Gauge-invariant operators of open bosonic string field theory in the low-energy limit

Yuji Okawa
The University of Tokyo, Komaba

In collaboration with Daiki Koyama and Nanami Suzuki

Online Conference
"2020 Workshop on String Field Theory and Related Aspects"
at ICTP-SAIFR/IFT-UNESP
June 8, 2020 in São Paulo, Brazil
(See related talks by Erbin and Vošmera.)

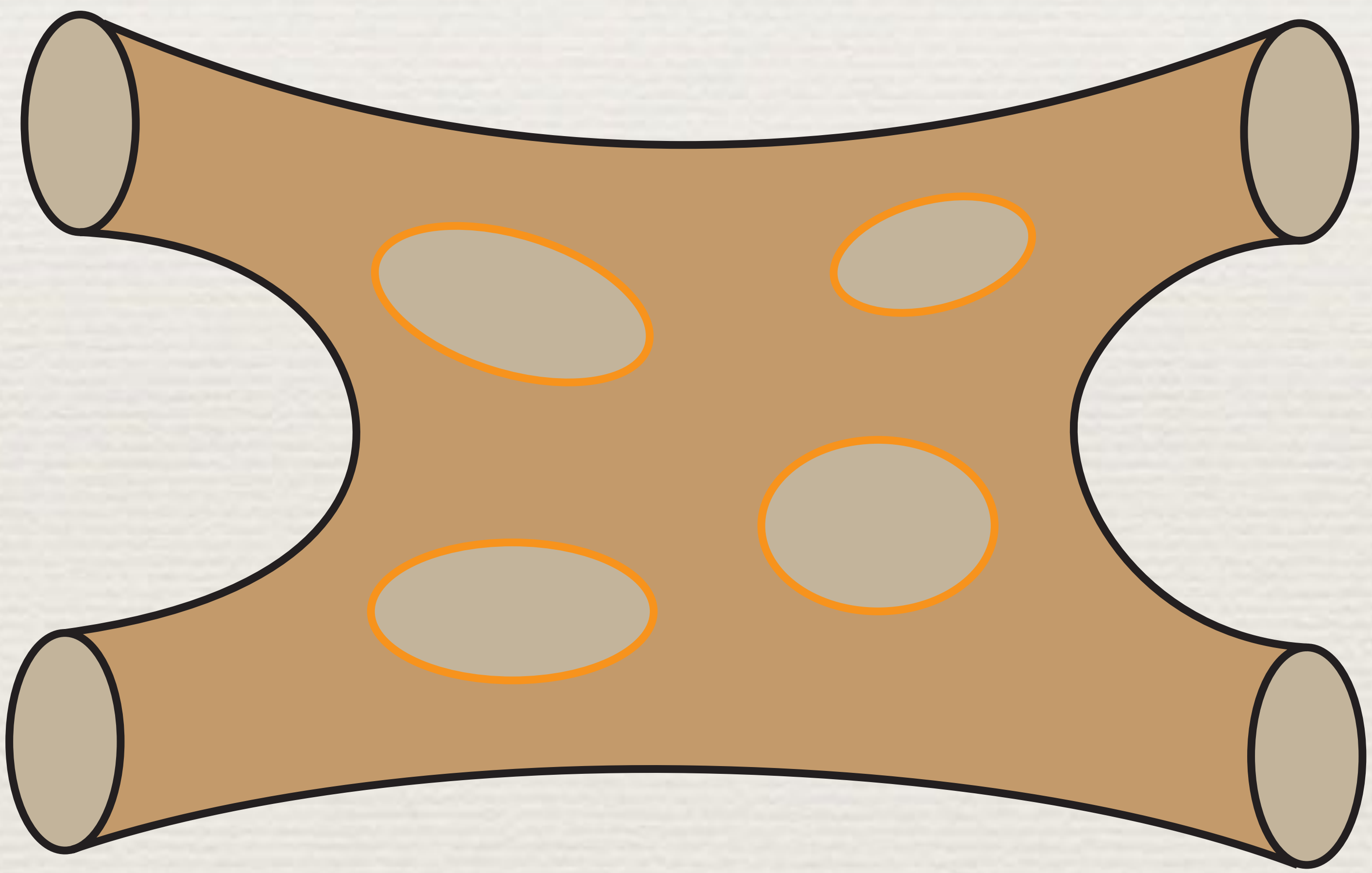
Introduction

The AdS/CFT correspondence can be thought of as providing a nonperturbative definition of closed string theory in terms of a gauge theory without containing gravity.

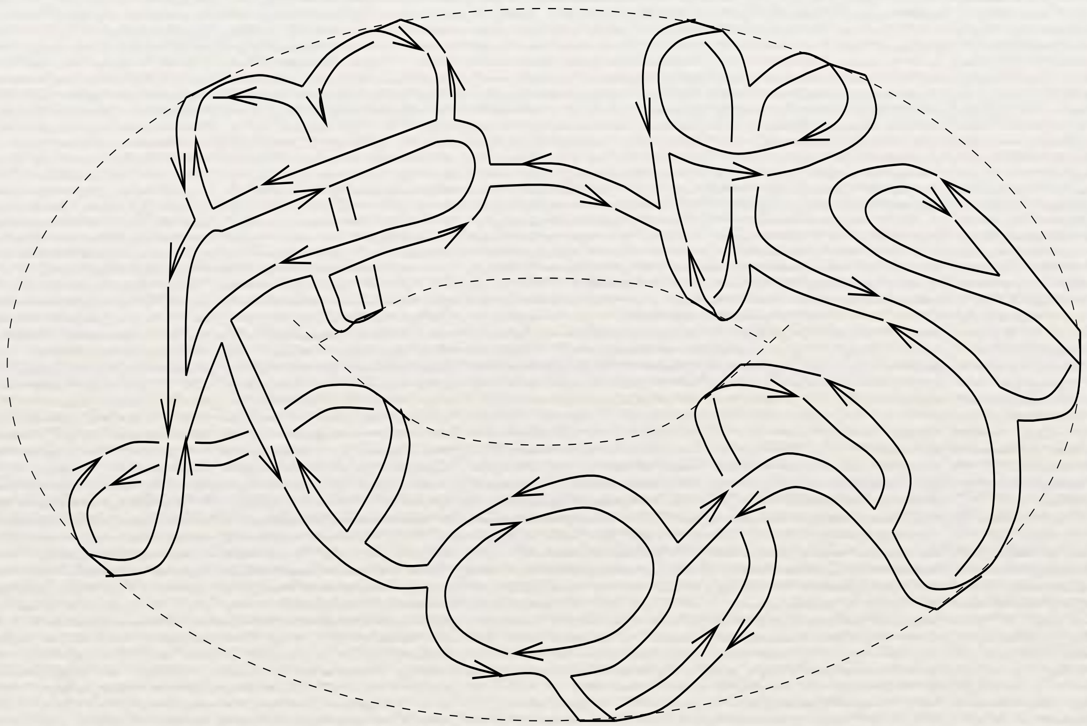
In the standard explanation of the AdS/CFT correspondence we consider the theory on D-branes and take the low-energy limit.

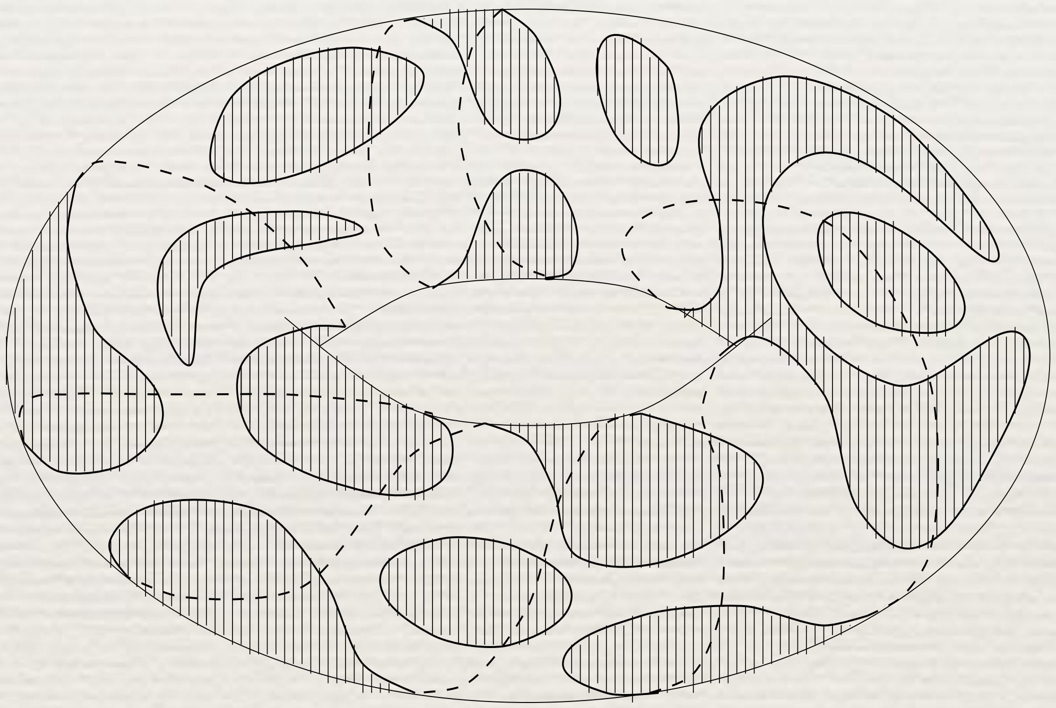
The gauge theory which provides a nonperturbative definition of closed string theory is obtained from the low-energy limit of the open string sector.

A key ingredient for proving the AdS/CFT correspondence is the equivalence of string theory with holes in the world-sheet and string theory on a different background without holes in the world-sheet.



This was discussed in the context of the large N duality of the topological string. hep-th/0205297, Ooguri and Vafa





Figures taken from hep-th/0205297 by Ooguri and Vafa

This was also discussed recently using the pure-spinor formalism.

arXiv:1903.08264, Berkovits

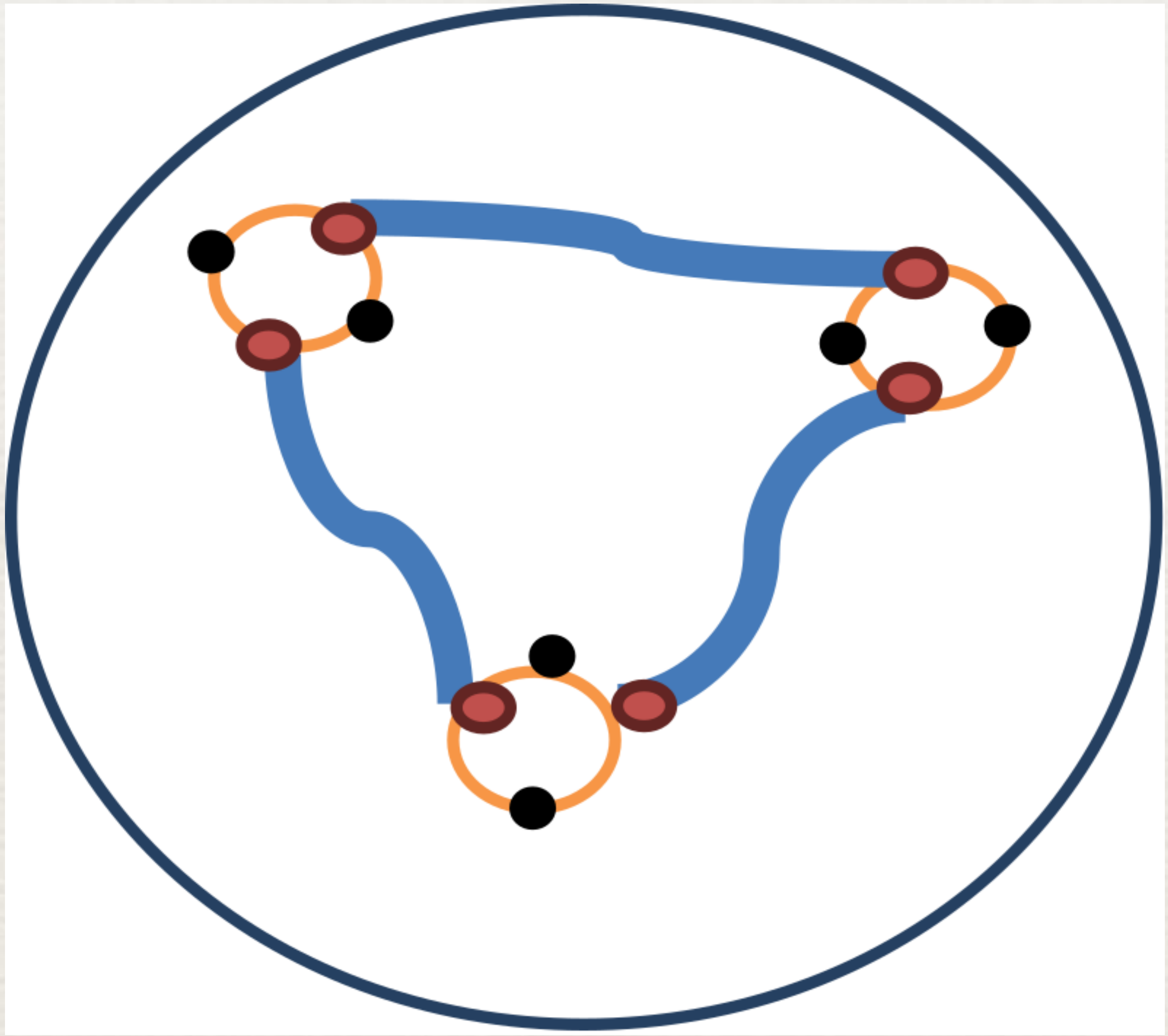


Figure taken from arXiv:1903.08264 by Berkovits

Even if we assume this equivalence, it is difficult to see gravity directly from the gauge theory, and one reason for this is that the world-sheet picture is gone after taking the low-energy limit.

This motivates us to consider *open string field theory* as a theory on D-branes before taking the low-energy limit.

In the AdS/CFT correspondence we consider correlation functions of gauge-invariant operators on the gauge theory side. We are therefore interested in correlation functions of gauge-invariant operators of open string field theory in this context.

While it is in general difficult to construct gauge-invariant operators in string field theory, a class of gauge-invariant operators have been constructed in open bosonic string field theory.

hep-th/0111092, Hashimoto and Itzhaki

hep-th/0111129, Gaiotto, Rastelli, Sen and Zwiebach

The action of open bosonic string field theory is given by

Witten, Nucl. Phys. B268 (1986) 253

$$S = -\frac{1}{2}\langle\, \Psi, Q\Psi \,\rangle - \frac{g}{3}\langle\, \Psi, \Psi * \Psi \,\rangle\,,$$

where g is the open string coupling constant, Q is the BRST operator, $\langle\, A, B \,\rangle$ and $A * B$ are the BPZ inner product and the star product, respectively, defined for a pair of states A and B.

The action is invariant under the gauge transformation given by

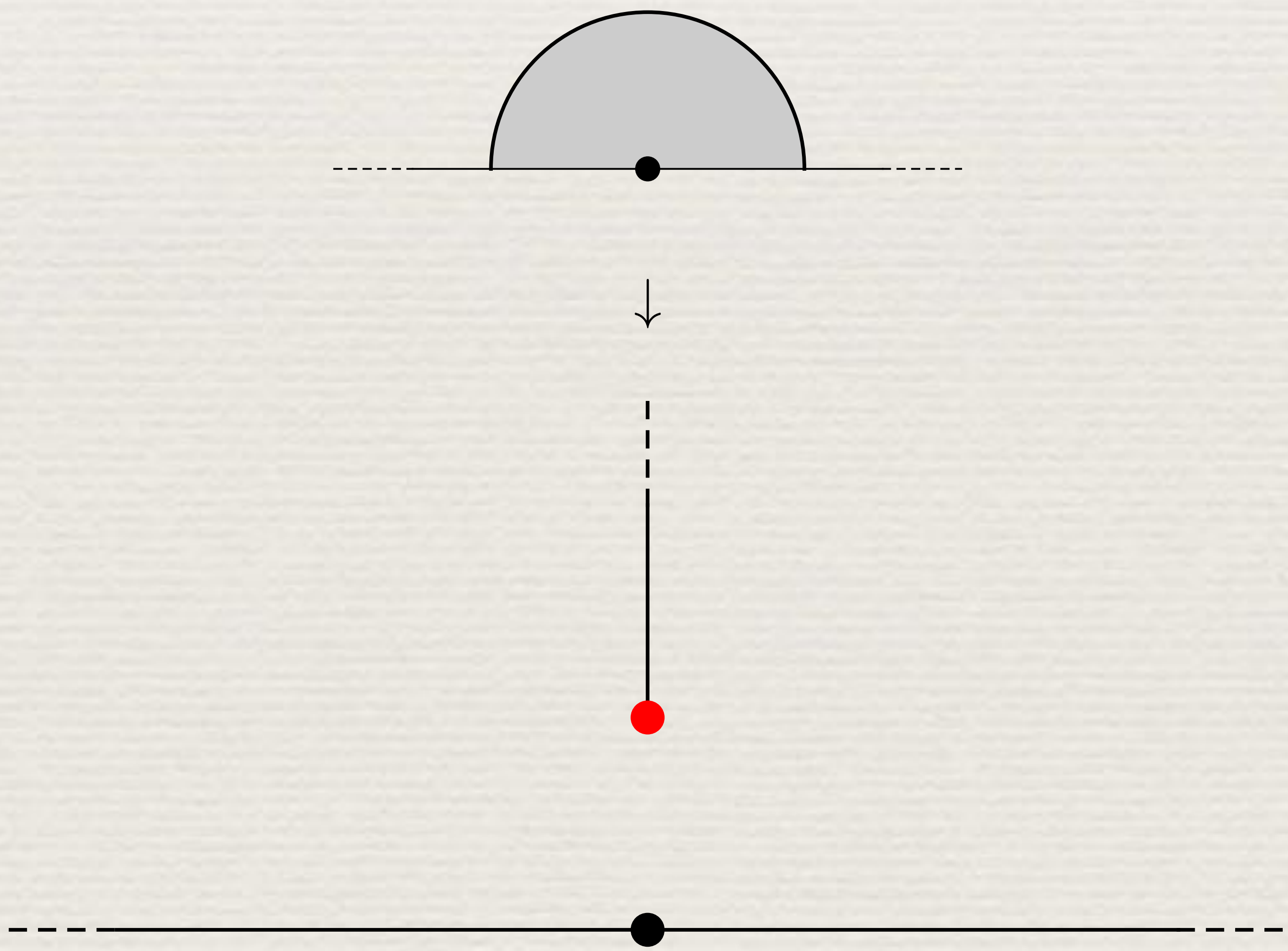
$$\delta_\Lambda \Psi = Q\Lambda + g\,(\, \Psi * \Lambda - \Lambda * \Psi \,)\,.$$

The gauge-invariant operator $\mathcal{A}_{\mathcal{V}}[\,\Psi\,]$ for an on-shell closed string vertex operator $\mathcal{V}$ is defined by

$$\mathcal{A}_{\mathcal{V}}[\,\Psi\,] = \langle\,\mathcal{V}(i)\,f_I \circ \Psi(0)\,\rangle_{\text{UHP}}$$

with

$$f_I(z) = \tan\Big(\,2\,\arctan z\,\Big) = \frac{2\,z}{1-z^2}\,.$$



These gauge-invariant operators have an interesting origin in open-closed string field theory.

A one-parameter family of formulations for open-closed bosonic string field theory were constructed, and it was observed that in a singular limit the action reduces to that of the cubic open bosonic string field theory with an additional vertex which couples one off-shell open string field and one on-shell closed string field.

hep-th/9202015, Zwiebach

$$S = -\frac{1}{2}\langle\, \Psi, Q\Psi \,\rangle - \frac{g}{3}\langle\, \Psi, \Psi * \Psi \,\rangle + \frac{1}{g}\langle\, J(\Phi), \Psi \,\rangle\,,$$

where Φ is the on-shell closed string field and $J(\Phi)$ is a map from a closed string field to an open string field.

The kinetic term of the closed string field is absent so that the resulting theory is no longer open-closed string field theory.

It is open string field theory with source terms for a set of gauge-invariant operators:

$$\langle\, J(\Phi), \Psi \,\rangle = \sum_\alpha \mathcal{G}_\alpha\, \mathcal{A}_{\mathcal{V}_\alpha}[\, \Psi \,]$$

with

$$\Phi = \sum_\alpha \mathcal{G}_\alpha\, \Phi_\alpha\,, \qquad Q\Phi = 0\,,$$

where Φ_α is the state corresponding to the on-shell vertex operator $\mathcal{V}_\alpha$.

We can show that the action with the source term is gauge-invariant using the following properties of $J(\Phi)$:

$$QJ(\Phi) = 0\,, \qquad J(\Phi) * A = A * J(\Phi)$$

for any open string field A.

An important consequence from this relation of the gauge-invariant operators and open-closed string field theory is that Feynman diagrams for correlation functions of the gauge-invariant operators are given by Riemann surfaces containing holes with bulk punctures and the moduli space of such Riemann surfaces is covered.

Let us now consider the theory on N coincident D-branes. If we evaluate correlation functions of the gauge-invariant operators in the $1/N$ expansion, by construction it reproduces the closed-string perturbation theory with holes in the world-sheet we mentioned before.

Thus to consider open string field theory as a theory before taking the low-energy limit can be a promising way for proving the AdS/CFT correspondence and for defining closed string theory nonperturbatively because we can keep track of the world-sheet picture in the limit.

See my talk last year in Florence for further details.

https://agenda.infn.it/event/18134/contributions/89052/attachments/62976/75690/SFT2019-Okawa.pdf

https://www.youtube.com/watch?v=TPkq_BS-2tw&list=PL1CFLtxeIrQqo-_lNROpUSBb07ON5xBvz&index=13

Of course, we need to extend the discussion to open superstring field theory, as quantization of open bosonic string field theory is formal because of tachyons in generic backgrounds.

While the action of open superstring field theory involving the Ramond sector had not been constructed for many years, this problem was recently overcome and we now have several formulations of open superstring field theory which are complete at the classical level.

arXiv:1508.00366, Kunitomo and Okawa
arXiv:1508.05387, Sen
arXiv:1602.02582, Erler, Okawa and Takezaki
arXiv:1602.02583, Konopka and Sachs

We consider that it is time to study open superstring field theory in this context.

On the other hand, it would be also useful to consider gauge-invariant operators of open bosonic string field theory in the noncritical string or in the topological string where tachyons are absent.

- It was shown that open bosonic string field theory on FZZT branes has been shown to reduce to the Kontsevich model.
  hep-th/0312196, Gaiotto and Rastelli

- Three-dimensional Chern-Simons gauge theory can be formulated as open string field theory in the topological string.
  hep-th/9207094, Witten

- The duality in the B-model topological string theory is also discussed recently. arXiv:1812.09257, Costello and Gaiotto

With the extension to open superstring field theory in mind, let us begin with the discussion in open bosonic string field theory.

The effective action for massless fields

Construction of the low-energy effective action of string field theory was discussed by Sen for closed superstring field theory.

arXiv:1609.00459, Sen

The string field projected onto the massless sector is used to describe the low-energy effective action, and it was shown that the gauge invariance of the low-energy effective action is inherited from that of the original theory.

The same strategy can be applied to open bosonic string field theory, and we consider the action including the source terms for the gauge-invariant operators.

In the case of open bosonic string field theory, however, we can integrate out massive fields only classically because the existence of tachyons in the open string and in the closed string renders the quantization formal.

One puzzling feature regarding gauge-invariant operators of open bosonic string field theory in our context is that they depend linearly on the open string field.

For example, the energy-momentum tensor is a typical example of the gauge-invariant operators we consider in the AdS/CFT correspondence, but the gauge-invariant operators of open bosonic string field theory do not resemble familiar energy-momentum tensors.

We will show that nonlinear dependence on the open string field is generated in the process of integrating out massive fields.

Although our discussion is in the bosonic theory and classical, the mechanism of generating nonlinear dependence can be easily understood in terms of Feynman diagrams, and we expect that the same mechanism will work in the quantum theory of the superstring.

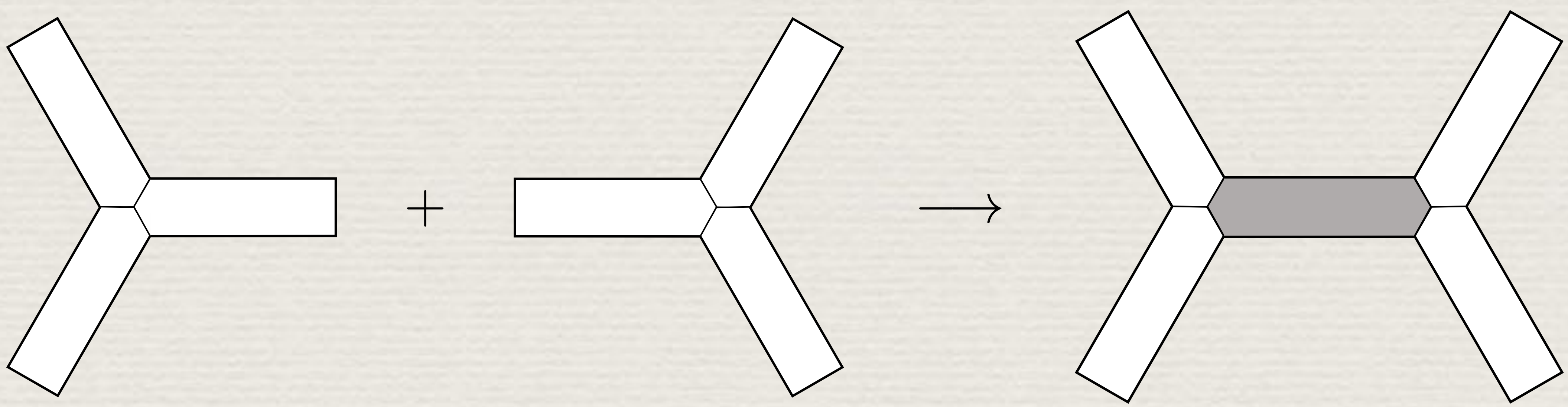
We denote the projection operator onto the massless sector by P. The string field Ψ for massless fields satisfies

$$P\,\Psi = \Psi\,.$$

The effective action for massless fields is given by

$$\begin{aligned} S = & -\frac{1}{2}\langle\, \Psi, Q\Psi\,\rangle - \frac{g}{3}\langle\, \Psi, \Psi * \Psi\,\rangle \\ & + \frac{g^2}{2}\langle\, \Psi * \Psi, \frac{b_0}{L_0}\,(1-P)\,(\Psi * \Psi)\,\rangle + O(g^3)\,, \end{aligned}$$

where b_0 is the zero mode of the b ghost and L_0 is the zero mode of the energy-momentum tensor.



We can show that the action is invariant up to $O(g^2)$ under the gauge transformation given by

$$\begin{aligned}\delta_\Lambda \Psi = {} & Q\Lambda + gP\,(\,\Psi * \Lambda - \Lambda * \Psi\,) \\ & + g^2 P\,[\,-h\,(\Psi * \Psi) * \Lambda + h\,(\Psi * \Lambda) * \Psi - h\,(\Lambda * \Psi) * \Psi \\ & \qquad - \Psi * h\,(\Psi * \Lambda) + \Psi * h\,(\Lambda * \Psi) + \Lambda * h\,(\Psi * \Psi)\,] \\ & + O(g^3)\,,\end{aligned}$$

where

$$h = \frac{b_0}{L_0}\,(1 - P)\,.$$

The action with the source term is given by

$$S = -\frac{1}{2}\langle\, \Psi, Q\Psi \,\rangle - \frac{g}{3}\langle\, \Psi, \Psi * \Psi \,\rangle + \frac{\kappa}{g}\langle\, J(\Phi), \Psi \,\rangle\,,$$

where we introduced the parameter κ to count the power of the source in $J(\Phi)$.

Let us add the coupling

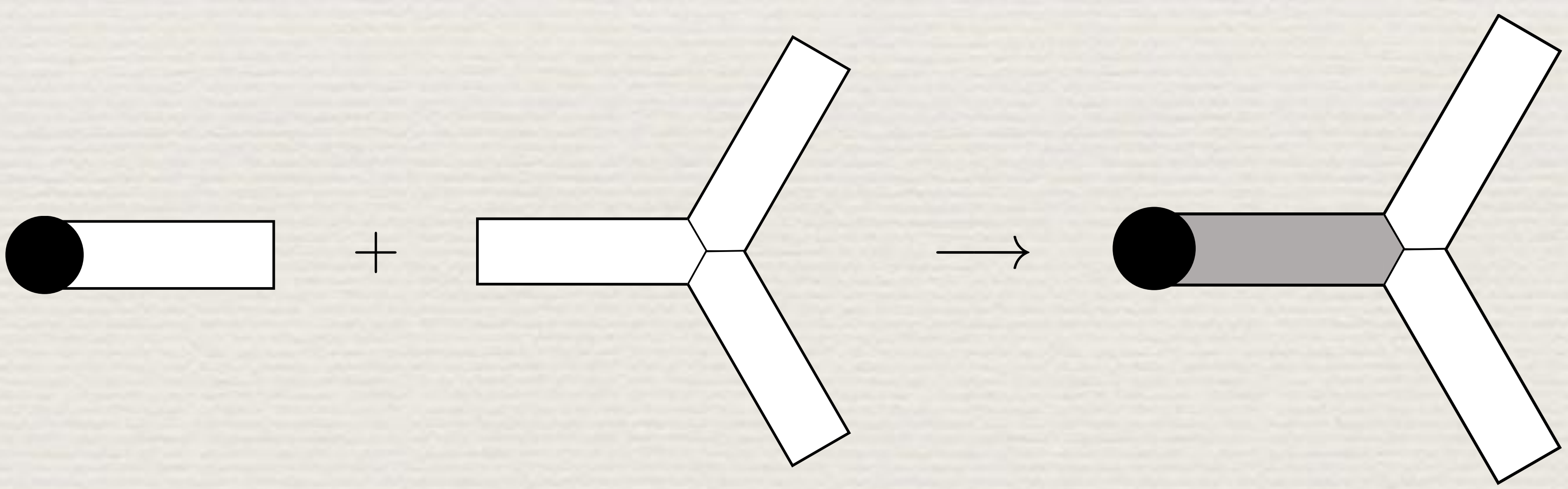
$$\frac{\kappa}{g}\langle\, J(\Phi), \Psi \,\rangle$$

to the effective action for massless fields:

$$S = -\frac{1}{2}\langle\, \Psi, Q\Psi \,\rangle - \frac{g}{3}\langle\, \Psi, \Psi * \Psi \,\rangle + \frac{g^2}{2}\langle\, \Psi * \Psi, h\,(\Psi * \Psi) \,\rangle + O(g^3)$$
$$+\,\kappa\left[\,\frac{1}{g}\langle\, J(\Phi), \Psi \,\rangle + O(g^0)\right] + O(\kappa^2)\,.$$

We find that the term $\langle\, J(\Phi), h\,(\Psi * \Psi)\,\rangle$ is required for gauge invariance:

$$
\begin{aligned}
S = & -\frac{1}{2}\,\langle\, \Psi, Q\Psi\,\rangle - \frac{g}{3}\,\langle\, \Psi, \Psi * \Psi\,\rangle + \frac{g^2}{2}\,\langle\, \Psi * \Psi, h\,(\Psi * \Psi)\,\rangle + O(g^3) \\
& + \kappa \left[\, \frac{1}{g}\,\langle\, J(\Phi), \Psi\,\rangle - \langle\, J(\Phi), h\,(\Psi * \Psi)\,\rangle + O(g) \right] + O(\kappa^2)\,.
\end{aligned}
$$



While the generation of nonlinear dependence of the gauge-invariant operators on the open string field is what we expected, an unexpected feature regarding the effective action is that the gauge transformation is modified as

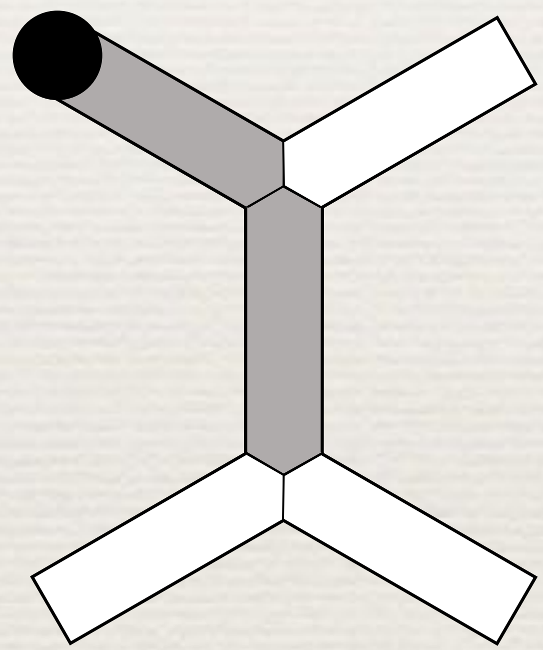
$$\begin{aligned}\delta_\Lambda \Psi = {} & Q\Lambda + gP\,(\,\Psi * \Lambda - \Lambda * \Psi\,) \\ & + g^2 P\,[\,-h\,(\Psi * \Psi) * \Lambda + h\,(\Psi * \Lambda) * \Psi - h\,(\Lambda * \Psi) * \Psi \\ & \qquad - \Psi * h\,(\Psi * \Lambda) + \Psi * h\,(\Lambda * \Psi) + \Lambda * h\,(\Psi * \Psi)\,] + O(g^3) \\ & + \kappa \Big[\, P\,[\,h\,J(\Phi) * \Lambda - \Lambda * h\,J(\Phi)\,] + O(g) \Big] + O(\kappa^2)\,,\end{aligned}$$

and gauge invariance requires terms which are nonlinear with respect to the sources.

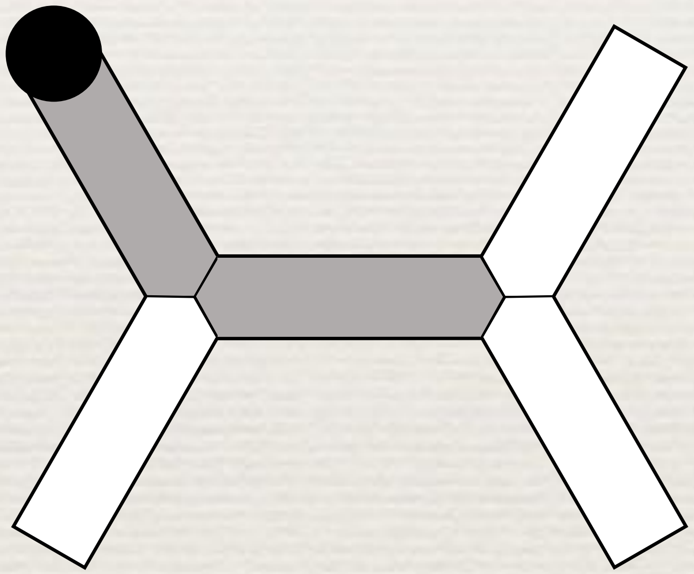
$$
\begin{aligned}
S = \quad & S_2^{(0)} + g\, S_3^{(0)} + g^2\, S_4^{(0)} + O(g^3) \\
& + \kappa \left[\frac{1}{g}\, S_1^{(1)} + S_2^{(1)} + g\, S_3^{(1)} + O(g^2) \right] \\
& + \kappa^2 \left[\frac{1}{g}\, S_1^{(2)} + S_2^{(2)} + O(g) \right] \\
& + \kappa^3 \left[\frac{1}{g}\, S_1^{(3)} + O(g^0) \right] \\
& + O(\kappa^4)\,,
\end{aligned}
$$

$$
\begin{aligned}
\delta_\Lambda \Psi = \quad & \delta_0^{(0)} \Psi + g\, \delta_1^{(0)} \Psi + g^2 \delta_2^{(0)} \Psi + O(g^3) \\
& + \kappa \left[\delta_0^{(1)} \Psi + g\, \delta_1^{(1)} \Psi + O(g^2) \right] \\
& + \kappa^2 \left[\delta_0^{(2)} \Psi + O(g) \right] \\
& + O(\kappa^3)\,.
\end{aligned}
$$

$S_3^{(1)} =$



$+$



There are some similarities between terms in the effective action and terms in the gauge transformation, and we find that the effective action and the modified gauge transformation can be written in terms of the same set of multi-string products which satisfy weak A_∞ relations.

(With hindsight, the original action including the source terms has a weak A_∞ structure in a rather trivial fashion, and the weak A_∞ structure of the effective action can be understood as being inherited from that of the original action.)

Weak A_∞ structure

Consider an action of the form:

$$S = -\frac{1}{g}\langle\,\Psi, V_0\,\rangle - \frac{1}{2}\langle\,\Psi, V_1(\Psi)\,\rangle - \frac{g}{3}\langle\,\Psi, V_2(\Psi,\Psi)\,\rangle$$
$$-\frac{g^2}{4}\langle\,\Psi, V_3(\Psi,\Psi,\Psi)\,\rangle - \frac{g^3}{5}\langle\,\Psi, V_4(\Psi,\Psi,\Psi,\Psi)\,\rangle + O(g^4)\,.$$

This action is invariant under the gauge transformation given by

$$\begin{aligned}\delta_\Lambda\Psi = {} & V_1(\Lambda) + g\,(\,V_2(\Psi,\Lambda) - V_2(\Lambda,\Psi)\,) \\ & + g^2\,(\,V_3(\Psi,\Psi,\Lambda) - V_3(\Psi,\Lambda,\Psi) + V_3(\Lambda,\Psi,\Psi)\,) \\ & + g^3\,(\,V_4(\Psi,\Psi,\Psi,\Lambda) - V_4(\Psi,\Psi,\Lambda,\Psi) \\ & \qquad + V_4(\Psi,\Lambda,\Psi,\Psi) - V_4(\Lambda,\Psi,\Psi,\Psi)\,) + O(g^4)\end{aligned}$$

if multi-string products satisfy a set of relations called weak A_∞ relations.

(In this talk all the discussions on cyclic properties are omitted.)

Weak A_∞ relations

$$V_1(V_0) = 0\,,$$
$$V_1(V_1(A_1)) - V_2(V_0, A_1) + V_2(A_1, V_0) = 0\,,$$
$$V_1(V_2(A_1, A_2)) - V_2(V_1(A_1), A_2) - (-1)^{A_1} V_2(A_1, V_1(A_2))$$
$$+ V_3(V_0, A_1, A_2) - V_3(A_1, V_0, A_2) + V_3(A_1, A_2, V_0) = 0\,,$$
$$V_1(V_3(A_1, A_2, A_3)) + V_3(V_1(A_1), A_2, A_3) + (-1)^{A_1} V_3(A_1, V_1(A_2), A_3)$$
$$+ (-1)^{A_1+A_2} V_3(A_1, A_2, V_1(A_3)) - V_2(V_2(A_1, A_2), A_3) + V_2(A_1, V_2(A_2, A_3))$$
$$- V_4(V_0, A_1, A_2, A_3) + V_4(A_1, V_0, A_2, A_3)$$
$$- V_4(A_1, A_2, V_0, A_3) + V_4(A_1, A_2, A_3, V_0) = 0\,.$$

Note

$V_0 \neq 0 \quad \rightarrow \quad$ weak A_∞ relations

$V_0 = 0 \quad \rightarrow \quad A_\infty$ relations

For example, the term $S_2^{(1)}$ in the action and $\delta_0^{(1)}\Psi$ in the gauge transformation given by

$$S_2^{(1)} = -\langle\, J(\Phi), h\,(\Psi * \Psi)\,\rangle\,, \qquad \delta_0^{(1)}\Psi = P\,[\, h\, J(\Phi) * \Lambda - \Lambda * h\, J(\Phi)\,]$$

can be written as

$$S_2^{(1)} = -\frac{1}{2}\,\langle\, \Psi, V_1^{(1)}(\Psi)\,\rangle\,, \qquad \delta_0^{(1)}\Psi = V_1^{(1)}(\Lambda)$$

in terms of the one-string product $V_1^{(1)}(A_1)$ given by

$$V_1^{(1)}(A_1) = P\,[\, h\, J(\Phi) * A_1 - (-1)^{A_1} A_1 * h\, J(\Phi)\,]\,.$$

An advantage of using the star product in open string field theory is that expressions for terms in the effective action are simpler and more explicit compared to closed string field theory.

However, expressions for terms in the effective action become rather lengthy at higher orders even in open bosonic string field theory based on the star product.

The weak A_∞ structure provides us with analytic control over terms in the effective action, and we present explicit expressions for the multi-string products to all orders.

Coalgebra representation

To simplify the description of the weak A_∞ structure, let us introduce *degree* defined by

$$\deg(A) = \epsilon(A) + 1 \mod 2\,,$$

where $\epsilon(A)$ is the Grassmann parity of A.

Associated with the star product, we define the two-string product $m_2(A_1, A_2)$ by

$$m_2(A_1, A_2) = (-1)^{\deg(A_1)} A_1 * A_2\,.$$

We also define the zero-string product w_0 by

$$w_0 = -\,J(\Phi)\,.$$

To describe the weak A_∞ structure to all orders, it is convenient to consider linear operators acting on the vector space $T\mathcal{H}$ defined by

$$T\mathcal{H} = \mathcal{H}^{\otimes 0} \oplus \mathcal{H} \oplus \mathcal{H}^{\otimes 2} \oplus \mathcal{H}^{\otimes 3} \oplus \ldots \,,$$

where we denoted the tensor product of n copies of the Hilbert space $\mathcal{H}$ by $\mathcal{H}^{\otimes n}$.

The weak A_∞ relations can be compactly expressed in terms of a linear operator $\mathbf{M}$ on $T\mathcal{H}$ which squares to zero:

$$\mathbf{M}^2 = 0\,.$$

Associated with the BRST operator Q, we define $\mathbf{Q}$ as follows:

$$
\begin{aligned}
\mathbf{Q}\,\mathbf{1} &= 0\,, \\
\mathbf{Q}\,A_1 &= Q\,A_1\,, \\
\mathbf{Q}\,(A_1 \otimes A_2) &= Q\,A_1 \otimes A_2 + (-1)^{\deg(A_1)} A_1 \otimes Q\,A_2\,, \\
\mathbf{Q}\,(A_1 \otimes A_2 \otimes A_3) &= Q\,A_1 \otimes A_2 \otimes A_3 + (-1)^{\deg(A_1)} A_1 \otimes Q\,A_2 \otimes A_3 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)} A_1 \otimes A_2 \otimes Q\,A_3\,, \\
&\ \ \vdots
\end{aligned}
$$

Associated with the two-string product m_2, we define $\boldsymbol{m}_2$ as follows:

$$
\begin{aligned}
\boldsymbol{m}_2\,\mathbf{1} &= 0\,, \\
\boldsymbol{m}_2\,A_1 &= 0\,, \\
\boldsymbol{m}_2\,(A_1 \otimes A_2) &= m_2(A_1, A_2)\,, \\
\boldsymbol{m}_2\,(A_1 \otimes A_2 \otimes A_3) &= m_2(A_1, A_2) \otimes A_3 \\
&\quad + (-1)^{\deg(A_1)} A_1 \otimes m_2(A_2, A_3)\,, \\
\boldsymbol{m}_2\,(A_1 \otimes A_2 \otimes A_3 \otimes A_4) &= m_2(A_1, A_2) \otimes A_3 \otimes A_4 \\
&\quad + (-1)^{\deg(A_1)} A_1 \otimes m_2(A_2, A_3) \otimes A_4 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)} A_1 \otimes A_2 \otimes m_2(A_3, A_4)\,, \\
&\vdots
\end{aligned}
$$

Associated with the zero-string product w_0, we define $\boldsymbol{w}_0$ as follows:

$$
\begin{aligned}
\boldsymbol{w}_0\,\mathbf{1} &= w_0\,, \\
\boldsymbol{w}_0\,A_1 &= w_0 \otimes A_1 + (-1)^{\deg(A_1)} A_1 \otimes w_0\,, \\
\boldsymbol{w}_0\,(A_1 \otimes A_2) &= w_0 \otimes A_1 \otimes A_2 + (-1)^{\deg(A_1)} A_1 \otimes w_0 \otimes A_2 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)} A_1 \otimes A_2 \otimes w_0\,, \\
\boldsymbol{w}_0\,(A_1 \otimes A_2 \otimes A_3) &= w_0 \otimes A_1 \otimes A_2 \otimes A_3 + (-1)^{\deg(A_1)} A_1 \otimes w_0 \otimes A_2 \otimes A_3 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)} A_1 \otimes A_2 \otimes w_0 \otimes A_3 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)+\deg(A_3)} A_1 \otimes A_2 \otimes A_3 \otimes w_0\,, \\
&\;\;\vdots
\end{aligned}
$$

For the action of open bosonic string field theory without introducing the source term for the gauge-invariant operators, the A_∞ structure can be described in terms of $\mathbf{M}$ given by

$$\mathbf{M} = \mathbf{Q} + \boldsymbol{m}_2 \,.$$

For the action of open bosonic string field theory including the source term for the gauge-invariant operators, the weak A_∞ structure can be described in terms of $\mathbf{M}$ given by

$$\mathbf{M} = \mathbf{Q} + \boldsymbol{m}_2 + \boldsymbol{w}_0 \,.$$

We denote the projection operator on $T\mathcal{H}$ by $\mathbf{P}$. The action of $\mathbf{P}$ is given by

$$\begin{aligned} \mathbf{P}\,\mathbf{1} &= \mathbf{1}\,, \\ \mathbf{P}\,A_1 &= PA_1\,, \\ \mathbf{P}\,(A_1 \otimes A_2) &= PA_1 \otimes PA_2\,, \\ \mathbf{P}\,(A_1 \otimes A_2 \otimes A_3) &= PA_1 \otimes PA_2 \otimes PA_3\,, \\ &\vdots \end{aligned}$$

Associated with the propagator h, we define $\boldsymbol{h}$ as follows:

$$
\begin{aligned}
\boldsymbol{h}\,\mathbf{1} &= 0\,, \\
\boldsymbol{h}\,A_1 &= h\,A_1\,, \\
\boldsymbol{h}\,(A_1 \otimes A_2) &= h\,A_1 \otimes P A_2 + (-1)^{\deg(A_1)} A_1 \otimes h\,A_2\,, \\
\boldsymbol{h}\,(A_1 \otimes A_2 \otimes A_3) &= h\,A_1 \otimes P A_2 \otimes P A_3 + (-1)^{\deg(A_1)} A_1 \otimes h\,A_2 \otimes P A_3 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)} A_1 \otimes A_2 \otimes h\,A_3\,, \\
\boldsymbol{h}\,(A_1 \otimes A_2 \otimes A_3 \otimes A_4) &= h\,A_1 \otimes P A_2 \otimes P A_3 \otimes P A_4 \\
&\quad + (-1)^{\deg(A_1)} A_1 \otimes h\,A_2 \otimes P A_3 \otimes P A_4 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)} A_1 \otimes A_2 \otimes h\,A_3 \otimes P A_4 \\
&\quad + (-1)^{\deg(A_1)+\deg(A_2)+\deg(A_3)} A_1 \otimes A_2 \otimes A_3 \otimes h\,A_4\,, \\
&\ \vdots
\end{aligned}
$$

For the effective action for massless fields without introducing the source term for the gauge-invariant operators, the A_∞ structure can be described in terms of $\mathbf{M}$ given by

$$\mathbf{M} = \mathbf{P}\,\mathbf{Q}\,\mathbf{P} + \mathbf{P}\,\boldsymbol{m}_2\,\frac{1}{\mathbf{I} + \boldsymbol{h}\,\boldsymbol{m}_2}\,\mathbf{P}\,.$$

The construction was used by Matsunaga in a different context.

arXiv:1901.08555, Matsunaga

For the effective action for massless fields including the source term for the gauge-invariant operators, the weak A_∞ structure can be described in terms of $\mathbf{M}$ given by

$$\mathbf{M} = \mathbf{P}\,\mathbf{Q}\,\mathbf{P} + \mathbf{P}\,(\,\boldsymbol{m}_2 + \boldsymbol{w}_0\,)\,\frac{1}{\mathbf{I} + \boldsymbol{h}\,(\,\boldsymbol{m}_2 + \boldsymbol{w}_0\,)}\,\mathbf{P}\,.$$

Discussion

Our discussion is motivated by the low-energy limit in the context of the AdS/CFT correspondence, and we are interested in the low-energy region compared to the scale determined by α' of the effective action for massless fields.

After taking this low-energy limit, the theory will be invariant under the ordinary gauge transformation.

Invariance under the ordinary gauge transformation requires familiar constraints, and, for example, the α' expansion of the effective action for the gauge field of the open string takes the form of a linear combination of gauge-invariant terms.

However, invariance under the ordinary gauge transformation does not constrain the coefficients in front of such gauge-invariant terms.

On the other hand, the effective action with an A_∞ structure does not take the form of a linear combination of gauge-invariant terms, and constraints from invariance under the gauge transformation associated with the A_∞ structure have a more dynamical flavor.

Furthermore, the insight we learned from our analysis indicates that correlation functions of gauge-invariant operators are similarly constrained from a weak A_∞ structure before strictly taking the low-energy limit.

Since the weak A_∞ structure is closely related to the world-sheet picture, we hope that the dynamics of gauge-invariant operators dictated by the weak A_∞ structure will help us reveal quantum gravity from the open string sector.