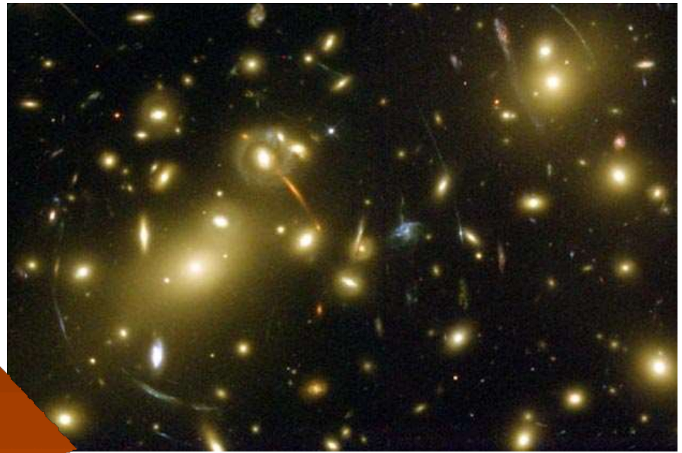
- Cosmic Microwave Background (CMB)
- Big bang nucleosynthesis (BBN)
- Supernovae (type Ia)
- Baryon acoustic oscillation (BAO)
- Gravitational lensing
- Number count of clusters of galaxies

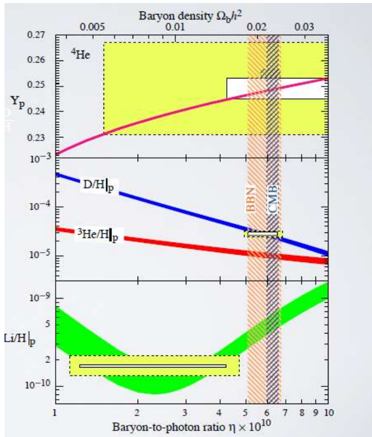
Cosmological probes

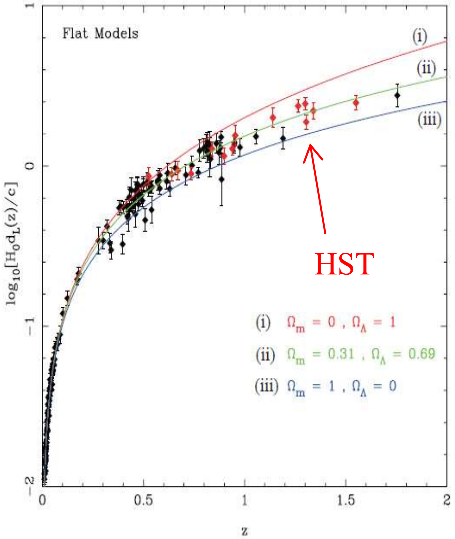


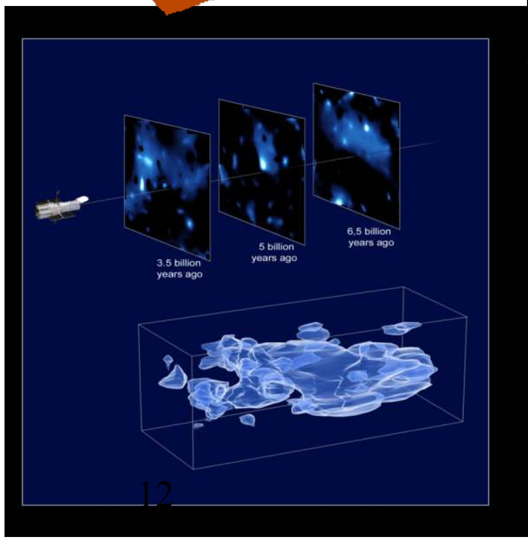


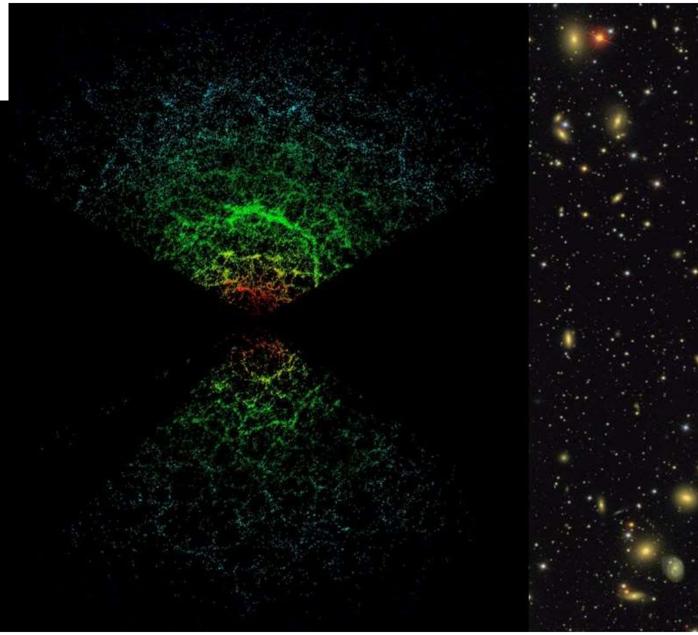













Consensus Cosmology
DARK MATTER
DARK ENERGY
INFLATION
Rests upon three mysterious pillars
All implicate new physics!
M. Turner 2013

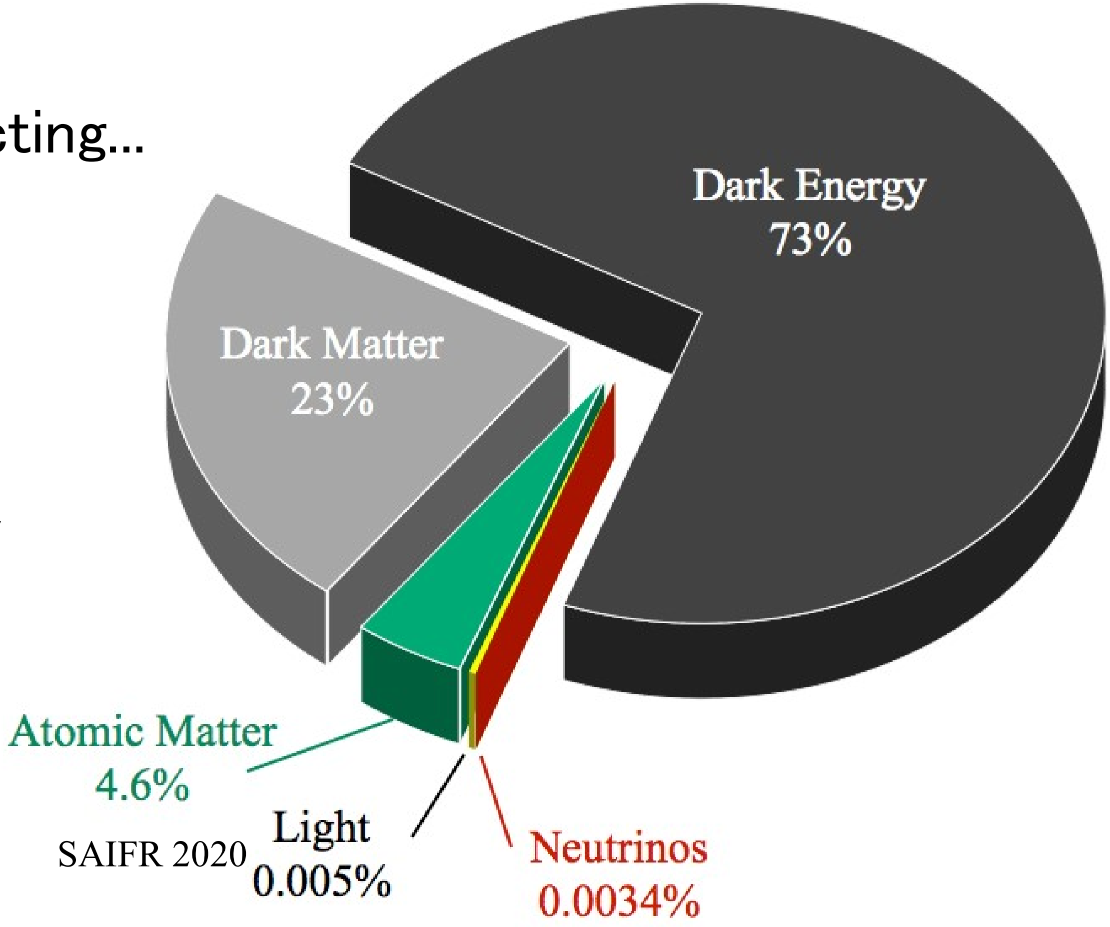
We know that we don't know what 95% of the Universe is made of:

What is dark matter?

Cold, warm, fuzzy, self-interacting…

What is dark energy?

New degree of freedom/MG:
Quintessence, galileon, f(R),
Hordensky, beyond Hordensky,
massive gravity, EFT of DE…
Does it interact with matter?
Does it cluster?



I.1– Brief Review of GR

General Relativity rules the Universe at large scales!
Classical description is sufficient in most cases.

I.1.0 – Classical field theory in a nutshell

Fields $\longrightarrow$ lagrangian $\longrightarrow$ action $\longrightarrow$ equations of motion

Fundamental degrees of freedom (not always physical)

Dynamics (use of symmetries)

$$\delta S = 0$$

I.1.1 – Einstein's equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

10 nonlinear differential equations. In general it must be solved numerically, eg gravitational waves from coalescence of binary black holes.

Fundamental field of gravity: metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

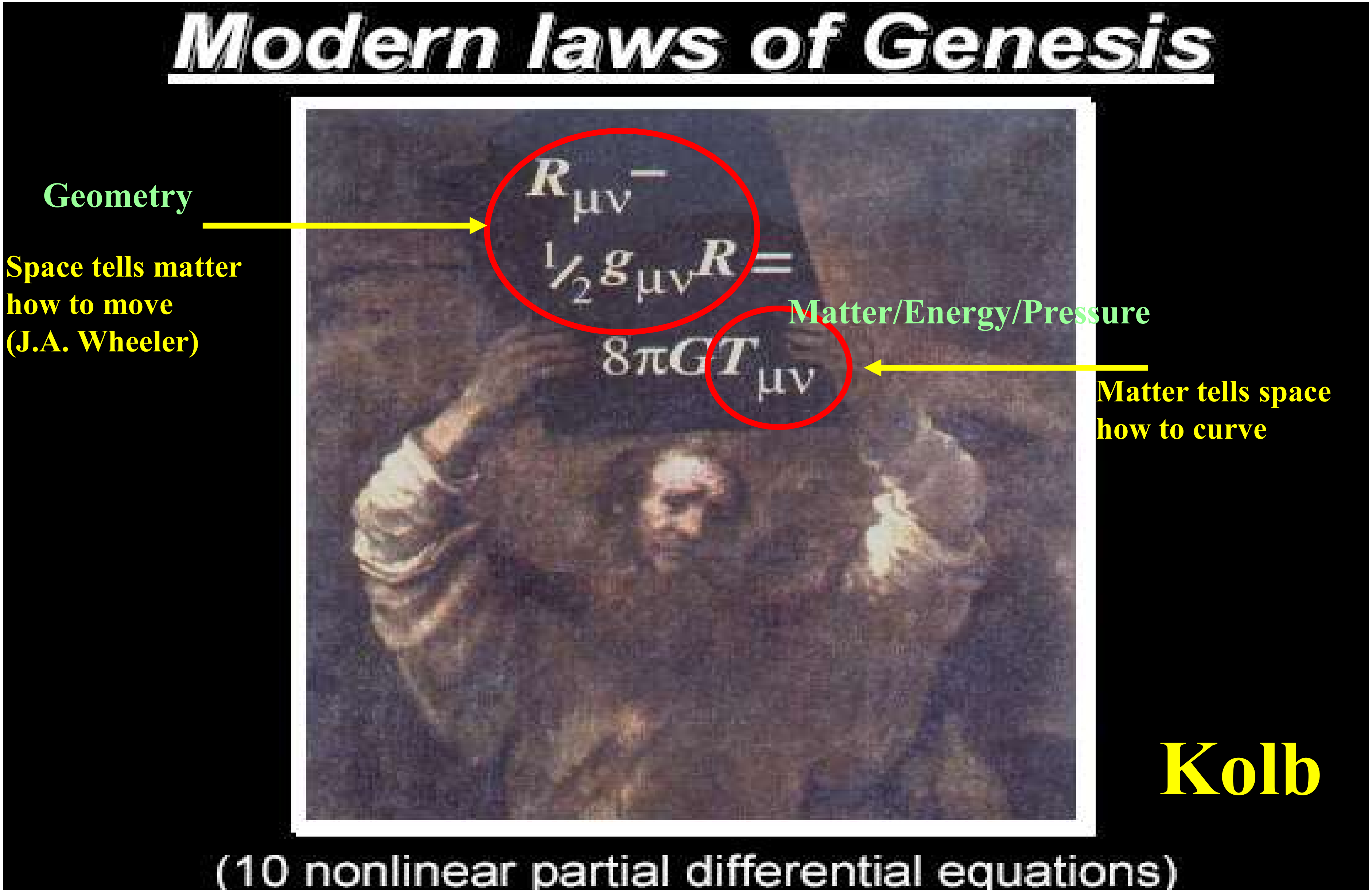
$$g_{\mu\alpha} g^{\alpha\nu} = \delta^\nu_\mu$$
$$g_{\mu\nu} g^{\mu\nu} = 4$$

Flat space-time – Minkwoski metric

$$p_\mu p^\mu = E^2 - (\vec{p})^2$$

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Standard Cosmological Model



Details of Einstein's equation:

• G: Newton's constant

$$G = \frac{1}{M_{\rm Pl}^2} \quad (\hbar = c = 1)$$
$$M_{\rm Pl} = 1.2 \times 10^{19}\ {\rm GeV}$$

Obs.: sometimes the *reduced* Planck mass is used:

$$\tilde{M}_{\rm Pl} = \frac{M_{\rm Pl}}{\sqrt{8\pi}} = 2.4 \times 10^{18}\ {\rm GeV}$$

Details of Einstein's equation:

• Christoffel symbols (aka metric conection, affine connection) – first derivative of the metric :

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left\{ \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right\}$$

• Ricci tensor – second derivative of the metric:

$$R_{\mu\nu} = \frac{\partial}{\partial x^{\alpha}} \Gamma^{\alpha}_{\mu\nu} - \frac{\partial}{\partial x^{\nu}} \Gamma^{\alpha}_{\alpha\mu} + \Gamma^{\alpha}_{\mu\nu} \Gamma^{\beta}_{\alpha\beta} - \Gamma^{\beta}_{\alpha\mu} \Gamma^{\alpha}_{\beta\nu}$$

• Ricci scalar: $R = g^{\mu\nu} R_{\mu\nu}$

I.1.2 – Einstein–Hilbert action

$$S_{\mathrm{E-H}}[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R[g_{\mu\nu}]$$

For the Hilbert-Einstein dispute see:
L. Corry, J. Renn, and J. Stachel, Science 278, 1270 (1997)
F. Winterberg, Z. Naturforsch. **59a,** 715 – 719 (2004)

- Action is invariant under general coordinate transformations:

$$x^\mu \to x'^\mu(x^\mu)$$

- $g = \det(g_{\mu\nu})$

・Dimensional analysis: $(\hbar = c = 1)$

$$[g]: \text{ dimensionless}; \quad [R]: \ E^2$$
$$[d^4x]: E^{-4}; \quad [S]: \text{ dimensionless}$$
$$\Rightarrow [G]: E^{-2}$$

$$G = \frac{1}{M_{\rm Pl}^2}$$

・Einstein equation in vacuum ($T_{\mu\nu} = 0$) obtained from:

$$\frac{\delta S_{\rm E-H}}{\delta g_{\mu\nu}} = 0$$

I.1.3 – The cosmological constant

February 1917 (~100 years ago): "Cosmological Considerations in the General Theory of Relativity" introduces the cosmological constant in the theory without violating symmetries:
a new constant of Nature!

It has an "anti-gravity" effect (repulsive force) and it was introduced to stabilize the Universe.

$$S_{\mathrm{E-H}} + S_{\Lambda} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \Lambda \right)$$

With the discovery of the expansion of the Universe (Hubble, 1929) it was no longer needed – "my biggest blunder".

43. "Cosmological Considerations in the General Theory of Relativity"

[*Einstein 1917b*]

SUBMITTED 8 February 1917
PUBLISHED 15 February 1917

IN: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1917): 142–152.

142 Sitzung der physikalisch-mathematischen Klasse vom 8. Februar 1917

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. EINSTEIN.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\rho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unend-

§ 4. On an Additional Term for the Field Equations of Gravitation

Poisson's equation given by equation (2). For on the left-hand side of field equation (13) we may add the fundamental tensor $g_{\mu\nu}$, multiplied by a universal constant, $-\lambda$, at present unknown, without destroying the general covariance. In place of field equation (13) we write

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T) \quad . \quad . \quad (13a)$$

George Gamow – My Worldline

correct, and changing it was a mistake. Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life. But this "blunder," rejected by Einstein, is still sometimes used by cosmologists even today, and the cosmological constant denoted by the Greek letter Λ rears its ugly head again and again and again.

I.1.4 – Modified gravity

Modified gravity is anything different from E–H (+ Λ) action (see 1601.06133)

Example: f(R) theories (see 1002.4928)

$$S[g_{\mu\nu}] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, f(R)$$

$$f(R) = a_0 + a_1 R + a_2 R^2 + \cdots + \frac{\alpha_1}{R} + \frac{\alpha_2}{R^2} + \cdots$$

cosmological constant

GR

Higher order derivatives

Issues with modified gravity:

• introduces new light degrees of freedom – new forces
Since there are stringent constraints one has to invoke “screening mechanisms” – chameleon, symmetron, Vainshtein, ...

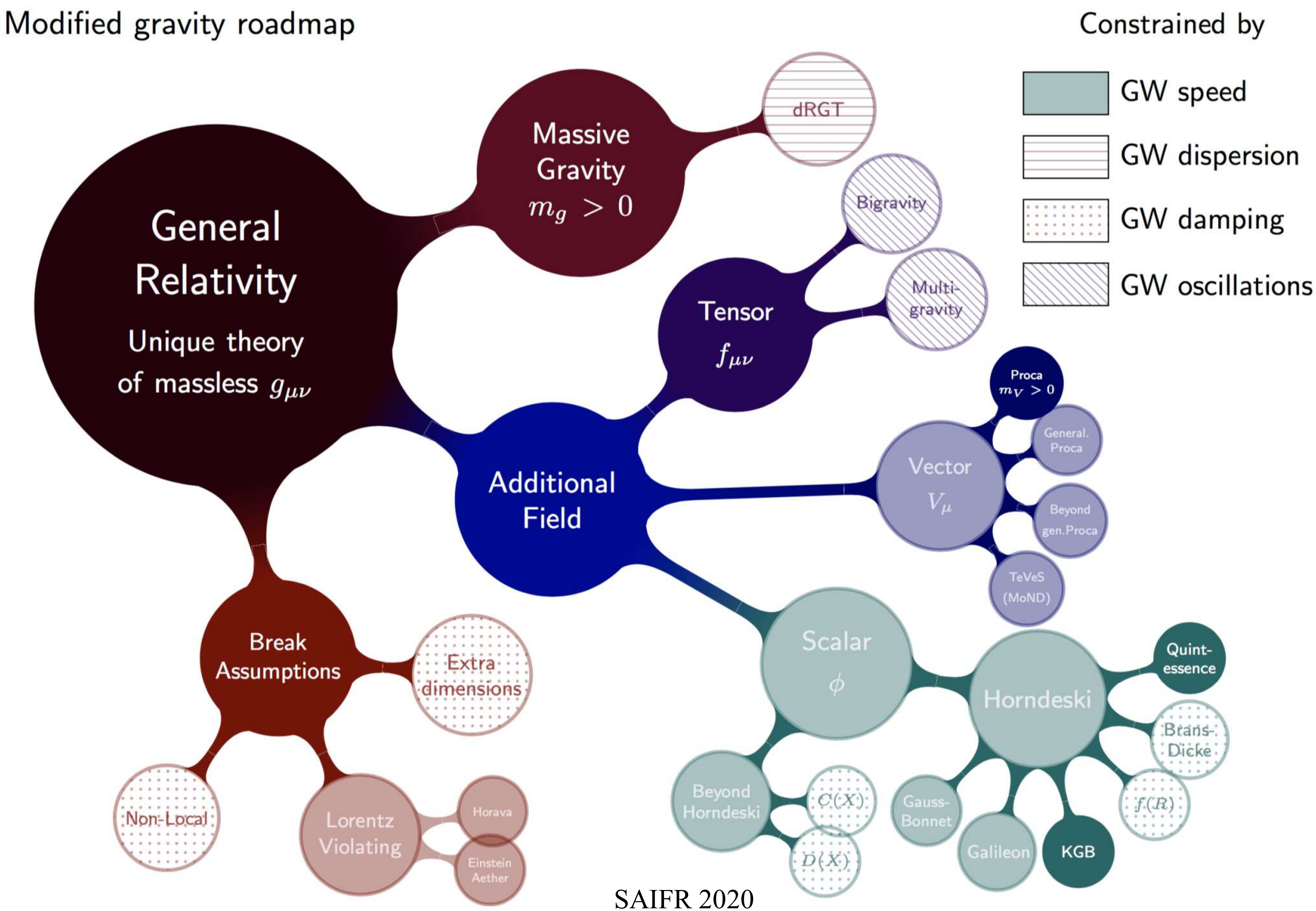
• may have classical instabilities due to higher derivatives in equations of motion (Ostrogradski instabilities)

• may have quantum instabilities – “ghosts”

• may be brought in the form of GR with a suitable change of coordinates (Jordan frame -> Einstein frame) introducing non-standard couplings in the matter sector

• search for MG: use simple parametrizations (more later)

Ezquiaga and Zumalacárregui – 1807.09241

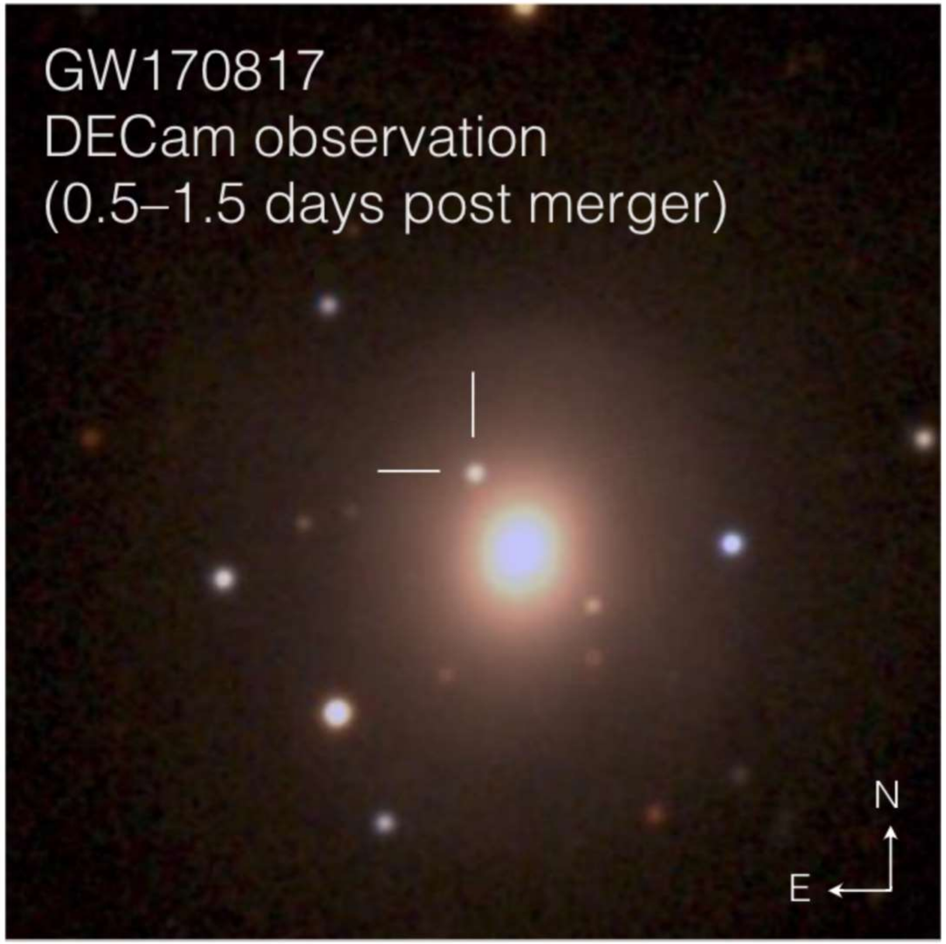


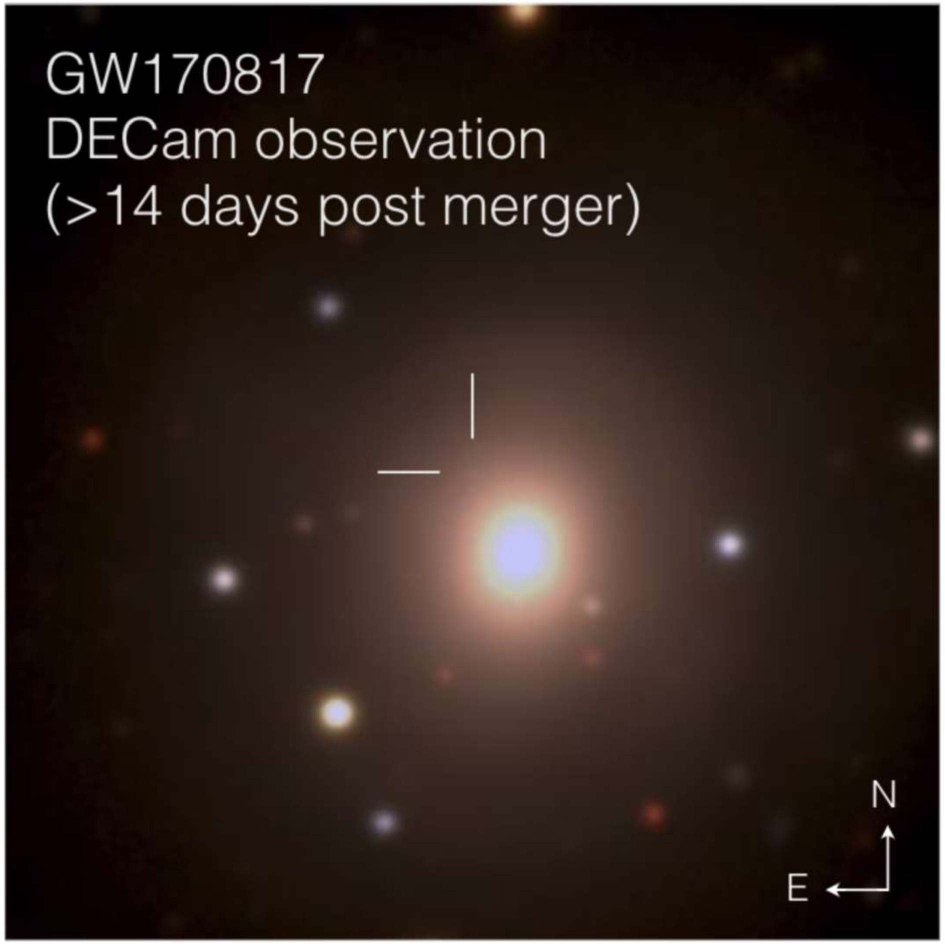
Some modified gravity theories predict that

$$\frac{\Delta c}{c} = \frac{c_g - c}{c} = \mathcal{O}(1)$$

In 2017 GW170817 was detected by LIGO and Virgo: the first detection of a merger of 2 neutron stars at a mere 130 million light-years from Earth.

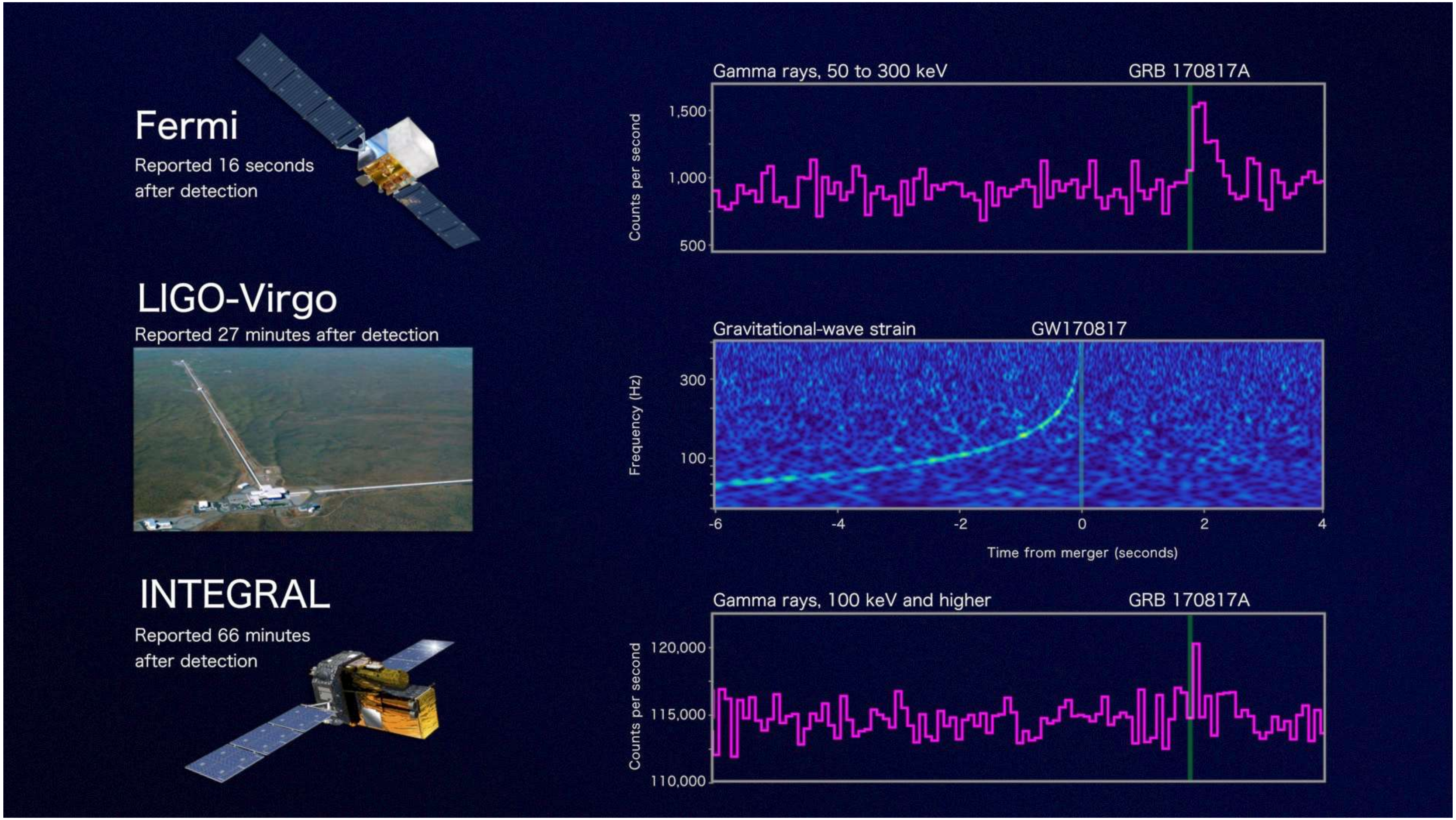
As opposed to black hole mergers, it also emitted light!!


GW170817
DECam observation
(0.5–1.5 days post merger)
N
E


GW170817
DECam observation
(>14 days post merger)
N
E

Also seen in gamma rays by Fermi and Integral with less than 2 seconds difference!!

Exercise 0: From this data estimate a bound on $$\frac{\Delta c}{c} = \frac{c_g - c}{c}$$



I.1.5 –Adding matter to the action

$$S_{\mathrm{matter}} = \int d^4x\sqrt{-g}\,\mathcal{L}_{\mathrm{matter}}$$

Examples:

Electromagnetism: $$S_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\sqrt{-g}\,g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

Real scalar field: $$S_\phi = \int d^4x\sqrt{-g}\,\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)\right]$$

I.1.6 – Energy–momentum tensor

Definition:

$$\delta S_{\mathrm{matter}} = \frac{1}{2}\int d^4x\sqrt{-g}\, T^{\mu\nu}(x)\delta g_{\mu\nu}$$

which implies

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{matter}}}{\delta g_{\mu\nu}}$$

Exercise 1: Show that for a real scalar field

$$T_\phi^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - \mathcal{L}_\phi g^{\mu\nu}$$

Exercise 2: Show that for a cosmological constant

$$T_\Lambda^{\mu\nu} = \Lambda g^{\mu\nu}$$

Hint: Use Ln(det(A)) = tr(LnA) to show that $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g^{\mu\nu}\delta g_{\mu\nu}$

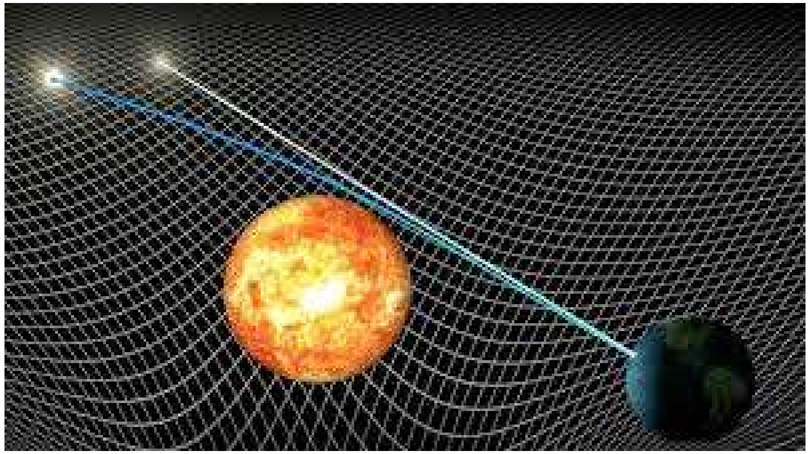
Finally, Einstein's equation for GR is obtained from the requirement:

$$\delta\left(S_{\mathrm{total}}\right) = \delta\left(S_{\mathrm{E-H}} + S_{\Lambda} + S_{\mathrm{matter}}\right) = 0$$

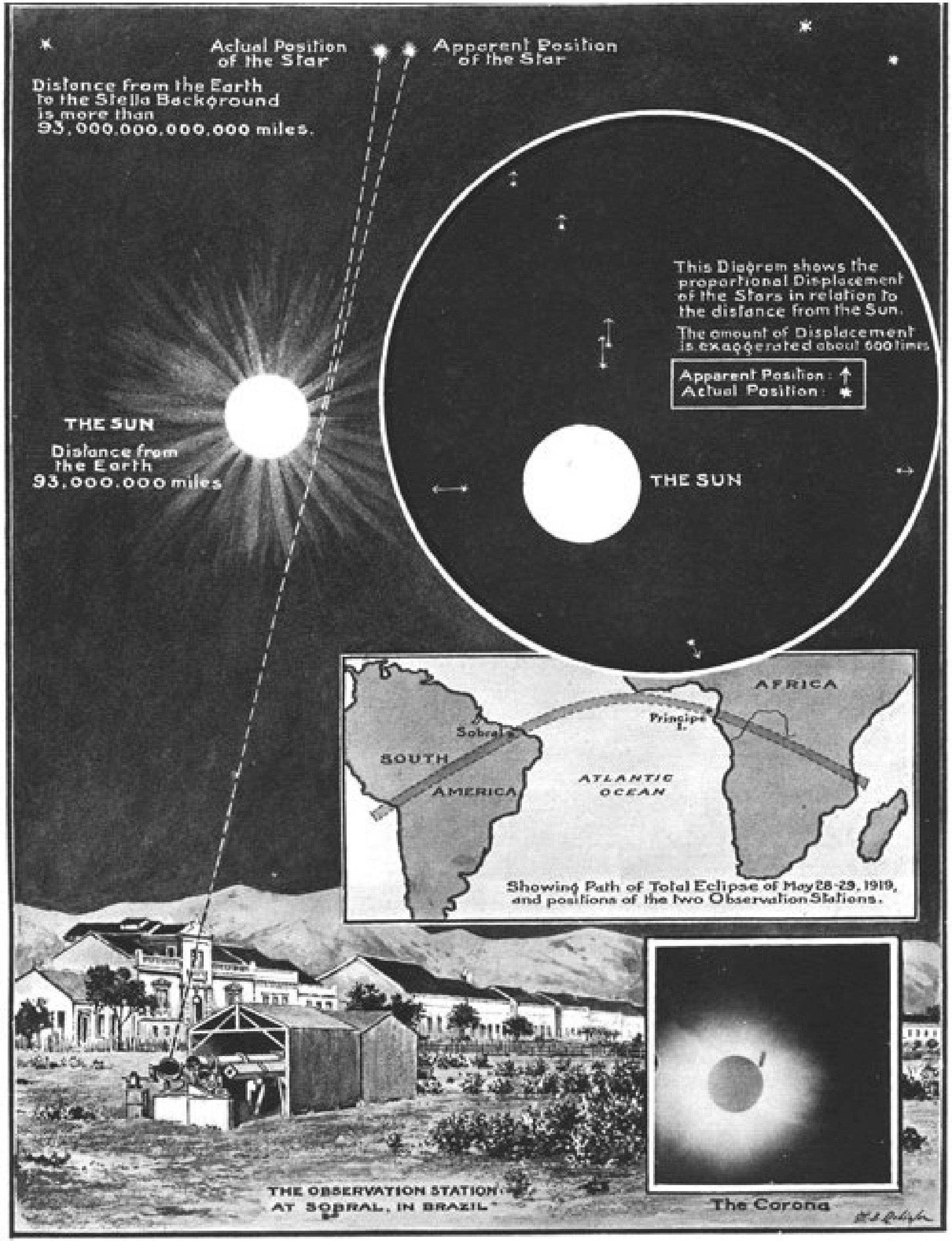
Historical interlude

100 years of the eclipse that confirmed GR

First test of GR: the solar eclipse in May 29, 1919



Two expeditions organized by the Royal Society:
Principe Island in Africa (Eddington)
Sobral in Brazil (Crommelin)



Actual Position of the Star
Apparent Position of the Star
Distance from the Earth to the Stella Background is more than 93,000,000,000,000 miles.
THE SUN
Distance from the Earth 93,000,000 miles
This Diagram shows the proportional Displacement of the Stars in relation to the distance from the Sun.
The amount of Displacement is exaggerated about 600 times
Apparent Position :
Actual Position :
THE SUN
AFRICA
Sobral
Principe I.
SOUTH AMERICA
ATLANTIC OCEAN
Showing Path of Total Eclipse of May 28-29, 1919, and positions of the two Observation Stations.
THE OBSERVATION STATION AT SOBRAL, IN BRAZIL
The Corona

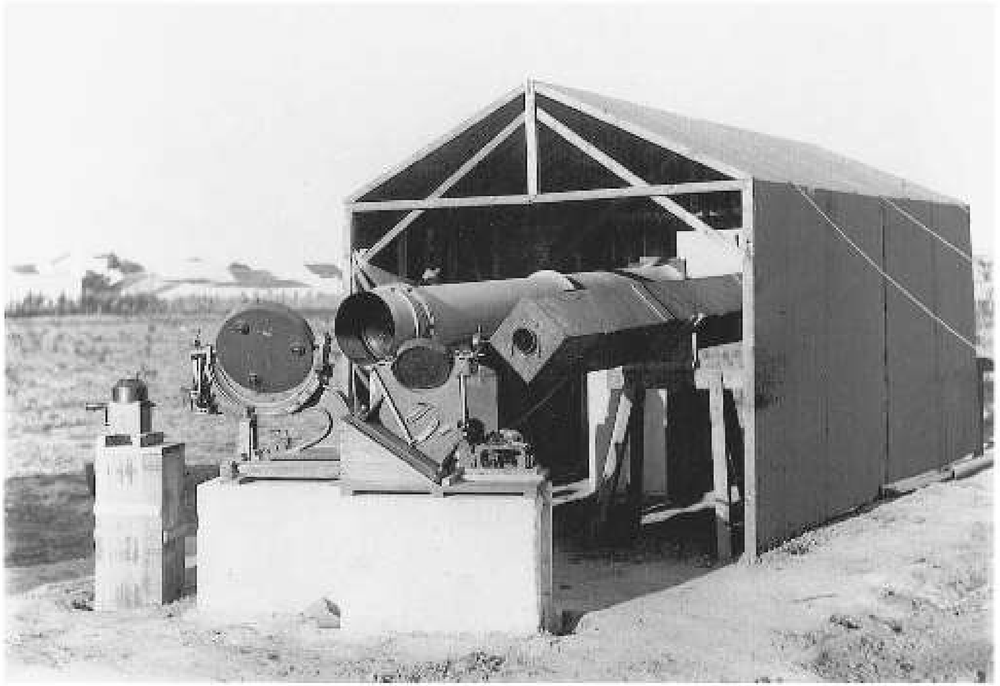


***Figure 1.** The eclipse observation equipment at Sobral. The troublesome coelostats can be seen in the foreground. Copyright Science Museum/Science and Society Picture Library. Inventory no. 1922-0277.*

Equipes brasileira e inglesa em Sobral, entre outras pessoas. Henrique Morize é o quarto, em pé, da esquerda para a direita. Os astrônomos ingleses estão sentados: A.C.D. Crommelin é o quarto da esquerda para a direita; C.R. Davidson é o quinto (Observatório Nacional/ MCT).

IX. *A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May* 29, 1919.

By Sir F. W. DYSON, *F.R.S., Astronomer Royal, Prof.* A. S. EDDINGTON, *F.R.S., and Mr.* C. DAVIDSON.

(*Communicated by the Joint Permanent Eclipse Committee.*)

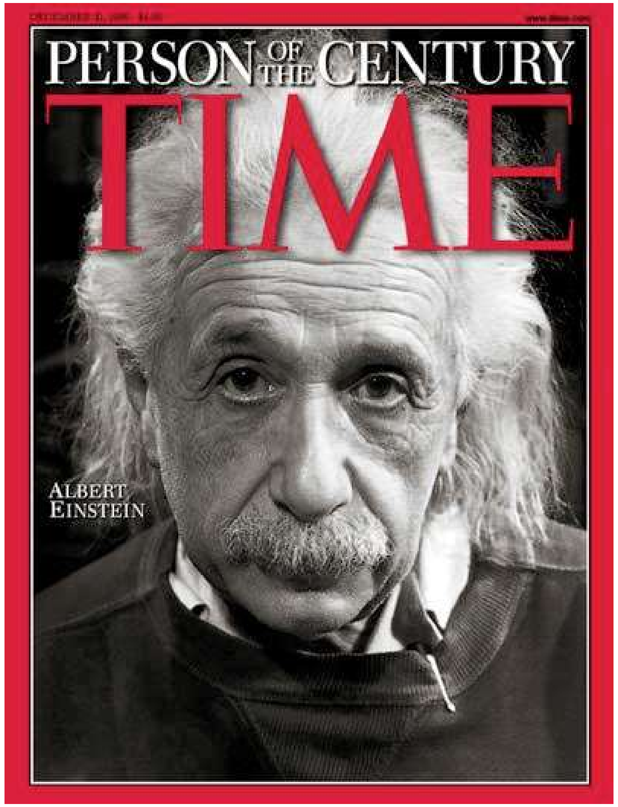
Received October 30,—Read November 6, 1919.

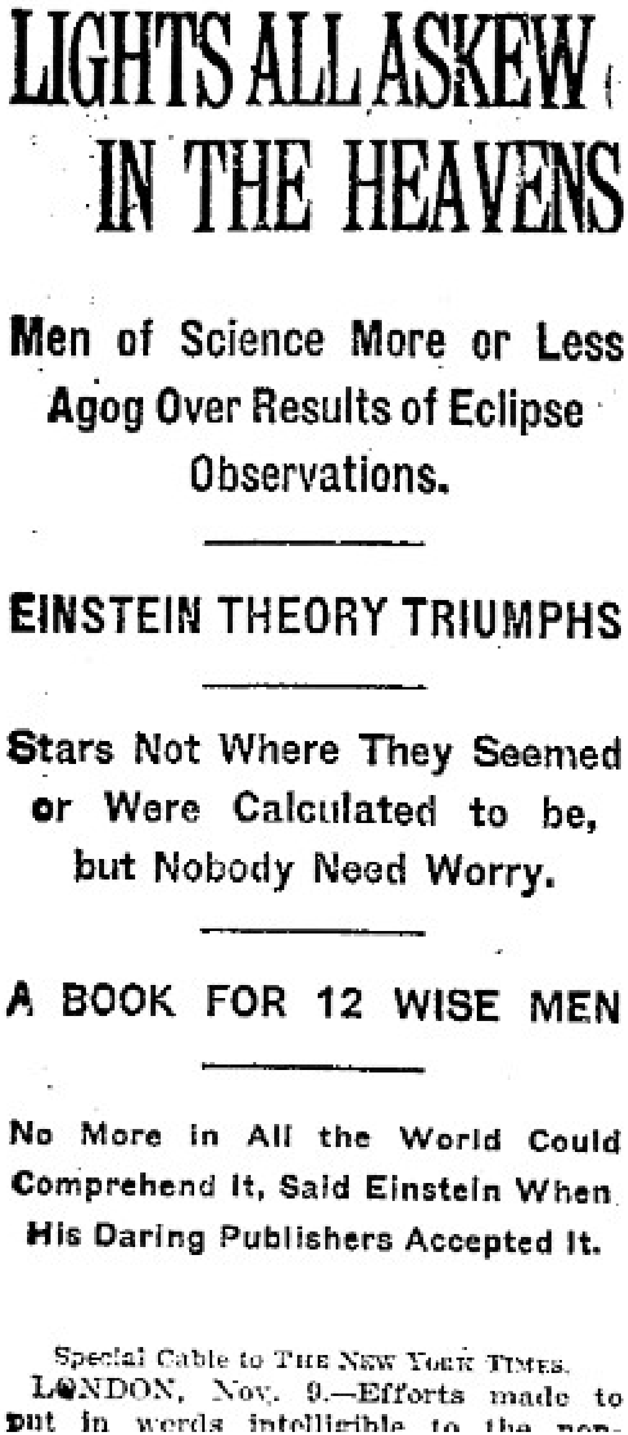
V. GENERAL CONCLUSIONS.

39. In summarising the results of the two expeditions, the greatest weight must be attached to those obtained with the 4-inch lens at Sobral. From the superiority of the images and the larger scale of the photographs it was recognised that these would prove to be much the most trustworthy. Further, the agreement of the results derived inde-

Thus the results of the expeditions to Sobral and Principe can leave little doubt that a deflection of light takes place in the neighbourhood of the sun and that it is of the amount demanded by EINSTEIN'S generalised theory of relativity, as attributable to the sun's gravitational field. But the observation is of such interest that it will probably be considered desirable to repeat it at future eclipses. The unusually favourable conditions of the 1919 eclipse will not recur, and it will be necessary to photograph fainter stars, and these will probably be at a greater distance from the sun.

"After a careful study of the plates I am prepared to say that there can be no doubt that they confirm Einstein's prediction. A very definite result has been obtained that light is deflected in accordance with Einstein's law of gravitation." 06/11/1919





LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

Special Cable to THE NEW YORK TIMES.

LONDON, Nov. 9.—Efforts made to put in words intelligible to the non-

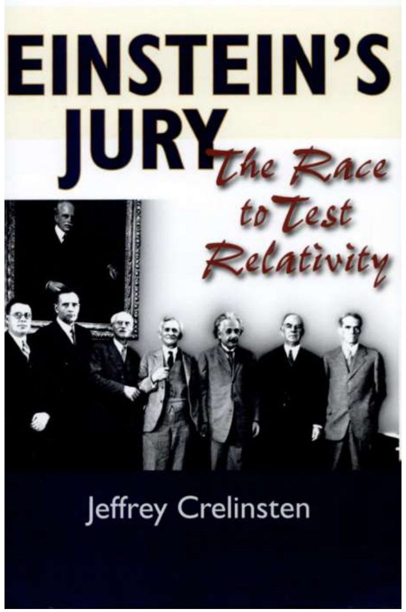
When Einstein visited Brazil in 1925, he declared to the local newspapers: "The idea that my mind conceived was proven in the sunny sky of Brazil."

Freundlich wrote Einstein that same night, offering to help develop ways to look for light bending near the Sun or the planet Jupiter. Back in Prague, Pollak told Einstein about the young Berlin astronomer, and Einstein gave him permission to send Freundlich proofs of his article. "Prof. Einstein has given me strict orders," wrote Pollak, "to inform you that he himself very much doubts that the experiments could be done successfully with anything except the Sun." He urged Freundlich "to send further reports to me, or perhaps to Prof. Einstein, about your views on an astronomical verification."[18]

Early attempt to test GR in 1912

As luck would have it, a visitor to the Berlin Observatory in November of that year opened for Freundlich another avenue of research on the problem. Charles Dillon Perrine, who had successfully resolved the "Vulcan problem" while at Lick Observatory, had left Lick in 1909 to become director of the Southern Hemisphere observatory in Cordoba. When Freundlich told him about Einstein's light-bending prediction, Perrine suggested that he write to various astronomers who might have old eclipse plates on which star images might be measured for deflection. Naturally, he mentioned the Lick Vulcan plates. Freundlich immediately drafted a circular letter, which he sent to several observatories, including Lick, asking for "support of astronomers, who possess eclipse-plates" to test Einstein's predicted light deflection by the Sun.[21]

In view of the likely unsuitability of the Vulcan plates for the task at hand, Campbell offered to lend the Vulcan cameras to Perrine to try Freundlich's problem at the eclipse in Brazil on October 9–10, 1912.



Perrine agreed to enlarge his eclipse program to include Freundlich's investigation and to take the photographs himself. Campbell sent the lenses down via the astronomer William Joseph Hussey. Perrine left Bue-

EARLY INVOLVEMENT, 1911–1914 59

nos Aires on September 13, 1912, and the eclipse took place on October 10. A few days later Campbell received a telegram from Edward C. Pickering of Harvard, the communication center for American astronomy: "Perrine cables from Brazil, rain."[31]

100
centenário do
ECLIPSE DE SOBRAL
1919–2019

Centenário do Eclipse Solar em Sobral/CE

End of historical interlude

I.2–Dynamics of the Universe

I.2.1 – Friedmann-Lemaître-Robertson-Walker

Universe is spatially homogeneous and isotropic on average.

It is described by the FLRW metric (for a spatially flat universe):

$$ds^2 = dt^2 - a(t)^2 \left[dx^2 + dy^2 + dz^2\right]$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{pmatrix}$$

LRW metric is determined by one time-dependent function:
ne so-called scale factor a(t).
istances in the universe are set by the scale factor.

cale factor is the key function to study how the average
niverse evolves with time.

convention: a=1 today
physical distances: $d(t) = a(t)\, d_0$

BS: conformal time

$$d\eta = \frac{dt}{a(t)}$$

$$ds^2 = a^2(t)[d\eta^2 - d\vec{r}^2]$$

Average evolution of the universe

– specified by one function: scale factor a(t)
– determines measurement of large scale distances, velocities and acceleration

$$a(t) \qquad \dot{a}(t) \qquad \ddot{a}(t)$$

– measured through standard candles (SNIa's) and standard rulers (position of CMB peak, BAO peak,...)

Redshift z: $$a(t) = \frac{1}{1+z}$$ z=0 today.

Expansion of the universe

Hubble parameter:

Expansion rate of the universe

$$H = \frac{\dot{a}(t)}{a(t)}$$

Hubble constant: Hubble parameter today (H_0)

Analogy of the expansion of the universe with a balloon:

Space itself expands and galaxies get a free “ride”.

Exercise 3: Show that for a spatially flat FLRW metric the Ricci tensor and the Ricci scalar are given by:

$$R_{00} = -3\frac{\ddot{a}}{a}; \;\; R_{ii} = a\ddot{a} + 2a^2;$$

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$$

Interlude: a quick tour to extra dimensions (geometry matte

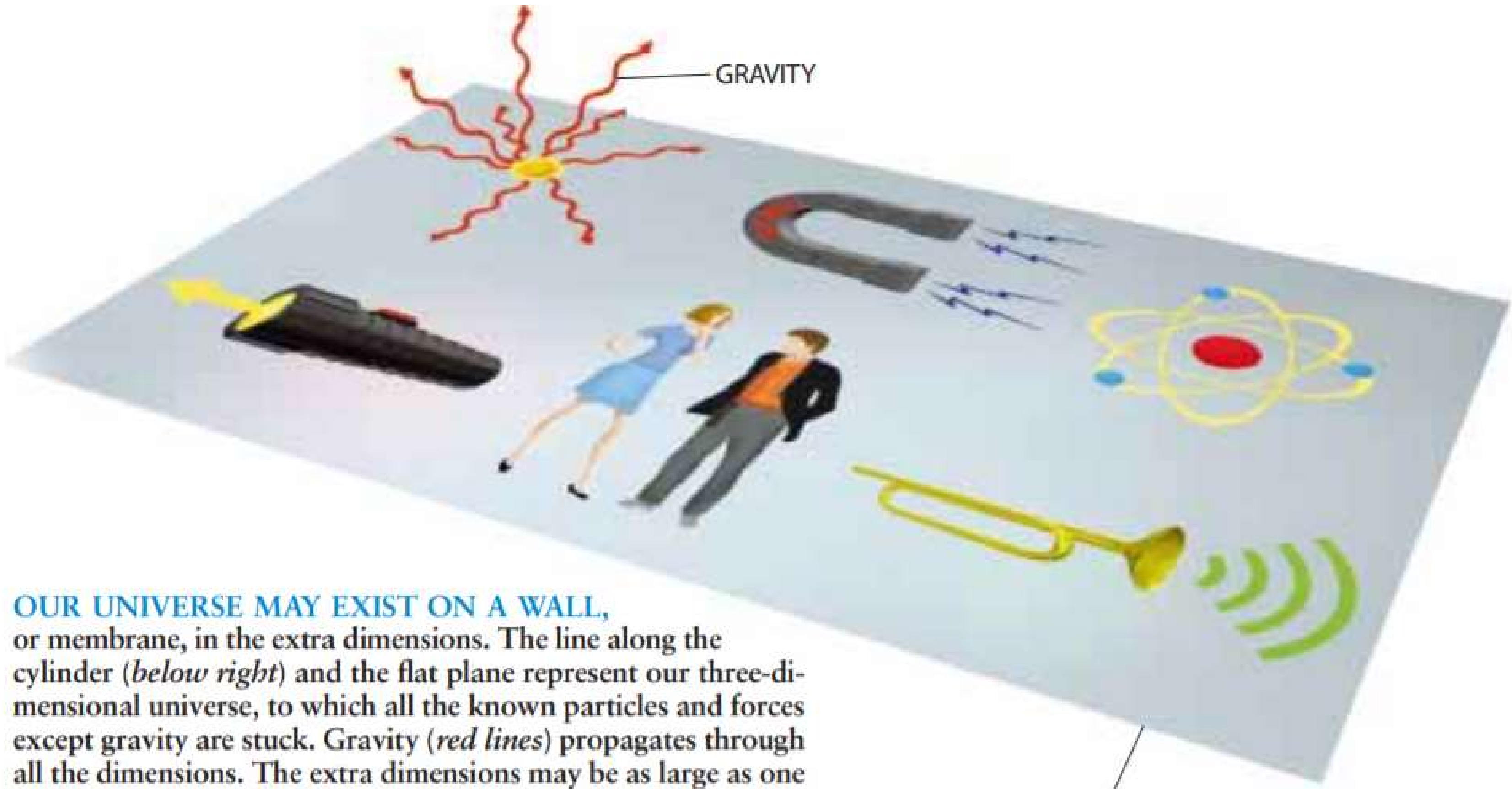
ph/0404096 hep-ph/0503177 (goes back to Kaluza and Klein in the 1920's

Flat extra dimensions (ADD - Arkani-Hamed, Dimopoulos and Dv 1998): n extra flat space-like dimensions compactified in a n-dimensional torus

$$S_{E-H} = \frac{1}{16\pi G_*} \int d^{4+n}x \sqrt{-g_{4+n}}\, R_{4+n}$$

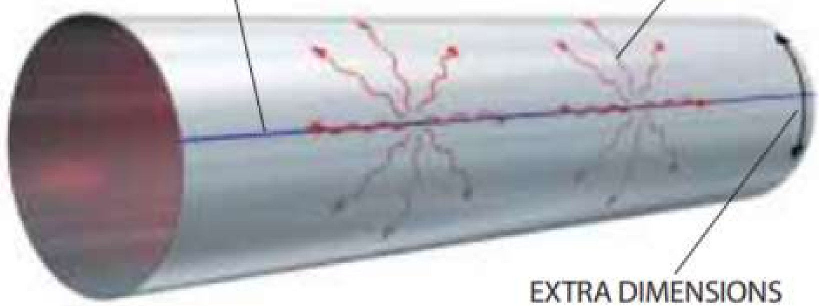
$$[G_*] : E^{-(n+2)}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu - \delta_{ab} dy^a dy^b$$



OUR UNIVERSE MAY EXIST ON A WALL, or membrane, in the extra dimensions. The line along the cylinder (*below right*) and the flat plane represent our three-dimensional universe, to which all the known particles and forces except gravity are stuck. Gravity (*red lines*) propagates through all the dimensions. The extra dimensions may be as large as one millimeter without violating any existing observations.

the full higher-dimensional space. In high-energy particle collisions, we expect to observe missing energy, the result of



ADD in SciAm

In this case one can show that

$$g_{4+n} = g_4 \qquad R_{4+n} = R_4$$

$$S_{E-H} = \frac{V_n}{16\pi G_*} \int d^4x \sqrt{-g_4}\, R_4$$

Effective Newton's constant is diluted by the volume of the extra dimensions:

$$G_N = \frac{G_*}{V_n}$$

Introducing a fundamental scale

$$M_* = \left(\frac{1}{G_*}\right)^{2+n} \qquad M_P^2 = M_*^{n+2} V_n$$

Exercise: assuming that $V_n = L^n$

Compute the size of the extra dimension L assuming that the fundamental scale if 1 TeV (to be relevant at colliders) for n=1 (~10^{11}m) and n=2 (~0.1mm).

Warped extra dimension (WED) – Randall and Sundrum (1999): one extra
npact dimension y (5th dimension). Each point in the 5th dimension has a 4
nensional Minkowski background. The metric can be parametrized as:

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^{\mu}dx^{\nu} - dy^2$$

ere A(y) is the curvature along the fifth dimension and is called the warp
tor.

$$0 \leq y \leq L$$

Let's consider the gravity action with a bulk cosmological constant:

$$S = \int d^4x \; dy \; \sqrt{-g_5} \left(\frac{1}{2k_*^2} R_5 - \Lambda_5 \right)$$

from which Einstein's equations follow:

$$G_{MN} = R_{MN} - \frac{1}{2} g_{MN} R_5 = k_*^2 \Lambda_5 g_{MN}$$

$[\Lambda_5] = E^5$

$[R_5] = E^2$

$[k_*^2] = E^{-3}$

$$k_*^2 = 8\pi G_* \equiv M_*^{-3}$$

Solving 5-5 componente of Einstein's equations:

$$G_{55} = -k_*^2 \, \Lambda_5 \; g_{55}$$

I tried EinsteinPy and MAXIMA without success. This is GR package for Mathematica

1 warped extra dimension (Randall-Sundrum)

In[33]:= (*Defining the covariant metric tensor*)

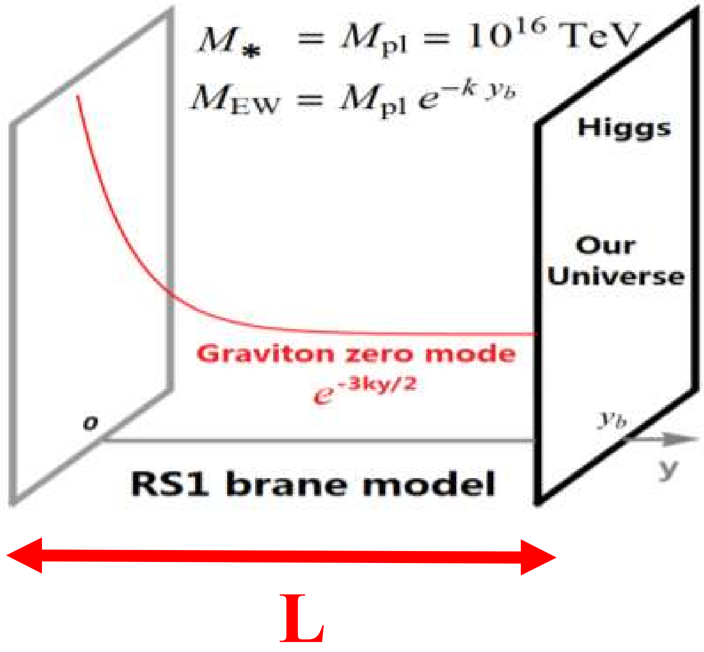
covMetricTensor = $\begin{pmatrix} e^{-2*A[u]} & 0 & 0 & 0 & 0 \\ 0 & -e^{-2*A[u]} & 0 & 0 & 0 \\ 0 & 0 & -e^{-2*A[u]} & 0 & 0 \\ 0 & 0 & 0 & -e^{-2*A[u]} & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$;

```
In[34]:= (*Initializing*)
Setup[coordinates, signature, covMetricTensor];

In[35]:= covEinsteinTensor[[5, 5]]
```

Out[35]= $6\,A'[u]^2$

$$A'(y) = \kappa \qquad \kappa = \sqrt{\frac{k_*^2 \Lambda_5}{6}}$$

$$A(y) = \kappa y \ \text{ assuming } \ A(0) = 0$$



Varping solves the hierarchy problem without large number (fine tuning):

$$M_{EW} = e^{-\kappa L} M_{Pl} \Rightarrow \kappa L \simeq 30$$

New particles are predict in models with exta dimensions: Kaluza-Klein xcitations or resonances. More details in Chacko's lectures.

nd of this interlude – back to 4 dimensions for the rest of ese lectures

I.2.2 – The right-hand side of Einstein equation: the energy-momentum tensor simplified

It is usually assumed that one can describe the components of the Universe as "perfect fluids": at every point in the medium there is a locally inertial frame (rest frame) in which the fluid is homogeneous and isotropic (consistent with FLRW metric):

$$T^{00} = \rho(t); \; T^{ij} = \delta^{ij} P(t); \; T^{0i} = 0$$

density   pressure   isotropy

Homegeneity: density and pressure depend only on time.

Energy-momentum in the rest frame (indices are important):

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

In a frame with a given 4-velocity:

$$T^{\mu\nu} = -Pg^{\mu\nu} + (\rho + P)u^{\mu}u^{\nu}$$

$$u^{\mu} = \gamma\,(1, \vec{v})$$

[Imperfect fluids: anisotropic stress, dissipation, etc.]

I.2.3 – Solving Einstein equation for the average Universe: Friedmann's equations

00 component:

$$R_{00} - \frac{1}{2} g_{00} R = 8\pi G T_{00} \implies$$

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho}$$

1st Friedmann equation

ii component:

$$R_{ii} - \frac{1}{2} g_{ii} R = 8\pi G T_{ii} \implies$$

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a} = -8\pi G P \implies$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

2nd Friedmann equation

I.2.4 – Evolution of different fluids

Exercise 4: take a time derivative of 1st Friedmann equation to derive the "continuity" equation :

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

Also follows from the conservation of the energy–momentum tensor and also from the 1st law of thermodynamics:

$$dU = -PdV,\ U = \rho V,\ V \propto a^{-3}$$

In order to study the evolution of a fluid we need to postulate a relation between density and pressure: the equation of state

Assume a simple relation:

$$P = \omega \rho$$

ω is called the equation of state parameter

Examples:

- Non-relativistic matter (dust): $P \ll \rho \longrightarrow \omega = 0$
- Relativistic matter (radiation): $\omega = 1/3$

- Cosmological constant: $\omega = -1$

$$T^{\mu}_{\nu,\Lambda} = \begin{pmatrix} \Lambda & 0 & 0 & 0 \\ 0 & \Lambda & 0 & 0 \\ 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & \Lambda \end{pmatrix} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

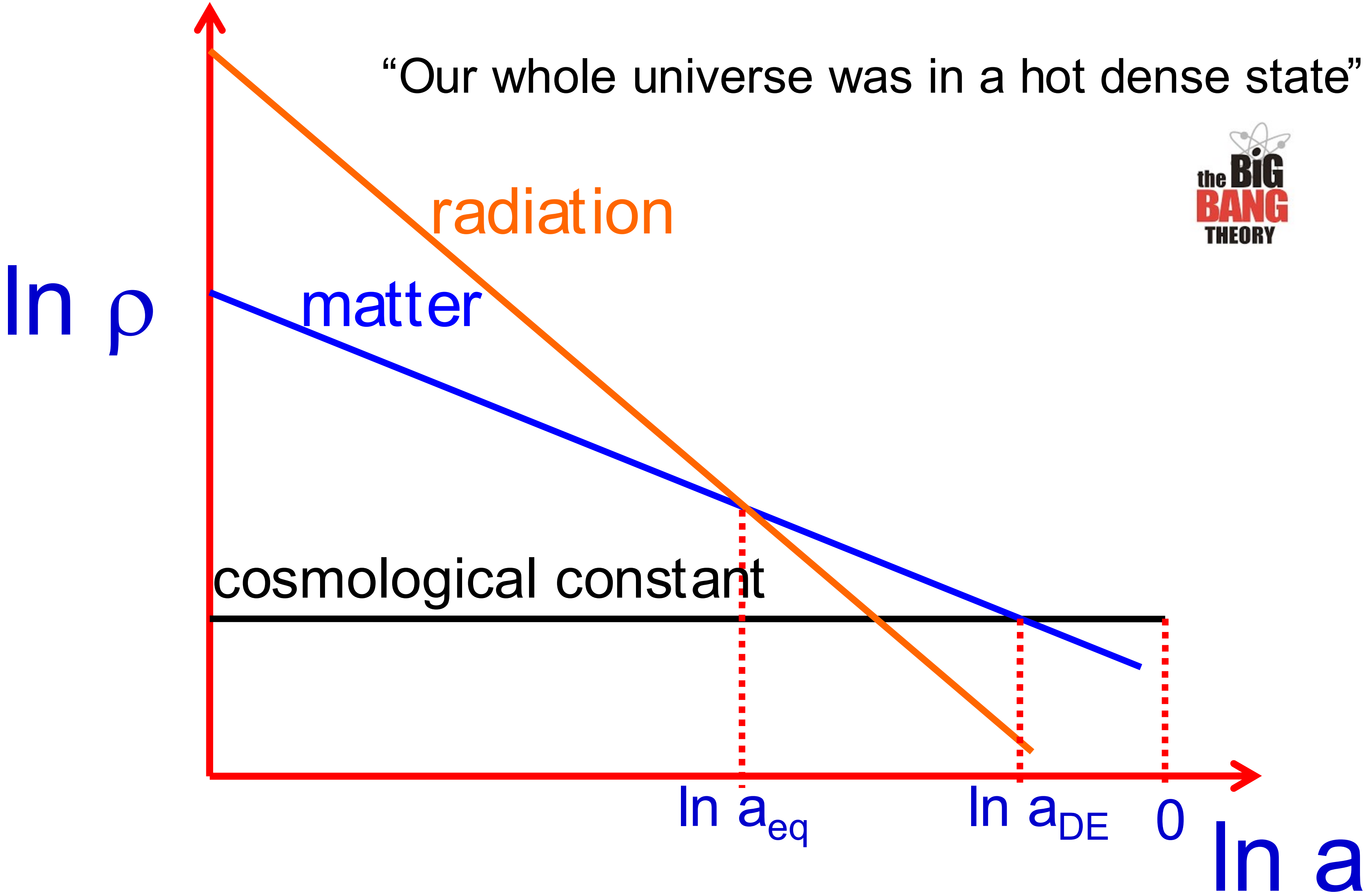
Exercise 5: From the continuity equation show that the evolution of the density for a constant equation of state is:

$$\rho(t) = \rho(t_i) \left(\frac{a(t)}{a(t_i)} \right)^{-3(1+\omega)}$$

OBS: It's easy to generalize to a time-dependente equation of state

- Non-relativistic matter (dust): $\rho \propto a^{-3}$
- Relativistic matter (radiation): $\rho \propto a^{-4}$
- Cosmological constant: $\rho \propto a^{0}$

“Our whole universe was in a hot dense state”



I.2.5 – Evolution of the scale factor

Exercise 6: Using 1st Friedmann equation and the result from last section:

$$\frac{\dot{a}}{a} \propto \sqrt{\rho}, \;\; \rho \propto a^{-3(1+\omega)}$$

show that:

$$a(t) \propto t^{\frac{2}{3(1+\omega)}} = \begin{cases} t^{2/3} \text{ (matter)} \\ t^{1/2} \text{ (radiation)} \end{cases}$$

but for the case of a cosmological constant one has an **exponential growth**:

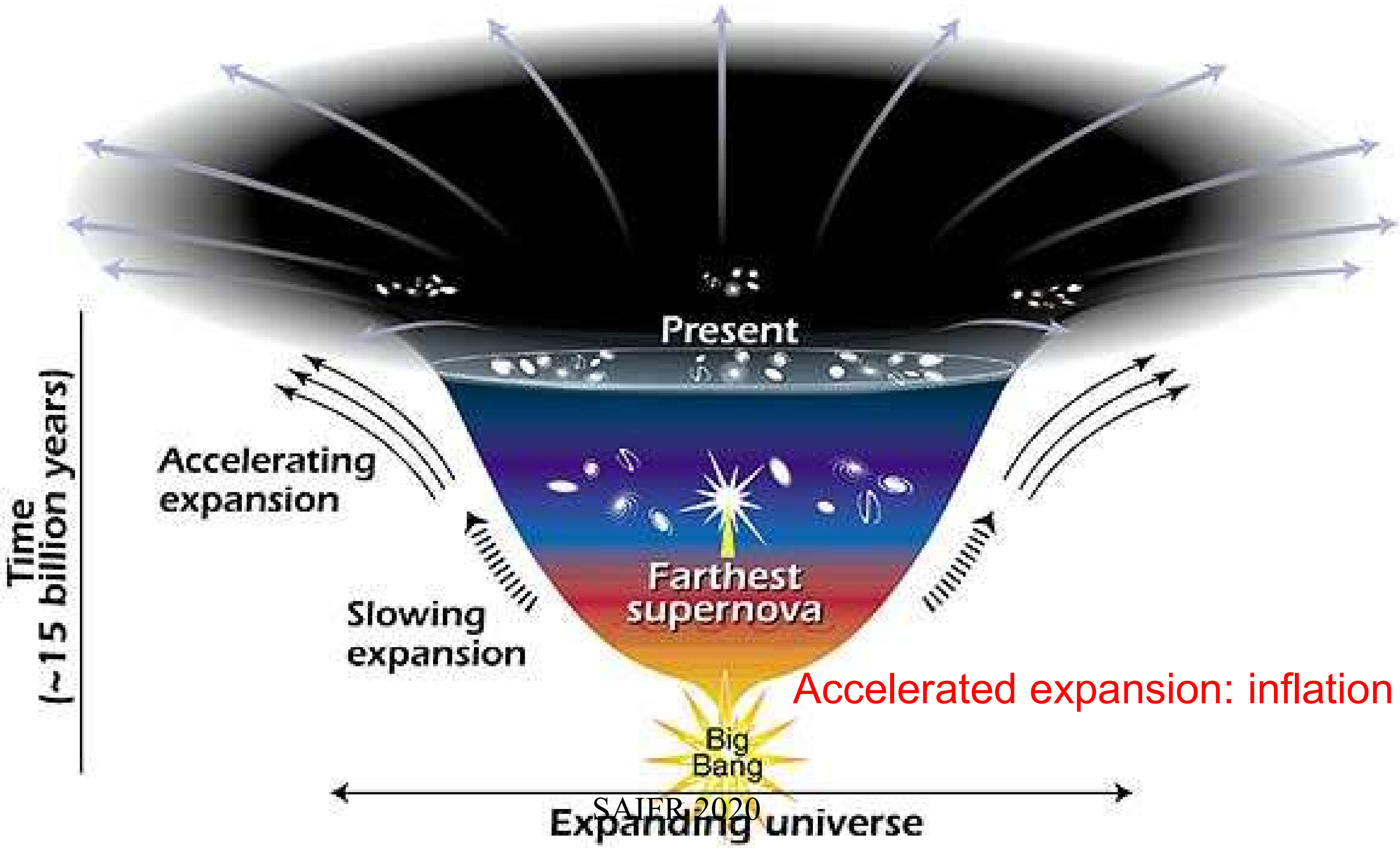
$$\frac{\dot{a}}{a} = \text{const.} = H \rightarrow a(t) \propto e^{Ht}$$

Exponential growth: universe is accelerating!

2nd Friedmann equation is (for w=−1):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = \frac{8\pi G}{3}\rho > 0$$

The Universe started to accelerate a couple of billion years ago. Before that there was a period of normal decelerated expansion, essential for the formation of galaxies.



Accelerated expansion: inflation

Curiosity: what happens if w<-1 ("phantom" dark energy)?

$$\omega = -1 - \delta \to \rho_{\rm phantom} \propto a^{-3(1+\omega)} = a^{3\delta}$$

Density increases with time. It can be shown that there is a singularity where $a \to \infty$ at finite time: the "big rip" [see astro-ph/0302506]

I.2.6 – Recipe of the Universe

Critical density: density at which the Universe is spatially flat.

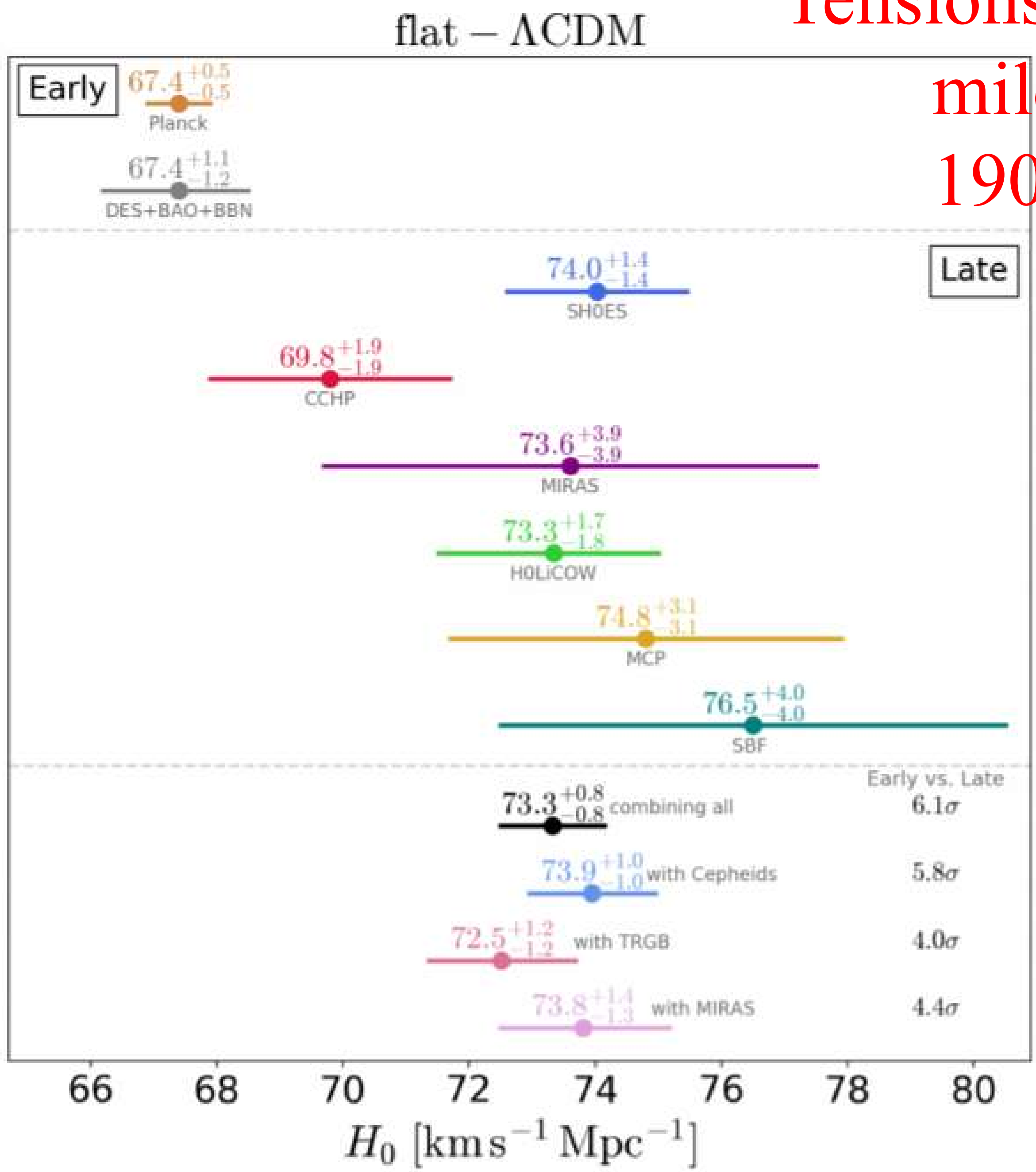
$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Hubble constant today has been measured with some precision and there is a mild tension:

$$H_0 = (67.8 \pm 0.9)\ \mathrm{km/s/Mpc}\ (\mathrm{Planck})$$
$$H_0 = (72.0 \pm 3)\ \mathrm{km/s/Mpc}\ (\mathrm{HST})$$

Exercise 6: estimate the critical density in units of protons/m^3

Tensions are not so mild anymore 1907.10625 – more later



Different contributions to the energy density budget of the Universe

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

Spatially flat universe:

$$\sum_i \Omega_i = 1$$

1st Friedmann equation:

$$\frac{H(t)^2}{H_0^2} = \sum_i \Omega_i^{(0)} a^{-3(1+\omega_i)}$$

Exercise 7: given a Universe with

$$\Omega_{\Lambda}^{(0)} = 0.7,\ \Omega_{\text{matter}}^{(0)} = 0.3,\ \Omega_{\text{rad}}^{(0)} = 5 \times 10^{-5}$$

compute:

a. H_0^{-1} in units of years
b. the age of the Universe
c. $(\rho_c)^{1/4}$ in units of eV – energy scale associated with the cosmological constant
d. H_0 in units of eV
e. the redshift z_{eq}
f. the redshift z_{Λ}

I.2.7 – Beyond Λ: dynamical dark energy

For a real homogeneous scalar field the energy-momentum tensor gives:

$$T_\phi^{00} = \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi);$$

$$T_\phi^{ii} = -g^{ii}P = -\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right)g^{ii}$$

and therefore the time-dependent equation of state in this case is:

$$\omega(t) = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \Rightarrow -1 \leq \omega \leq 1$$

If potential energy dominates w~-1 and scalar field resembles a cosmological constant: quintessence field. Can be ultralight (~H_0)!

Some examples of dark energy models:

Cosmological Constant $p_\Lambda = -\rho_\Lambda$

Canonical Scalar Field: Quintessence $\mathcal{L}_Q = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)$

Perfect Fluid $p_0 = w\rho_0$ $\delta p = c_{\text{eff}}^2\delta\rho$

Chaplygin Gas $\rho_{Ch} = -A\rho_{Ch}^{-\alpha}$

K-essence $\mathcal{L} = F(X,\varphi)$ $X = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi$

e.g. Tachyon, Born-Infeld $\mathcal{L}_T = V(\varphi)\sqrt{1 - \partial^\mu\varphi\partial_\mu\varphi}$

Klein-Gordon equation for a scalar field in an arbitrary metric

$$\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

$$S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi, \; \delta S_\phi = 0 \Rightarrow$$

$$\frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} \; \partial^\mu \phi] + \frac{dV}{d\phi} = 0$$

lein-Gordon equation for a homogeneous scalar field in FRWL metric

$$\sqrt{-g} = a^3(t)$$

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{dV}{d\phi} = 0$$

I.2.8 – Vacuum energy: the elephant in the room

Quantum mechanics – zero point energy of a harmonic oscillator:

$$E = \hbar\omega(n + 1/2)$$

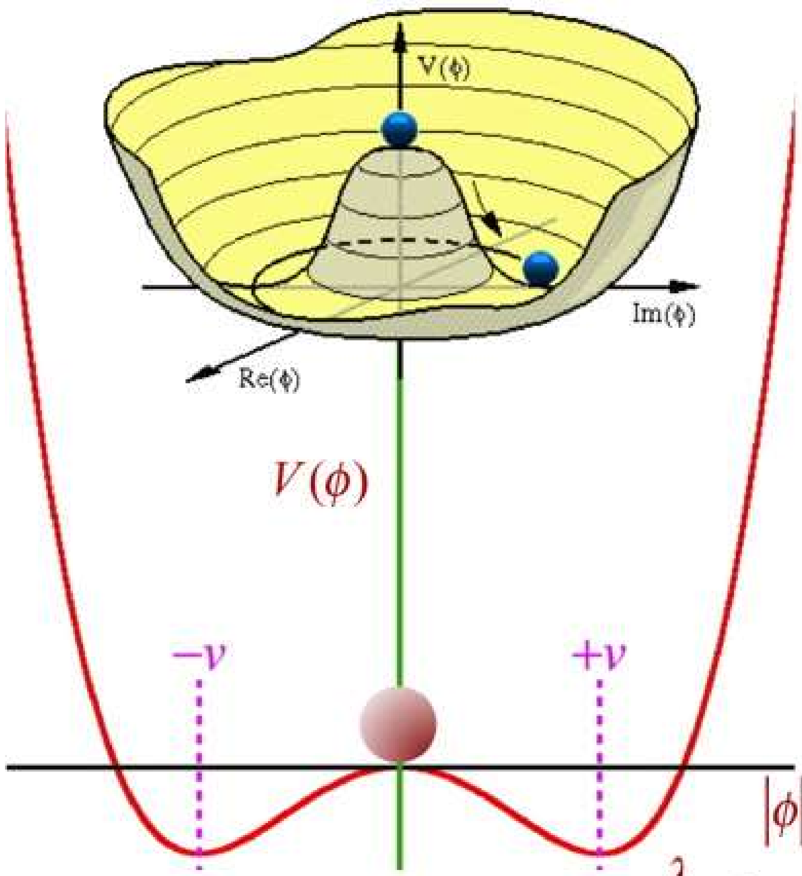
In Quantum Field Theory, the energy density of the vacuum is (free scalar field of mass m):

$$\rho_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2}$$

and is infinite! Integral must be cut-off at some physical energy scale - goes as $(\text{cut-off})^4$.

If integral is cutoff at the Planck scale, disagreement of ~ 10^{120} with data. This is know as the cosmological constant problem.

The Higgs field and the vacuum energy:



$$V(\phi) = \tfrac{1}{2}\mu^2\phi^\dagger\phi + \tfrac{1}{4}\lambda(\phi^\dagger\phi)^2$$

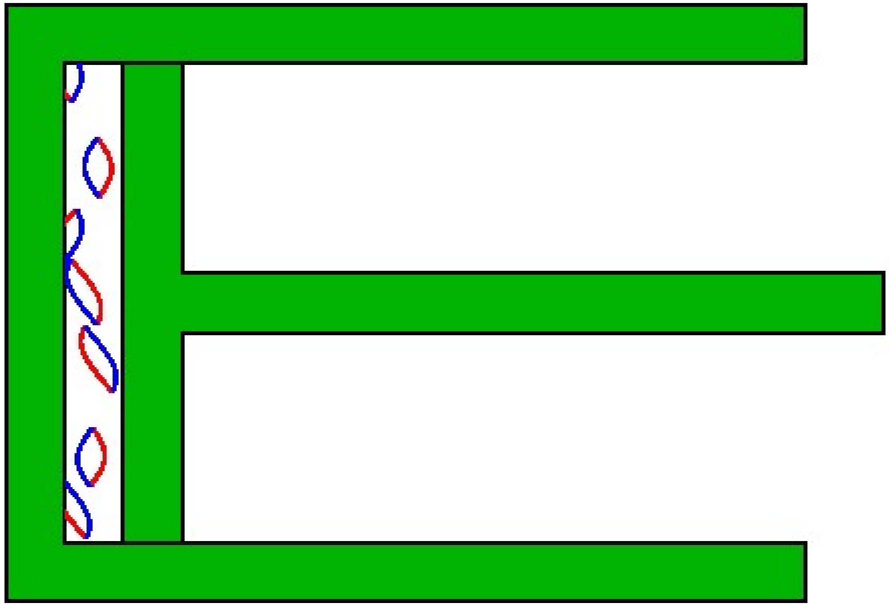
Groundstate at $|\phi_0| = \sqrt{\frac{-\mu^2}{\lambda}} \equiv \upsilon$

$$|\phi| = \sqrt{\phi^\dagger\phi} = \sqrt{\phi^{+\dagger}\phi^+ + \phi^{0\dagger}\phi^0}$$

$$V(\phi_0) = -\frac{\lambda}{4}\upsilon^4$$

Vacuum energy

$$\rho_\Lambda \propto \text{constant}$$



E. L. Wright

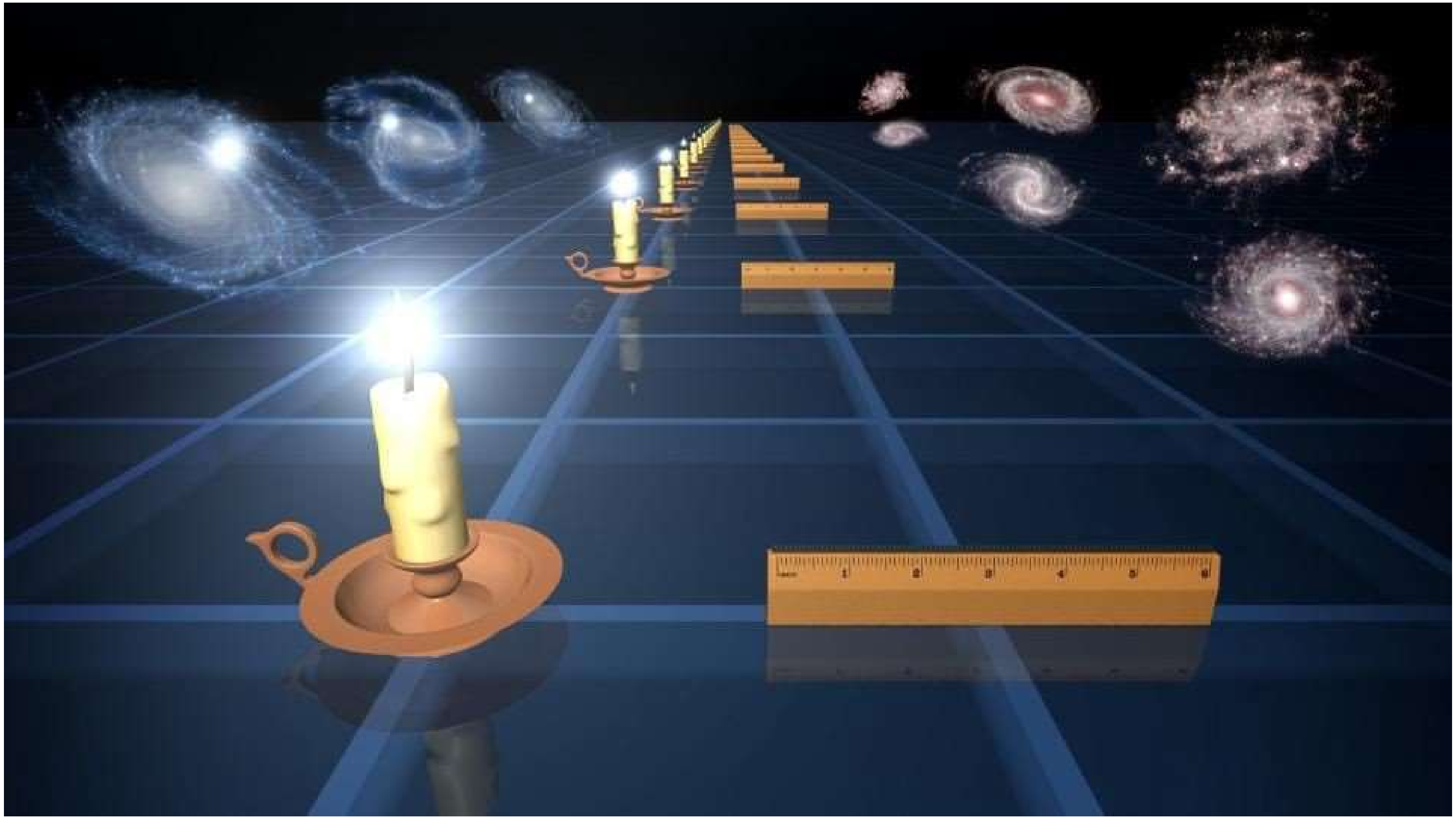
$$dE = -pdV \Rightarrow p_\Lambda < 0$$

I.2.9 – Distances in the Universe

here are 2 ways to measure large distances in the Universe:

from known luminosities (standard candles – eg SNIa) – **luminosity distance**

from known scales (standard rulers – eg BAO) - **angular diameter distance**



Let's recall that:
physical distance = a(t) comoving distance
and discuss some other typical distances in the Universe.

a. Comoving distance between us (z=0) and an object at redshift z:

$$ds^2 = 0 \Rightarrow dt^2 = a(t)^2 d\chi^2 \Rightarrow$$

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}$$

b. Comoving particle horizon: largest region in causal contact since the Big Bang:

$$\chi(z)_{\rm phys} = \int_0^t \frac{dt'}{a(t')}$$

c. Luminosity distance (d_L):

$$F = \frac{L}{4\pi d_L^2}$$

Flux of photons

Known luminosity of the source

In FLRW there are 2 extra source of dilution of the flux:

- redshift of photons (1/(1+z))
- rate of arrival decrease by (1/(1+z)) – time dilation

Therefore:

$$d_L = (1+z)\chi(z)$$

Exercise 8a: Recovering Hubble's law

Show that by expanding $H(z) = H_0 + \frac{dH}{dz} z$

To first order in redshift one finds

$$d_L = \frac{z}{H_0}$$

Therefore: $$d_L = (1+z)\chi(z)$$

Exercise 8: plot $d_L(z)$ for $0<z<2$ for a flat Universe with

α. $\Omega_m = 1$ and $\Omega_\Lambda = 0$
β. $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$

We will use the code “Core Cosmology Library” CCL that was developed for LSST.

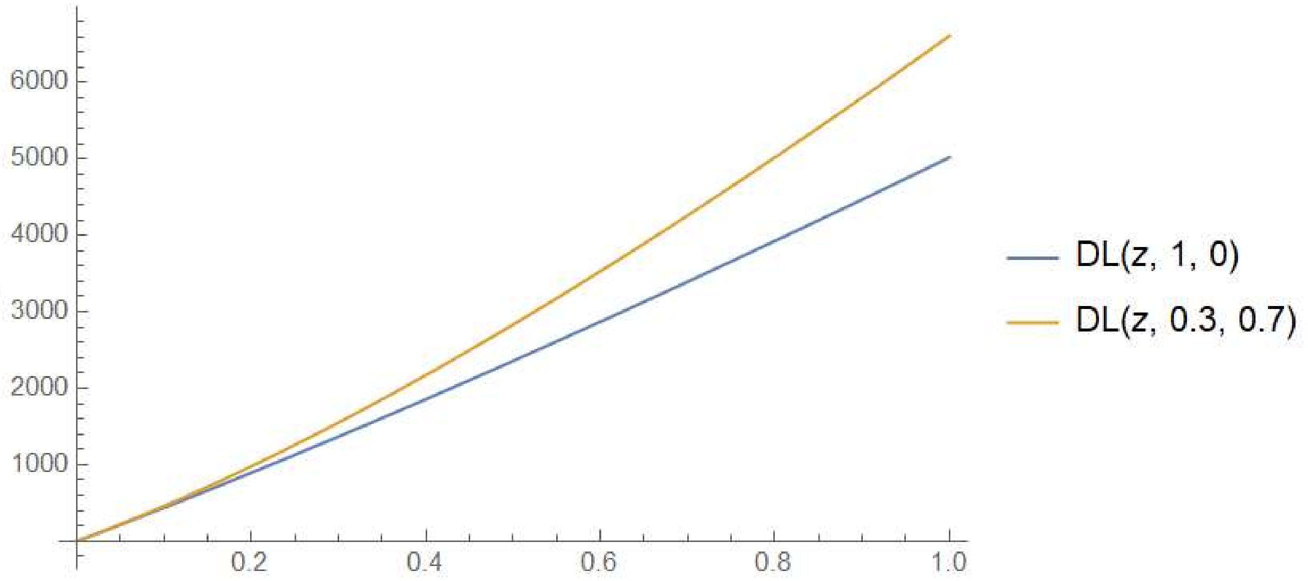
d_L is larger for a Universe with Λ -> objects with same z look fainter

This is how the accelerated expansion of the Universe was discovered in 1998 using SNIa

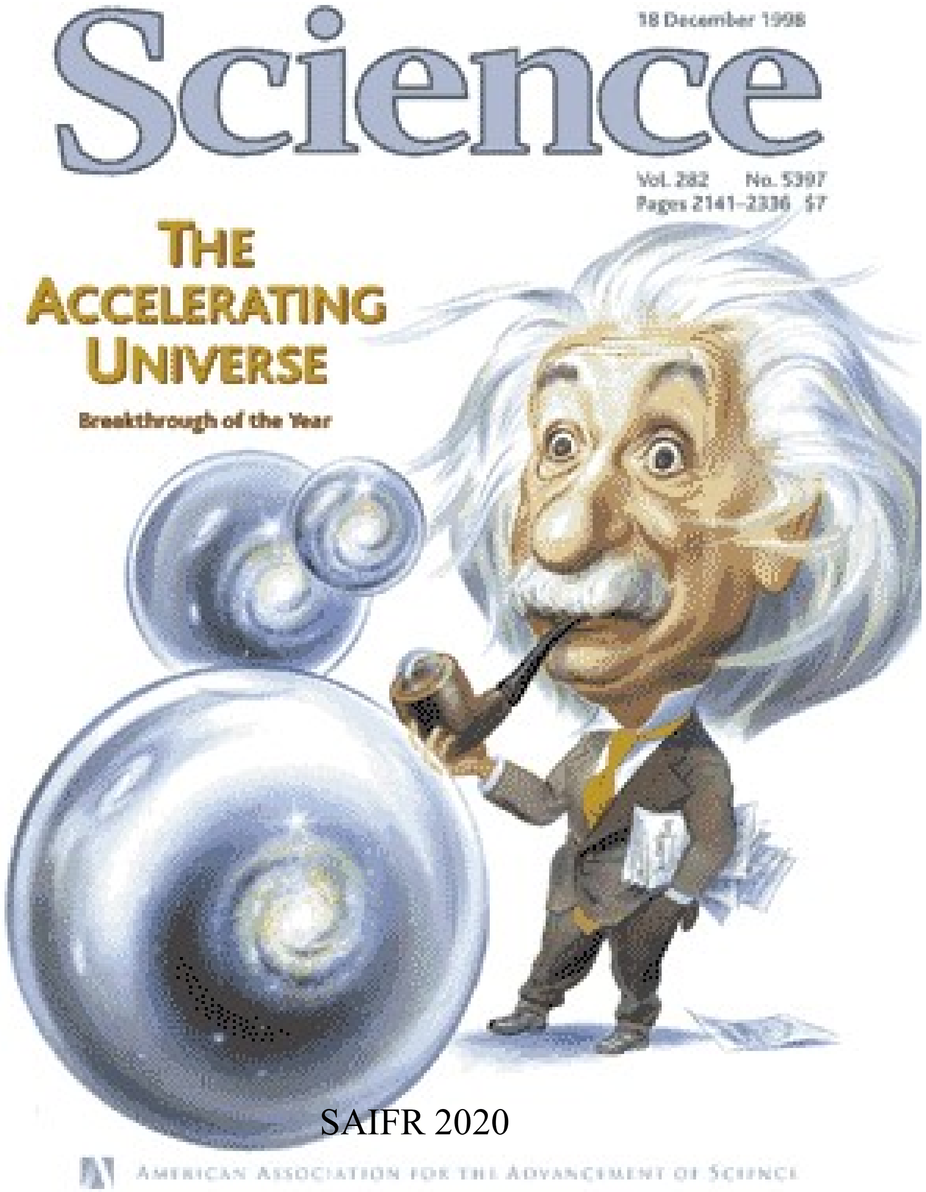
Comoving distance, luminosity distance, sound horizon at decoupling

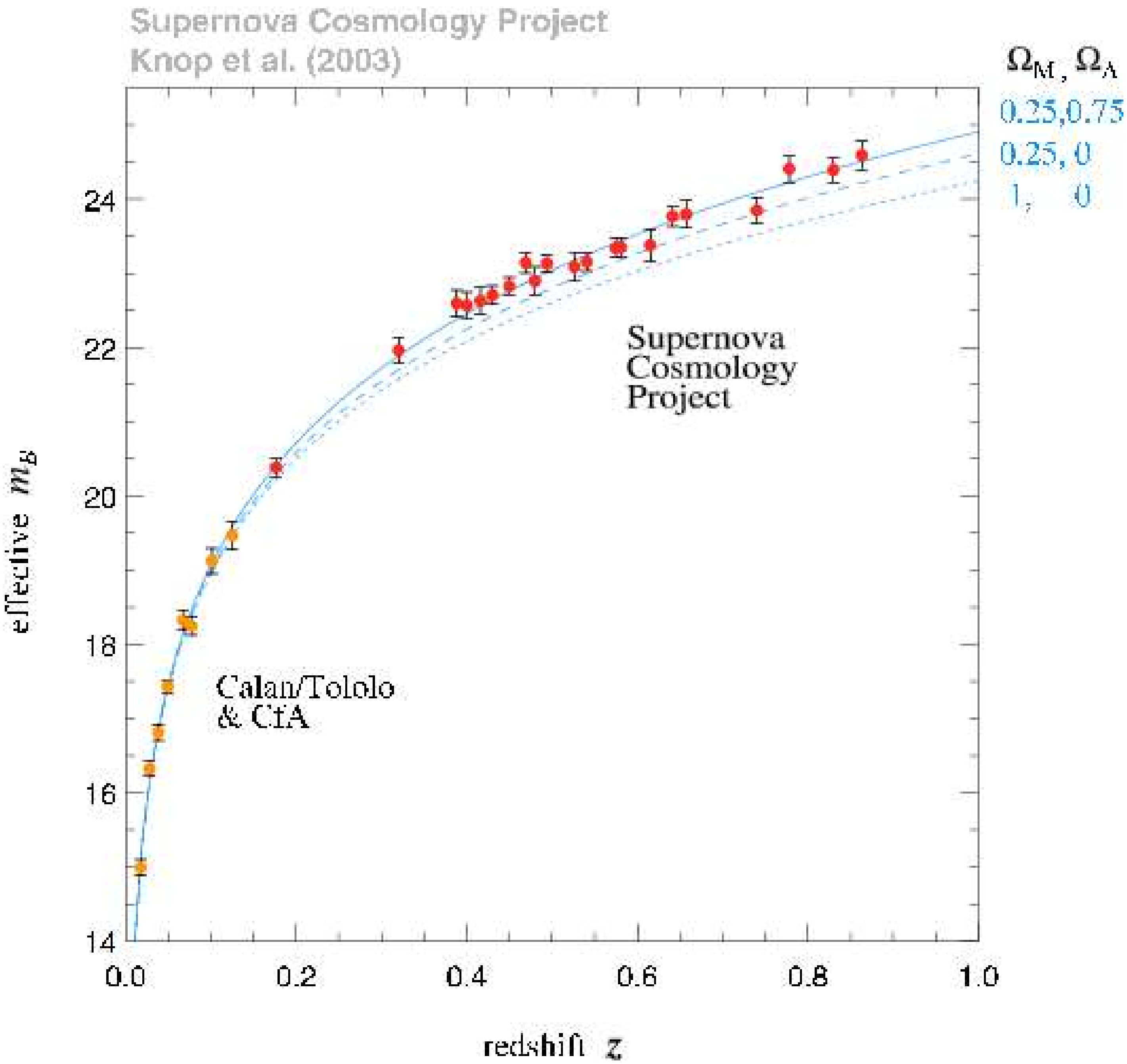
```
In[127]:= H0 = 70. (* km/s/Mpc *);
          c = 300000. (*km/s*);

In[130]:= H[z_, Om_, OΛ_] := H0 √(Om (1 + z)^3 + OΛ)

In[131]:= DL[z_, Om_, OΛ_] := c (1 + z) NIntegrate[1 / H[zp, Om, OΛ], {zp, 0, z}]
                                        integra numéricamente

In[137]:= Plot[{DL[z, 1, 0], DL[z, 0.3, 0.7]}, {z, 0, 1}, PlotLegends → "Expressions"]
          gráfico                                            legenda do gráfico
```

Out[137]=



The big surprise in 1998:




Supernova Cosmology Project
Knop et al. (2003)
Ω_M, Ω_Λ
0.25,0.75
0.25, 0
1, 0
Supernova Cosmology Project
Calan/Tololo & CfA
effective m_B
redshift z
14
16
18
20
22
24
0.0
0.2
0.4
0.6
0.8
1.0

The Nobel Prize in Physics 2011
Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

Share this: 290

The Nobel Prize in Physics 2011

Photo: U. Montan

Saul Perlmutter

Prize share: 1/2

Photo: U. Montan

Brian P. Schmidt

Prize share: 1/4

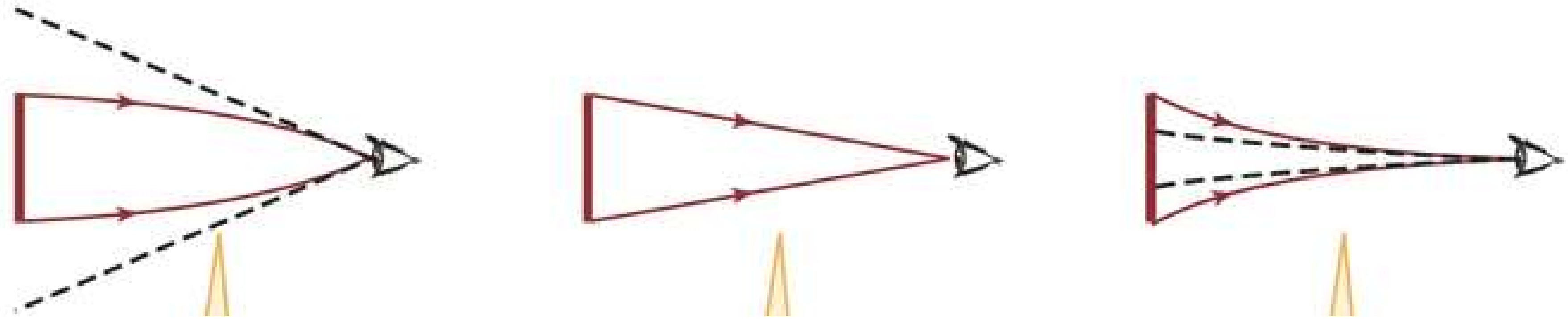
Photo: U. Montan

Adam G. Riess

Prize share: 1/4

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".*

d. Angular diameter distance (d_A):
related to the angle subentended by a physical lenght (l)



Closed Universe Flat Universe Open Universe

$$d_A = \frac{l}{\delta\theta},\ l = a\chi\delta\theta \Rightarrow$$

$$\boxed{d_A = \frac{1}{1+z}\chi(z)}$$

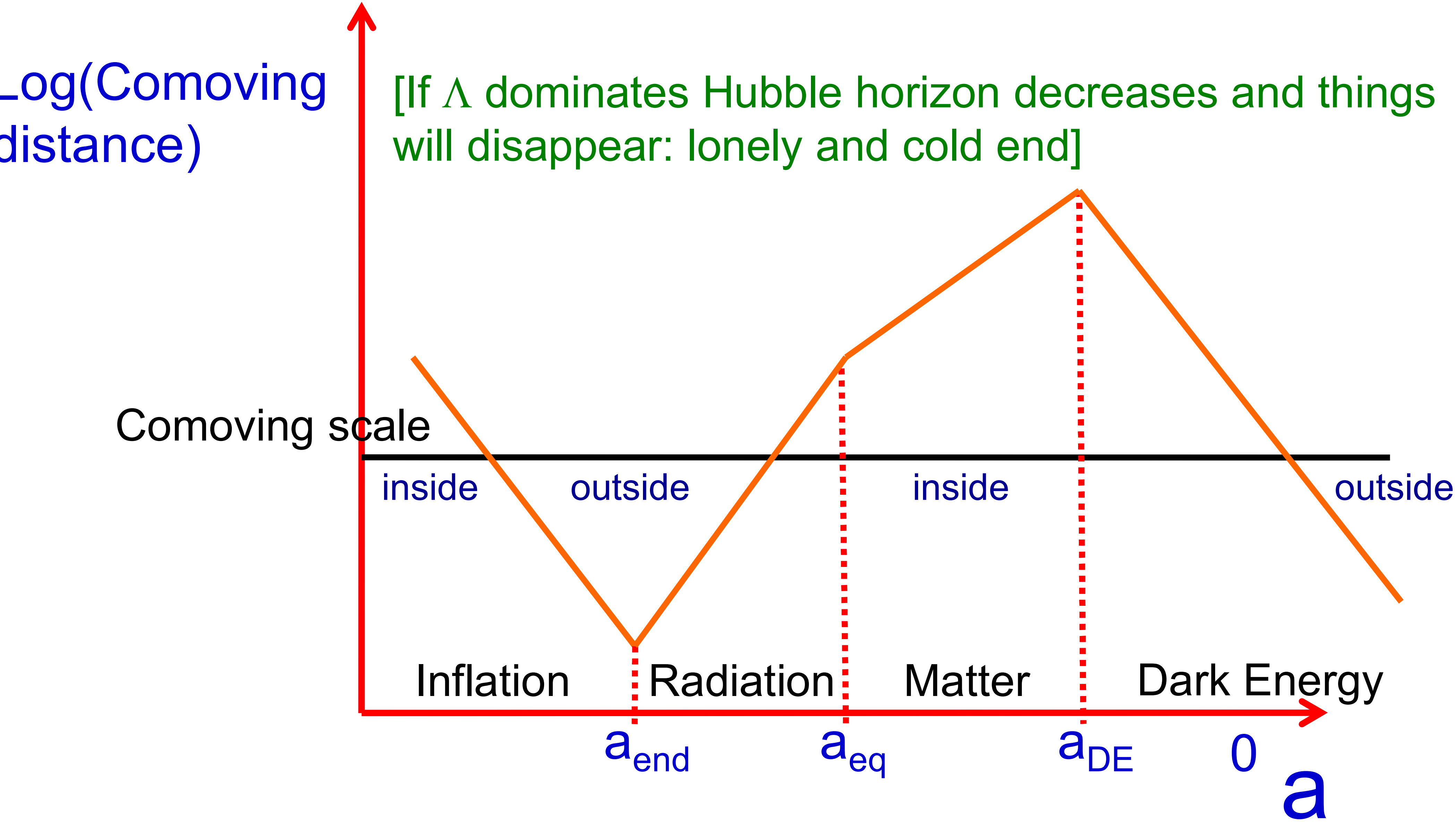
e. Hubble radius: distance particles can travel in a Hubble time (c=1)

$$R_H = \frac{1}{H(t)}$$

Comoving Hubble radius: $$r_H = \frac{1}{aH} = \frac{1}{\dot{a}}$$

- Radiation dominated $r_H \propto a$
- Matter dominated $r_H \propto a^{1/2}$
- Λ dominated $r_H \propto 1/a$

Comoving Hubble radius during the evolution of the Universe



I.3- Thermal history of the Universe

I.3.1 – Brief review of thermodynamics

Quick way to derive relation between temperature and scale factor:

$$\rho_r \propto T^4;\ \rho_r \propto a^{-4} \Rightarrow a \propto T^{-1}$$

Stefan–Boltzmann law

More formally the number density and energy density are:

$$n = \frac{g}{(2\pi)^3} \int d^3p \, f(\vec{p})$$

$$\rho = \frac{g}{(2\pi)^3} \int d^3p \, E(\vec{p}) f(\vec{p})$$

$$E = \sqrt{|\vec{p}|^2 + m^2}$$

E is the energy of a state, f(p) is the phase-space distribution and g is number of internal degrees of freedom (eg g=2 for photons, g=16 for gluons, g=12 for quarks, etc).

Phase–space distribution (+ for FD, – for BE), k_B=1,
μ chemical potential:

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

Relativistic limit (T>>m) and T>>μ

$$\rho = \left(\frac{\pi^2}{30}\right) g T^4 \begin{cases} 1 \text{ (Bose – Einstein)} \\ \frac{7}{8} \text{ (Fermi – Dirac)} \end{cases}$$

$$n = \frac{\zeta(3)}{\pi^2} g T^3 \begin{cases} 1 \text{ (Bose – Einstein)} \\ \frac{3}{4} \text{ (Fermi – Dirac)} \end{cases} \qquad \zeta(3) = 1.202\cdots$$

Exercise 9: compute the number of CMB photons (T=2.73 K) in 1 cm^3

Non-relativistic limit (T<<m) and μ=0 [same for B-E and F-D]

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$\rho = mn$$

Exponential Boltzmann suppression

Density of relativistic particles in the Universe is set by the effective number of relativistic degrees of freedom g_*:

$$\rho_r = \frac{\pi^2}{30} g_* T^4$$

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{\text{fermions}} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^4$$

$g_*(T)$ changes when mass thresholds are crossed as T decreases and Particles become non-relativistic.
At high T (>200 GeV) $g_*^{(SM)} \sim 100$.

tive number of relativistic degrees of freedom g_* during the evolution of the Univers

$$g_\gamma = 2$$

$$= 2 \times 3 = 6$$

$$g_{e^\pm} = 4$$

$$4g_*$$

Temperature	New Particles	$4N(T)$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < T_c^\dagger$	π's	69
$T_c < T < m_{\text{strange}}$	π's + $u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{\text{charm}}$	$s, \bar{s}$	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{\text{bottom}}$	$\tau^\pm$	303
$m_b < T < m_{\text{W,Z}}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{\text{Higgs}}$	$W^\pm, Z$	381
$m_H < T < m_{\text{top}}$	H^0	385
$m_t < T$	$t, \bar{t}$	427

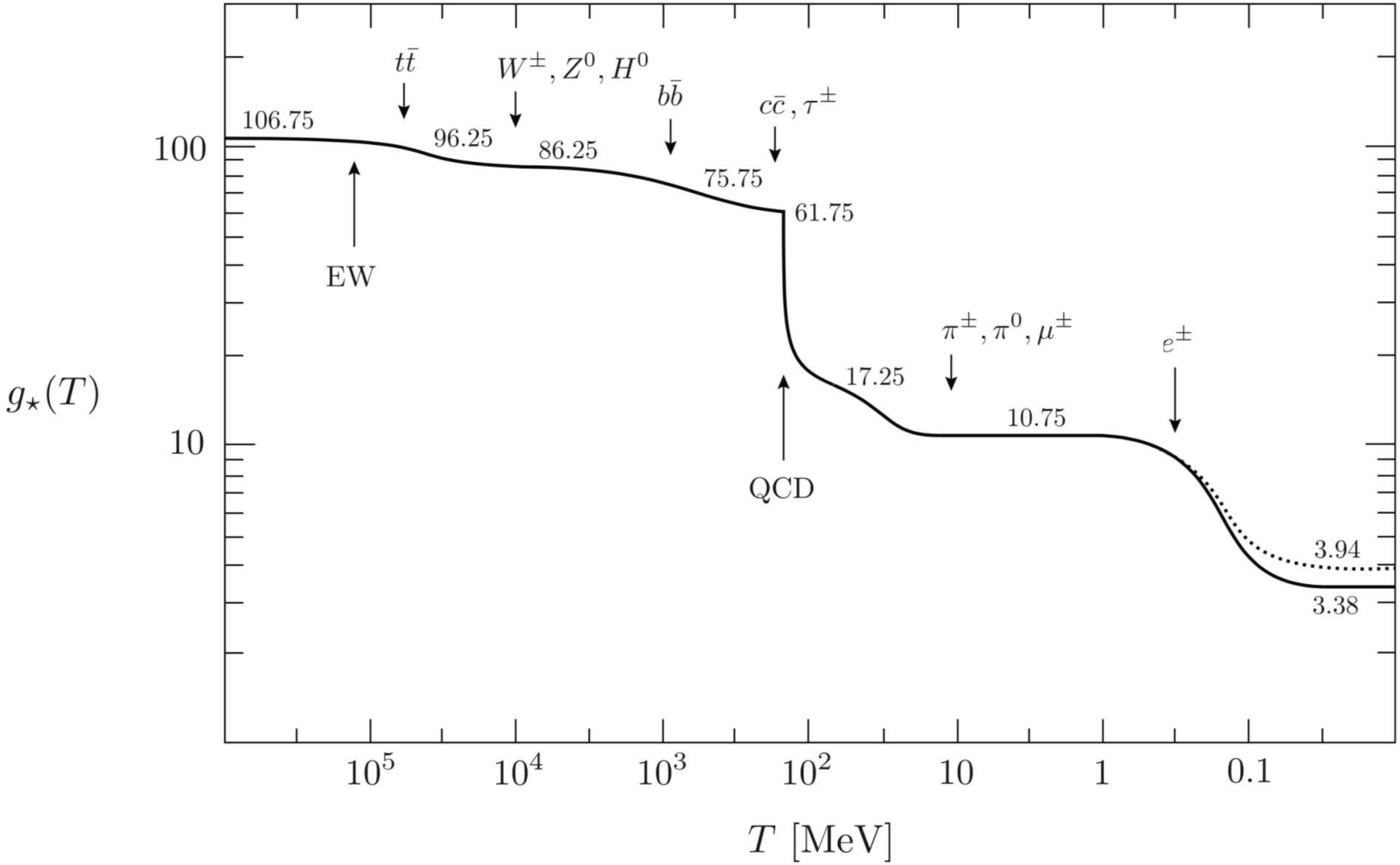


Figure 3.4: *Evolution of relativistic degrees of freedom $g_\star(T)$ assuming the Standard Model particle content. The dotted line stands for the number of effective degrees of freedom in entropy $g_{\star S}(T)$.*

I.3.2 – Temperature-time relationship

From Friedmann's 1st equation for a radiation-dominated era:

$$H = \sqrt{\frac{\rho_r}{3\tilde{M}_{\rm Pl}^2}} \sim \frac{T^2}{M_{\rm Pl}}$$

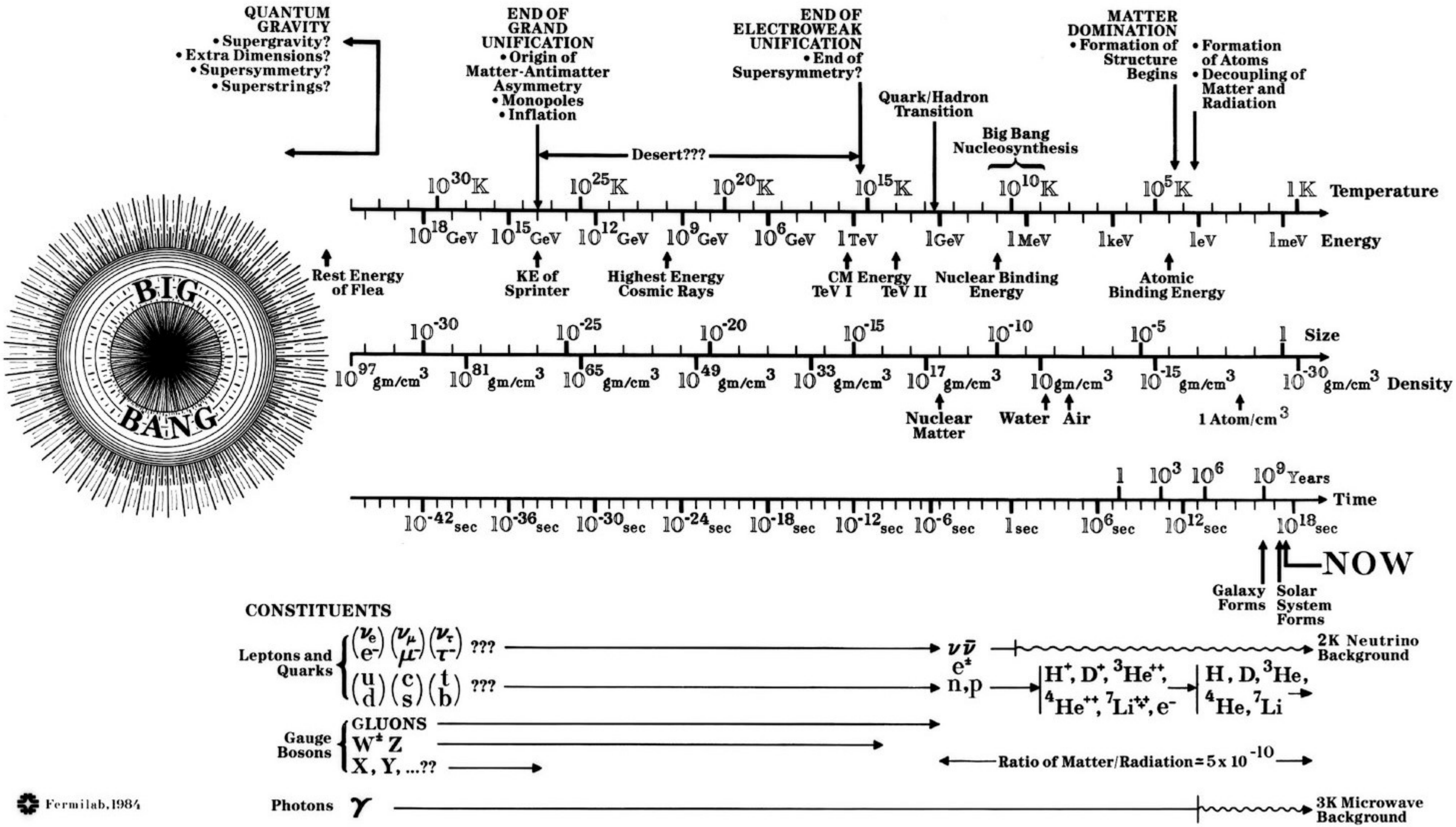
and $H = \dfrac{\dot{a}}{a} \propto t^{-1}$

one finds: $T \propto t^{-1/2}$

Putting numbers: $T({\rm MeV}) \simeq 1.5 g_*^{-1/4} t({\rm s})^{-1/2}$

Thermal history of the Universe

Kolb & Turner



I.3.3 – Decoupling of species

Different particles are in thermal equilibrium when they can interact efficientlyThere are 2 typical rates that can be compared:

- rate of particle interactions:

$$\Gamma(T) = n\langle\sigma v\rangle$$

Number density

Thermal averaged cross section x velocity

- expansion rate of the Universe:

$$H(T)$$

When

$$\Gamma(T) \gg H(T)$$

particles are in thermal equilibrium.

As a first estimate particles decouple when $\Gamma(T) \sim H(T)$

More precise estimate requires solving a Boltzmann equation (more later)

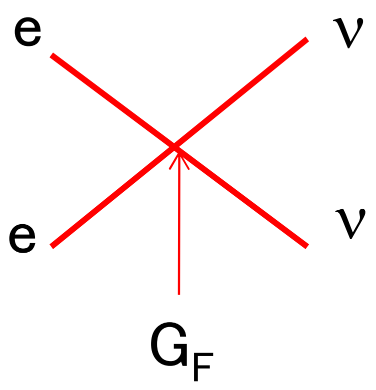
Example: decoupling of neutrinos from the thermal bath

Weak interactions:

$$\nu_e + \bar{\nu}_e \leftrightarrow e^+ + e^-$$

$$e^- + \bar{\nu}_e \leftrightarrow e^- + \bar{\nu}_e$$

Low energy cross section (4–fermi interaction):



$G_F = 10^{-5}$ GeV^{-2}: Fermi constant

$$\sigma \sim G_F^2 T^2;\ n_\nu \sim T^3 \Rightarrow \Gamma_\nu(T) \sim G_F^2 T^5$$

$$H(T) \sim T^2/M_{\rm Pl}$$

$$T_{\nu,\rm dec} = \left(\frac{1}{G_F^2 M_{\rm Pl}}\right)^{1/3} \sim 1\,\rm MeV$$

r decoupling neutrinos cool down as T α 1/a.

y would have the same temperature as photons except for the that photons get heated up by the annihilation of e^+e^- at around 5 MeV. Hence neutrinos are a bit cooler (T_n=1.95 K)

s.1: Today there is a cosmic ν background which is very difficult detect – experiment Ptolomy is being designed for this search.

bs.2: In SM n’s are massless and only ν_L exist.
ome extensions of the SM postulate the existence
f ν_R to explain n masses. This is a new degree of freedom and is
auge singlet under SM interactions [eg 1303.6912] – sterile neutri

hese states were never in thermal equilibrium and are usually hea
owever there are models where these sterile neutrinos can be wa
ark matter.

Obs.3: Experiments such as Planck are sensitive to the number of relativistic degrees of freedom present at the time of CMB. This is characterized by the so-called N_{eff} parameter.

$$\rho_r = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{eff} \right]$$

In the SM, $N_{eff} = 3.046$ (ν's are heated by e^+e^- annihilation). There were some measurements giving a larger N_{eff} which prompted many papers postulating new relativistic degrees of freedom dubbed **"dark radiation"**.

In 2018 Planck measured $N_{eff} = 2.99 +/- 0.17$ and most people are now happy.

Obs.4: If massless $\rho_\nu \sim \rho_\gamma$. If massive (Lee–Weinberg bound):

$$\rho_\nu \simeq \sum_i m_{\nu,i} n_{\nu,i} \Rightarrow \Omega_\nu^{(0)} = \frac{\sum_i m_{\nu,i}}{94\mathrm{eV}}$$

Use:

$$n_\nu = \frac{T_\nu^3}{T_\gamma^3}\frac{3}{4} n_\gamma = \frac{3}{11} n_\gamma; \quad \text{since } T_\nu = \left(\frac{4}{11}\right)^3 T_\gamma$$

$$n_\gamma^0 = 422\ cm^{-3} \Rightarrow n_\nu^0 = 115\ cm^{-3}$$

$$\rho_c \approx 1h^2 10^4 \frac{eV}{cm^3} \Rightarrow \Omega_\nu h^2 \approx \frac{\sum m_\nu}{90\ eV}$$

Obs.5: Most stringent bounds on ν masses comes from cosmology
More later

I.3.4 – Decoupling of matter and radiation

Decoupling occurs when the Universe cools down and hydrogen can be formed.

Naivelly one may think that this happens at a temperature T_{dec}=13.6 eV

However, one must take into account that there are many more photons than protons in the Universe!

Should find temperature of decoupling for the reaction:

$$H + \gamma \longleftrightarrow e^- + p^+$$

rect treatment: solve a Boltzmann equation. Here we will use
pler, more physical estimate: simply find the temperature at
ch the number density of photons with energy larger than the
rogen ionization energy equal the number density of protons

$$n_\gamma^{E>E_i}(z) = n_p(z)$$

$$n_\gamma^{E>E_i}(z) = \frac{2}{(2\pi)^3}\int_{E_i}^{\infty} dE \frac{E^2}{e^{E/T(z)}-1} \qquad T(z) = T_0(1+z)$$

$$n_p(z) = n_p^0(1+z)^3$$

$$n_p^0 = \frac{1}{m_p}\rho_c\Omega_b \qquad \rho_c = \frac{3H_0^2}{8\pi G} = \frac{3H_0^2}{8\pi}M_{Pl}^2$$

Decoupling of matter and radiation using units of GeV

```
Mproton = 0.938;
Mplanck = 1.22 × 10.^(19);
H0 = 2.1 × 0.75 × 10.^(-42);
T0 = 2.37 × 10^(-13);
OmegaB = 0.048;
```

In[122]:= sol = FindRoot[$\int_{xi}^{\infty} \frac{x^2}{e^x - 1} dx == \frac{3\,\text{OmegaB}}{2\,\pi^2} \frac{\text{H0^2 Mplanck^2}}{\text{T0^3 Mproton}}$, {xi, 30}]

encontra raiz

Out[122]= {xi → 29.065}

In[121]:= $\frac{3\,\text{OmegaB}}{2\,\pi^2} \frac{\text{H0^2 Mplanck^2}}{\text{T0^3 Mproton}}$

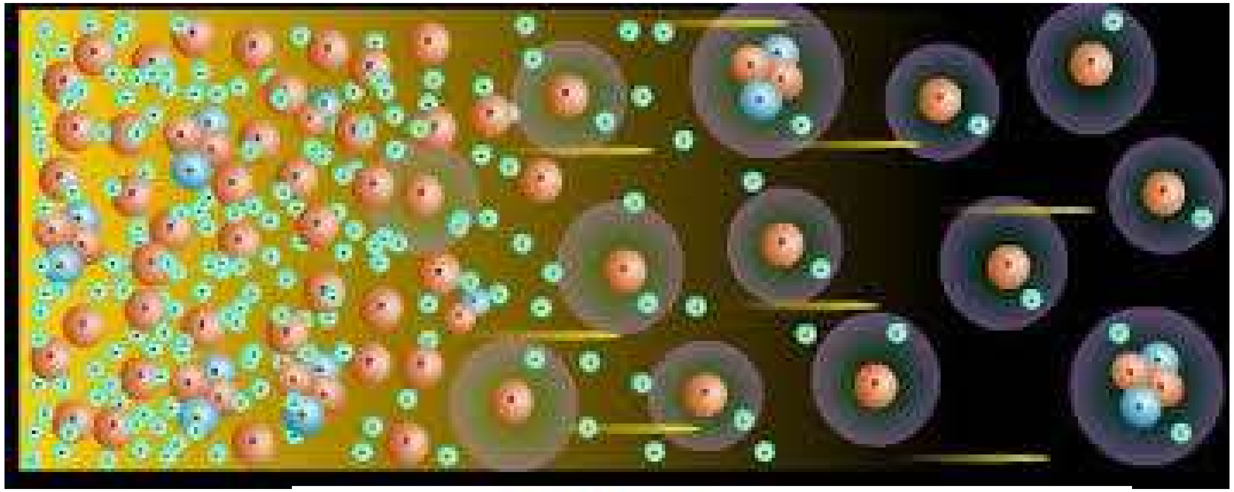
Out[121]= 2.15708×10^{-10}

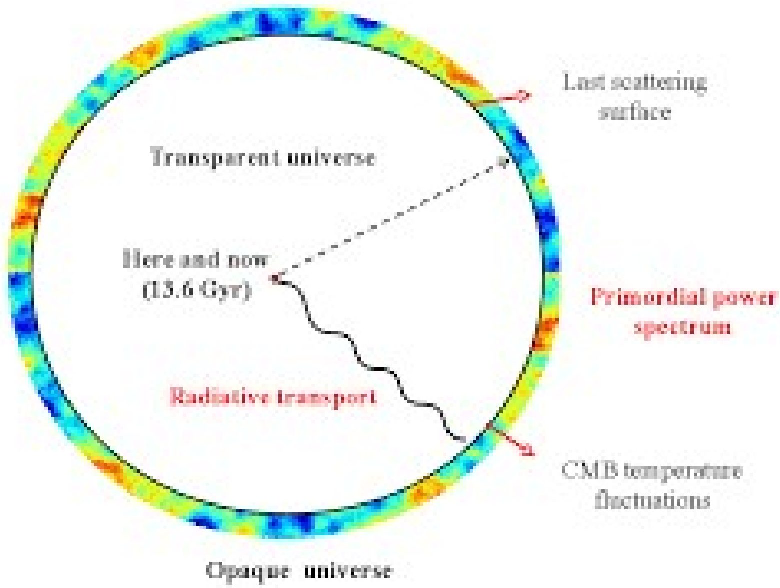
$$xi = \frac{E_i}{T_{dec}} \Rightarrow T_{dec} = 13.6/29 = 0.47 \quad \text{eV}$$

Correct result: T_{dec} = 0.26 eV = 0.26/8.6 10^{-5} = 3000 K

$z_{dec} = T_{dec}/T_0 = 1100$

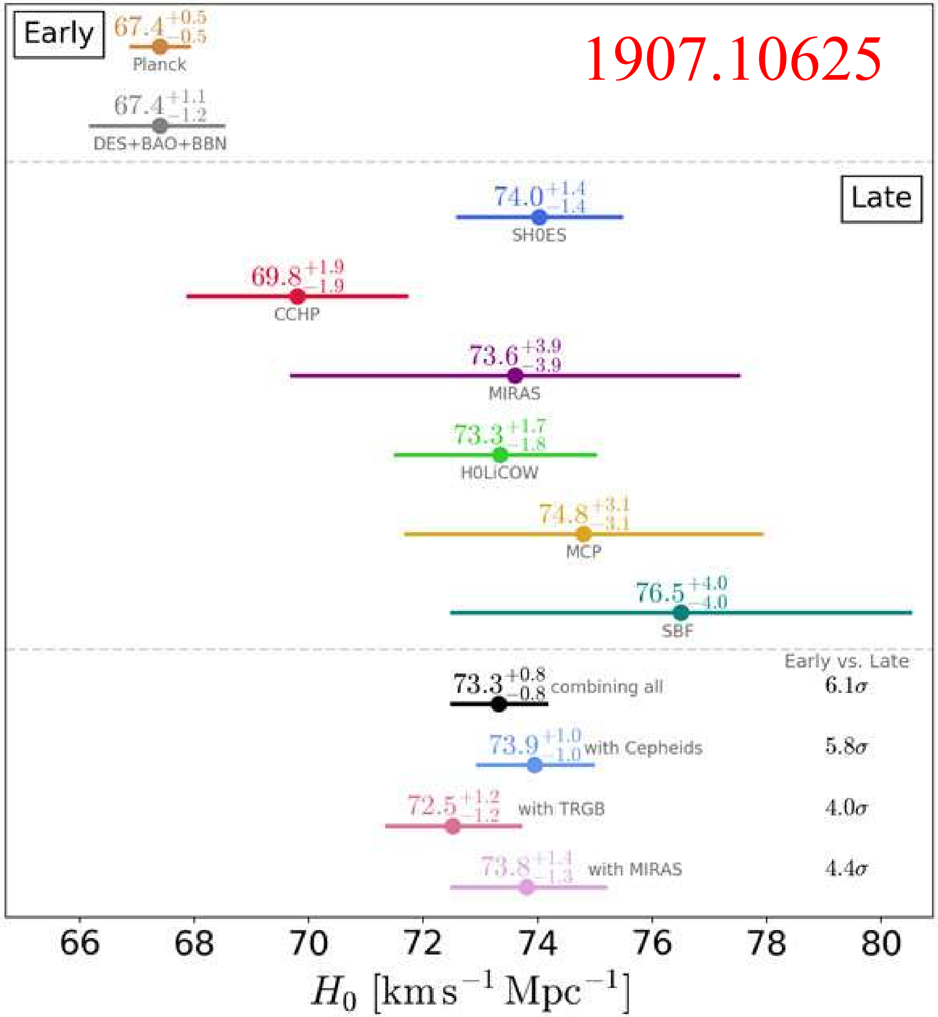
er decoupling the Universe is neutral and becomes transparente to otons: “last scattering suface” at z=1100.





I.3.5 – Hubble tension (or crisis?)

First crack in the standard ΛCDM model?



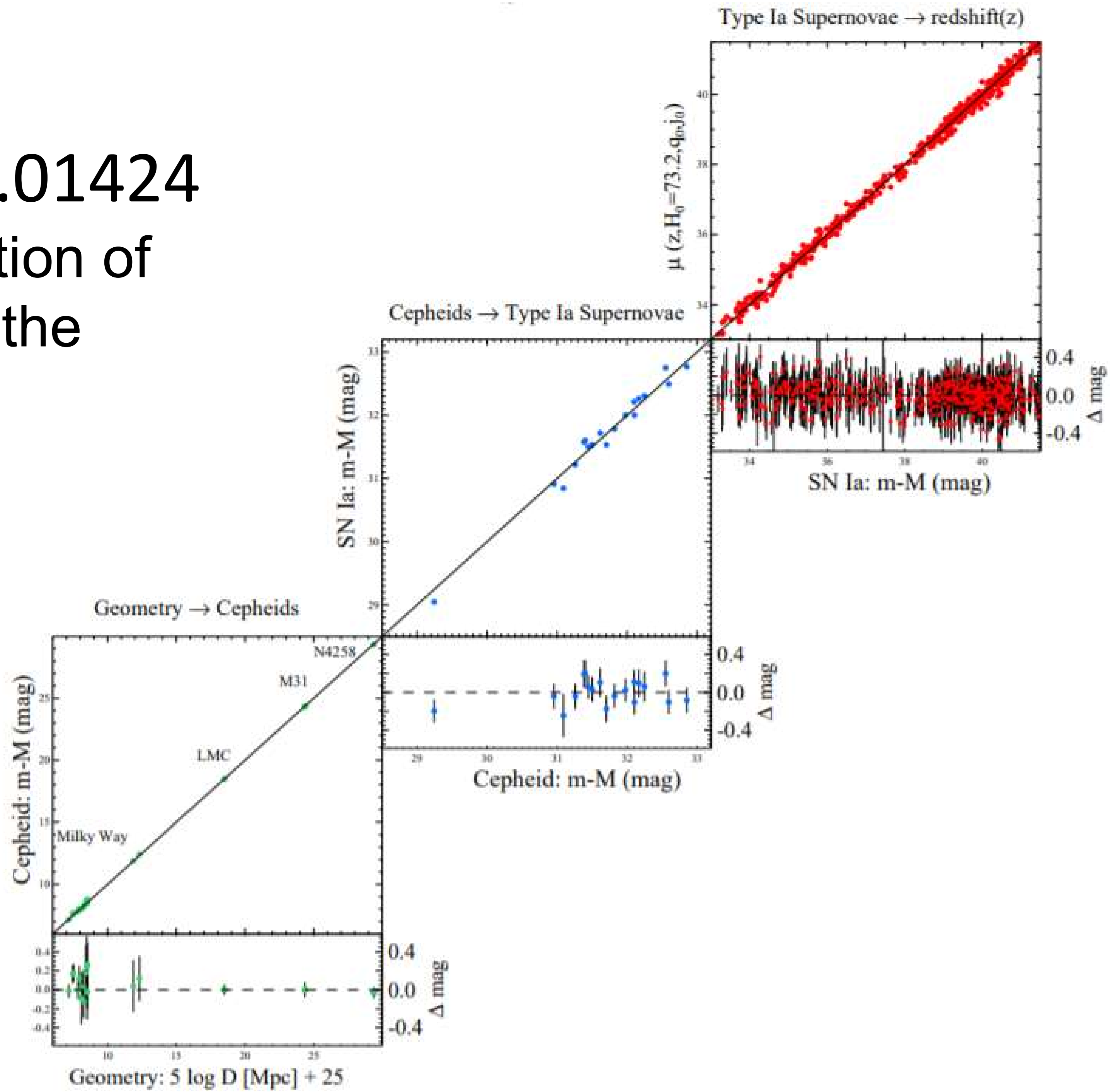
H_0 measurements from SNIa are direct:
Hubble's law

Depends on distance measurements: difficult
Calibration using Cepheid stars are importante
Systematic effects are dominant

Distance ladder

Riess et al 1604.01424

A 2.4% Determination of the Local Value of the Hubble Constant



MB does not measure H_0 directly! It's a result of a mplicated fit to the CMB angular power spectrum th several parameters.

owever, we can have an idea: the quantity measured CMB is the angular scale of the CMB θ_*

MB is a standard ruler in the sky! But what is the ysical scale of CMB? The sound horizon at decouplin he sound horizon at decoupling: r_s

$$r_s = \int_{z_{dec}}^{\infty} \frac{c_s}{H(z)}$$

c_s: speed of perturbations in the coupled baryon-photon fluid – for relativistic fluids

$$c_s = \frac{\delta P}{\delta \rho} = \frac{1}{\sqrt{3}}$$

$$\theta_* = \frac{r_s}{d_A(z_{dec})}$$

r_s ← Early times

$d_A(z_{dec})$ ← Late times

ecall that in flat ΛCDM model:

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda}$$

there is an extra contribution with respect to the ΛCDM model to H(z) around the recombination era: in order to keep r_s fixed requires **a lower value of H_0**

Early dark energy - 1811.04083, 2003.07355
Decaying dark matter - 1903.06220, 2004.07709, 2004.06114
...