New Physics Beyond the Standard Model

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- Neutral Naturalness

The Axion Solution to the Strong CP Problem

Why Physics Beyond the Standard Model?

The Standard Model (SM) does an excellent job of explaining most experimental data. Nevertheless there is now conclusive evidence of the existence of physics beyond the Standard Model.

① Neutrino masses and mixings cannot be accommodated in the SM.

② SM does not explain the observed excess of matter over antimatter.

③ Astrophysical and cosmological observations indicate that about 80% of the matter in the universe is some form of non-luminous cold dark matter. No particle in the SM can play this role.

④ SM does not explain the origin of the apparently acausal density perturbations that seeded the growth of structure in the early universe.

Apart from these observations that require new physics to explain, there are also more theoretical reasons to expect physics beyond the SM.

① The SM cannot describe physics at energies above the Planck scale.

② The SM cannot explain the large hierarchy between the value of the cosmological constant $(\sim (10^{-3}\,eV)^4)$, and other scales in physics such as the QCD scale $(\sim GeV^4)$ and the electroweak scale $(\sim (100\,GeV)^4)$. These higher scales feed into the cosmological constant through quantum loops, and so we are left with a fine tuning problem of order $(100\,GeV/10^{-3}\,eV)^4 \sim 10^{56}$. Anthropics?

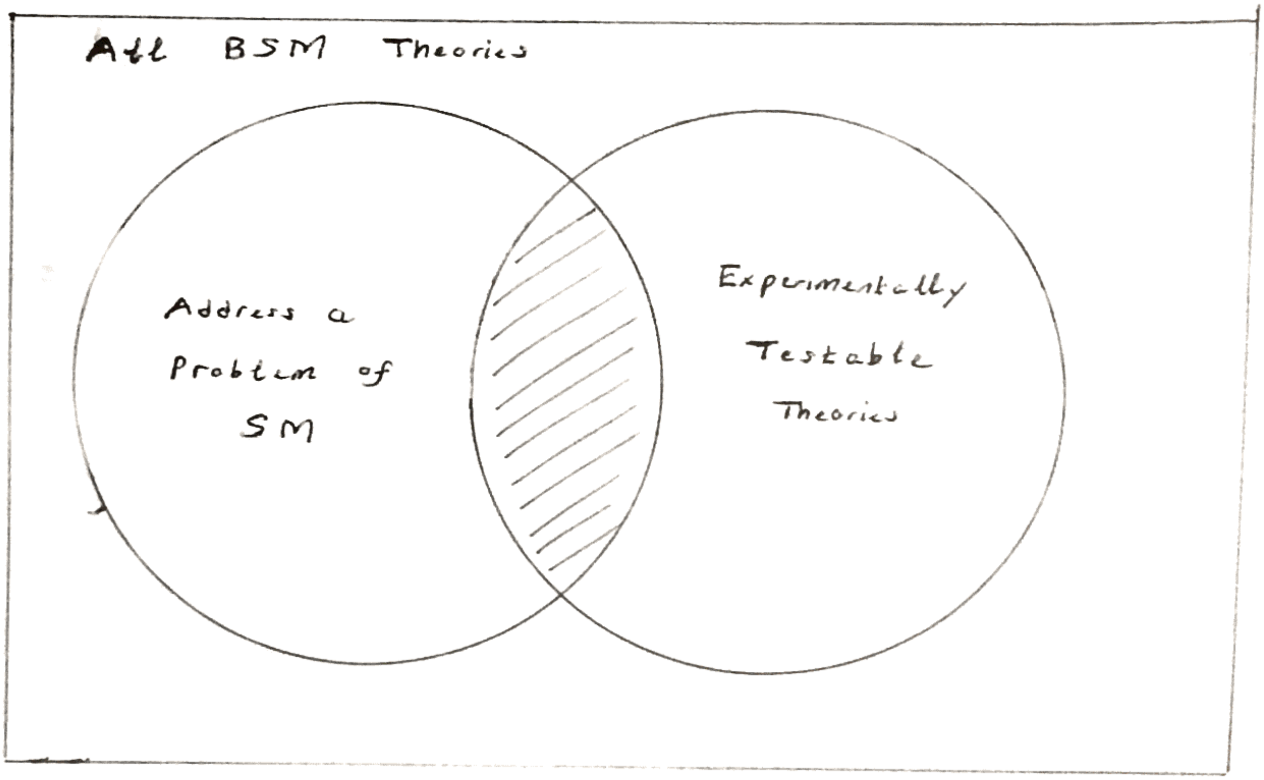
③ The SM cannot explain the hierarchy between the electroweak scale $(\sim 100\,GeV)$ and the Planck scale $(\sim 10^{19}\,GeV)$, or indeed the hierarchy between the electroweak scale and any other higher scale that may be present, such as the grand unification scale $(\sim 10^{16}\,GeV)$ or the scale of inflation. Quantum corrections from the Planck scale feed into the electroweak scale leading to a fine tuning problem at the level of $(10^{19}\,GeV/10^2\,GeV)^2 \sim 10^{34}$. Anthropics?

④ The smallness of the electric dipole moment of the neutron is not explained in the SM.
Radiative corrections from high scales would be expected to induce a value about 10^{10} larger than the observed upper bound.
No known anthropic explanation!

Apart from these, there are some other features of the SM that seem to require explanation.

① The pattern of fermion masses and mixings seem to exhibit a hierarchical structure.
Although radiatively stable, this would seem to hint at some under-lying structure.

② The quantization of electric charge is not explained in the SM. Even though $U(1)_Y$ charges of the SM particles are only constrained by anomaly cancellation, ~~they~~ their electric charges are all integer multiples of $(1/3)$.



In studying physics beyond the SM, our major focus is on theories that lie in the shaded region. These are theories that address a problem of the SM, and are also experimentally testable. Unfortunately, many well motivated theories are simply not testable, or very difficult to test. These include the high scale seesaw model for neutrino masses and many models that explain the matter-antimatter asymmetry.

Historically, much attention has been paid to models ~~the~~ that address the hierarchy between the Planck and electroweak scales, since these theories are often testable.

Constraints on New Physics

For a model of new physics to be viable, it needs to be consistent with every experimental fact known to man. These include limits from direct searches for new particles, and also indirect bounds from new physics contributions to the rates for various processes. The range of constraints is vast.

I want to focus on some characteristic features of the SM that are not generally shared by extensions of the SM. These are accidental baryon and lepton number symmetries, suppression of CP violation, suppression of flavor changing neutral currents (FCNC's) and custodial SU(2) symmetry. These are often the source of the strongest constraints on new physics. In what follows, we consider them in turn.

Accidental Symmetries of the SM

Baryon number is an "accidental symmetry" of the SM. What this means is that, given the particle content and gauge symmetries of the SM, it is not possible to write a renormalizable operator that violates baryon number.

In exactly the same way, the SM possesses an accidental lepton number symmetry. However, in this case, electron number, muon number and tau number are all conserved separately, and correspond to accidental symmetries. There are therefore three accidental symmetries in the lepton sector of the SM, one for each flavor of lepton, and overall lepton number is the diagonal subgroup.

Non-perturbative effects in the SM violate these global symmetries, and only a $U(1)_{B-L}$ subgroup is preserved. However, these non-perturbative effects are exponentially suppressed at zero temperature, and can be neglected.

In general, these accidental symmetries need not be symmetries in extensions of the SM. Bounds on processes that violate these accidental symmetries, such as proton decay and $\mu \to e\gamma$, can be used to place powerful constraints on many models of new physics.

Suppression of CP Violation

In the SM the only source of CP violation is the phase in the CKM matrix. Recall that the CKM matrix captures the flavor structure in the couplings of quarks to W bosons,

$$\frac{g}{\sqrt{2}} \bar{u}_{L\alpha} \gamma^\mu V_{\alpha\beta} d_{L\beta} W_\mu^+ + \text{h.c}$$

The factor of $\frac{1}{\sqrt{2}}$ is from the normalization of $W^\pm$ relative to $W^{1,2,3}$. Here α and β are flavor indices.

The CKM matrix has 4 physical parameters. These are the three mixing angles and one phase. In the standard parametrization,

$$V_{CKM} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

where θ_{12}, θ_{13} and θ_{23} are the 3 mixing angles and δ the phase. From data, $\sin\theta_{12} \approx .22$, $\sin\theta_{23} \approx .04$, $\sin\theta_{13} \approx .004$ and $\delta \approx 66^\circ$.

CP violation vanishes in the limit that any of the three mixing angles vanishes. Even though the phase δ is large, all the mixing angles are small, especially θ_{13}. Hence CP violation is suppressed in the SM. New physics that contributes to CP violation is very constrained.

Suppression of Flavor Changing Neutral Currents

Consider the following decays.

$Br(K^+ \to \mu^+ \nu) = 0.64$ $\quad Br(K_L \to \mu^+\mu^-) = 7 \times 10^{-9}$

In both cases the initial state is hadronic and the final state is leptonic, and the decay violates quark flavor symmetries. But the rate for one is suppressed with respect to the other by many orders of magnitude.

The crucial difference is that in the first case there is a net flow of electric charge into the leptons, so it is a "charged current" process, while in the second case there is no such flow of charge, so it is a "neutral current" process. In the SM flavor changing neutral current processes are suppressed. To give another example,

$Br(B^- \to D^0 \ell \bar{\nu}) = 0.02$ $\quad Br(B^- \to K^{*-} \ell^+ \ell^-) = 5 \times 10^{-7}$

Once again the charged current process is sizable while the neutral current process is very suppressed. Note that this suppression only applies to flavor violating decays. For example,

$Br(J/\psi \to \mu^+\mu^-) = 0.06$

There are two reasons for the suppression of flavor changing neutral currents (FCNC's) in the SM. Firstly, there are no tree level FCNCs in the SM. Secondly, although there are FCNCs at loop level, they are suppressed by the "GIM mechanism". We consider these in turn.

The particles that can mediate neutral currents in the SM are the photon, the gluon, the Higgs and the Z. The photon and the gluon cannot mediate flavor change because the unbroken gauge symmetry ensures that the gauge kinetic terms of the fermions are always flavor diagonal. In the case of the Higgs, its couplings are always diagonal in the mass basis. Consider,

$$Y_{\alpha\beta}\, Q_\alpha H d^c_\beta \rightarrow Y_{\alpha\beta}\, Q_\alpha \frac{1}{\sqrt{2}}(v+h)\, d^c_\beta$$

When we go to a basis in which the masses of the quarks are diagonal, the couplings of the Higgs will also be diagonal. Note that this is not necessarily true in extensions of the SM with more than one Higgs doublet. In these theories additional structure is required to avoid FCNCs.

The reason that the Z does not mediate FCNCs is that it couples to the fermions of the different generations in exactly the same way. Consider the couplings of the Z to the up-type quarks,

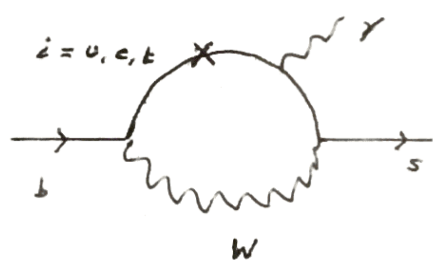
$$\frac{g}{\cos\theta_W}\left\{\bar{U}_{L\alpha}\gamma^\mu\underbrace{\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)}_{\text{independent of flavor index }\alpha}U_{L\alpha}+\bar{U}_{R\alpha}\gamma^\mu\underbrace{\left(-\frac{2}{3}\sin^2\theta_W\right)}_{\text{independent of flavor index }\alpha}U_{R\alpha}\right\}Z_\mu$$

When we transform to a basis in which the quark masses are diagonal, the fact that the couplings of the Z are proportional to the identity in flavor space means that the couplings of the Z will continue to be flavor diagonal and proportional to the identity in the new basis.

It follows from this that in the mass basis, the only source of flavor violation in the SM is the CKM matrix. Therefore tree level flavor violation must involve the W boson, and is therefore associated with a charged current.

We see that in the SM, FCNCs can only arise at loop level, and are therefore suppressed. However, as we now show, even at loop level there is additional suppression because of the GIM mechanism and because of the form of the CKM matrix.

Consider the process $b \to s\gamma$. This proceeds through penguin diagrams,



If we neglect the masses of the up-type quarks in the loop, this amplitude is proportional to

$$\sum_{i=u,c,t} V_{ib}^* V_{is} = 0$$

and vanishes because of unitarity of the CKM. The amplitude therefore scales as,

$$M \sim \sum_i V_{ib}^* V_{is} \, f(m_i)$$

where the function $f(m_i)$ scales as

$$f(m_i) \sim \frac{m_i^2}{m_W^2} \quad \text{for} \quad m_i^2 \ll m_W^2$$

$$\sim \text{constant} \quad \text{for} \quad m_i^2 \gg m_W^2$$

At this point it is useful to recall the flavor structure of the CKM matrix,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

where $\lambda \sim 0.2$.

Consider the charm quark contribution to amplitude,

$$M_c \sim \underbrace{V_{cb}^* V_{cs}}_{\substack{\sim \lambda^2 \\ \text{CKM suppression}}} \left(\frac{m_c^2}{m_W^2}\right)$$

GIM suppression

This contribution is suppressed by the CKM mixing angles and also by the GIM mechanism. The up quark contribution is even more suppressed. For the top quark,

$$M_t \sim \underbrace{V_{tb}^* V_{ts}}_{\sim \lambda^2}$$

the contribution is CKM suppressed, but not GIM suppressed.

These features are general. All contributions to FCNCs are GIM suppressed, unless they involve the top quark. But since the mixing angles of the top are small, there is always some additional suppression, especially for processes that only involve external light quarks.

Custodial SU(2) Symmetry

In the limit that the hypercharge gauge interactions and the Yukawa couplings are turned off, the Higgs sector of the SM has an enhanced symmetry. To see this, first let us consider the potential for the Higgs.

$$V(H) = -m^2 |H|^2 + \lambda |H|^4$$

Let us write this out explicitly in terms of real fields.

$$H = \begin{pmatrix} H_1 + i H_2 \\ H_0 + i H_3 \end{pmatrix}$$

Then $\quad |H|^2 = H_0^2 + H_1^2 + H_2^2 + H_3^2$

Since $V(H)$ depends on H only through $|H|^2$, we see that the Higgs potential exhibits an $SO(4)$ global symmetry under which the 4 real fields H_0, H_1, H_2, H_3 rotate into each other. When H_0 acquires a VEV, the $SO(4)$ symmetry is broken to $SO(3)$. The fields H_1, H_2 and H_3 rotate into each other under the unbroken $SO(3)$. We see that the Higgs potential has a bigger symmetry than $SU(2) \times U(1)$ and even after electroweak breaking, some part of this remains.

The $SU(2)_L$ gauge interactions also respect this global symmetry. To see this, recall that the Lie algebra of $SO(4)$ is the same as that of $SU(2) \times SU(2)$. This allows us to write the Lagrangian in a way that keeps the custodial symmetry manifest.

Consider the Higgs doublet, $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$. Now $(i\sigma_2) H^* = \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix}$ is also a doublet of $SU(2)$ but with the opposite hypercharge. Then,

$$D_\mu H = \partial_\mu H - i g W_\mu H - \frac{i g'}{2} B_\mu H$$

where $W_\mu = W_\mu^a \frac{\sigma^a}{2}$. Similarly,

$$D_\mu (i\sigma_2 H^*) = \partial_\mu (i\sigma_2 H^*) - i g W_\mu (i\sigma_2 H^*) + \frac{i g'}{2} B_\mu (i\sigma_2 H^*)$$

We now define a bi-doublet field,

$$\phi = \frac{1}{\sqrt{2}} (H, \, i\sigma_2 H^*) = \frac{1}{\sqrt{2}} \begin{pmatrix} H^+ & H^{0*} \\ H^0 & -H^- \end{pmatrix}$$

We can write the Higgs Lagrangian in terms of ϕ.

$$\phi^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} H^+ \\ -H^T (i\sigma_2) \end{pmatrix}$$

$$\mathrm{Tr}(\phi^+ \phi) = \frac{1}{2} \{ H^+ H + H^T H^* \} = |H|^2$$

This allows us to write the potential for the Higgs,

$$V(H) = -m^2 \, Tr(\phi^+ \phi) + \lambda \{ Tr(\phi^+ \phi) \}^2$$

We can write the gauge covariant kinetic term for the Higgs as

$$Tr \{ (D_\mu \phi)^+ D^\mu \phi \}$$

where $D_\mu \phi$ is given by,

$$D_\mu \phi = \left(\partial_\mu \phi - i g W_\mu^a \frac{\sigma^a}{2} \phi - \frac{i g'}{2} B_\mu \phi \sigma_3 \right)$$

Expanding this out we find,

$$D_\mu \phi = \frac{1}{\sqrt{2}} \left(D_\mu H , D_\mu (i \sigma_2 H^*) \right)$$

$$= \frac{1}{\sqrt{2}} \left(D_\mu H , i \sigma_2 (D_\mu H^+)^T \right)$$

$$(D_\mu \phi)^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} D_\mu H^+ \\ - (D_\mu H^+)^* (i \sigma_2) \end{pmatrix}$$

Then $Tr \{ (D_\mu \phi)^+ D^\mu \phi \} = D_\mu H^+ D^\mu H$

as required.

This Lagrangian is of course invariant under $SU(2)_L$ and $U(1)_Y$. Under $SU(2)_L$ we have

$$\phi \rightarrow L \phi$$

$$D_\mu \phi \longrightarrow L \, D_\mu \phi$$

Then $\quad \mathrm{Tr}(\phi^\dagger \phi) \to \mathrm{Tr}(\phi^\dagger L^\dagger L \phi) = \mathrm{Tr}(\phi^\dagger \phi)$

Similarly $\quad \mathrm{Tr}\{(D_\mu \phi)^\dagger D^\mu \phi\} \to \mathrm{Tr}\{(D_\mu \phi)^\dagger L^\dagger L\, D^\mu \phi\}$

$$= \mathrm{Tr}\{(D_\mu \phi)^\dagger D^\mu \phi\}$$

Under $U(1)_Y$, ϕ transforms as

$$\phi \to \phi\, e^{i\theta/2\, \sigma_3}$$

This corresponds to

$$(H, i\sigma_2 H^*) \to (H, i\sigma_2 H^*)\, e^{i\theta/2\, \sigma_3}$$

$$\to (H, i\sigma_2 H^*) \begin{pmatrix} e^{i\theta/2} & \\ & e^{-i\theta/2} \end{pmatrix}$$

$$\to \left(H e^{i\theta/2},\ i\sigma_2 (H e^{i\theta/2})^*\right)$$

We see that this leads to $(H \to H e^{i\theta/2})$ as required.

The Higgs has charge $1/2$ under $Y/2$

The crucial point is that the Higgs Lagrangian has an additional $SU(2)$ symmetry in the limit that $g' = 0$. We call this symmetry $SU(2)_R$. Under $SU(2)_R$

$$\phi \rightarrow \phi R^{+}$$

Then

$$Tr(\phi^{+}\phi) \rightarrow Tr(R\phi^{+}\phi R) = Tr(\phi^{+}\phi)$$

When $g' = 0$, $\quad D_\mu \phi \rightarrow (D_\mu \phi) R^{+}$

$$Tr\{(D_\mu\phi)^{+} D^\mu \phi\} \rightarrow Tr\{R(D_\mu\phi)^{+}(D^\mu\phi)R^{+}\}$$
$$\rightarrow Tr\{(D_\mu\phi)^{+} D^\mu\phi\}$$

We see that in the limit that $g' = 0$ and the Yukawa couplings are turned off, the Higgs sector is invariant under an $SU(2)_L \times SU(2)_R$ global symmetry under which

$$\phi \rightarrow L\phi R^{+}$$

Clearly, $U(1)_Y$ is a subgroup of $SU(2)_R$.

When the Higgs acquires a vev, we have

$$\langle \phi \rangle = \frac{1}{2}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$$

Clearly both $SU(2)_L$ and $SU(2)_R$ are broken.

However, the subgroup $SU(2)_{L+R}$ corresponding to rotations with $L = R$ is unbroken.

$$L \langle \phi \rangle R^{+} = \langle \phi \rangle \quad \text{if} \quad L = R$$

Therefore we have

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_{L+R}$$

The unbroken $SU(2)_{L+R}$ is called "custodial $SU(2)$ symmetry". (Some authors call $SU(2)_R$ by this name.)

The 3 Goldstone bosons are eaten by the Higgs mechanism to give masses to the W and Z. Now, under $SU(2)_L$ the W_μ^a transform as

$$W_\mu \rightarrow L W_\mu L^{+}$$

while W_μ is a singlet under $SU(2)_R$. Hence the W_μ transform as a triplet under the unbroken custodial symmetry. This implies that in the limit that g' and the Yukawa couplings are set to zero, the $W^{\pm}$ and Z must have exactly the same mass. This is true to all orders in perturbation theory.

Since the W and Z boson masses have been very well measured, new physics models that violate custodial $SU(2)$ symmetry are very constrained.

Precision Electroweak Measurements

The precision electroweak measurements are a set of accurate data on flavor diagonal processes involving the electroweak gauge bosons. It consists of order a few dozen measurements. Taken together, these observations overdetermine the ~~electroweak~~ parameters in the electroweak sector of the SM, and therefore constitute a powerful check on the consistency of the theory. They can also be used to place strong constraints on new physics.

The precision electroweak data consists of the following observations.

1. Z-pole measurements at LEP 1 (CERN) and SLC (SLAC). This data includes the Z mass, Z width, branching ratios of the Z into quarks and leptons, forward-backward asymmetries and left-right asymmetries. This data achieved very high statistics, and is often the most relevant for constraining new physics.

2. The W mass, obtained at the Tevatron (Fermilab), LEP 2 and the LHC. This is very accurately measured even though much of the data is from hadron machines.

(3) Measurements above the Z-pole at LEP 2. This includes $e^+e^- \rightarrow$ fermions at energies above the Z-pole and also $e^+e^- \rightarrow W^+W^-$ data.

(4) Low energy measurements from a diverse range of experiments. The most precisely measured are the fine structure constant α and the Fermi constant G_F. However, there is also data on neutrino scattering on nucleons and electrons, atomic parity violation and several other processes.

(5) The Higgs mass and top mass from the Tevatron and LHC.

To illustrate the power of precision electroweak observations, consider the following observables.

$$\alpha^{-1} = 137.035999139 \; (31)$$

$$G_F = 1.1663787 \; (6) \times 10^{-5} \; GeV^{-2}$$

$$m_Z = 91.1876 \; (21) \; GeV$$

$$m_W = 80.387 \; (16) \; GeV$$

$$A_e = .15138 \; (216)$$

$$\Gamma_Z = 2.4952 \; (23) \; GeV$$

Here α is the fine structure constant measured from the anomalous magnetic moment of the electron and

G_F is the Fermi constant obtained from muon decay. m_Z and Γ_Z are the mass and width of the Z boson, measured at the Z-pole. m_W is the mass of the W boson. The left-right asymmetry of the electron A_e is defined as

$$A_e = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

where σ_L is the cross section for $(e^+e^- \rightarrow e^+e^-)$ at the Z-pole when the initial electron has left-handed polarization. σ_R is the same cross section but for right-handed electron in the initial state.

Now the point is that, at tree level in the SM, these six observables depend only on the 3 parameters g, g' and v. Here g is the gauge coupling of $SU(2)_L$, g' is the gauge coupling of $U(1)_Y$ and v is the VEV of the Higgs. Hence this set of observables is overdetermined, and can be used to test the consistency of the SM (at tree level). Since all of these six are very accurately measured, this constitutes a powerful test of the SM.

When the new physics is heavy compared to the scales at which the precision electroweak measurements are made, we can integrate out the new particles and construct an effective theory in terms of the SM fields only. Then

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i a_i O_i$$

where the operators O_i are higher dimensional operators composed only of the SM fields. We will restrict to dimension 6.

The S and T Parameters

There is a special class of dimension 6 operators that arise in extensions of the SM in which the new physics does not couple to fermions, but only to the SM gauge fields and the Higgs doublet. These operators are referred to as "universal" or "oblique" because they affect the SM quarks and leptons only through their couplings to the SM gauge fields.

Let us consider the dimension 6 operators that contain only the gauge and Higgs fields. There is only one operator with six Higgs fields, which is $(H^\dagger H)^3$. Coming to operators with only four Higgs fields, we have

$$(H^\dagger H)(D_\mu H^\dagger D^\mu H) \quad \text{and} \quad |(H^\dagger D_\mu H)|^2$$

Other terms with four Higgs fields can be rewritten in terms of these operators using the equations of motion and integration by parts.

There are several terms with two Higgs fields, most of which are of the form

$$(H^\dagger H) F_{\mu\nu} F^{\mu\nu} \quad \text{or} \quad (H^\dagger H) F_{\mu\nu} \tilde{F}^{\mu\nu}$$

where $F_{\mu\nu}$ is one of $G_{\mu\nu}$, $W_{\mu\nu}$ or $B_{\mu\nu}$. However, there are also two terms that involve different gauge fields,

$$(H^\dagger I^a H) W^a_{\mu\nu} B^{\mu\nu}$$

$$(H^\dagger I^a H) \tilde{W}^a_{\mu\nu} B^{\mu\nu}$$

We will not consider any operators that only involve gauge fields.

Of the operators we have listed, two are very tightly constrained by experiment, and therefore very important. These are

$$O_S = (H^\dagger \tau^a H) W^a_{\mu\nu} B^{\mu\nu}$$

$$O_T = |(H^\dagger D_\mu H)|^2$$

where τ^a are the Pauli matrices. When H is replaced by its VEV, the operator O_S induces mixing between the photon and Z of the SM. The operator O_T violates the "custodial symmetry" of the SM that ensures the tree level relation between the W and Z boson masses, $m_W = m_Z \cos\theta_W$. When H is replaced by its VEV, we obtain a correction to the Z mass but not to the W mass.

The other dimension six terms we have listed are not strongly constrained by the data. When the Higgs is replaced by its VEV, their only effect is to alter the values of parameters that are already present in the SM. Although these operators affect the properties of the Higgs, at present they are only weakly constrained.

We therefore consider the SM Lagrangian to which we add the two operators O_S and O_T

$$\mathcal{L} = \mathcal{L}_{SM} + a_S O_S + a_T O_T$$

There is a one-to-one correspondence between the operators O_S and O_T and the S and T parameters of Peskin and Takeuchi, in the basis that we are employing. The S and T parameters are related to the coefficients a_S and a_T as

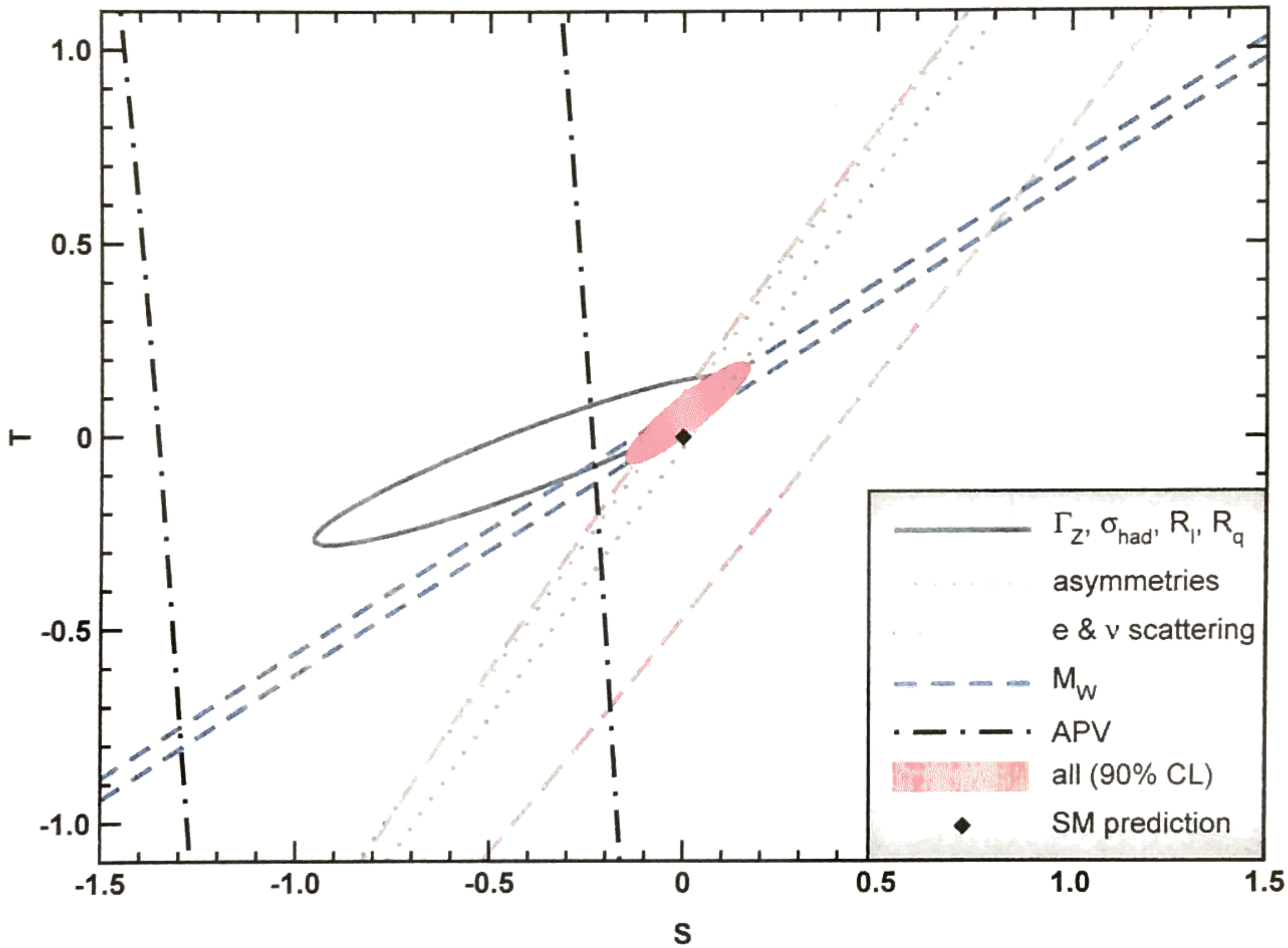
$$S = \frac{4 \sin\theta_W \cos\theta_W \, v^2}{\alpha} \, a_S$$

$$T = - \frac{v^2}{2\alpha} \, a_T$$

where $v = 246$ GeV is the Higgs vev and θ_W is the Weinberg angle. α is the fine structure constant.

Peskin and Takeuchi also introduced a third parameter called U. In theories with a light Higgs this corresponds to a dimension 8 operator, and so we do not consider it.

The experimentally allowed range of the S and T parameters is shown in the figure. The region inside the ellipse is consistent with the data.



1.0
0.5
0
-0.5
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T
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-1.0
-0.5
0
0.5
1.0
1.5
S
Γ_Z, σ_{had}, R_l, R_q
asymmetries
e & ν scattering
M_W
APV
all (90% CL)
SM prediction