Chiral strings and their vertex operators

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Outline

- Bosonic ambitwistor string
- Zero-momentum spectrum and tensile deformation
- Geometrical interpretation
- Building left and right movers
- Tree level amplitudes

- Chiral string model underpinning the CHY formulæ.

- Action and BRST charge:

$$S_{bos} = \frac{1}{2\pi}\int d^2z\{P_m\bar{\partial}X^m + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c}\}, \quad (1)$$

$$Q_0 = \oint\{cT - bc\partial c + \tfrac{1}{2}\tilde{c}\underbrace{P^m P_m}_{\text{massless}}\}. \quad (2)$$

- Physical spectrum non-unitary (higher derivatives).
- Integrated vertex operators:

$$V = \overbrace{\bar{\delta}(k\cdot P)}^{\text{localization}}\ b_{-1}\tilde{b}_{-1}U. \quad (3)$$

- Not an ordinary tension (zero or infinity) limit of the string.

- Turning on background fields = deforming the BRST charge (but free action).

- Zero-momentum cohomology at ghost number 2:

$$U_G = c\tilde{c}P_m P_n, \quad (4)$$
$$U_{\mathcal{T}} = \tfrac{1}{2}c\tilde{c}\partial X^m \partial X_m - bc\tilde{c}\partial\tilde{c} - \tfrac{3}{2}\tilde{c}\partial^2\tilde{c}. \quad (5)$$

- Deformations of the BRST charge:

$$b_{-1}U_G = \tilde{c}P_m P_n, \quad (6)$$
$$b_{-1}U_{\mathcal{T}} = \tfrac{1}{2}\tilde{c}\partial X^m \partial X_m + b\tilde{c}\partial\tilde{c}. \quad (7)$$

- Dimensions:

$$\frac{[U_G]}{[U_{\mathcal{T}}]} = \ell^{-4} \sim \mathcal{T}^2. \quad (8)$$

- $U_{\mathcal{T}}$ is the tension field $\Rightarrow$ tensile deformation:

$$Q = Q_0 + \mathcal{T}^2 \oint \tilde{c}(\tfrac{1}{2}\partial X^m \partial X_m - b\partial\tilde{c}). \tag{9}$$

- $P^2 = 0$ is replaced by the constraint $\mathcal{H} = 0$:

$$\mathcal{H} = \tfrac{1}{2}P^2 + \tfrac{\mathcal{T}^2}{2}(\partial X)^2. \tag{10}$$

- $\mathbb{Z}_2$ symmetry:

$$\mathcal{T} \to -\mathcal{T}. \tag{11}$$

Also a symmetry of the physical spectrum!

- Massless spectrum (ϕ, $H = db$, g):

$$\Box\phi = 0, \tag{12}$$
$$\partial_p H^{mnp} = 0, \tag{13}$$
$$\Box g^{mn} - \partial_p \partial^{(m} g^{n)p} = \partial^m \partial^n \left(\tfrac{2}{\mathcal{T}}\phi - g^{pq}\eta_{pq}\right). \tag{14}$$

- Massive spectrum (spin 2, $m^2 = \pm 4\mathcal{T}$):

$$(\Box \mp 4\mathcal{T}) h^{mn}_{\pm} = 0, \tag{15}$$
$$\partial_n h^{mn}_{\pm} = 0, \tag{16}$$
$$h^{mn}_{\pm}\eta_{mn} = 0. \tag{17}$$

The 1st order Polyakov action:

$$S = \frac{1}{2\pi}\int d^2\sigma\{P_m\partial_\tau X^m + \tfrac{1}{4\mathcal{T}}[e_+(P+\mathcal{T}\partial_\sigma X)^2 + e_-(P-\mathcal{T}\partial_\sigma X)^2]\}. \quad (18)$$

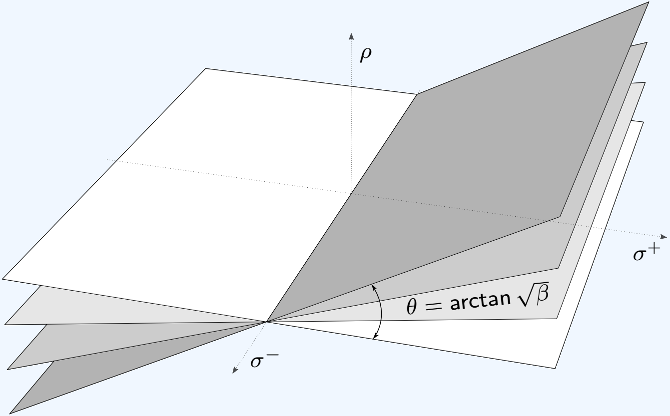
Reparametrization symmetry:

$$e_\pm = \frac{\pm\beta - 1}{\beta + 1}. \quad (19)$$

- any finite β is equivalent to the conformal gauge $\beta = 0$.
- the (singular) chiral gauge corresponds to the limit $\beta \to \infty$.

In terms of the worldsheet metric,

$$ds^2 = -d\sigma^+ d\sigma^- + \beta(d\sigma^+)^2 :$$



Building left and right movers

- the tension enables a sectorization of the chiral model.
- redefinition of the ghosts:

$$c_\pm \equiv c \mp \mathcal{T}\tilde{c}, \tag{20}$$
$$b_\pm \equiv \frac{1}{2\mathcal{T}}(\mathcal{T}b \mp \tilde{b}). \tag{21}$$

- natural rewriting of the BRST charge as $Q = Q^+ + Q^-$:

$$Q^\pm \equiv \oint \{c_\pm T_\pm - b_\pm c_\pm \partial c_\pm\}, \tag{22}$$
$$\{Q, b_\pm\} \equiv T_\pm,$$
$$= \mp\frac{1}{4\mathcal{T}}P_\pm^2 - b_\pm \partial c_\pm - \partial(b_\pm c_\pm), \tag{23}$$
$$P_\pm^m = P^m \pm \mathcal{T}\partial X^m. \tag{24}$$

- energy-momentum tensor: $T = T_+ + T_-$.

- generalized particle-like Hamiltonian:

$$\begin{aligned} \mathcal{H} &\equiv \mathcal{T}(T_- - T_+), \\ &= \tfrac{1}{2}P^2 + \tfrac{\mathcal{T}^2}{2}(\partial X)^2 + \text{ghosts}. \end{aligned} \tag{25}$$

Some examples of interest include:

- the spinning string.
- the pure spinor superstring.
- purely bosonic models in $d < 26$ with current algebras: $DF^2 + YM$ [Oliver's talk].

All contain graviton excitations (either type II, heterotic or bosonic), with or without ghosts.

Vertex operators

- Unintegrated vertex operators:

$$U(z;k^m) = U_+(z)U_-(z)e^{ik\cdot X(z)}, \tag{26}$$

- Integrated vertex operators:

$$V(z;k^m) \overset{?}{\equiv} (b_-)_{-1}\cdot(b_+)_{-1}\cdot U(z;k^m), \tag{27}$$

  Problem: wrong conformal weight $(2,0)$ and $\bar{\partial}V = 0$.

A possible solution is to build a BRST closed operator $\bar{\delta}(\mathcal{H}_{-1})$, such that the $\bar{\delta}(\mathcal{H}_{-1})\cdot\mathcal{H}_{-1}$ "vanishes",

$$V(z;k^m) \equiv (b_-)_{-1}\cdot(b_+)_{-1}\cdot\bar{\delta}(\mathcal{H}_{-1})\cdot U(z;k^m), \tag{28}$$

such that $[Q, V(z;k^m)] = \frac{\partial}{\partial z}(\dots)$.

The sector splitting operator

Consider the BRST-closed operator

$$\bar{\Delta} = e^{-\alpha \mathcal{H}_{-1}}, \tag{29}$$

where $\mathcal{H}_{-1}$ is the -1 mode of $\mathcal{H}$.

- artificially introduces a anti-holomorphic dependence.
- analogous to

$$e^{zL_{-1}} \mathcal{O}(0) = \mathcal{O}(z). \tag{30}$$

- point splitting:

$$\bar{\Delta} \cdot \mathcal{O}^{\pm}(z) = \mathcal{O}^{\pm}(z \pm \mathcal{T}\alpha), \tag{31}$$

- new coordinates:

$$z^{\pm} \equiv z \pm \mathcal{T}\alpha. \tag{32}$$

The only non-trivial operation is the action of $\bar{\Delta}$ on X^m,

$$\bar{X}_m(z,\alpha) \equiv \bar{\Delta} \cdot X_m(z), \tag{33}$$

satisfying

$$\partial_+ \partial_- \bar{X}_m = 0. \tag{34}$$

In fact, $\bar{X}^m$ behaves like an (almost) ordinary worldsheet scalar:

$$\bar{X}^m(z^+, z^-)\,\bar{X}^n(y^+, y^-) \sim -\tfrac{\eta^{mn}}{2\mathcal{T}} \ln(z^+ - y^+) + \tfrac{\eta^{mn}}{2\mathcal{T}} \ln(z^- - y^-). \tag{35}$$

The localization operators $\bar{\delta}(k \cdot P)$ are naturally reproduced:

$$\lim_{\mathcal{T} \to 0} \bar{\Delta} \cdot e^{ik\cdot X} =: e^{ik\cdot X} e^{i\alpha(k\cdot P)} : . \tag{36}$$

With the α integration, the second exponential can be thought of as a representation for $\bar{\delta}(k \cdot P)$.

Vertex operators with $\bar{\Delta}$

- Unintegrated vertex operators:

$$\begin{aligned} \bar{U}(z,\alpha;k) &\equiv \bar{\Delta}\cdot U(z;k), \\ &= U_+(z^+)U_-(z^-)e^{ik\cdot\bar{X}(z^+,z^-)}, \end{aligned} \tag{37}$$

  such that

$$U(z;k) = \lim_{\alpha\to 0}\bar{U}(z^+,z^-;k). \tag{38}$$

- The integrated vertex operator is defined as

$$V(z,\alpha) = (b_-)_{-1}\cdot(b_+)_{-1}\cdot\bar{\Delta}\cdot U, \tag{39}$$

  and satisfies

$$[Q,V] = \partial_+(\ldots) + \partial_-(\ldots). \tag{40}$$

Tree level amplitudes

At tree level, N-point amplitudes can be cast as

$$\begin{aligned} \mathcal{A}_N(k^1,\dots,k^N) &= \left\langle \prod_{i=1}^{3} \bar{U}_i(z_i^+,z_i^-;k^i) \prod_{j=4}^{N} \mathcal{V}_j(k^j) \right\rangle , \\ &= \left\langle \prod_{i=1}^{3} U_i(z_i;k^i) \prod_{j=4}^{N} \mathcal{V}_j(k^j) \right\rangle , \end{aligned} \tag{41}$$

where

$$\begin{aligned} \mathcal{V}_j(k^j) &\equiv \int dz_j d\alpha_j \, V(z_j,\alpha_j;k^j), \\ &= \frac{1}{2\mathcal{T}} \int_{S^2} dz_j^+ dz_j^- V'(z_j^+,z_j^-;k^j), \end{aligned} \tag{42}$$

and $V' = V$ when $z_j^\pm = z_j \pm \mathcal{T}\alpha_j$.

$\mathcal{A}_N$ can be easily computed using the sign-flipped XX OPE.

However, there is an ambitwistor-like solution. The equation of motion for P^m in the amplitude reads

$$\frac{1}{2\pi}\bar{\partial}P^m = i\sum_{i=1}^{3} k_i^m \delta^2(z-z_i) + \frac{i}{2}\sum_{j=4}^{N} k_j^m[\delta^2(z-z_j-\mathcal{T}\alpha_j)+\delta^2(z-z_j+\mathcal{T}\alpha_j)]. \quad (43)$$

On the Riemann sphere, there is a unique solution for this equation, given by

$$P^m(z) = i\sum_{i=1}^{3}\frac{k_i^m}{(z-z_i)} + i\sum_{j=4}^{N}\left(\frac{\frac{1}{2}k_j^m}{(z-z_j-\mathcal{T}\alpha_j)}+\frac{\frac{1}{2}k_j^m}{(z-z_j+\mathcal{T}\alpha_j)}\right), \quad (44)$$

which has the expected tensionless limit.

With this result, it is straighforwrd to show that

$$\left\langle \prod_{i=1}^{N} : e^{ik^i \cdot \bar{X}(z_i^+, z_i^-)} : \right\rangle \propto \delta^d \left(\sum k \right) \prod_{i>j}^{N} \left(\frac{z_{ij}^+}{z_{ij}^-} \right)^{\frac{(k_i \cdot k_j)}{2\mathcal{T}}} , \qquad (45)$$

clearly showing the outcome of the sign flip in the $\bar{X}\bar{X}$ OPE.

- Möbius invariance.
- well-defined for only specific values of $k_i \cdot k_j$ (branch cuts).
- adapted KLT + "analytic continuation".
- finite spectrum $\leftrightarrow$ finite number of poles.

$$\mathcal{A}_4 \approx \int d^2z\, z^{m-2}(1-z)^{n-2}\bar{z}^{\bar{m}-2}(1-\bar{z})^{\bar{n}-2} \left(\frac{\bar{z}}{z}\right)^{\frac{u}{4\mathcal{T}}} \left(\frac{1-\bar{z}}{1-z}\right)^{\frac{t}{4\mathcal{T}}}, \quad (46)$$

where $\{m,n\} = 0,\ldots,4$, with $m+n \leq 4$.

Mandelstam variables s, t and u, with $s+t+u=0$.

$$\mathcal{A}_4 \propto \pi \frac{\Gamma(3-\bar{m}-\bar{n}+S)}{\Gamma(m+n-2+S)} \frac{\Gamma(m-1-U)}{\Gamma(2-\bar{m}-U)} \frac{\Gamma(n-1-T)}{\Gamma(2-\bar{n}-T)}, \quad (47)$$

$$S \equiv \tfrac{s}{4\mathcal{T}}, \quad T \equiv \tfrac{u}{4\mathcal{T}}, \quad U \equiv \tfrac{u}{4\mathcal{T}}.$$

Final remarks

- ambitwistor string: tensionless limit + singular metric gauge;
- construction of IVO's directly from the chiral model;
- reproduce the chiral amplitudes from Siegel et al;
- loops: modular invariance unlikely;
- SFT: tensionless limit from the field theory point of view;
- heterotic model (SUSY + tachyons);

Thank you!

Extra: $DF^2 + YM$

Massless+massive vector in the physical spectrum:

$$S_0 = 2\mathcal{T}\int d^dx\{G_{ma}(\Box G^m_a + 4\mathcal{T}G^m_a - \partial^m\partial_n G^n_a) - H_{ma}(\Box H^m_a - \partial^m\partial_n H^n_a)\}.$$

Field redefinition:

$$A^m_a \equiv H^m_a + G^m_a, \qquad B^m_a \equiv \mathcal{T}(H^m_a - G^m_a),$$

Then:

$$S_0 = \int d^dX\,\{2B_{ma}\partial_n F^{mn}_a + 2(B^m_a - \mathcal{T}A^m_a)(B_{ma} - \mathcal{T}A_{ma})\},$$

with $F^{mn}_a = \partial^m A^n_a - \partial^n A^m_a$.

Note that B_a^m has an algebraic equation of motion:

$$B_a^m = \mathcal{T} A_a^m + \tfrac{1}{2}\partial_n F_a^{nm}.$$

Plugging it back in the action, one obtains

$$S_0|_B = \int d^d X \left\{ \mathcal{T} F_a^{mn} F_{mna} - \tfrac{1}{2}\partial_n F_a^{mn} \partial^p F_{mpa} \right\}.$$

This is the kinetic part of the $(DF)^2 + YM$ theory (Johansson-Nohle).

Its tensionless (ambitwistor) limit is the DF^2 theory, with higher derivatives.