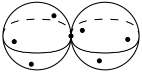
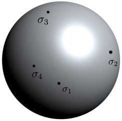
Twistorial Ambitwistor strings
II. Amplitudes

Yvonne Geyer

Chulalongkorn University, Bangkok

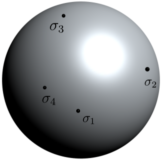




String field theory
Workshop
ICTP/SAIFR

arXiv:2001.05928 with G. Albonico, L.Mason
arXiv:1812.05548 with L. Mason

Worldsheet representations of FT amplitudes



D dimensions

- ambitwistor string, CHY
- localization: $P^2 = 0$
- [Cachazo-He-Yuan, Mason-Skinner, Berkovits]

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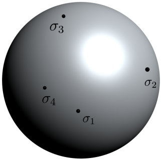

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$D = 4$ dimensions

- twistor and ambitw. string
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  $[\tilde{\lambda}(\sigma_-)\, \tilde{\kappa}_-] = 0$
- [Berkovits-Witten, Roiban-Spradlin-Volovich, Skinner]

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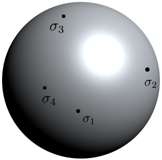

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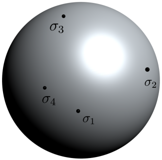

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$D = 6$ dimensions

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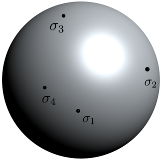

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TODAY

$D = 6$ dimensions

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CHY amplitudes [Cachazo-He-Yuan]

S-matrix for massless QFTs

$$\mathscr{M}_n = \int_{\mathfrak{M}_{0,n}} \frac{\mathrm{d}^n\sigma}{\mathrm{vol}\,\mathrm{SL}(2,\mathbb{C})} \prod_{i=1}^{n}{}' \,\bar{\delta}(\mathcal{E}_i)\;\; \mathscr{I}_n(\sigma_i, k_i, q_i)$$

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S-matrix for massless QFTs

D-dim momenta k_i

$k_i^2 = 0$

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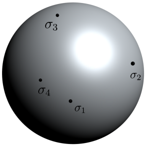
moduli space $\mathfrak{M}_{0,n}$

$\sigma_i \in \mathbb{CP}^1$



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holom. δ-fns
$\bar{\delta}(x) \equiv \bar{\partial}\left(\frac{1}{2i\pi\, x}\right)$

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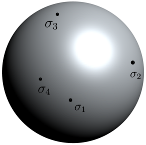

scattering equations $\mathcal{E}_i$

- Construction: $P_\mu = \sum_{i=1}^{n} \frac{k_{i\,\mu}}{\sigma-\sigma_i}\mathrm{d}\sigma$

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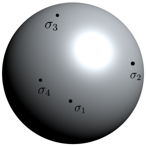

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- geometric interpretation: $P^2 = 0$
- fully localized

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S-matrix for massless QFTs

Integrand $\mathscr{I}_n$

- 'data' $q_i{:}T_i^{a_i}$, ϵ_i,
- theory-specific
  $\mathscr{I}_n = \mathscr{I}_n^{1/2}\,\tilde{\mathscr{I}}_n^{1/2}$

$$\mathscr{M}_n = \int_{\mathfrak{M}_{0,n}} \frac{d^n\sigma}{\text{vol}\,SL(2,\mathbb{C})} \prod_{i=1}^{n}{}' \,\bar{\delta}(\mathcal{E}_i)\;\; \mathscr{I}_n(\sigma_i, k_i, q_i)$$

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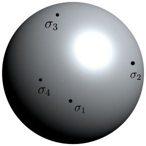

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CHY vs Spinorial

	CHY	RSVW (4d twistor string)
any D	✓	×
various theories	✓	(✓)
worldsheet theory	✓	✓
loop amplitudes	✓	
manifest susy	×	✓

CHY vs Spinorial

	CHY	RSVW (4d twistor string)
any D	✓	×
various theories	✓	(✓)
worldsheet theory	✓	✓
loop amplitudes	✓	
manifest susy	×	✓

Question:

Spinorial models/formulas
beyond $D = 4, 10$?

6D spinor-helicity

- Vectors: $k_\mu = \gamma^{AB} k_{AB}\,, \qquad k_{AB} = k_{[AB]} = \frac{1}{2}\varepsilon_{ABCD}k^{CD}$

6D spinor-helicity

$\mu = 0, \ldots, 5$

$\mathrm{Spin}(6, \mathbb{C}) \simeq \mathrm{SL}(4, \mathbb{C})$
$A, B = 0, \ldots, 3$

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$$\kappa_A^a : \qquad k_{AB} = \kappa_A^a \kappa_B^b\, \varepsilon_{ab} =: \langle \kappa_A \kappa_B \rangle$$
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little group:
$\text{Spin}(4,\mathbb{C}) \simeq \text{SL}(2,\mathbb{C}) \times \text{SL}(2,\mathbb{C})$
$a = 0, 1, \;\; \dot a = \dot 0, \dot 1$

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Momentum eigenstates:

- $F_A^B = \epsilon_A \epsilon^B$
- EoM: $\epsilon_A k^{AB} = 0 \;\Rightarrow\; \epsilon_A = \epsilon_a \kappa_A^a$
  $\epsilon^A k_{AB} = 0 \;\Rightarrow\; \epsilon^A = \epsilon_{\dot a} \kappa^{A\dot a}$

Polarized scattering equations

- Recall
  Scattering equs: $\mathcal{E}_i = \mathrm{Res}_{\sigma_i} P^2(\sigma) = k_i \cdot P(\sigma_i)$, $P(\sigma) = \sum_i \frac{k_i}{\sigma - \sigma_i}$

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polarized SE

$$u_{ia}\, \lambda^a_A\,(\sigma_i) = \; v_{ia}\, \kappa^a_{iA}$$

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$\exists \ (u_i, v_i)$
normal.: $\langle v_i \epsilon_i \rangle = 1$

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Ansatz:
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- Properties:
  - Ansatz correct: $\mathrm{Res}_{\sigma_i}\langle \lambda_A(\sigma)\lambda_B(\sigma)\rangle = \epsilon_{i[A}\langle u_i \lambda_{B]}(\sigma_i)\rangle = k_{iAB}$

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  - natural measure $d\mu^{\mathrm{pol}}_n$

6D supersymmetry

- $(N, \tilde{N})$ susy: $\{\, Q_{AI}\,, Q_{BJ}\} = k_{AB}\, \Omega_{IJ}$ $\{Q^A_{\dot{I}}, Q^B_{\dot{J}}\} = k^{AB} \Omega_{\dot{I}\dot{J}}$

6D supersymmetry

susy generators

$\mathrm{Sp}(N) \times \mathrm{Sp}(\tilde{N})$ R-sym. metric $\Omega_{IJ} = \Omega_{[IJ]}$, $I, J = 1, \ldots, 2N$

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- on-shell susy:
  - momentum eigenstates: $Q_{AI} = \kappa_A^a Q_{aI}$, $Q^A_I = \kappa^{A\dot a} Q_{\dot a I}$
  - further reduction: manifest little group [Cachazo-Guevara-Heydeman-Mizera-Schwarz-Wen]
    manifest R-symmetry [Albonico-YG-Mason]

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supermomenta

$$Q_{aI} = \xi_a\, q_I + \epsilon_a \Omega_{IJ} \frac{\partial}{\partial q_J}$$

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little group inv.
not manifest
$\langle \xi \epsilon \rangle = 1$

supermomenta

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supermomenta

- super YM: $\mathcal{F} := (\,F^B_A\,, \psi^B_I,\ \tilde{\psi}_{A\dot{I}}\,, \phi_{I\dot{I}})$

$$Q_{CJ}\,\mathcal{F} = (k_{AC}\psi^B_J, \Omega_{JI}F^A_C, k_{AC}\phi_{J\dot{I}}, \Omega_{JI}\tilde{\psi}_{C\dot{I}})$$

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$$F^B_A = \left(\epsilon_A + q^2\langle\xi\kappa_A\rangle\right)\left(\epsilon^B + \tilde{q}^2[\xi\kappa^B]\right) \qquad \tilde{\psi}_{A\dot{I}} = \tilde{q}_{\dot{I}}\left(\epsilon_A + q^2\langle\xi\kappa_A\rangle\right)$$

Amplitudes

related work: [Cachazo-Guevara-Heydeman-Mizera-Schwarz-Wen]

$$\mathscr{M}_n = \int d\mu_n^{\mathrm{pol}} \;\; \mathscr{I}_n^{1/2}\, \tilde{\mathscr{I}}_n^{1/2} \;\; e^{F_N + \tilde{F}_{\tilde{N}}}$$

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polarized measure

$$d\mu_n^{\mathrm{pol}} = d\mu_{uv\sigma} \prod_{i=1}^{n} \bar{\delta}(\langle v_i \epsilon_i \rangle - 1)\, \delta^4(\mathcal{E}_{iA})$$

$$d\mu_{uv\sigma} = \frac{\prod_j d^2 u_j \, d^2 v_j \, d\sigma_j}{\mathrm{vol}\,\mathrm{SL}(2,\mathbb{C})_\sigma \times \mathrm{vol}\,\mathrm{SL}(2,\mathbb{C})_u}$$

Amplitudes

related work: [Cachazo-Guevara-Heydeman-Mizera-Schwarz-Wen]

$$\mathscr{M}_n = \int d\mu_n^{\mathrm{pol}} \; \mathscr{I}_n^{1/2}\, \tilde{\mathscr{I}}_n^{1/2} \; e^{F_N + \tilde{F}_{\tilde{N}}}$$

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- fully localized
- DoF in $d\mu_n^{\text{pol}}$:
  $5n + (3+3) - 5n = 6$ mom. conservation

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supersymmetry factor

- $F_N = \sum_{i<j} \frac{\langle u_i u_j \rangle}{\sigma_{ij}} q_{iI} q_j^I - \frac{1}{2} \sum_i \langle \xi_i v_i \rangle q_i^2$
- susy invariance: $Q_{AI} e^{F_N} = 0$ on $\mathcal{E}_{iA}$

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brane integrands, see: [Heydeman-Schwarz-Wen]

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  - spin-1: $\mathcal{I}^{\text{kin}} = \det{}' H$ with $H_{ij} = \frac{\epsilon_{iA}\,\epsilon_j^A}{\sigma_{ij}}$

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reduced determinant:

on polarized scatt. equs.: $\sum_i u_i^a H_{ij} = 0$

$\Rightarrow \; \det{}' H = \langle u_i u_{ij} \rangle^{-1} \, [\tilde{u}_i \tilde{u}_j]^{-1} \det H^{[ij]}$

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- <u>double-copy matrix of theories</u>

	PT	$\det{}' A$	$\det{}' H\, e^{F_1 + \tilde{F}_1}$	$\frac{\mathrm{Pf}\,' A}{\mathrm{Pf}\, U}\, e^{F_2}$
PT	Bi-adjoint scalar	NLSM	$\mathcal{N} = (1,1)$ sYM	•
$\det{}' A$		Galileon	$\mathcal{N} = (1,1)$ D5	$\mathcal{N} = (2,0)$ M5
$\det{}' H\, e^{F_1 + \tilde{F}_1}$			$\mathcal{N} = (2,2)$ sugra	•
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Question:

Connection to worldsheet model?

Twistorial ambitwistor string [Lionel's talk]

- Chiral 2D CFT:

ambitwistor string

$$S_{\mathbb{A}} = \frac{1}{2\pi} \int_{\Sigma} \varepsilon_{ab}\, Z^a \cdot \bar{\partial} Z^b + A_{ab}\, Z^a \cdot Z^b$$

super ambi-twistors $Z_a = (\lambda_{Aa}, \mu_a^A, \eta_a^I) \in \Omega^0(K_\Sigma^{1/2}),\ A_{ab} \in \Omega^{0,1}$.

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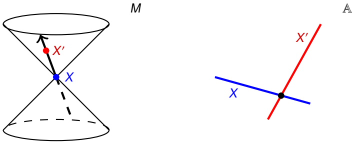
- c.f. 4D twistor and ambitwistor string
- target space: $\mathbb{A}$ = phase space of complexified null geodesics



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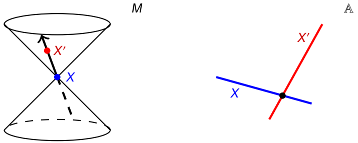

- Vertex operators:

$$V_i = \int d^2u_i\, d^2v_i\, \bar\delta(\langle v_i \epsilon_i\rangle - 1)\, \delta^4\left(\langle u_i \lambda_A\rangle - \langle v_i \kappa_{iA}\rangle\right)\, w\, e^{i\left(\langle u_i \mu^A\rangle \epsilon_{iA} + \langle u_i \eta^I\rangle q_{iI} - \frac{1}{2}\langle \xi_i v_i\rangle q_i^2\right)}$$

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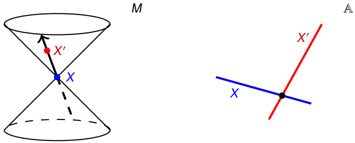

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$\Rightarrow$ worldsheet theory for QFT amplitudes

Correlator

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$$\mathscr{M}_n = \left\langle V_1\, V_2 \ldots V_n \right\rangle$$

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$\Rightarrow$ Supersymmetry rep e^{F_N}

$$F_N \supset \sum_{i<j} \frac{\langle u_i u_j\rangle}{\sigma_{ij}} q_{iI} q^I_j$$

BCFW recursion [Britto-Cachazo-Feng-Witten]

- Deformation:

$$\hat{k}_1 = k_1 + z\,q$$
$$\hat{k}_n = k_n - z\,q$$

BCFW recursion

[Britto-Cachazo-Feng-Witten]

- **Deformation:**

$$\hat{k}_1 = k_1 + z\, q \qquad \hat{k}_n = k_n - z\, q$$

On-shell:

$$0 = \hat{k}_1^2 = \hat{k}_n^2$$
$$0 = q^2 = q \cdot k_{1,n}$$

BCFW recursion [Britto-Cachazo-Feng-Witten]

- <u>Deformation:</u>
$$\hat{k}_1 = k_1 + z\,q$$
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- <u>Cauchy:</u>
$$\mathcal{M} = \mathcal{M}(z=0) = \oint_{\substack{\circlearrowleft \\ |z|=\varepsilon}} \frac{1}{z}\mathcal{M}(z) = -\oint_{\substack{\circlearrowright \\ |z|=\varepsilon}} \frac{1}{z}\mathcal{M}(z)$$

BCFW recursion [Britto-Cachazo-Feng-Witten]

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Poles: $0 = \hat{k}_L^2 = k_L^2 + 2z\,q\cdot k_L$, $\quad k_L = \sum_{i\in L} k_i$

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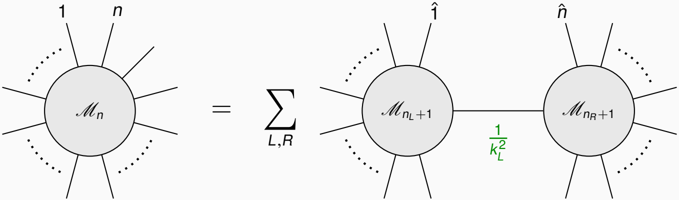

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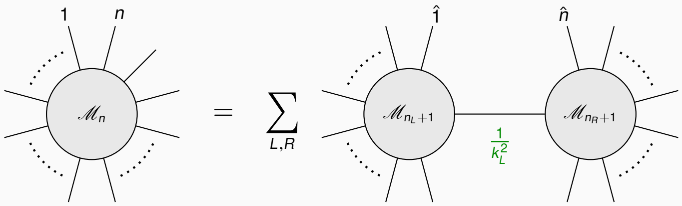
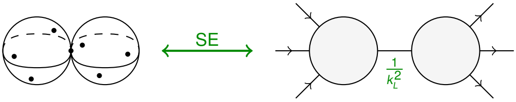


- **Proof of worldsheet formula:** [Albonico-YG-Mason]



boundary of $\mathfrak{M}_{0,n}$

factorisation channel

Summary and Outlook

- Summary

	CHY	twistorial
any D	✓	$D = 4, 5, 6, 10$
various theories	✓	✓
worldsheet theory	✓	✓
loop amplitudes	✓	
manifest susy	×	✓

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- Future directions
  - worldsheet models in 5d [WiP with D.Skinner and L. Mason]
    also: 6d c.f. Lionel's talk
  - loop amplitudes? [c.f. Wen-Zhang]