New results on localizing SFT effective actions

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A tale of two stories

1. Can we generalize the story of tree-level bosonic OSFT effective action with gauge-invariant sourcing to open superstring field theory?

   Can we actually evaluate the corresponding tree-level on-shell open-closed amplitudes?

   [see also Yuji's and Harold's talks on Monday]

2. Can we compute the tree-level quartic effective potential in heterotic Yang-Mills from heterotic SFT without having to know the closed string bosonic quartic vertex?

   [Erbin, Maccaferri, JV: 1912.05463]

Gauge-invariants for A_∞ SFTs

General A_∞ SFT: given a cyclic A_∞ structure $(\mathcal{H}, \{m_k\}_{k=1}^{\infty}, \omega)$, we have

$$S(\Psi) = \sum_{k=1}^{\infty} \frac{1}{k+1} \omega(\Psi, m_k(\Psi^{\otimes k}))$$

$$\sum_{l=1}^{k-1} m_l m_{k-l} = 0$$

$$\omega(\Psi_1, m_{k-1}(\Psi_2, \ldots, \Psi_k)) = = -(-1)^{d(\Psi_1)} \omega(m_{k-1}(\Psi_1, \ldots, \Psi_{k-1}), \Psi_k)$$

[see Hiroshige Kajiura's talk for intro]

Tensor coalgebra language: [Gaberdiel, Zwiebach: hep-th/9705038; Erler: 1505.02069]

$$S(\Psi) = \int_0^1 dt\, \langle \omega | \pi_1 \boldsymbol{\partial}_t \frac{1}{1-\Psi(t)} \otimes \pi_1 \mathbf{m} \frac{1}{1-\Psi(t)}$$

$$\mathbf{m}^2 = 0$$

$$\langle \omega | \pi_2 \mathbf{m} = 0$$

where $\Psi(0) \equiv 0$, $\Psi(1) \equiv \Psi$ and $\mathbf{m} \equiv \mathbf{m}_1 + \mathbf{m}_2 + \ldots$

Gauge transformation: given an odd cyclic coderivation $\boldsymbol{\Lambda}$, we have

$$\delta_{\text{gauge}} \Psi = \pi_1 [\mathbf{m}, \boldsymbol{\Lambda}] \frac{1}{1-\Psi}$$

[see e.g. Erler: 1610.03251]

Gauge-invariants: for an odd cyclic coderivation $\mathbf{e}$,

$$\mathcal{E}(\Psi) \equiv \int_0^1 dt\, \langle \omega | \pi_1 \boldsymbol{\partial}_t \frac{1}{1-\Psi(t)} \otimes \pi_1 \mathbf{e} \frac{1}{1-\Psi(t)}$$

is gauge-invariant (up to pieces that vanish on-shell) whenever $[\mathbf{e}, \mathbf{m}] = 0$

Ellwood invariant in cubic OSFT: a small recap (1)

Cubic OSFT action augmented by Ellwood invariant:

[see Harold's and Yuji's talks for intro]

$$\mathfrak{S}(\Psi;\mu) = \mu\omega(\Psi, e_0) + \frac{1}{2}\omega(\Psi, m_1(\Psi)) + \frac{1}{3}\omega(\Psi, m_2(\Psi,\Psi))$$

e_0	midpoint insertion of an on-shell $(h,\bar{h}) = (0,0)$ closed string primary state on identity string field [Hashimoto, Itzhaki: hep-th/0111092; Gaiotto, Rastelli, Sen, Zwiebach: hep-th/0111129]
m_1, m_2	BRST operator Q, Witten's star product
ω	BPZ inner product (a.k.a. symplectic form)

Tensor coalgebra language:

$$\mathfrak{S}(\Psi;\mu) = \int_0^1 dt\, \langle\omega|\pi_1 \boldsymbol{\partial}_t \frac{1}{1-\Psi(t)} \otimes \pi_1 \mathfrak{m}(\mu) \frac{1}{1-\Psi(t)}$$

where $\mathfrak{m}(\mu) \equiv \mathbf{m} + \mu\mathbf{e}$ with $\mathbf{m} \equiv \mathbf{m}_1 + \mathbf{m}_2$, $\mathbf{e} \equiv \mathbf{e}_0$ and

$$0 = \mathbf{m}^2$$

$$0 = [\mathbf{m}, \mathbf{e}] \qquad (\implies \text{gauge-invariant } \mathcal{E}(\Psi))$$

$$0 = \mathbf{e}^2$$

so that $\mathfrak{m}(\mu)^2 = \mathbf{m}^2 + \mu[\mathbf{m},\mathbf{e}] + \mu^2\mathbf{e}^2 = 0 \quad \implies$ (special) weak A_∞ struct.

Ellwood invariant in cubic OSFT: a small recap (2)

Tree-level effective dynamics:

[Kajiura: math/0306332; Sen: 1609.00459; Yuji's talk]

1. split $\Psi \in \mathcal{H}$ using a projector $P_0 \equiv I_0 \Pi_0$ (such that im $P_0 =$ ker L_0) as

$$\Psi \equiv P_0 \Psi + (1 - P_0)\Psi \equiv \psi + R \,, \qquad b_0 R = 0$$

here $I_0 : P_0\mathcal{H} \to \mathcal{H}$ and $\Pi_0 : \mathcal{H} \to P_0\mathcal{H}$ are the canonical projection and inclusion

2. integrate R out using the propagator $h_0 \equiv -(b_0/L_0)\bar{P}_0$ satisfying the HK decomposition + annihilation conditions (→ SDR [Harold's & Hiroshige's talks])

$$Qh_0 + h_0 Q = P_0 - 1 \,, \qquad \Pi_0 h_0 = h_0 I_0 = (h_0)^2 = 0$$

Case $\mu = 0$: eff. A_∞ structure

[Konopka: 1507.08250; Erler: 1610.03251; Matsunaga: 1901.08555]

$$\tilde{\mathbf{m}} \equiv \tilde{\mathbf{\Pi}}_0 \mathbf{m} \tilde{\mathbf{I}}_0$$

→ $\mathbf{m}_2$-perturbed homotopy-equivalence data

$$\tilde{\mathbf{I}}_0 \equiv \frac{1}{\mathbf{1}_{T\mathcal{H}} - \mathbf{h}_0 \mathbf{m}_2} \mathbf{I}_0 \,, \quad \tilde{\mathbf{\Pi}}_0 \equiv \mathbf{\Pi}_0 \frac{1}{\mathbf{1}_{T\mathcal{H}} - \mathbf{m}_2 \mathbf{h}_0} \,, \quad \tilde{\mathbf{h}}_0 \equiv \frac{1}{\mathbf{1}_{T\mathcal{H}} - \mathbf{h}_0 \mathbf{m}_2} \mathbf{h}_0$$

→ unpackaged products: $\tilde{m}_k \equiv \pi_1 \tilde{\mathbf{m}} \pi_k$, where $\tilde{m}_1 = P_0 m_1$, $\tilde{m}_2 = P_0 m_2$,

$$\tilde{m}_3(\psi_1, \psi_2, \psi_3) = P_0 m_2(h_0 m_2(\psi_1, \psi_2), \psi_3) + P_0 m_2(\psi_1, h_0 m_2(\psi_2, \psi_3)) \,, \ldots$$

[Kajiura: hep-th/0112228]

Ellwood invariant in cubic OSFT: a small recap (3)

Cyclicity: BPZ properties of $h_0 \implies \tilde{\mathbf{m}}, \tilde{\mathbf{I}}_0$ cyclic w.r.t. $\langle\tilde{\omega}|\pi_2 \equiv \langle\omega|\pi_2\mathbf{I}_0$

Effective Ellwood invariant: defining

$$\tilde{\mathbf{e}} \equiv \tilde{\mathbf{\Pi}}_0\mathbf{e}\tilde{\mathbf{I}}_0 \quad \implies \quad \tilde{\mathcal{E}}(\psi) = \int_0^1 dt\, \langle\tilde{\omega}|\pi_1\boldsymbol{\partial}_t \frac{1}{1-\psi(t)} \otimes \pi_1\tilde{\mathbf{e}}\frac{1}{1-\psi(t)}$$

can show $[\tilde{\mathbf{e}}, \tilde{\mathbf{m}}] = 0 \implies \tilde{\mathcal{E}}(\psi)$ gauge-invariant for the effective SFT at $\mu = 0$, given by cyclic products $\tilde{e}_k \equiv \pi_1\tilde{\mathbf{e}}\pi_k$, where $\tilde{e}_0 = P_0 e_0$ and for $k > 0$

$$\tilde{e}_k(\psi^{\otimes k}) = \sum_{l=0}^{k-1} \tilde{m}_k(\psi^{\otimes l}, h_0 e_0, \psi^{\otimes k-1-l})$$

Case $\mu \neq 0$: effective weak cyclic A_∞ structure using "vertical composition"
[see Yuji's and Harold's talks]

$$\tilde{\mathfrak{m}}(\mu) \equiv \tilde{\mathbf{m}} + \mu\tilde{\mathbf{e}} + \sum_{\alpha=2}^{\infty} \mu^\alpha \tilde{\mathbf{\Pi}}_0\mathbf{e}(\tilde{\mathbf{h}}_0\mathbf{e})^{\alpha-1}\tilde{\mathbf{I}}_0$$

→ insufficient to deform $\tilde{\mathbf{m}}$ by $\mu\tilde{\mathbf{e}}$ since $(\tilde{\mathbf{m}} + \mu\tilde{\mathbf{e}})^2 \neq 0$ (because $\tilde{\mathbf{e}}^2 \neq 0$)

→ need to consider amplitudes with arbitrary number of (on-shell) closed strings

→ unpackaged products $\tilde{\mathfrak{m}}_k \equiv \pi_1\tilde{\mathfrak{m}}\pi_k$ (except $\tilde{\mathfrak{m}}_0$, which starts as $\mu P_0 e_0 + \dots$)

$$\tilde{\mathfrak{m}}_k(\psi^{\otimes k}) = \sum_{\alpha=0}^{\infty} \sum_{\sum_{i=1}^{\alpha+1} l_i = k} \mu^\alpha \tilde{m}_{k+\alpha}(\psi^{\otimes l_1}, h_0 e_0, \psi^{\otimes l_2}, \dots, \psi^{\otimes l_\alpha}, h_0 e_0, \psi^{\otimes l_{\alpha+1}})$$

[see also Masuda, Matsunaga: 2003.05021]

Ellwood invariant in cubic OSFT: a small recap (4)

Vacuum shift: true vac. $\Psi_{\rm v}(\mu) \equiv \sum_{\alpha=1}^{\infty} \mu^\alpha \Psi_\alpha$ of the full OSFT with Ellwood

$$\begin{aligned} \Psi_1 &= h_0 e_0 + \psi_1 \,, \\ \Psi_2 &= h_0 m_2(h_0 e_0 + \psi_1, h_0 e_0 + \psi_1) + \psi_2 \,, \\ &\vdots \end{aligned}$$

→ $\psi_\alpha \in \ker L_0$ possible corrections at each order [Sen: 1411.7478]

→ can be obstructed ($\Longrightarrow$ BCFT unable to adapt to μ-deformation)

$$\begin{aligned} O_1 &\equiv P_0 e_0 + P_0 m_1(\psi_1) \,, \\ O_2 &\equiv P_0 m_2(h_0 e_0 + \psi_1, h_0 e_0 + \psi_1) + P_0 m_1(\psi_2) \,, \\ &\vdots \end{aligned}$$

Coalgebra description: denoting $\psi_{\rm v}(\mu) \equiv \sum_{\alpha=1}^{\infty} \mu^\alpha \psi_\alpha$, we obtain

$$\begin{aligned} \sum_{\alpha=1}^{\infty} \mu^\alpha \Psi_\alpha &= \pi_1 \frac{1}{\mathbf{1}_{T\mathcal{H}} - \mathbf{h}_0(\mu \mathbf{e} + \mathbf{m}_2)} \frac{1}{1 - \psi_{\rm v}(\mu)} \,, \\ \sum_{\alpha=1}^{\infty} \mu^\alpha O_\alpha &= \pi_1 \tilde{\mathbf{m}}(\mu) \frac{1}{1 - \psi_{\rm v}(\mu)} \,, \end{aligned}$$

→ corrections $\psi_k \in \ker L_0$ determine the true vacuum $\psi_{\rm v}(\mu)$ of the eff. SFT

→ full SFT obstructions O_α coincide with the EOMs for $\psi_{\rm v}(\mu)$

→ in most examples $\psi_\alpha = 0 \implies$ obstructions O_α determine the eff. tadpole

Munich A_∞ open SFT: NS sector (1)

Action and products: [Erler, Konopka, Sachs: 1312.2948]

$$S(\Psi) = \int_0^1 dt\, \langle \omega_{\mathsf{S}} | \pi_1 \boldsymbol{\partial}_t \frac{1}{1-\Psi(t)} \otimes \pi_1 \mathbf{M} \frac{1}{1-\Psi(t)}$$

→ define $\mathbf{M} \equiv \sum_{n=0}^{\infty} \mathbf{M}_{n+1}^{(n)}$ where $\#_{\mathsf{pic}}(M_{n+1}^{(n)}) = n$, $\#_{\mathsf{gh}}(M_{n+1}^{(n)}) = 1-n$

→ start with bosonic products $M_1^{(0)} \equiv Q$, $M_2^{(0)} \equiv m_2$ and $M_k^{(0)} \equiv 0$ for $k > 2$, define recursively

$$\mathbf{M}_{n+1}^{(n-1)} \equiv \frac{1}{n-1} \Big([\mathbf{M}_2^{(0)}, \boldsymbol{\mu}_n^{(n-1)}] + [\mathbf{M}_3^{(1)}, \boldsymbol{\mu}_{n-1}^{(n-2)}] + \ldots + [\mathbf{M}_n^{(n-2)}, \boldsymbol{\mu}_2^{(1)}] \Big) \,,$$

$$\mu_{n+1}^{(n)} \equiv \frac{1}{n+2} \Big(\xi_0 M_{n+1}^{(n-1)} - M_{n+1}^{(n-1)} \sum_{k=0}^{n} 1^{\otimes k} \otimes \xi_0 \otimes 1^{\otimes n-k} \Big) \,,$$

$$\mathbf{M}_{n+1}^{(n)} \equiv \frac{1}{n} \Big([\mathbf{M}_1^{(0)}, \boldsymbol{\mu}_{n+1}^{(n)}] + [\mathbf{M}_2^{(1)}, \boldsymbol{\mu}_n^{(n-1)}] + \ldots + [\mathbf{M}_n^{(n-1)}, \boldsymbol{\mu}_2^{(1)}] \Big) \,,$$

→ both $\mathbf{M}_n^{(p)}$ and the gauge products $\boldsymbol{\mu}_n^{(p)}$ are cyclic

Munich A_∞ open SFT: NS sector (2)

Generating functions: [see Hiroshi Kunitomo's talk for the heterotic L_∞ version]

$$\mathbf{M}(s,t) \equiv \sum_{n=0}^{\infty} t^n (\mathbf{M}_{n+1}^{(n)} + s\mathbf{M}_{n+2}^{(n)})\,, \qquad \boldsymbol{\mu}(t) \equiv \sum_{n=0}^{\infty} t^n \boldsymbol{\mu}_{n+2}^{(n+1)}$$

→ satisfy differential equations (with ICs $\mathbf{M}(1,0) = \mathbf{m}$ and $\mathbf{M}(0,1) = \mathbf{M}$)

$$\frac{\partial}{\partial t}\mathbf{M}(s,t) = [\mathbf{M}(s,t), \boldsymbol{\mu}(t)]\,, \qquad \frac{\partial}{\partial s}\mathbf{M}(s,t) = [\boldsymbol{\eta}, \boldsymbol{\mu}(t)]\,,$$

→ these imply $\mathbf{M}^2 = [\boldsymbol{\eta}, \mathbf{M}] = 0$

→ can solve to obtain

$$\mathbf{M} = \mathbf{G}^{-1}\mathbf{Q}\mathbf{G}$$

where we have introduced cyclic cohomomorphism

$$\mathbf{G} \equiv \overleftarrow{\mathcal{P}}_t \exp\left(\int_0^1 dt\, \boldsymbol{\mu}(t)\right)$$

Ellwood invariant for Munich A_∞ open SFT

Definition: starting with $e_0 \in \mathcal{H}$ given by a local midpoint insertion of a weight $(0,0)$ on-shell bulk primary with $\#_{\text{gh}} = 2$, $\#_{\text{pic}} = -1$, we define higher products

$$\mathbf{E}_0 \equiv \mathbf{e}_0 \,, \qquad \mathbf{E}_k \equiv \frac{1}{k}\left([\mathbf{E}_0, \boldsymbol{\mu}_{k+1}^{(k)}] + \ldots + [\mathbf{E}_{k-1}, \boldsymbol{\mu}_2^{(1)}] \right)$$

Generating function:

$$\mathbf{E}(t) \equiv \sum_{k=0}^{\infty} t^k \mathbf{E}_k \,, \qquad \frac{\partial}{\partial t}\mathbf{E}(t) = [\mathbf{E}(t), \boldsymbol{\mu}(t)]$$

→ implies $[\mathbf{E}, \mathbf{M}] = \mathbf{E}^2 = [\boldsymbol{\eta}, \mathbf{E}] = 0$, also $\mathbf{E}$ manifestly cyclic

→ can solve to obtain $\mathbf{E} = \mathbf{G}^{-1}\mathbf{e}_0\mathbf{G}$

→ gauge-invariant quantity (up to pieces that vanish on-shell)

$$\begin{aligned}
\mathcal{E}(\Psi) &\equiv + \int_0^1 dt\, \langle \omega_{\mathsf{S}} | \pi_1 \boldsymbol{\partial}_t \frac{1}{1-\Psi(t)} \otimes \pi_1 \mathbf{E} \frac{1}{1-\Psi(t)} \\
&= + \int_0^1 dt\, \langle \omega_{\mathsf{L}} | \pi_1 \mathbf{G} \boldsymbol{\xi}_t \frac{1}{1-\Psi(t)} \otimes \pi_1 \mathbf{e}_0 \\
&= - \int_0^1 dt\, \langle \omega_{\mathsf{L}} | \tilde{A}_t (\xi_0 \tilde{\Psi}(t)) \otimes e_0 \qquad \text{[Erler: 1308.4400]}
\end{aligned}$$

⟹ t-Ellwood invariant of WZW-like SFT (using the field redefinition of [Erler, Okawa, Takezaki: 1505.01659; very useful discussions with Ted Erler])

Effective Munich SFT with Ellwood invariant (1)

Completely parallel to the bosonic OSFT case. Differences:

$\rightarrow$ $M_k \neq 0$ for $k > 2$

$\rightarrow$ $E_k \neq 0$ for $k > 0$

μ-deformed products: $\mathfrak{M}(\mu) \equiv \mathbf{M} + \mu \mathbf{E}$

$$\mathfrak{M}(\mu)^2 = \mathbf{M}^2 + \mu[\mathbf{E}, \mathbf{M}] + \mu^2 \mathbf{E}^2 = 0 \qquad \Longrightarrow \qquad \text{weak } A_\infty$$

Eff. structure at $\mu = 0$: cyclic A_∞ structure

$$\tilde{\mathbf{M}} \equiv \tilde{\mathbf{\Pi}}_0 \mathbf{M} \tilde{\mathbf{I}}_0$$

$\rightarrow$ $\delta\mathbf{M}$-perturbed homotopy-equivalence data ($\delta\mathbf{M} \equiv \sum_{k=2}^{\infty} \mathbf{M}_k$)

$$\tilde{\mathbf{I}}_0 \equiv \frac{1}{\mathbf{1}_{T\mathcal{H}} - \mathbf{h}_0 \delta\mathbf{M}} \mathbf{I}_0 \,, \quad \tilde{\mathbf{\Pi}}_0 \equiv \mathbf{\Pi}_0 \frac{1}{\mathbf{1}_{T\mathcal{H}} - \delta\mathbf{M} \mathbf{h}_0} \,, \quad \tilde{\mathbf{h}}_0 \equiv \frac{1}{\mathbf{1}_{T\mathcal{H}} - \mathbf{h}_0 \delta\mathbf{M}} \mathbf{h}_0$$

Eff. structure at $\mu \neq 0$: weak cyclic A_∞ structure ($\tilde{\mathbf{E}} \equiv \mathbf{\Pi}_0 \mathbf{E} \mathbf{I}_0$ & $[\tilde{\mathbf{E}}, \tilde{\mathbf{M}}] = 0$)

$$\tilde{\mathfrak{M}}(\mu) \equiv \tilde{\mathbf{M}} + \mu \tilde{\mathbf{E}} + \sum_{\alpha=2}^{\infty} \mu^\alpha \tilde{\mathbf{\Pi}}_0 \mathbf{E} (\tilde{\mathbf{h}}_0 \mathbf{E})^{\alpha-1} \tilde{\mathbf{I}}_0$$

Vacuum shift: obstructions to the full SFT vac. shift given by the equations of motion for the true vacuum of the effective SFT (see the cubic case for details)

Effective Munich SFT with Ellwood invariant (2)

Effective action:

$$\tilde{\mathfrak{S}}(\psi;\mu) = \int_0^1 dt\, \langle\tilde{\omega}|\pi_1 \boldsymbol{\partial}_t \frac{1}{1-\psi(t)} \otimes \pi_1 \tilde{\boldsymbol{\mathfrak{M}}}(\mu) \frac{1}{1-\psi(t)}$$

Double expansion of couplings:

$$\tilde{\mathfrak{S}}(\psi;\mu) = \sum_{k=0}^{\infty} \sum_{\alpha=0}^{\infty} \underbrace{\frac{1}{k+1} \mu^\alpha \tilde{\omega}(\psi, \tilde{\mathfrak{M}}_{k\alpha}(\psi^{\otimes k}))}_{\equiv \tilde{\mathfrak{S}}_{k\alpha}}$$

→ $k+1$ counts the number of open-string insertions
→ α counts the number of (on-shell) closed-string insertions

$$\tilde{\mathfrak{M}}_{k0} = \tilde{M}_k\,, \qquad \tilde{\mathfrak{M}}_{k\alpha} \equiv \pi_1 \tilde{\mathbf{\Pi}}_0 \mathbf{E} (\tilde{\mathbf{h}}_0 \mathbf{E})^{\alpha-1} \tilde{\mathbf{I}}_0 \pi_k \quad \text{for } \alpha > 0$$

Low-order expressions: have $\tilde{\mathfrak{M}}_{00} = 0$, $\tilde{\mathfrak{M}}_{10} = P_0 M_1$ and

$\tilde{\mathfrak{M}}_{01}(\psi^{\otimes 0}) = P_0 e_0$ → tadpole at $\mathcal{O}(\mu^1)$

$\tilde{\mathfrak{M}}_{11}(\psi^{\otimes 1}) = P_0 e_1(\psi) + P_0 M_2(h_0 e_0, \psi) + P_0 M_2(\psi, h_0 e_0)$ → mass-shift at $\mathcal{O}(\mu^1)$

$\tilde{\mathfrak{M}}_{20}(\psi^{\otimes 2}) = P_0 M_2(\psi, \psi)$ → cubic coupling at $\mathcal{O}(\mu^0)$

$\tilde{\mathfrak{M}}_{30}(\psi^{\otimes 3}) = P_0 M_3(\psi^{\otimes 3}) + P_0 M_2(\psi, h_0 M_2(\psi,\psi)) + P_0 M_2(h_0 M_2(\psi,\psi), \psi)$

→ quartic coupling at $\mathcal{O}(\mu^0)$

$\mathcal{N} = 2$ localization (1)

Closed-string state:

$$e_0 = \varepsilon_{ij} c\bar{c}(X_0 + \bar{X}_0)\mathbb{U}^i_{\frac{1}{2}}\bar{\mathbb{U}}^j_{\frac{1}{2}} e^{-\phi-\bar{\phi}}(i)I \qquad (e_0 \text{ is on-shell})$$

Open-string state:

$$\psi = c\mathbb{V}_{\frac{1}{2}} e^{-\phi} \qquad \Longrightarrow \qquad 0 = \tilde{\mathfrak{M}}_{10}(\psi^{\otimes 1}) \equiv Q\psi \quad (\psi \text{ is on-shell})$$

Assume from now on that the background at hand supports a global $\mathcal{N} = 2$ worldsheet superconformal symmetry. Both ψ and e_0 at zero momentum.

R-charge decomposition: [Sen: 1508.02481; Maccaferri, Merlano: 1801.07607]

$$\mathbb{V}_{\frac{1}{2}} = \mathbb{V}^+_{\frac{1}{2}} + \mathbb{V}^-_{\frac{1}{2}}\,,$$
$$\mathbb{U}^i_{\frac{1}{2}} = (\mathbb{U}^i_{\frac{1}{2}})^+ + (\mathbb{U}^i_{\frac{1}{2}})^-\,,$$
$$\bar{\mathbb{U}}^j_{\frac{1}{2}} = (\bar{\mathbb{U}}^j_{\frac{1}{2}})^+ + (\bar{\mathbb{U}}^j_{\frac{1}{2}})^-\,,$$

Projector conditions: $\mathcal{N} = 2$ worldsheet SCA implies that

[Maccaferri, Merlano: 1801.07607, 1905.04958; Mattiello, Sachs: 1902.10955; JV: 1910.00538]

$$0 = \tilde{\mathfrak{M}}_{20}(\psi^{\otimes 2}) \equiv P_0 M_2(\psi, \psi)\,, \qquad (\to \text{no eff. cubic coupling at } \mathcal{O}(\mu^0))$$
$$0 = \tilde{\mathfrak{M}}_{01}(\psi^{\otimes 0}) \equiv P_0 e_0\,, \qquad (\to \text{eff. action tadpole-free at } \mathcal{O}(\mu^1))$$

$\mathcal{N}=2$ localization (2)

Auxiliary fields: bulk-boundary and boundary OPEs

$$\mathbb{G}_1^{\pm} \equiv \varepsilon_{ij} \lim_{z\to\bar z} \left[(\mathbb{U}^i_{\frac12})^{\pm}(z)(\bar{\mathbb{U}}^j_{\frac12})^{\pm}(\bar z) \right] ,$$

$$\mathbb{G}_0 \equiv \varepsilon_{ij} \lim_{z\to\bar z} \left[2z \left((\mathbb{U}^i_{\frac12})^{-}(z)(\bar{\mathbb{U}}^j_{\frac12})^{+}(\bar z) - (\mathbb{U}^i_{\frac12})^{+}(z)(\bar{\mathbb{U}}^j_{\frac12})^{-}(\bar z) \right) \right] ,$$

$$\mathbb{H}_1^{\pm} \equiv \lim_{z\to 0} \left[\mathbb{V}^{\pm}_{\frac12}(z)\mathbb{V}^{\pm}_{\frac12}(-z) \right] ,$$

$$\mathbb{H}_0 \equiv \lim_{z\to 0} \left[2z \left(\mathbb{V}^{-}_{\frac12}(z)\mathbb{V}^{+}_{\frac12}(-z) - \mathbb{V}^{+}_{\frac12}(z)\mathbb{V}^{-}_{\frac12}(-z) \right) \right] ,$$

$\tilde{\mathfrak{S}}_{30}$ coupling: use the results of [Maccaferri, Merlano: 1801.07607]

$$\tilde{\mathfrak{S}}_{30} = \langle \mathbb{H}_1^{+} | \mathbb{H}_1^{-} \rangle + \frac14 \langle \mathbb{H}_0 | \mathbb{H}_0 \rangle \qquad (\to \text{quartic eff. potential at } \mathcal{O}(\mu^0))$$

$\tilde{\mathfrak{S}}_{11}$ coupling: can use R-charge conservation and c-ghost saturation to show

$$\tilde{\mathfrak{S}}_{11} = 2\langle \mathbb{G}_1^{-} | \mathbb{H}_1^{+} \rangle + 2\langle \mathbb{G}_1^{+} | \mathbb{H}_1^{-} \rangle + \langle \mathbb{G}_0 | \mathbb{H}_0 \rangle \qquad (\to \text{mass-shift at } \mathcal{O}(\mu^1))$$

→ no integrations over bosonic moduli in $\tilde{\mathfrak{S}}_{30}$ and $\tilde{\mathfrak{S}}_{11}$

→ fully localized on the bdy of the moduli space (propagator → ∞-long strip)

$\mathcal{N}=2$ localization (3)

Leading-order effective potential:

$$\tilde{\mathfrak{V}} - \tilde{\mathfrak{V}}_{\text{min}} = -\langle \mathbb{H}_1^+ + 2\mu\mathbb{G}_1^+ | \mathbb{H}_1^- + 2\mu\mathbb{G}_1^- \rangle - \frac{1}{4}\langle \mathbb{H}_0 + 2\mu\mathbb{G}_0 | \mathbb{H}_0 + 2\mu\mathbb{G}_0 \rangle$$

→ terms on r.h.s. can be shown positive definite

→ true vacuum of the effective theory: $\psi_{\mathsf{v}}(\mu) = 0 + \mathcal{O}(\mu^2)$

→ global minima at

$$\mathbb{H}_1^\pm = -2\mu\mathbb{G}_1^\pm \,, \qquad \mathbb{H}_0 = -2\mu\mathbb{G}_0 \,, \qquad (\clubsuit)$$

→ depth of the global minima

$$\mathfrak{V}_{\text{min}} = 4\Big(\big\langle \mathbb{G}_1^+ \big| \mathbb{G}_1^- \big\rangle + \frac{1}{4}\big\langle \mathbb{G}_0 \big| \mathbb{G}_0 \big\rangle\Big)$$

$\Longrightarrow$ $\tilde{\mathfrak{V}}$ is an eff. tachyon potential for modes $\in \ker L_0$ which become relevant for $\mu \neq 0 \rightarrow$ condensation into a bound state with mass defect $\tilde{\mathfrak{V}}_{\text{min}}$

Example: D(-1)/D3 system in a B-field [level-trunc. → David: hep-th/0007235]

→ (♣) give non-commutative ADHM eqns [Nekrasov, Schwarz: hep-th/9802068]

→ $\tilde{\mathfrak{V}}_{\text{min}}$ reproduces the 1/4-BPS bound-state mass-defect computed from supersymmetry [Obers, Pioline: hep-th/9809039]

A tale of two stories

1. Can we generalize the story of tree-level bosonic OSFT effective action with gauge-invariant sourcing to open superstring field theory?

   Can we actually evaluate the corresponding tree-level on-shell open-closed amplitudes?

   [see also Yuji's and Harold's talks on Monday]

2. Can we compute the tree-level quartic effective potential in heterotic Yang-Mills from heterotic SFT without having to know the closed string bosonic quartic vertex?

   [Erbin, Maccaferri, JV: 1912.05463]

WZW-like heterotic SFT: NS sector

Expanded action: [Berkovits, Okawa, Zwiebach: hep-th/0406212, hep-th/0409018]

$$S(\Phi) = \frac{1}{2}\langle \eta_0 \Phi, Q\Phi \rangle + \frac{\kappa}{3!}\langle \eta_0 \Phi, [\Phi, Q\Phi] \rangle + \\ + \frac{\kappa^2}{4!}\Big(\langle \eta_0 \Phi, [\Phi, [\Phi, Q\Phi]] \rangle + \langle \eta_0 \Phi, [\Phi, Q\Phi, Q\Phi] \rangle \Big) + \mathcal{O}(\kappa^3)$$

→ Φ in the large Hilbert space, $\#_{\text{gh}}(\Phi) = +1$, $\#_{\text{pic}}(\Phi) = 0$

$$\Phi \in \underbrace{\text{SCFT}_{\text{m}}}_{c_{\text{m}}=15} \otimes \text{CFT}_{bc} \otimes \text{CFT}_{\beta\gamma} \otimes \underbrace{\overline{\text{CFT}}_{\text{m}}}_{\bar{c}_{\text{m}}=26} \otimes \overline{\text{CFT}}_{bc}$$

→ level-matching $b_0^- \Phi = L_0^- \Phi = 0$

→ inner product $\langle \Phi_1, \Phi_2 \rangle \equiv \langle \Phi_1 | c_0^- | \Phi_2 \rangle$

→ cyclic symmetric bosonic CSFT products $[\Phi_1, \dots, \Phi_k]$ satisfy L_∞ relations

Tree-level effective WZW-like SFT

Kernel of L_0^+: for all $\varphi \in \ker L_0^+$ have

$$\varphi = \varphi_A + \varphi_D + \text{unphysical fields}$$

→ $\varphi_A \equiv c\gamma^{-1}\mathcal{V}_{\frac{1}{2},1}\bar{c}$ with $\mathcal{V}_{\frac{1}{2},1} \equiv \varepsilon_{ik}\mathbb{V}^i_{\frac{1}{2}}\bar{\mathbb{W}}^k_1 \implies$ "usual" physical fields

→ $\varphi_D \equiv D\xi Y Q(\partial c - \bar{\partial}\bar{c})$ with $D \in \mathbb{R}$, Y the inverse PCO $\implies$ ghost dilaton

Effective action: for physical fields $\varphi_{\mathsf{p}} \equiv \varphi_A + \varphi_D \in \ker L_0^+$

$$S(\varphi_{\mathsf{p}}) = \frac{1}{2}\langle \eta_0\varphi_{\mathsf{p}}, Q\varphi_{\mathsf{p}}\rangle + \frac{1}{3!}\langle \eta_0\varphi_{\mathsf{p}}, [\varphi_{\mathsf{p}}, Q\varphi_{\mathsf{p}}]\rangle +$$
$$+ \frac{1}{4!}\Big(\langle \eta_0\varphi_{\mathsf{p}}, [\varphi_{\mathsf{p}}, [\varphi_{\mathsf{p}}, Q\varphi_{\mathsf{p}}]]\rangle + \langle \eta_0\Phi, [\varphi_{\mathsf{p}}, Q\varphi_{\mathsf{p}}, Q\varphi_{\mathsf{p}}]\rangle\Big) +$$
$$+ \frac{1}{8}\left\langle [\eta_0\varphi_{\mathsf{p}}, Q\varphi_{\mathsf{p}}], \frac{b_0^+}{L_0^+}\xi_0\overline{P}_0[\eta_0\varphi_{\mathsf{p}}, Q\varphi_{\mathsf{p}}]\right\rangle + \mathcal{O}(\varphi_{\mathsf{p}}^5)$$

→ we fixed Siegel gauge to integrate out modes outside $\ker L_0^+$

→ out-of-Siegel equations trivialized by EOM for φ [see Harold's talk]

→ unphysical fields in $\ker L_0^+$ decouple up to quartic order

$\mathcal{N} = 2$ localization (1)

Assume global $\mathcal{N} = 2$ worldsheet superconf. symm. in the holomorphic sector.

Effective couplings at zero momentum for φ_A only (so that $\eta_0 Q \varphi_A = 0$).

R-charge decomposition: $\varphi_A \equiv \varepsilon_{ik} c \gamma^{-1} \mathbb{V}^i_{\frac{1}{2}} \bar{\mathbb{W}}^k_1 \bar{c}$ where

$$\mathbb{V}^i_{\frac{1}{2}} = (\mathbb{V}^i_{\frac{1}{2}})^+ + (\mathbb{V}^i_{\frac{1}{2}})^-$$
$$\varphi_A = \varphi^+ + \varphi^-$$

Projector condition: $\mathcal{N} = 2$ global SCA implies

$$P_0[\eta_0 \varphi_A, Q\varphi_A] = 0 \qquad (\rightarrow \text{zero eff. cubic coupling})$$

Quartic eff. coupling: R-charge conservation & c-ghost saturation $\Longrightarrow$

$$\tilde{S}_4(\varphi_A) = -\frac{1}{8}\left\langle [\eta_0 \varphi^-, \varphi^-], P_0[\varphi^+, Q\varphi^+] \right\rangle - \frac{1}{8}\left\langle [\eta_0 \varphi^+, \varphi^+], P_0[\varphi^-, Q\varphi^-] \right\rangle$$
$$- \frac{1}{8}\left\langle [\varphi^-, \varphi^+], P_0[\eta_0 \varphi^-, Q\varphi^+] \right\rangle - \frac{1}{8}\left\langle [\varphi^+, \varphi^-], P_0[\eta_0 \varphi^+, Q\varphi^-] \right\rangle$$

$\rightarrow$ fully localized on the bdy of the moduli space (propagator $\rightarrow \infty$-long tube)

$\rightarrow$ fundamental bosonic quartic vertex drops out

$\mathcal{N}=2$ localization (2)

Auxiliary fields: bulk OPEs

$$\mathbb{H}^{\pm}_{1,1} = \lim_{\substack{z\to 0\\ \bar z\to 0}} \Big[(2\bar z)\mathcal{V}^{\pm}_{\frac{1}{2},1}(z,\bar z)\mathcal{V}^{\pm}_{\frac{1}{2},1}(-z,-\bar z)\Big]$$

$$\mathbb{H}_{0,1} = \lim_{\substack{z\to 0\\ \bar z\to 0}} \Big[\pm|2\bar z|^2\mathcal{V}^{\pm}_{\frac{1}{2},1}(z,\bar z)\mathcal{V}^{\mp}_{\frac{1}{2},1}(-z,-\bar z)\Big]$$

Leading-order effective potential:

$$\tilde V(\varphi_A) = -\frac{1}{4}\Big(\langle \mathbb{H}^{+}_{1,1}|\mathbb{H}^{-}_{1,1}\rangle + \langle \mathbb{H}_{0,1}|\mathbb{H}_{0,1}\rangle\Big) \qquad (\spadesuit)$$

$\rightarrow$ global minima at $\mathbb{H}^{\pm}_{1,1} = \mathbb{H}_{0,1} = 0 \implies$ moduli space for given background

Yang-Mills in flat 10d space:

$$\varphi_A = (g_{\mu\nu}+B_{\mu\nu})\xi c\psi^{\mu}e^{-\phi}\bar c\, i\bar\partial\bar X^{\nu} + A_{\mu i}\xi c\psi^{\mu}e^{-\phi}\bar c\bar J^{i}\,,$$

$\rightarrow$ $\bar J_i$ for $i=1,\ldots,\dim\mathfrak{g}$ are the $k=1$ affine KM currents for given heterotic gauge group $G=SO(32)\,, E_8\times E_8$

$\rightarrow$ ($\spadesuit$) becomes (with C the Dynkin index)

$$\tilde V(g,B,A) = \frac{1}{16}\frac{1}{C}\mathrm{tr}\,\big[[A_\mu,A_\nu][A^\mu,A^\nu]\big]$$

$\rightarrow$ algebraic YM coupling, no dependence on g, B

Conclusions and outlook

Summary:

$\rightarrow$ we defined a gauge-invariant quantity for the Munich A_∞ open SFT

$\rightarrow$ we used it to study deformations of open superstring field theory

$\rightarrow$ we examined efficient methods (localization) of evaluating certain low-order tree-level on-shell Neveu-Schwarz open-closed and heterotic amplitudes at zero momentum

Future directions:

$\rightarrow$ investigate the fate of the WZW-like structure at low energies

$\rightarrow$ localization: in the Ramond sector? at higher orders in perturbation theory? at loop level? for non-zero momentum? for ghost-dilaton couplings?

$\rightarrow$ effect of RR deformations on open superstring backgrounds?

$\rightarrow$ Ellwood invariant from a limit of open-closed superstring field theory?

Thank you!