Bosonic Tachyons from the Supersymmetric Point of View

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2. Embedding of Bosonic theories
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Tachyons in String Theory

- Open tachyon condensation well understood.
- Local closed tachyons as well (decay of compact dimensions, orbifold defects)
- What about the bulk tachyon?
Results of the 90s and early 2000s

\[ V(t) = -t^2 + \frac{6561}{4096} t^3 - 3.0172 t^4 + 9.924 t^5 \]  

- Adding higher orders and higher level fields leads to oscillating behavior.
- Cubic order \( \rightarrow \) minimum, Quartic order \( \rightarrow \) run-away
- Quintic order \( \rightarrow \) minimum ... 
- Seams to converge to a minimum at \( t \approx 0.05 \)  
  (Moeller, Yang 2006)
Potential of the Bosonic Closed String Tachyon (cubic order)
Potential of the Bosonic Closed String Tachyon (quartic order)
Introduction
Embedding of Bosonic theories
Construction of SFT Around the Embedding
$L_\infty$ and Quartic order

Idea

- The bosonic string can be embedded in the superstring (Berkovits, Vafa 1993)
- This adds additional d.o.f which exactly cancel at the bosonic point.
- What happens if one deforms the CFT away from the bosonic point?
The Theory Space of SFT

- Introduction
- Embedding of Bosonic theories
- Construction of SFT Around the Embedding
  \( L_\infty \) and Quartic order

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Bosonic Tachyons from the Supersymmetric Point of View
Steps in the Calculation

- Embed the bosonic string in the superstring.
- Use superstring field theory.
- Apply numerical methods (Rastelli, Zwiebach, Moeller).
- Calculate higher dimensional potential (in field space).
- Solve the resulting EOMs.
Berkovits’ and Vafa’s embedding (1993)

- Hidden $\mathcal{N} = 2$ SUSY in bosonic string (requires choice of current).
- Add spin-shifted fermionic $b'c'$ ghost system with $h = (3/2, -1/2)$ to the matter system.
- Correct spin-statistic.

\[
T = 26 \times T_X + T_{b'c'} + T_{bc} + T_{\beta\gamma} \quad (2)
\]

\[
T_{b'c'} = -3/2b'\partial c' - 1/2\partial b'c' + 1/2\partial^2(c'\partial c') \quad (3)
\]
Berkovits’ and Vafa’s Embedding (1993)

\[ T_{\mathcal{N}=1} = 26 \times T_X + T_{b'c'} + T_{bc} + T_{\beta\gamma} \]  \hspace{1cm} (4)

- \( \mathcal{N} = 1 \) string, equivalent to the bosonic string.
- \( \beta\gamma \) and \( b'c' \) contributions to amplitudes cancel.
- Exact endpoint of the type 0 tachyon condensation. (S. Hellerman, I. Swanson 2008)
- Can be extended to \( \mathcal{N} = 2 \):

\[ T_{\mathcal{N}=2} = 26 \times T_X + 2T_{b'c'} + T_{Q=1}^{\phi} + T_{bc} + 2T_{\beta\gamma} + T_{\eta\xi} \]  \hspace{1cm} (5)

- \( \mathcal{N} = 2 \) has hidden \( \mathcal{N} = 4 \) symmetry → can in principle be continued
Steps in the computation

1. Construct level truncated Hilbert space of the CFT.
2. Rewrite the SFT potential as a sum of string functions (requires b-insertions, PCO prescriptions...).
3. Evaluate the string functions (conservation laws, ghost number conservations, conformal maps to the n-punctured sphere).
4. Solve the resulting EOMs.
1. Hilbert space

- $L_0 - \overline{L}_0 |\psi\rangle = 0$
- $b_0 - \overline{b}_0 |\psi\rangle = 0$
- picture -1 in NS, -3/2 or -1/2 in R
- no ghost number constraints
- usual treatment of $\beta\gamma$ system ($\eta \xi + \phi$)
- lowest lying state in NS sector:

$$|t\rangle = t(x)c_1\overline{c}_1c'_1\overline{c}'_1\overline{e}^{-\phi - \overline{\phi}}|0\rangle, \quad h = -2$$

16 additional tachyons $B_i$ with weight $h = -1$

121 massless fields
Bosonic String Field

Applying the same rules to the usual bosonic string field one obtains:

\[ \psi_{bos} = t(x)c_1 \bar{c}_1 |0\rangle + d_1(x)c_{-1}c_1 \bar{c}_{-1} \bar{c}_1 |0\rangle + d_2(x)c_{-1}c_1 |0\rangle \\
+ d_3(x)\bar{c}_{-1} \bar{c}_1 |0\rangle + g_{\mu\nu}(x)\alpha_{-1}^\mu c_1 \bar{\alpha}_{-1}^\nu \bar{c}_1 |0\rangle \\
+ A_{\mu,1}(x)\alpha_{-1}^\mu c_1 \bar{c}_{-1} \bar{c}_1 |0\rangle + A_{\mu,2}(x)c_{-1}c_1 \bar{c}_1 \bar{\alpha}_{-1}^\mu |0\rangle \\
+ A_{\mu,3}(x)\alpha_{-1}^\mu c_1 |0\rangle + A_{\mu,4}(x)\bar{c}_1 \bar{\alpha}_{-1}^\mu |0\rangle \\
+ I(x) |0\rangle . \]

Demanding ghost number \( g = 2 \) would eliminate the additional massless fields.
**Bosonic Action (Qubic order, level 2)**

\[ V(\psi) = -t^2 + \frac{27}{32} td_2 d_3 + \frac{6561}{4096} t^3 + \frac{27}{32} ltd_1 + \frac{27}{16} g_{\mu\nu} g^{\mu\nu} t . \]  

- Not in the usual twist symmetric basis.
- A field redefinition brings it to the usual form:

\[
|d\rangle = |d_2\rangle - |d_3\rangle = (c_{-1} c_1 - \bar{c}_{-1} \bar{c}_1) |0\rangle \tag{8}
\]

\[
|d_g\rangle = |d_2\rangle + |d_3\rangle = (c_{-1} c_1 + \bar{c}_{-1} \bar{c}_1) |0\rangle , \tag{9}
\]
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$\mathcal{N} = 1$ Action, (EKS)

\[ S_{NS} = -\frac{1}{2} \langle \psi | c_0^+ c_0^- L_0^+ | \psi \rangle + \frac{\omega \left( \Phi, L_2^{(1,1)}(\Phi, \Phi) \right)}{6} + \frac{\omega \left( \Phi, L_3^{(2,2)}(\Phi, \Phi, \Phi) \right)}{24} \]

\[ \omega \left( \Phi, L_2^{(1,1)}(\Phi, \Phi, \Phi) \right) = \{ f_1 \circ \psi(0), f_2 \circ \psi(0), f_3 \circ X\overline{X} \psi(0) \} + 8 \text{ permutations} \]

\[ \overline{X} XV(y) = \oint \frac{dz}{2\pi iz} \oint \frac{dw}{2\pi iw} \overline{X}(z) X(w) V(y) . \]
Quadratic terms

\[ S_{\text{kin},NS} = \frac{1}{2} \langle \psi | c^+ c^- L_0 | \psi \rangle = \frac{h}{2} \langle \psi | c^+ c^- | \psi \rangle . \]  \hspace{1cm} (10)

These can be evaluated using the BPZ inner product. With normalization

\[ \langle 0 | c_{-1} \bar{c}_{-1} c'_{-1/2} \bar{c}'_{-1/2} c_0 \bar{c}_0 c_1 \bar{c}_1 c'_{1/2} \bar{c}'_{1/2} e^{-2\Phi-2\bar{\Phi}} | 0 \rangle = 2 , \]  \hspace{1cm} (11)

one obtains:

\[ V_2^{(2)} = -t^2 + B_2 B_5 - B_4 B_7 + B_{10} B_{13} + B_{11} B_{16} . \]  \hspace{1cm} (12)

Diagonalizing the fields gives 4 additional tachyons, 8 massless fields and 4 massive fields.
Qubic terms

\[ \{\{\psi^3\}\} = \{f_1 \circ \psi(0), f_2 \circ \psi(0), X \overline{X} f_3 \circ \psi(0)\} + 8 \text{ permutations} \quad (13) \]

- Conformal transformations known. \((-\sqrt{3}, 0, \sqrt{3})\) convention.
- Eliminate all creation operators using conservation laws.

\[ \langle V_3| \sum_{i=1}^{3} \int_{C_i} dz_i \phi^{(i)}(z_i) O(z_i) = 0 . \quad (14) \]

- \(\phi\) has dimension \(1 - h(O)\)
- Only \(c_1\) and \(c_{1/2}'\) remain
Qubic terms

- Rewrite closed amplitudes into open.
- Signs form commuting operators cancel.

\[
\langle 0 | c_1^{(1)} c_1^{(2)} c_1^{(3)} c_1^{(1)} c_1^{(2)} c_1^{(3)} c_{1/2}^{(1)} c_{1/2}^{(2)} c_{1/2}^{(3)} e^{a_1 \Phi^{(1)} + a_2 \Phi^{(2)} + a_3 \Phi^{(3)} + b_1 \Phi^{(1)} + b_2 \Phi^{(2)} + b_3 \Phi^{(3)}} | 0 \rangle = \\
2 \langle 0 | c_1^{(1)} c_1^{(2)} c_1^{(3)} | 0 \rangle_o \langle 0 | c_1^{(1)} c_1^{(2)} c_1^{(3)} | 0 \rangle_o \langle 0 | c_{1/2}^{(1)} c_{1/2}^{(2)} c_{1/2}^{(3)} | 0 \rangle_o \langle 0 | c_{1/2}^{(1)} c_{1/2}^{(2)} c_{1/2}^{(3)} | 0 \rangle_o \cdot \langle 0 | e^{a_1 \Phi^{(1)} + a_2 \Phi^{(2)} + a_3 \Phi^{(3)} + b_1 \Phi^{(1)} + b_2 \Phi^{(2)} + b_3 \Phi^{(3)}} | 0 \rangle_o
\]

\[
\langle 0 | c_1(z_1) c_1(z_2) c_1(z_3) | 0 \rangle = (z_1 - z_2)(z_1 - z_3)(z_2 - z_3), \quad \text{(15)}
\]

\[
\langle 0 | c_{1/2}^{(1)} c_{1/2}^{(2)}(z_1) | 0 \rangle = (z_1 - z_2) \quad \text{(16)}
\]

\[
\langle 0 | e^{a_1 \Phi^{(1)} + a_2 \Phi^{(2)} + a_3 \Phi^{(3)}} | 0 \rangle = \delta(a_1 + a_2 + a_3 + 2) \prod_{i<j} (z_i - z_j)^{-a_i \cdot a_j} \quad \text{(17)}
\]
Results

- Tachyonic part of the potential.
- Massless also evaluated, but too long to show here.

\[
V_1^{(3)} = -\frac{243 B_1^2 t}{256} + \frac{243}{128} B_1 B_2 t + \frac{243}{128} B_1 B_5 t - \frac{243}{128} B_2 B_5 t \\
- \frac{1215}{512} B_1 B_6 t - \frac{243}{128} B_{12} B_{15} t + \frac{243}{128} B_{11} B_{16} t + \frac{6561 t^3}{4096}
\]
Solutions to EOMs

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**Table:** The values of the tachyon, the potential and the masses of the fields in the 4 different solutions at cubic order. This excludes the 4 massless fields which do not appear at cubic order.
Potential
Evaluation of Quartic Terms

- Much more complicated when cubic terms.
- All parts of the computation in principle known, but amount of terms to compute explodes.
- Conformal maps only known numerically. (but solved by Moeller)
EKS solution of $L_\infty$-relations

\[ L_n^{(p,q)} = \frac{1}{p+q} \sum_{k=0}^{n} \left( \sum_{r,s} \left[ L_{n-k+1}^{(r,s)}, \lambda_{k+2}^{(p-r,q-s)} \right] + \sum_{r,s} \left[ L_{n-k+1}^{(r,s)}, \bar{\lambda}_{k+2}^{(p-r,q-s)} \right] \right). \]

\[ \lambda_{n+2}^{(p+1,q)} = \frac{n-p+1}{n+3} \left( \xi_0 L_{n+2}^{(p,q)} - L_{n+2}^{(p,q)} (\xi_0 \Pi_{N+1}) \right), \quad (18) \]

\[ \bar{\lambda}_{n+2}^{(p+1,q)} = \frac{n-q+1}{n+3} \left( \bar{\xi}_0 L_{n+2}^{(p,q)} - L_{n+2}^{(p,q)} (\bar{\xi}_0 \Pi_{N+1}) \right). \quad (19) \]

- Express the superstring brackets as combinations of the bosonic brackets.
- Triple sum $\rightarrow$ terms add up fast.
- All parts of the computation in principle known, but explodes in number of term.
EKS solution of $L_\infty$-relations

- Explicit form very long, $\mathcal{O}(200)$ terms for $L_3^{(2,2)}$.
- Includes terms of the form $L_2[L_2[\psi_1, \psi_2], \psi_2]$
- These turn out to be difficult to evaluate.
Explicit Evaluation of $L_2[L_2[\psi_1, \psi_2], \psi_2]$

Two possibilities:

- Use conservation laws and reflector state. Works but slow.
  \[ L_2[V_1, V_2] = \langle V_{123'} | \left( |V_1\rangle_{(1)} \bigotimes |R_{33'}\rangle \bigotimes |V_2\rangle_{(2)} \right) \]  
  (20)

- Explicit formula using surface states with insertions not at the origin. Problems with $e^{-\phi}$ terms.
But if the ghost structure of the vertex operators is
\[ |V\rangle = V(\alpha, \phi)c_1 \bar{c}_1 c'_1 \bar{c}'_1 e^{-\Phi - \bar{\Phi}} |0\rangle, \]
drastic simplifications happen.
For heterotic string:

\[
\frac{1}{4!} \omega \left( \Phi, L_3^{(2)}(\Phi, \Phi, \Phi) \right) = \\
\frac{5}{108} \omega_L \left( \xi_0 X_0 \Phi, L_2^{(0)}(\Phi, \xi_0 L_2^{(0)}(\Phi, \Phi)) \right) + \frac{1}{216} \omega_L \left( \xi_0 X_0 \Phi, L_2^{(0)}(\Phi, L_2^{(0)}(\Phi, \xi_0 \Phi)) \right) \\
+ \frac{1}{108} \omega_L \left( \xi_0 \Phi, L_2^{(0)}(\Phi, \xi_0 X_0 L_2^{(0)}(\Phi, \Phi)) \right) - \frac{1}{48} \omega_L \left( \xi_0 X_0 \Phi, L_2^{(0)}(\xi_0 \Phi, L_2^{(0)}(\Phi, \Phi)) \right) \\
+ \frac{1}{96} \omega_L \left( \xi_0 X_0^2 \Phi, L_3^{(0)}(\Phi, \Phi, \Phi) \right) + \frac{1}{32} \omega_L \left( \xi_0 X_0 \Phi, L_3^{(0)}(\Phi, \Phi, X_0 \Phi) \right). \tag{21}
\]
**Introduction**

Embedding of Bosonic theories

**Construction of SFT Around the Embedding**

$L_\infty$ and Quartic order

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**Simplification for Special Ghost Structure**

$L_2[L_2[\psi_1, \psi_2], \psi_2]$ comes always with 2 $\xi_0$ insertions and one PCO insertion.

$$X = \{Q, \xi\} = e^\phi (b' + c'(T_m + \partial(c')b') + \frac{5}{2} \partial^2 c')$$

$$+ 2\partial(\eta)e^{2\phi} b + \eta \partial(e^{2\phi} b) + c\partial\xi.$$

- Due to b’c’ ghost number conservation only the first terms contribute.
- $L_2[L_2[\psi_1, \psi_2], \psi_2]$ terms vanish for ghost structure $c_1\overline{c}_1c'_1/2\overline{c}'_1/2e^{-\Phi-\overline{\Phi}}!$
- This includes all fields of the bosonic theory.
- Truncated to this structure the SFT looks like bosonic SFT with 1 additional time.
The additional tachyons do not have this structure, hard to compute.

Beyond the massless level the b-insertions become much more complicated.

Compared to the bosonic theory the number of fields increases much faster with the level.

Work in Progress.
Outlook

- Further computations require better computational methods. (quartic perhaps already doable)
- Include torus contributions.
- Extension to the $\mathcal{N} = 2$ embedding.
- Application to open SFT, much easier computations.