Construction of Covariant Vertex Operators in the Pure Spinor Formalism

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(Based on work in collaboration with S. Chakrabarti and M. Verma)

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Plan of the talk

- **Three Parts**

  1. **Some facts and basic assumptions**
  2. **Illustration by re-derivation of unintegrated vertex operator at first massive level of open superstring**
  3. **Integrated vertex Operator and Generalization to all massive vertex operators .**

Part I

Some Facts and Assumptions

- Any string amplitude is of the form

$$\underbrace{\left(\int \prod_i d\tau_i\right)}_{\text{Moduli integration}} \left\langle V_1 \cdots (b_1, \mu_1) \cdots \left(\int dz_1 U_1\right) \cdots \right\rangle$$

- V_i, U_i are the unintegrated and integrated vertex operators respectively.
- b_i are b-ghosts inserted by using the μ_i the Beltrami differential.
- In the pure spinor formulation of superstrings, b have $\bar{\lambda}\lambda$ poles that provide divergences in $\bar{\lambda}\lambda \to 0$.
- Are there other sources for such divergences? Want to avoid them as much as possible.
- Yes and no.
- It depends on how we choose to express our vertex operators.

- The unintegrated vertex operators are found by solving for a ghost number 1 and confromal weight 0 object V via

$$QV = 0, \quad V \simeq V + Q\Omega$$

- Q is the BRST-charge and Ω characterize some freedom of choosing V.
- Ω can be used to eliminate the unphysical degrees of freedom (d.o.f).
- By unphysical d.o.f we mean e.g. superfluous d.o.f that can be eliminated by going to a special frame of reference.
- Is there a procedure that automatically takes care of Ω?
- Yes. Working exclusively with physical d.o.f, from the very beginning, implicitly assumes Ω has been taken care of.

III

- Consider

$$D_\alpha S = T_\alpha$$

- Above S and T_α are some superfields and D_α is super-covariant derivative.

- Can we strip off D_α from S?

- Yes, we can

$$S = -\frac{1}{m^2}(\not\gamma)^{\alpha\beta} D_\beta T_\alpha$$

- But, only for $m^2 \neq 0$.

Conclusions from slide I

- To avoid $\bar{\lambda}\lambda$ poles in V we work the in minimal gauge.
- In the pure spinor formalism no natural way to define integrated vertex operator.
- From the RNS formalism we know $U(z) = \oint dw b(w)V(z)$ or $QU = \partial V$ where ∂ is worldsheet derivative.
- First form uses b ghost explicitly so, can potentially give $\bar{\lambda}\lambda$ poles.
- Second form involves V and Q neither have such poles. We use this relation to solve for U.

Conclusion from slide II

We know the physical d.o.f at any mass level from RNS formalism.

Conclusions from slide III

We saw

$$D_\alpha S = T_\alpha \implies S = -\frac{1}{m^2}(\not{\gamma})^{\alpha\beta} D_\beta T_\alpha$$

We shall assume this kind of inversion is always possible. Hence, our analysis is valid for all massive states.

The Pure Spinor Formalism

- The action in the in $10\ d$ flat spacetime (for left movers) [Berkovits, 2000]

$$S = \frac{1}{2\pi\alpha'} \int d^2z \, [\underbrace{\partial X^m \bar{\partial} X_m + p_\alpha \bar{\partial}\theta^\alpha}_{Matter} + \underbrace{w_\alpha \bar{\partial}\lambda^\alpha}_{Ghost}]$$

- (X^m, θ^α) form $\mathcal{N} = 1$ supersapce in $10\ d$.

- To keep spacetime SUSY manifest, we work with supersymmetic momenta

$$\begin{aligned} \Pi^m &= \partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta) \\ d_\alpha &= p_\alpha - \frac{1}{2}\partial X^m(\gamma_m\theta)_\alpha - \frac{1}{8}(\gamma_m\theta)_\alpha(\theta\gamma^m\partial\theta) \end{aligned}$$

- λ^α satisfies the pure spinor constraint

$$\lambda\gamma^m\lambda = 0 \quad \underset{Trans}{\overset{Gauge}{\Longrightarrow}} \quad \delta_\epsilon w_\alpha = \epsilon_m(\gamma^m\lambda)_\alpha$$

- To keep Gauge invariance manifest, instead of w_α, we work with

$$J = (w\lambda) \quad \text{and} \quad N^{mn} = \frac{1}{2}(w\gamma^{mn}\lambda)$$

The Pure Spinor Formalism

- The vertex operators come in two varieties unintegrated and integrated vertex V and U respectively.

- The physical states lie in the cohomology of the BRST charge Q with ghost number 1 and zero conformal weight

$$Q \equiv \oint dz \lambda^{\alpha}(z) d_{\alpha}(z) \quad \rightarrow \quad QV = 0, \quad V \sim V + Q\Omega, \quad QU = \partial V$$

- We shall take the vertex operators in the plane wave basis

$$V := \hat{V} e^{ik.X} \quad , \quad U := \hat{U} e^{ik.X}$$

- $\hat{V}$ has conformal weight n and $\hat{U}$ has conformal weight $n+1$ as $[e^{ik.X}] = \alpha' k^2 = -n$ at n^{th} excited level of open strings.

Important Identity

$$I \equiv : N^{mn} \lambda^{\alpha} : (\gamma_m)_{\alpha\beta} - \frac{1}{2} : J \lambda^{\alpha} : \gamma^n_{\alpha\beta} - \alpha' \gamma^n_{\alpha\beta} \partial \lambda^{\alpha} = 0$$

I. Pure Spinor Formalism - Important OPE's

- Some OPE's which we shall require are (V is arbitrary superfield)

$$d_\alpha(z)d_\beta(w) = -\frac{\alpha'}{2(z-w)}\gamma^m_{\alpha\beta}\Pi_m(w) + \cdots$$ where $\cdots$ are non-singular pieces of OPE.

$$d_\alpha(z)V(w) = \frac{\alpha'}{2(z-w)}D_\alpha(w) + \cdots$$ where, $D_\alpha \equiv \frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\alpha\partial_m$

Part II

Unintegrated Vertex Operator at $m^2 = \frac{1}{\alpha'}$

Construction of Vertex Operators

- States are zero weight conformal primary operators lying in the BRST cohomology

- **Goal**: Find an algorithm to compute conformal primary, zero weight operators appearing at 1st excited level of superstring.

- **In other words**: Solve for $[V] = 0$ with ghost number 1 and $[U] = 1$ with ghost number 0 satisfying

$$QV = 0, \quad V \sim V + Q\Omega, \quad QU = \partial V$$

constructed out of

Field/Operator	Conformal Weight	Ghost Number
Π^m	1	0
d_α	1	0
$\partial\theta^\alpha$	1	0
N^{mn}	1	0
J	1	0
λ^α	0	1

States at the first excited level of open superstring

- The first unintegrated massive vertex operator is known [Berkovits-Chandia,2002].

- We rederive it is to illustrate our methodology which can be generalized to construct any vertex operator [S. Chakrabarti thesis].

- At this level we have states of mass$^2 = \frac{1}{\alpha'}$ and they form a supermultiplet with 128 bosonic and 128 ferimonic d.o.f.

- The total 128 bosonic d.o.f are captured by a 2nd rank symmetric-traceless tensor g_{mn} and a three form field b_{mnp}

- g_{mn} and b_{mnp} satisfy

$$g_{mn} = g_{nm}, \quad \eta^{mn} g_{mn} = 0, \quad \partial^m g_{mn} = 0 \implies \mathbf{44}$$
$$b_{mnp} = -b_{nmp} = -b_{pnm} = -b_{mpn} = 0, \quad \partial^m b_{mnp} = 0 \quad \implies \mathbf{84}$$

- The fermionic d.o.f are captured by a tensor-spinor field $\psi_{m\alpha}$

$$\partial^m \psi_{m\alpha} = 0, \quad \gamma^{m\alpha\beta} \psi_{m\beta} = 0 \implies \mathbf{128}$$

Construction of Unintegrated Vertex Operator at First Massive level

- Recall our vertex operators are of the form

$$V = \hat{V} e^{ik.X}$$

- In rest of the talk we drop $e^{ik.X}$ and also for simplicity of notation drop the $\hat{}$ in $\hat{V}$

- At first excited level we need to solve for

$$QV = 0 \qquad \text{with} \quad [V] = 1, \quad \text{subject to} \quad V \sim V + Q\Omega$$

- The most general ghost number 1 and conformal weight zero operator is

$$\begin{aligned} V \quad = \quad & \partial\lambda^a A_a(X,\theta) + \lambda^\alpha \partial\theta^\alpha B_{\alpha\beta}(X,\theta) + d_\beta \lambda^\alpha C^\beta{}_\alpha(X,\theta) \\ & + \Pi^m \lambda_\alpha H_{ma}(X,\theta) + J\lambda^a E_\alpha(X,\theta) + N^{mn}\lambda^\alpha F_{\alpha mn}(X,\theta) \end{aligned}$$

- The superfields $A_\alpha, B_{\alpha\beta}, \cdots$ contain the spacetime fields.

- Ω can be used to eliminate all the gauge degrees of freedom and restrict the form of superfields in V e.g.

$$B_{\alpha\beta} = \gamma_{\alpha\beta}^{mnp} B_{mnp} \quad \text{i.e.} \quad 256 \to 120$$

- Berkovits-Chandia showed that if one solves $QV = 0$ subject to $V \simeq V + Q\Omega$, one finds the same states described earlier.
- We **assume** that we already know the spectrum at a given mass level.
- Our goal is not to show that pure spinor has same spectrum as that of NSR or GS formalisms.
- Our goal is find a (simple?) algorithm that gives covariant expressions for the vertex operators.
- Our strategy is to work directly with the physical superfields.
- In rest of the talk we shall see how do we can do this.

- Its important to note that if we have made complete use of Ω we shall be left with just physical fields.

- Introduce physical superfields corresponding to each physical field such that[1]

$$G_{mn}\Big|_{\theta=0} = g_{mn}, \quad B_{mnp}\Big|_{\theta=0} = b_{mnp}, \quad \Psi_{n\alpha}\Big|_{\theta=0} = \psi_{n\alpha}$$

- We further demand that other conditions satisfied by physical fields are also satisfied by the corresponding physical superfields. For example for g_{mn}

$$\begin{aligned} & g_{mn} = g_{nm}, \quad \eta^{mn} g_{mn} = 0, \quad \partial^m g_{mn} = 0 \\ \Longrightarrow \quad & G_{mn} = G_{nm}, \quad \eta^{mn} G_{mn} = 0, \quad \partial^m G_{mn} = 0 \end{aligned}$$

- For $\psi_{m\alpha}$

$$\partial^m \psi_{m\alpha} = 0, \quad \gamma^{m\alpha\beta}\psi_{m\beta} = 0 \quad \Longrightarrow \quad \partial^m \Psi_{m\alpha} = 0, \quad \gamma^{m\alpha\beta}\Psi_{m\beta} = 0$$

- For the 3-form field b_{mnp}

$$\partial^m b_{mnp} = 0 \quad \Longrightarrow \partial^m B_{mnp} = 0$$

[1] Apparently Rhenomic formulation of supersymmetric theories uses these ideas as pointed out to us by Ashoke few months back. We thank him for bringing this to notice.

- Next we expand all the unfixed superfields appearing in the unintegrated vertex operator as linear combination of the physical superfields $G_{mn}, B_{mnp}, \Psi_{m\alpha}$
- Lets take an example

$$F_{\alpha mn} = a_1 \; k_{[m}\Psi_{n]\alpha} + a_2 \; k^s \left(\gamma_{s[m}\Psi_{n]}\right)_\alpha$$

- To see if we have not missed anything we can do a rest frame analysis

$$F_{\alpha mn} = \begin{cases} F_{\alpha 0i} & \Longrightarrow \; \mathbf{16} \; \otimes \mathbf{9} = \mathbf{16} \oplus \mathbf{128} \\ F_{\alpha ij} & \Longrightarrow \; \mathbf{16} \; \otimes \mathbf{36} = \mathbf{16} \oplus \mathbf{128} \oplus \mathbf{432} \end{cases}$$

Hence, $F_{\alpha mn}$ is reducible to the following irreps.

$$\mathbf{16} \oplus \mathbf{128} + \mathbf{16} \oplus \mathbf{128} \oplus \mathbf{432}$$

- Thus, we have two physically relevant irreps $\mathbf{128}$ and we keep them.
- We throw away the unphysical d.o.f.

- We repeat this procedure for $A_\alpha, B_{\alpha\beta}, C^\beta_{\ \alpha}, E_\alpha$ and $H_{m\alpha}$ as well.
- Its absolutely trivial to see that A_α and E_α must vanish. Berkovits-Chandia find same conclusion after gauge fixing.
- We denote by a_i the coefficients that relate superfields in V to $G_{mn}, B_{mnp}, \Psi_{m\alpha}$.
- QV produces terms that contain the supercovariant derivatives

$$D_\alpha H_{m\alpha}, \quad D_\alpha B_{\beta\sigma}, \quad D_\alpha C^\beta{}_\sigma, \quad D_\alpha F_{\beta mn}$$

- But, all such terms are expressible in terms of the supercovariant derivatives of the physical superfields

$$D_\alpha G_{mn}, \quad D_\alpha B_{mnp} \quad \text{and} \quad D_\alpha \Psi_{m\beta}$$

e.g.

$$D_\alpha F_{\beta mn} = a_1\ k_{[m} D_\alpha \Psi_{n]\beta} + a_2\ k^s \left(\gamma_{s[m}\right)_\beta{}^\sigma D_\alpha \Psi_{n]\sigma}$$

- How do we determine $D_\alpha G_{mn}, D_\alpha B_{mnp}$ and $D_\alpha \Psi_{m\beta}$?

- Determination of the supercovariant derivative of physical superfields is our next major step.

- We employ the same strategy to write these in terms of physical superfields e.g.

$$D_\alpha \Psi_{m\beta} = b_1 \gamma^s_{\alpha\beta} G_{sm} + \gamma^{stu}_{\alpha\beta} \left(b_2 k_{[s} B_{tu]m} + b_3 k_m B_{stu} \right) + b_4 (\gamma_m{}^{stuv})_{\alpha\beta} k_s B_{tuv}$$

- Similarly for $D_\alpha G_{mn}$ and $D_\alpha B_{mnp}$.

- This introduces a fresh set of undermined constants $\{b_i\}$.

- Once again the e.o.m obtained by $QV = 0$ will determine these.

- There is one further complication that introduces a third set of undetermined coefficients we collectively denote by $\{c_i\}$.

- Not all of the operators in QV are independent e.g.

$$I^n_\beta \equiv N^{mn}\lambda^\alpha(\gamma_m)_{\alpha\beta} - \frac{1}{2}J\lambda^\alpha\gamma^n_{\alpha\beta} - \alpha'\gamma^n_{\alpha\beta}\partial\lambda^\alpha = 0$$

can be used to express some operators in terms of others.

- Notice that I^n_β is carries ghost number 1 and conformal weight 1.

- I^n_β generates constraints at various ghost number and conformal weights e.g.

$$N^{st}\lambda^\alpha\lambda^\beta\gamma_{s\beta\gamma} - \frac{1}{2}J\lambda^\alpha\lambda^\beta\gamma_{t\beta\gamma} - \frac{5\alpha'}{4}\lambda^\alpha\partial\lambda^\beta\gamma_{t\beta\gamma} - \frac{\alpha'}{4}\lambda^\gamma\partial\lambda^\beta(\gamma)^\alpha{}_\delta\gamma^s_{\beta\gamma} = 0$$

is at ghost number 2 and conformal weight 1.

- This can be written as

$$K \equiv -N_{st}\lambda^\alpha\lambda^\beta(\gamma^{vwxy}\gamma^{[s})_{\alpha\beta}K^{t]}_{vwxy} + J\lambda^\alpha\lambda^\beta(\gamma^{vwxy}\gamma_s)_{\alpha\beta}K^s_{vwxy}$$
$$+\alpha'\lambda^\alpha\partial\lambda^\beta\left[2\gamma^{vwxys}_{\alpha\beta}\eta_{st}K^t_{vwxy} + 16\gamma^{wxy}_{\alpha\beta}K^s_{wxys}\right] = 0$$

Relevant for this talk.

- We can re-express the Lagrange multiplier superfield in terms of the phsysical superfields

$$K_{mnpqr} = c_1 \ k_m k_{[n} B_{pqr]} + c_2 \ \eta_{m[n} B_{pqr]}$$

- Now we have expressed all unknown superfields and differential relations in terms of the physical superfield.

- Now we solve for

$$QV + K = 0$$

- We can now freely set the coefficients of each of the basis operators to zero because of the Lagrange multipliers.

- Now we get a set of algebraic equation involving the $\{a_i, b_i, c_i\}$.

- Solving these linear set of equations determines all the superfields appearing in the vertex operators, the Lagrange multipliers and the Differential relations in terms of the physical superfields.

Result - Unintegrated Vertex

- We find that the unintegrated vertex operator is writable as

$$V \;=: \partial\theta^\beta \lambda^\alpha B_{\alpha\beta} :+: d_\beta \lambda^\alpha C_\alpha^\beta :+: \Pi^m \lambda^\alpha H_{m\alpha} :+: N^{mn} \lambda^\alpha F_{\alpha mn} :$$

  where,

$$B_{\alpha\beta} = (\gamma^{mnp})_{\alpha\beta} B_{mnp} \quad ; \quad C_\alpha^\beta = (\gamma^{mnpq})_\alpha^\beta C_{mnpq} \quad ; \quad H_{m\alpha} = -72\Psi_{m\alpha}$$

$$C_{mnpq} = \frac{1}{2}\partial_{[m} B_{npq]} \quad ; \quad F_{\alpha mn} = \frac{1}{8}\left(7\partial_{[m} H_{n]\alpha} + \partial^q (\gamma_{q[m})_\alpha{}^\beta H_{n]\beta}\right)$$

- This agrees with Berkovits-Chandia.
- This complete the general methodology and is applicable for construction of the integrated vertex operators.
- We point out some important new features that arise.

Part III

Integrated Vertex Operator and Generalization

Construction of the Integrated Vertex Operator

- Having obtained V, we can determine the corresponding integrated vertex operator by

$$QU - \partial V = 0 \qquad \text{ghost no. 1 and cnf. weight 2.}$$

- U is the only unknown in the above equation and we can employ the method we used to solve for V.

- Most of the subtleties appear in three kinds of identities at this level.

  1. Follows from I^n_β by taking world-sheet partial derivatives and composition with other weight one operators.

  2. New kinds of constraints true by reordering of operators appear e.g.

  $$d_\alpha d_\beta + d_\beta d_\alpha = -\frac{\alpha'}{2} \partial \Pi_m \gamma^m_{\alpha\beta}$$

  3. It happens that there are some coefficients that are not fixed by above procedure. This only means that the corresponding operator vanishes identically e.g. $N^{mn} N^{pq} \eta_{mp} G_{nq} = 0$.

- After taking care of all these we find

Result

$$U = \,:\Pi^m\Pi^n F_{mn}: + :\Pi^m d_\alpha F_m{}^\alpha: + :\Pi^m \partial\theta^\alpha G_{m\alpha}: + :\Pi^m N^{pq} F_{mpq}:$$
$$+ :d_\alpha d_\beta K^{\alpha\beta}: + :d_\alpha \partial\theta^\beta F^\alpha{}_\beta: + :d_\alpha N^{mn} G^\alpha{}_{mn}: + :\partial\theta^\alpha\partial\theta^\beta H_{\alpha\beta}:$$
$$+ :\partial\theta^\alpha N^{mn} H_{mn\alpha}: + :N^{mn} N^{pq} G_{mnpq}:$$

where,

$$F_{mn} = -\frac{18}{\alpha'} G_{mn} \quad , \quad F_m^\alpha = \frac{288}{\alpha'} (\gamma^r)^{\alpha\beta} \partial_r \Psi_{m\beta} \quad , \quad G_{m\alpha} = -\frac{432}{\alpha'} \Psi_{m\alpha}$$

$$F_{mpq} = \frac{12}{(\alpha')^2} B_{mpq} - \frac{36}{\alpha'} \partial_{[p} G_{q]m} \quad , \quad K^{\alpha\beta} = -\frac{1}{(\alpha')^2}\, \gamma^{\alpha\beta}_{mnp} B^{mnp}$$

$$F^\alpha{}_\beta = -\frac{4}{\alpha'} (\gamma^{mnpq})^\alpha{}_\beta \partial_m B_{npq} \quad , \quad G^\alpha_{mn} = \frac{48}{(\alpha')^2} \gamma^{\alpha\sigma}_{[m} \Psi_{n]\sigma} + \frac{192}{\alpha'} \gamma^{\alpha\sigma}_r \partial^r \partial_{[m} \Psi_{n]\sigma}$$

$$H_{\alpha\beta} = \frac{2}{\alpha'} \gamma^{mnp}_{\alpha\beta} B_{mnp} \quad , \quad H_{mn\alpha} = -\frac{576}{\alpha'}\, \partial_{[m} \Psi_{n]\alpha} - \frac{144}{\alpha'} \partial^q (\gamma_{q[m})_\alpha{}^\sigma \Psi_{n]\sigma}$$

$$G_{mnpq} = \frac{4}{(\alpha')^2} \partial_{[m} B_{n]pq} + \frac{4}{(\alpha')^2} \partial_{[p} B_{q]mn} - \frac{12}{\alpha'} \partial_{[p} \partial_{[m} G_{n]q]}$$

[S.P.K, S. Chakrabarti and M. Verma - 2018]

Generalization to all vertex operators

- We first construct the unintegrated vertex operator and then using this solve for the corresponding integrated operator.
- Steps for Unintegrated vertex operator construction

STEP I Identify the fields that capture particle content at the given mass level and introduce superfields whose θ independent component are these field e.g. for a f_A

$$F_A(X^m, \theta) := f_A(X^m) + f_{A\alpha_1}(X^m)\theta^{\alpha_1} + \cdots + f_{A\alpha_1\cdots\alpha_{16}}\theta^{\alpha_1}\cdots\theta^{\alpha_{16}}$$

STEP II Constrain the superfields to satisfy all the constraints that the cooresponding fields satisfy e.g. if $f_A = \psi_{s\alpha}$

$$\partial^m \psi_{m\alpha} = 0 \quad \overset{impose}{\Longrightarrow} \quad \partial^m \Psi_{m\alpha} = 0$$

$$\gamma^{m\alpha\beta}\psi_{m\beta} = 0 \quad \overset{impose}{\Longrightarrow} \quad \gamma^{m\alpha\beta}\Psi_{m\beta} = 0$$

STEP III Ansatz for unintegrated vertex operator:

$$V = \sum_A B_A S^A$$

where, B_A are the basis operators at conformal weight n and ghost number 1.

STEP IV a Find out all of the constraints at the required mass level and ghost number by taking OPE's with the original constraint identity

$$I := \ : N^{mn}\lambda^{\alpha} : (\gamma_m)_{\alpha\beta} - \frac{1}{2} : J\lambda^{\alpha} : \gamma^n_{\alpha\beta} - \alpha' \gamma^n_{\alpha\beta} \partial\lambda^{\alpha} = 0$$

STEP IV b Find out all the constraints that are true by trivial reordering of operators eg.

$$d_\alpha d_\beta + d_\beta d_\alpha = -\frac{\alpha'}{2} \partial\Pi_m \gamma^m_{\alpha\beta}$$

STEP IV c Drop terms that are identically zero that appear in the equation eg.

$$: N^{mn} N^{pq} \eta_{np} G_{mq} := 0$$

STEP V Introduce the Lagrange multiplier superfields K_A. Use group decomposition to write the superfields S_A appearing in V and them as general linear combination of physical superfields introduced in **STEP I**

$$S_A = \sum_B c_{AB} F_B \quad , \quad K_A = \sum_B d_{AB} F_B$$

II. Construction of the Vertex Operators

STEP VI Compute QV. This will give rise to terms of the form

$$D_\alpha S_A$$

where, D_α is the supercovariant derivative. By making use of group theory decomposition write

$$D_\alpha S_A = \sum_B g_{\alpha AB} F_B$$

STEP VII Solve $QV = 0$ respecting the constraints by method of elimination or Lagrange multipliers. This determines c_A, d_A and g_{cAB} and we have constructed our unintegrated vertex operator.

- Now we are ready for the construction of the integrated vertex operator.
- We need to follow the same steps but this time we need to solve for

$$QU = \partial V$$

- The solution to the above equation gives the integrated vertex operator.

Applications

- As a by product of this procedure we are able to get relationship between the physical superfields that can be easily used to perform θ expansion and hence do amplitude computations [Subhroneel's talk].
- We also used the integrated vertex operator to compute the mass renormalization at one loop for stable non-BPS the massive states at first excited level in Heterotic strings [to appear - in collaboration with Mritunjay].
- The above result matches with the one obtained earlier using RNS formalism [Ashoke]
- Can use the integrated vertex operator to perform computations at tree level and one loop level to see if structural relations/identities found in the case of massless case hold true (O. Schlotterer's Talk).

THANK YOU