Twistorial ambitwistor-strings: 1. Models

Lionel Mason

The Mathematical Institute, Oxford
lmason@maths.ox.ac.uk

Workshop on fundamental aspects of String Theory, ICTP SAIFR

With Yvonne Geyer, arxiv:1812.05548, 1901.00134, & Giulia Albonico 2001.05928 & D Skinner 200?.????.

Cf work by: Cachazo, Guevara, Heydeman, Mizera, Schwarz, Wen, arxiv:1710.02170, 1805.11111, 1812.06111, 1907.03485.

and related to models by Bandos et. al.

Take-home messages

- Ambitwistor-strings give wide-ranging generalization of Twistor-strings.
- Original model has clear parallels to standard RNS string.
- Target is *ambitwistor-space* $\mathbb{A}$, space of complex null geodesics.
- Twistorial representations of ambitwistor spaces exist in higher dimensions, here particularly 6d, also 4/5/10/11d.
- Ambitwistor strings are chiral strings whose vertex operators are built from Penrose transform of space-time fields on $\mathbb{A}$ in terms of $H^1_{\bar\partial}(\mathbb{A})$.
- Quantizing ambitwistor-strings in these representations give rise to new manifestly supersymmetric formulae for *field theory* amplitudes based on *polarized scattering equs*. (These are discussed in detail in Yvonne Geyer's talk).

Outline

- Review of RNS models and geometry of ambitwistor spaces.
- Twistor representation of ambitwistor space in 4d $\rightsquigarrow$ Witten-Berkovits-Skinner twistor strings & 4d ambitwistor string.
- Twistor representation of $\mathbb{A}$ in 6d.
- String models and vertex operators.
- Models in 5d and massive ambitwistor strings in 4d.
- Survey of approaches in 10/11d.

Bosonic ambitwistor string action:

$$S_B = \int_\Sigma P_\mu \bar\partial X^\mu - e\, P^2/2\,.$$

- Σ Riemann surface, coordinate $\sigma \in \mathbb{C}$
- Complexify space-time (M, g), coords $X \in \mathbb{C}^d$, g hol.
- $X : \Sigma \to M, \qquad P \in \Omega^{1,0}_\Sigma \otimes X^*(T^*M)$.

Underlying geometry:

- Lagrange multiplier $e \in \Omega^{0,1} \otimes T^{1,0}\Sigma$ forces $P^2 = 0$,
- e is also worldsheet gauge field for Hamiltonian flow of P^2:

$$\delta(X, P, e) = (\alpha P, 0, 2\bar\partial\alpha).$$

Target reduces to

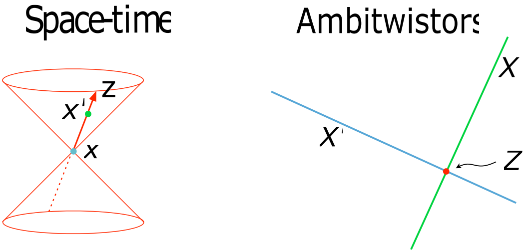
$$\mathbb{A} = T^*M|_{P^2=0}/\{\text{gauge}\}.$$

This is *Ambitwistor space*, space of complexified light rays.
It is holomorphic symplectic with potential $\theta = P_\mu dX^\mu$.

The geometry of space of complex light rays

Ambitwistor space $\mathbb{A}$ is space of complexified light rays.

- Light rays primary, an event $x \leftrightarrow$ its lightcone $X \subset \mathbb{A}$.
- Space-time $M =$ space of such $X \subset \mathbb{A}$.



Space-time geometry is encoded in complex structure of $\mathbb{A}$.

Theorem (LeBrun 1983 following Penrose 1976)

Complex structure of $\mathbb{A}$ determines $(M, [g])$. Correspondence stable under deformations of $P\mathbb{A}$ that preserve $\theta = P_\mu \mathrm{d}X^\mu$.

Quantize bosonic ambitwistor string:

- $(X, P) : \Sigma \to T^*M$,

$$S_B = \int_\Sigma P_\mu(\bar\partial + \tilde e \partial)X^\mu - e\, P^2/2\,.$$

- Gauge fix $\tilde e = e = 0$, $\rightsquigarrow$ ghosts & BRST Q
- $Q^2 = 0 \Leftrightarrow D = 26$ (10 with worldsheet SUSY).
- Amplitudes are computed as correlators of vertex ops

$$\mathcal{M}_n = \langle V_1 \dots V_n \rangle$$

- For appropriate choices of world sheet supersymmetry and matter we obtain full range of CHY formulae.

Need appropriate vertex operators.

Vertex operators and worldsheet matter

- Field perturbations $\leftrightarrow$ deformation of $\mathbb{C}$-structure of $\mathbb{A}$.
- Vertex ops: $V_i = \delta(\theta) \in H^1_{\bar\partial}(\mathbb{A}, L)$ from field perturbation.
- General structure:

$$V_i = w_1 w_2 \delta(k_i \cdot P) e^{ik_i \cdot x} \,,$$

w_1, w_2 are currents from some worldsheet matter system.

- E.g. current algebra or worldsheet susy

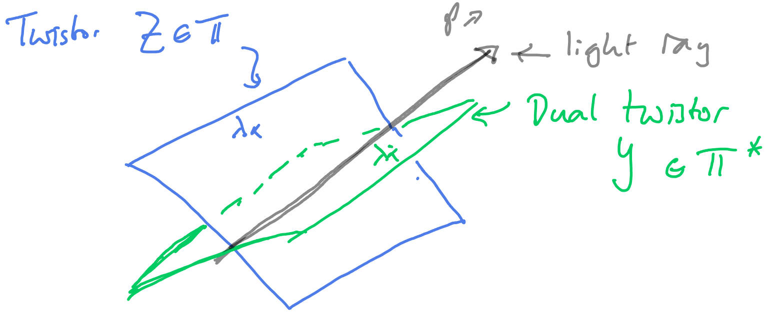
$$S_\Psi = \int_\Sigma \Psi_\mu \bar\partial \Psi^\mu + \chi P \cdot \Psi \,.$$

Proposition

With $w_{1/2} = (\epsilon \cdot P + \epsilon \cdot \Psi k \cdot \Psi)$, $\langle V_1 \ldots V_n \rangle$ gives CHY gravity formulae etc..

From twistors to light rays in 4d

Solve constraint $P^2 = 0$ with $P_{\alpha\dot{\alpha}} = \lambda_\alpha \lambda_{\dot{\alpha}}$:



4D Twistor-strings and supersymmetry

Super-twistor space $\mathbb{T} = \mathbb{C}^{4|4}$; spinors of superconformal group.

$$\mathbb{A} = \{(Y, Z) \in \mathbb{T}^* \times \mathbb{T} | Y \cdot Z = 0\} / \{Y \cdot \partial_Y - Z \cdot \partial_Z\} .$$

This representation leads to

- the twistor-string $(Y, Z) \in \mathbb{T}^*(-d-2) \times \mathbb{T}(d)$:

$$S_{\mathbb{T}} = \int Y \cdot \bar{\partial} Z + AY \cdot Z ,$$

  [Witten, Berkovits 2003/4, Skinner 2013] $\rightsquigarrow$ RSVW SYM and Cachazo-Skinner SUGRA formulae.

- 4d ambitwistor string $(Y, Z) \in \mathbb{T}^*(-1) \times \mathbb{T}(-1)$:

$$S_{\mathbb{A}} = \int Y \cdot \bar{\partial} Z - Z \cdot \bar{\partial} Y + AY \cdot Z$$

  [Geyer, Lipstein, M. 2014] $\rightsquigarrow$ new ambidextrous formulae.

Spinors, little groups and polarization data in 6d

6d spinors:

$$Spin(6,\mathbb{C}) = SL(4,\mathbb{C}),$$

Use indices $\mu = 1,\ldots,6$, $A = 1,\ldots,4$ identified by

$$k_\mu \leftrightarrow k_{AB} = k_{[AB]} = \frac{1}{2}\varepsilon_{ABCD}k^{CD}, \qquad \varepsilon_{ABCD} = \varepsilon_{[ABCD]}\,.$$

Little group and its spinors:

$$Spin(4,\mathbb{C}) = SL(2)\times SL(2) \subset \{\text{ stabilizer of null } k\}.$$

- k null $\Leftrightarrow k^{AB}k^{CD}\varepsilon_{ABCD} = 0 \Leftrightarrow k_{AB}$ rank-2 so define

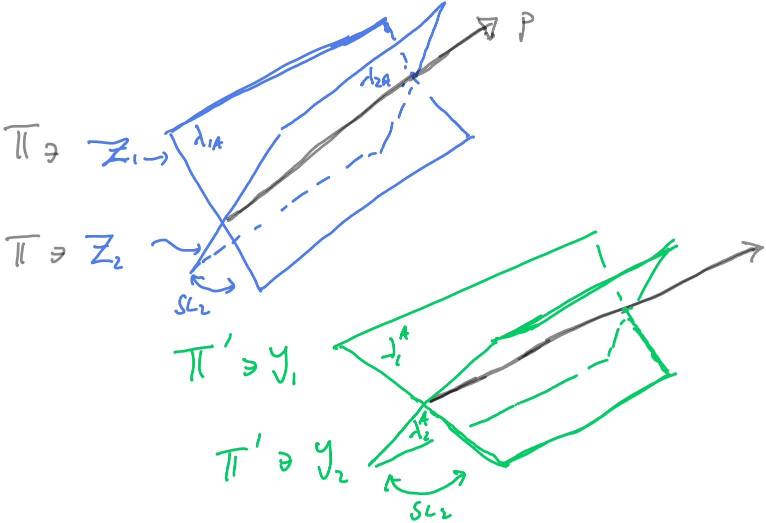
$$\kappa^A_{\dot a}: \qquad k^{AB} = \varepsilon^{\dot a\dot b}\kappa^A_{\dot a}\kappa^B_{\dot b} =: \left[\kappa^A\kappa^B\right],$$

$$\kappa_{aA}: \qquad k_{AB} = \kappa^a_C\kappa^b_D\varepsilon_{ab} =: \langle\kappa_A\kappa_B\rangle\,,$$

where, $a = 0,1$, $\dot a = \dot 0, \dot 1$ are $SL(2)$ indices.

Twistors and light rays in 6d

Solve $P^2 = 0$ with $P_{AB} = \lambda_C^a \lambda_D^b \varepsilon_{ab} =: \langle \lambda_A \lambda_B \rangle$ or $P^{AB} = \varepsilon^{\dot{a}\dot{b}} \lambda_{\dot{a}}^A \lambda_{\dot{b}}^B =: [\lambda^A \lambda^B]$,



Super-Twistors and ambitwistors-strings

- Twistors are pure spinors for conformal group $SO(8|2N)$

$$\mathbb{T} = \{\mathcal{Z} := (\lambda_A, \mu^A, \eta^I) \in \mathbb{C}^{8|2N} | \mathcal{Z} \cdot \mathcal{Z} = 0\} .$$

where $\mathcal{Z} \cdot \mathcal{Z} := \mu^A \lambda_A + \mu^A \lambda_A + \Omega_{IJ}\, \eta^I \eta^J$.

- $(N, 0)$-superspace-time $= \mathbb{C}^{6|8N}$ with coords (x^{AB}, θ^I_A).
- Super-twistors are totally null self-dual $3|6N$-planes:

$$\mu^A = x^{AB}\lambda_B + \theta^{IA}\eta_I , \quad \eta^I = \theta^{IA}\lambda_A .$$

Super-ambitwistors from pairs of twistors:

- Pair $\mathcal{Z}_a = (\mathcal{Z}_1, \mathcal{Z}_2)$ intersect $\Leftrightarrow \mathcal{Z}_1 \cdot \mathcal{Z}_2 = 0$
- $\rightsquigarrow$ twistors meet along super light ray.
- so with $a = 1, 2$:

$$\mathbb{A} = \{\mathcal{Z}_a | \mathcal{Z}_a \cdot \mathcal{Z}_b = 0\}/SL(2) .$$

- Ambitwistor-string action for $\mathcal{Z}_a \in \sqrt{\Omega^{1,0}_\Sigma}, A_{ab} \in \Omega^{0,1}$:

$$S_{\mathcal{Z}} = \int_\Sigma \epsilon_{ab}\mathcal{Z}^a \bar{\partial} \mathcal{Z}^b + A_{ab}\mathcal{Z}^a \cdot \mathcal{Z}^b .$$

Super-ambitwistors from primed $(0, \tilde{N})$ super-twistors:

- with $\mathcal{Y} = (\lambda^A, \mu_A, \eta_{\tilde{I}})$, $\dot{a} = 1, 2$:

$$\mathbb{A} = \{\mathcal{Y}_{\dot{a}} | \mathcal{Y}_{\dot{a}} \cdot \mathcal{Y}_{\dot{b}} = 0\}/SL(2)\,.$$

- Ambitwistor-string action for $\mathcal{Y}_{\dot{a}} \in \sqrt{\Omega^{1,0}_{\Sigma}}, A_{\dot{a}\dot{b}} \in \Omega^{0,1}$:

$$S_{\mathcal{Y}} = \int_{\Sigma} \epsilon_{\dot{a}\dot{b}} \mathcal{Y}^{\dot{a}} \bar{\partial} \mathcal{Y}^{\dot{b}} + A_{\dot{a}\dot{b}} \mathcal{Y}^{\dot{a}} \cdot \mathcal{Y}^{\dot{b}}\,.$$

Conformal invariance: $\mathbb{M}^6 =$ projective quadric $Q \subset \mathbb{P}^7$

- $\mathbb{P}^7$ has homogensous coords $\mathcal{X} \in \mathbb{C}^8$,
- quadric $Q = \{[\mathcal{X}] \in \mathbb{P}^7 | \mathcal{X} \cdot \mathcal{X} = 0\}$.
- $\exists$ light ray thru $\mathcal{X}_1, \mathcal{X}_2$ if $\mathcal{X}_1 \cdot \mathcal{X}_2 = 0$ so

$$S_{\mathcal{X}} = \int_{\Sigma} \epsilon_{ij} \mathcal{X}^i \bar{\partial} \mathcal{X}^j + A_{ij} \mathcal{X}^i \cdot \mathcal{X}^j\,.$$

Twisted analogue of 6d model by Adamo, Monteiro & Paulos.

Vertex operators and worldsheet matter

- Recall vertex operators $\leftrightarrow H^1_{\bar\partial}(\mathbb{A}, L) \ni w_1 w_2 \bar\delta(k \cdot P)\mathrm{e}^{ik\cdot x}$
- w_i world-sheet currents (world-sheet matter).
- In twistor variables we have

$$\bar\delta(k\cdot P)e^{ik\cdot x} = \int d^2u d^2v\, \bar\delta\left(\langle v\epsilon\rangle - 1\right) \bar\delta^4\Big(\langle u\lambda_A\rangle - \langle v\kappa_A\rangle\Big)\, \mathrm{e}^{iu^a\mu_a^A\langle\epsilon\kappa_A\rangle}.$$

  where $(u_a, v_a) \in \mathbb{C}^4$ and ϵ_a a given little group spinor.
- Main idea: $k \cdot P = 0 \Leftrightarrow \exists (u_a, v_a) \neq 0$ s.t.

$$\langle u\lambda_A\rangle = \langle v\kappa_A\rangle\,, \qquad \textit{Polarised Scattering equs..}$$

  These underpin all amplitude formulae.

For w_i = currents in pair of current algebras, get manifestly conformally invariant biadjoint-scalar amplitudes in 6d.

Analogue of worldsheet supersymmetry

For SYM and SUGRA need analogue of worldsheet SUSY:

- Let $\rho^A, \tilde{\rho}_A \in \sqrt{\Omega^{1,0}_\Sigma}$, and gauge fields $\chi^a, \tilde{\chi}^{\dot{a}} \in \Omega^{0,1}_\Sigma$

$$S_{\tilde{\rho},\rho} = \int_\Sigma \rho^A \bar{\partial} \tilde{\rho}_A + \chi^a \lambda_{aA} \rho^A + \tilde{\chi}^{\dot{a}} \lambda^A_{\dot{a}} \tilde{\rho}_A \,,$$

- Then contribution to vertex operator is

$$w = \epsilon \cdot \langle \lambda\lambda \rangle + F^B_A \rho^A \tilde{\rho}_B,$$

  where $\epsilon =$ polarization, $F = \epsilon \wedge k$.
- **Problem:** λ_{aA} and $\lambda^A_{\dot{a}}$ live in different models!
- However, in 5d models these are identified.

In 5d, no chirality, 'twistors = primed twistors', $\lambda^A_{\dot{a}} = \omega^{AB}\lambda_{aB}$.

- Gauge translation symmetry in $\omega^{AB}\partial/\partial x^{AB}$ direction $\rightsquigarrow$
- $P_{AB} = \langle\lambda_A\lambda_B\rangle$ generates translations: $a \in \Omega^{0,1}_\Sigma$ gauge field

$$S^B_{\mathcal{Z},5d} = S_{\mathcal{Z}} + \int_\Sigma a\,\omega^{AB}\langle\lambda_A\lambda_B\rangle$$

- For maximal SYM now have model

$$S^{SYM}_{\mathcal{Z},5d} = S^B_{\mathcal{Z},5d} + S_{\tilde{\rho},\rho} + S_{\text{Current Alg}}$$

- For maximal SUGRA

$$S^{SUGRA}_{\mathcal{Z},5d} = S^B_{\mathcal{Z},5d} + S_{\tilde{\rho}_1,\rho_1} + S_{\tilde{\rho}_2,\rho_2}$$

- All models have vanishing gauge anomalies.
- Critical when extra 5 dims are included + usual groups.

In fact amplitude formulae all live in 6d (see Yvonne's talk).

Further reduction to 4d can be lifted to introduce masses.

- gauge another direction $\tilde{\omega}^{AB}$ with $\tilde{a} \in \Omega^{0,1}_{\Sigma}$

$$S_{4d} = S_{\mathcal{Z},5d} + \int_{\Sigma} \tilde{a}(\tilde{\omega}^{AB}\langle\lambda_A\lambda_B\rangle + J)\,.$$

- J is a current from theory that generates masses.
- $5d$ spinor index A reduces to Dirac spinor index in $4d$.
- Little group index reduces to 4d massive little group.
- Higgsed maximal SYM has heterotic model

$$S^{SYM\ Higgs}_{\mathcal{Z},4d} = S^{B}_{4d} + S_{\tilde{\rho},\rho} + S_{\text{Current Alg}}$$

where J is current in Lie algebra for Higgs field.

11D SUGRA:

- Spinors indices for $SO(11)$, $\mathfrak{a} = 1, \ldots, 32$
- Little group $SO(9)$, $\alpha = 1, \ldots, 16$ stabilizing null $P_\mu \rightsquigarrow$

$$\lambda_{\alpha\mathfrak{a}}\lambda^{\alpha}_{\mathfrak{b}} = \Gamma^{\mu}_{\mathfrak{a}\mathfrak{b}}P_\mu \,, \qquad \Gamma^{\mathfrak{a}\mathfrak{b}}_{\mu}\lambda^{\alpha}_{\mathfrak{a}}\lambda^{\beta}_{\mathfrak{b}} = -2P_\mu\delta^{\alpha\beta} \,.$$

- Super-twistors $\mathcal{Z} \in \mathbb{T} = \mathbb{C}^{64|1}$ with skew inner product $\epsilon(,)$.
- Model

$$S = \int_\Sigma \epsilon(\mathcal{Z}_\alpha, \bar\partial \mathcal{Z}^\alpha) + A^{\alpha\beta}_M(\mathcal{Z}_\alpha \Gamma^M \mathcal{Z}_\beta) \,.$$

- incomplete as far as worldsheet matter goes.
- Good amplitude formulae exist.
- close to models of Bandos 1404.1299, 1908.07482; not quite ambitwistor spaces as he relaxes some constraints to incorporate M-theory modes.

Principle: Linear geometric realizations of $\mathbb{A}$ lead to different supersymmetric ambitwistor-string models, trying to live in 10d. Some incomplete stories:

- See 1908.06899 [Berkovits, Guillen & M.] for impure 10d super-twistor model. [Non-covariant amplitudes, using light cone gauge.]
- See 1905.03737 [Berkovits, Casali, Guillen & M.] pure spinor 11d model (obstruction to vertex operator).
- No chiral 6D worldsheet model is complete for SYM or SUGRA. Using both chiralities gives awkward constraints.
- Can now do SUSY formulae in all relevant dimensions, 10d type IIA, IIB, heterotic, DBI, & 11d but incomplete models.
- Links to pure spinor strings and so on.
- Ambitwistor string field theory???

Thank You!