A simple solution for static backgrounds in cubic superstring field theory

Ruggero Noris

Politecnico di Torino, INFN

Workshop on Fundamental Aspects of String Theory
June 09, 2020

Based on: T. Erler, C. Maccaferri, R.N. [1901.08038,JHEP06(2019)027]
+In progress

Content

Intertwining solution

- In 2014 and 2019, T.Erler and C.Maccaferri studied a set of solutions of the equations of motions of cubic OSFT, showing that strings attached to a given D-brane can rearrange themselves to create other D-branes sharing the same closed string background.
- Such intertwining solution relies on a tachyon vacuum solution and on the so-called *intertwining fields* $(\Sigma, \bar{\Sigma})$ satisfying

$$Q_{\rm tv}\Sigma = Q_{\rm tv}\bar{\Sigma} = 0, \qquad \bar{\Sigma}\Sigma = 1.$$

- The explicit expression is given by

$$\Psi_* = \Psi_{\rm tv} - \Sigma\Psi_{\rm tv}\bar{\Sigma}.$$

- The intertwining fields can be built in terms of flag states, which are in general complicated. However, in the particular case of static backgrounds, $(\Sigma, \overline{\Sigma})$ can be built as

$$\Sigma = Q_{\rm tv}(\sqrt{H}\sigma B\sqrt{H}), \qquad \overline{\Sigma} = Q_{\rm tv}(\sqrt{H}B\overline{\sigma}\sqrt{H}),$$

  with $(\sigma, \overline{\sigma})$ being insertions of weight zero (super)conformal primaries.

- Indeed, when the time component of the BCFT remains unaltered, such operator insertions can be defined in terms of matter (super)conformal primaries, $(\sigma^{(h)}, \overline{\sigma}^{(h)})$, as

$$\begin{aligned} \sigma(x) &= e^{i\sqrt{h}X^0}\sigma^{(h)}(x), \\ \overline{\sigma}(x) &= e^{-i\sqrt{h}X^0}\overline{\sigma}^{(h)}(x), \end{aligned} \qquad \text{such that} \qquad \begin{cases} \lim_{x\to 0} & \overline{\sigma}(x)\sigma(0) = 1, \\ \lim_{x\to 0} & \sigma(x)\overline{\sigma}(0) = \frac{g_*}{g_0} \end{cases}.$$

  where $g_{*,0} = \langle 1 \rangle_{BCFT_{*,0}}$ are the disk partition functions in the respective $BCFT$.

- In terms of flag states, they correspond to the limit $\ell \to 0$ of the height of the horizontal strip. This is possible only when operator insertions have $h = 0$.

Examples

By going through the tachyon vacuum, thus annihilating the starting D-brane system, one can build another one.

- For example, one can describe the translation of a D-brane in a certain spacetime direction (for example X^1) over a distance d. This can be achieved by using boundary condition changing operators fixing the endpoints of open strings to different values on that direction. One then has

$$\sigma(x) = e^{id(X^0+\tilde{X}^1)}(x),$$
$$\overline{\sigma}(x) = e^{-id(X^0+\tilde{X}^1)}(x),$$

  where $\tilde{X}^1 = X^1(z) - \bar{X}^1(\bar{z})|_{z=\bar{z}=x}$.

- Another example is given by the creation of D-branes of codimension $(2n)$: Dp-$D(p\pm 2n)$. In these cases, the boundary condition changing operators are given in terms of twist fields and (bosonised) spin fields

$$\sigma(x) = e^{i\sqrt{\frac{n}{4}}X^0}\Delta\; e^{\frac{i}{2}\sum_{i=1}^{n} H_i}(x),$$
$$\overline{\sigma}(x) = e^{-i\sqrt{\frac{n}{4}}X^0}\overline{\Delta}\; e^{-\frac{i}{2}\sum_{i=1}^{n} H_i}(x).$$

The explicit form of the solution

Given a tachyon vacuum $\Psi_{\text{tv}} = \sqrt{F(K)}\left(c\frac{B}{H}c + B\gamma^2\right)\sqrt{F(K)}$, the intertwining solution can be expressed as

$$\begin{aligned}\Psi_* = &\sqrt{F}\left(c\frac{B}{H}c + B\gamma^2\right)\sqrt{F} - \sqrt{H}\sigma\sqrt{\frac{F}{H}}\left(c\frac{B}{H}c + B\gamma^2\right)\sqrt{\frac{F}{H}}\bar{\sigma}\sqrt{H} + \sqrt{H}Q\sigma BFQ\bar{\sigma}\sqrt{H}\\ &-\left(\sqrt{H}Q\sigma B\sqrt{\frac{F}{H}}c\sqrt{\frac{F}{H}}\bar{\sigma}\sqrt{H} + \text{conj.}\right) - \left(\sqrt{H}\left[\sqrt{\frac{F}{H}}c\sqrt{\frac{F}{H}},\sigma\right]B\sqrt{\frac{F}{H}}c\sqrt{\frac{F}{H}}\bar{\sigma}\sqrt{H} + \text{conj.}\right)\\ &-\left(\sqrt{H}Q\sigma BF\left[\bar{\sigma},\sqrt{\frac{F}{H}}c\sqrt{\frac{F}{H}}\right]\sqrt{H} + \text{conj.}\right) - \sqrt{H}\left[\sqrt{\frac{F}{H}}c\sqrt{\frac{F}{H}},\sigma\right]BF\left[\bar{\sigma},\sqrt{\frac{F}{H}}c\sqrt{\frac{F}{H}}\right]\sqrt{H},\end{aligned}$$

where "conj" means reality conjugation.
Here $H(K) = \frac{1-F(K)}{K}$ is the homotopy string field, which trivializes the cohomology, $Q_{\text{tv}}(BH) = 1$ and $F(K)$ satisfies

$$F(0) = 1, \quad F'(0) < 0, \quad F(\infty) = 0, \quad F(K) < 1.$$

Analysis of the solution

- By using

$$Q\sigma = c\partial\sigma + \gamma\delta\sigma = c[1+K,\sigma] + \gamma\delta\sigma$$

  one gets the explicit form of the solution.

- This solution, by construction, satisfies the equations of motion. In order to prove that it is well-defined as a string field, one has to check that it does not contain OPE divergences and that the Ψ^2 term in the equation of motion does not contain the triple product $\sigma\overline{\sigma}\sigma$. Indeed this string field leads to an anomaly of the star product

$$\frac{g_*}{g_0}\sigma = (\sigma\overline{\sigma})\sigma \neq \sigma(\overline{\sigma}\sigma) = \sigma,$$

  as in general $g_* \neq g_0$!

Assumptions

In order to fully address these potential problems, we make the following assumptions

$$\sigma(s)\overline{\sigma}(0) = \text{regular}, \qquad \sigma(s)\delta\overline{\sigma}(0) = \text{regular},$$

where $\delta\overline{\sigma}$ is the susy variation of the boundary condition changing operator. The first condition is satisfied by construction and the second one has been explicitly checked in all the examples previously mentioned. As a consequence of these assumptions, one has that

$$\begin{aligned}
\sigma(s)\partial\overline{\sigma}(0) &\sim \text{ less singular than simple pole,} \\
\partial\sigma(s)\partial\overline{\sigma}(0) &\sim \text{ less singular than double pole,} \\
\delta\sigma(s)\delta\overline{\sigma}(0) &\sim \text{ less singular than simple pole,} \\
\partial\sigma(s)\delta\overline{\sigma}(0) &\sim \text{ less singular than simple pole.}
\end{aligned}$$

Results

The previous assumptions lead to conditions on the fuction $F(K)$ appearing in the tachyon vacuum: to see this, take for example a one-parameter family of functions

$$F(K) = \left(1 - \frac{1}{\nu}K\right)^{\nu}.$$

Here $\nu < 0$ represents the leading level in the dual $\mathcal{L}^-$ expansion ($K \to \infty$): in particular $\nu = -1$ corresponds to the simple solution, while $\nu \to -\infty$ corresponds to Schnabl's solution.
In particular, the operator $\frac{1}{2}\mathcal{L}^- = \frac{1}{2}(\mathcal{L}_0 - \mathcal{L}_0^*)$ allows to control the behaviour towards the identity string field, which is responsible for worldsheet collisions.

- One can rigorously prove, using the $\mathcal{L}^-$ expansion, that a solution is free from OPE divergences and associativity anomalies provided that $\nu < -1$. This upper bound excludes the simple solution (for the superstring).

Issues at $\nu = -1$: the simple solution

What happens exactly at $\nu = -1$, in the simple solution case? The following two terms show up

$$-\frac{1}{\sqrt{1+K}}\sigma B\gamma^2\overline{\sigma}\frac{1}{\sqrt{1+K}} + \frac{1}{\sqrt{1+K}}\gamma\delta\sigma\frac{B}{1+K}\gamma\delta\overline{\sigma}\frac{1}{\sqrt{1+K}}.$$

Both terms, while being finite from the OPE point of view, lead to the ill-defined triple product $\overline{\sigma}\sigma\overline{\sigma}$ when computing Ψ^2 in the equations of motion. To see this, consider its product with the bosonic term

$$\frac{1}{\sqrt{1+K}}c(1+K)\sigma\frac{B}{1+K}\overline{\sigma}(1+K)c\frac{1}{\sqrt{1+K}}.$$

Supersymmetries transformations and new string fields

One way to solve this problem is to embrace worldsheet supersymmetry and to explicitly consider

$$\delta\cdot = [G, \cdot],$$

as it is done for worldsheet derivatives $\partial\cdot = [K, \cdot]$.

- Here G is given by the insertion in an infinitesimal width strip of the infinite vertical line $\int_{-i\infty}^{+i\infty} \frac{dz}{2\pi i} T_F(z)$ [T. Erler 2011].

Notice that the product γG does not change the GSO sector of the boundary condition changing operators

$$\sigma \in \mathrm{GSO}(\pm) \implies \gamma[G, \sigma] \in \mathrm{GSO}(\pm)$$

Algebra relations

- The newly introduced field satisfies the following relations

$$G^2 = K, \quad [G, B] = [G, K] = 0, \quad QG = 0.$$

The first one represents the known fact that supersymmetry transformations are the "square root" of translations.

- Here the bracket and the action of the BRST operator are defined as an extension of the ones appearing in GSO$(+)$ case:

$$[\Psi, \Phi] = \Psi\Phi - (-1)^{Grass(\Psi)Grass(\Phi)}\Phi\Psi,$$
$$Q(\Psi\Phi) = Q\Psi\Phi + (-1)^{Grass(\Psi)}\Psi Q\Phi.$$

Alternatively, one can tensor the whole algebra with internal Chan-Paton factors and use effective grassmanality $E = Grass + F$, but this is not strictly needed here.

Anticipation on the simple solution

Before checking the full solution in the generic $F(K)$ case, let us focus on the two terms we showed previously in the simple case $F(K) = \frac{1}{1+K}$

$$
\begin{aligned}
& -\frac{1}{\sqrt{1+K}}\left(\sigma B\gamma^2\bar{\sigma} - \gamma\delta\sigma\frac{B}{1+K}\delta\bar{\sigma}\gamma\right)\frac{1}{\sqrt{1+K}} = \\
& = -\frac{1}{\sqrt{1+K}}\left(\sigma B\gamma^2\bar{\sigma} - \gamma[G,\sigma]\frac{B}{1+K}[G,\bar{\sigma}]\gamma\right)\frac{1}{\sqrt{1+K}} = \\
& = -\frac{1}{\sqrt{1+K}}\left(\sigma B\gamma^2\bar{\sigma} - \gamma\sigma\frac{BG^2}{1+K}\bar{\sigma}\gamma + \dots\right)\frac{1}{\sqrt{1+K}} = \\
& = -\frac{1}{\sqrt{1+K}}\left(\sigma B\gamma^2\bar{\sigma} - \gamma\sigma\frac{BK}{1+K}\bar{\sigma}\gamma + \dots\right)\frac{1}{\sqrt{1+K}} = \\
& = -\frac{1}{\sqrt{1+K}}\left(\cancel{\sigma B\gamma^2\bar{\sigma}} - \cancel{\sigma B\gamma^2\bar{\sigma}} + \gamma\sigma\frac{B}{1+K}\bar{\sigma}\gamma + \dots\right)\frac{1}{\sqrt{1+K}}.
\end{aligned}
$$

We see that the introduction of G allows to cancel the ambiguous terms.

The generic $F(K)$ solution: new bound on the dual $\mathcal{L}^-$ level

The introduction of the string field G allows to write the complete solution as

$$\Psi_* = \sqrt{F}\left(c\frac{B}{H}c + B\gamma^2\right)\sqrt{F} - \sqrt{H}c\frac{1}{H}\sigma BF\overline{\sigma}\frac{1}{H}c\sqrt{H} - \sqrt{H}\sigma\left(\sqrt{\frac{F}{H}}\gamma B\gamma\sqrt{\frac{F}{H}} - \gamma\sqrt{\frac{F}{H}}B\sqrt{\frac{F}{H}}\gamma + \gamma\sqrt{\frac{F}{H}}BF\sqrt{\frac{F}{H}}\gamma\right)\overline{\sigma}\sqrt{H}+$$

$$+\sqrt{H}\gamma\left(G\sigma BFG\overline{\sigma} + \sigma GBF\overline{\sigma}G - G\sigma BF\overline{\sigma}G\right)\gamma\sqrt{H} - \sqrt{H}\sigma\left[\sqrt{\frac{F}{H}},c\right]\frac{B}{H}\left[c,\sqrt{\frac{F}{H}}\right]\overline{\sigma}\sqrt{H}+$$

$$-\left(\sqrt{H}\gamma G\sigma BF\overline{\sigma}\frac{1}{H}c\sqrt{H} - \sqrt{H}\gamma\sigma GBF\overline{\sigma}\frac{1}{H}c\sqrt{H} + \text{conj.}\right) + \left(\sqrt{H}c\frac{B}{H}\sigma\sqrt{\frac{F}{H}}\left[\sqrt{\frac{F}{H}},c\right]\overline{\sigma}\sqrt{H} + \text{conj.}\right)$$

$$+\left(\sqrt{H}c\frac{B}{H}\sigma F\left[\overline{\sigma},\sqrt{\frac{F}{H}}\left[\sqrt{\frac{F}{H}},c\right]\right]\sqrt{H}+\text{conj.}\right) - \left(\sqrt{H}\gamma G\sigma B\sqrt{\frac{F}{H}}\left[c,\sqrt{\frac{F}{H}}\right]\overline{\sigma}\sqrt{H} - \sqrt{H}\gamma\sigma GB\sqrt{\frac{F}{H}}\left[c,\sqrt{\frac{F}{H}}\right]\overline{\sigma}\sqrt{H} + \text{conj.}\right)$$

$$+\left(\sqrt{H}\gamma G\sigma BF\left[\overline{\sigma},\sqrt{\frac{F}{H}}\left[\sqrt{\frac{F}{H}},c\right]\right]\sqrt{H} - \sqrt{H}\gamma\sigma GBF\left[\overline{\sigma},\sqrt{\frac{F}{H}}\left[\sqrt{\frac{F}{H}},c\right]\right]\sqrt{H}+\text{conj.}\right)$$

$$+\left(\sqrt{H}\sigma\left[\sqrt{\frac{F}{H}},c\right]B\sqrt{\frac{F}{H}}\left[\overline{\sigma},\sqrt{\frac{F}{H}}\left[\sqrt{\frac{F}{H}},c\right]\right]\sqrt{H} + \text{conj.}\right) - \sqrt{H}\left[\left[c,\sqrt{\frac{F}{H}}\right]\sqrt{\frac{F}{H}},\sigma\right]BF\left[\overline{\sigma},\sqrt{\frac{F}{H}}\left[\sqrt{\frac{F}{H}},c\right]\right]\sqrt{H}$$

The terms in red are exactly the ones that cancel out in the simple solution. Furthermore, the analysis of the solution shows that it is well-defined provided that $\nu < -1/2$, which is a milder bound.

The solution at level $\nu = -1$

From previous slides we see that the bound on ν has been lifted from $\nu < -1$ to $\nu < -1/2$. The simple solution can now be reached and it is natural to ask ourselves how the full solution looks like.

$$\Psi_* = \frac{1}{\sqrt{1+K}}\left(cB(1+K)c + B\gamma^2\right)\frac{1}{\sqrt{1+K}} - \frac{1}{\sqrt{1+K}}\sigma\gamma\frac{B}{1+K}\gamma\bar{\sigma}\frac{1}{\sqrt{1+K}} + \\ - \frac{1}{\sqrt{1+K}}\left\{\chi\sigma\frac{B}{1+K}\bar{\sigma}\bar{\chi} - \gamma\sigma G\frac{B}{1+K}\bar{\sigma}\bar{\chi} - \chi\sigma\frac{B}{1+K}G\bar{\sigma}\gamma\right\}\frac{1}{\sqrt{1+K}},$$

where

$$\chi = c(1+K) + \gamma G, \qquad \bar{\chi} = (1+K)c + G\gamma.$$

Consistency checks

At last, one can double check that the obtained solution indeed satisfies the equations of motion. The proof relies on the following identities

$$Q\sigma = [\chi, \sigma] = [\bar{\chi}, \sigma], \quad Q\overline{\sigma} = [\chi, \overline{\sigma}] = [\bar{\chi}, \overline{\sigma}],$$
$$[\chi, B] = [\bar{\chi}, B] = 1 + K, \qquad \bar{\chi} - \chi = \frac{1}{2}\partial c,$$

which allow to show that $\overline{\Sigma}\Sigma = 1$ and on the supersymmetric variations

$$\delta c = -2\gamma,$$
$$\delta\gamma = -\frac{1}{2}\partial c.$$

Conclusion

- Just as realising that $\partial\sigma = [1+K, \sigma]$ allows to improve the behaviour of the bosonic solution from $\nu_{\text{bos}} < -2$ to $\nu_{\text{bos}} < 0$, here, in the superstring case, realising that $\delta\sigma = [G, \sigma]$ allows to lift the bound from $\nu_{\text{super}} < -1$ to $\nu_{\text{super}} < -1/2$.
  We can now study a simple solution, free from OPE divergences and anomalies.
- It would be now interesting to investigate more cases of D-brane transitions and to further understand the flag solution for the superstring. Another fascinating idea would be to try and upgrade this solution to the WZW-Berkovits OSFT, since solutions of cubic OSFT are the first step to obtain solutions of the non-polynomial equations of motion.

Thank you for the attention!

Backup slides: Chan-Paton case

The standard way to introduce string fields in the GSO$-$ sector is to tensor the whole algebra with Pauli matrices in the following way

$$K \to K \otimes \mathbb{I}, \quad B \to B \otimes \sigma_3 \quad c \to c \otimes \sigma_3$$
$$G \to G \otimes \sigma_1, \quad \gamma \to \gamma \otimes \sigma_2, \quad \sigma, \overline{\sigma} \to \sigma, \overline{\sigma} \otimes \mathbb{I}.$$

The derivation and algebraic relations are then given by [T. Erler 2011]

$$[\![\Psi, \Phi]\!] = \Psi\Phi - (-1)^{E(\Psi)E(\Phi)+F(\Psi)F(\Phi)}\Phi\Psi,$$
$$Q(\Psi\Phi) = Q\Psi\Phi + (-1)^{E(\Psi)}\Psi Q\Phi,$$

where $E = Grass + F$.

Backup slides: Chan-Paton case

The whole analysis is analogous to the previous one and the map between the two choices is given by

$$\begin{array}{ccc} \text{No Chan Paton} & \to & \text{Chan Paton} \\ \gamma ... G & \to & i\gamma ... G \\ G ... \gamma & \to & -iG ... \gamma \\ \gamma B \gamma & \to & -\gamma B \gamma \end{array}$$

where ... may indicate the presence of a $\sigma, \overline{\sigma}$ or nothing. These rules follow from standard Pauli matrices algebra.
The simple solution takes then the form

$$\Psi_* = \frac{1}{\sqrt{1+K}}\left(cB(1+K)c + B\gamma^2\right)\frac{1}{\sqrt{1+K}} + \frac{1}{\sqrt{1+K}}\sigma\gamma\frac{B}{1+K}\gamma\overline{\sigma}\frac{1}{\sqrt{1+K}} + \\ - \frac{1}{\sqrt{1+K}}\left\{\chi\sigma\frac{B}{1+K}\overline{\sigma}\bar{\chi} - i\gamma\sigma G\frac{B}{1+K}\overline{\sigma}\bar{\chi} + i\chi\sigma\frac{B}{1+K}G\overline{\sigma}\gamma\right\}\frac{1}{\sqrt{1+K}},$$

where

$$\chi = c(1+K) + i\gamma G, \qquad \bar{\chi} = (1+K)c - iG\gamma.$$