The pure spinor b ghost in curved backgrounds

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Plan of the talk

- Review of the pure spinor string in a flat background
- Pure spinor string in a curved background
- Non-minimal variables in a curved background
- Construction of the b ghost in the heterotic string in a curved background
- Final remarks

Review of pure spinor in a flat background

(Berkovits, hep-th/0001035)
Given the superspace coordinates in ten dimensions (X^m, θ^α), their momenta and a pair of conjugate pure spinor variables.

$$S = \int d^2z \, \frac{1}{2}\partial X_m \bar{\partial} X^m + p_\alpha \bar{\partial}\theta^\alpha + \omega_\alpha \bar{\partial}\lambda^\alpha,$$

$$Q = \oint \lambda^\alpha \left(p_\alpha - \frac{1}{2}(\gamma_m\theta)_\alpha \partial X^m - \frac{1}{8}(\gamma_m\theta)_\alpha(\theta\gamma^m\partial\theta) \right) \equiv \oint \lambda^\alpha d_\alpha.$$

Q is nilpotent because the pure spinor constraint $\lambda\gamma^m\lambda = 0$

$$Q^2 = \oint (\lambda\gamma_m\lambda)\Pi^m, \quad \left(\Pi^m = \partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta) \right)$$

$\Rightarrow$ Q is declared as the BRST charge of the theory.
Note that ω_α is defined up to $(\lambda\gamma_m)_\alpha\Lambda^m$.

Review of pure spinor in a flat background

It reproduces the superstring physical spectrum through the cohomology of Q.

$$Q(\lambda^\alpha A_\alpha(X,\theta)) = 0, \quad \lambda^\alpha A_\alpha(X,\theta) \sim \lambda^\alpha A_\alpha(X,\theta) + \lambda^\alpha D_\alpha \Omega.$$

For example,

$$A_\alpha(X,\theta) = (\gamma^m \theta)_\alpha a_m(X) + (\theta\gamma^{mnp}\theta)(\gamma_{mnp}\psi(X))_\alpha + \cdots ,$$

a_m describes a photon and ψ a photino.

This model is conformal, spacetime supersymmetric and BRST invariant.

This is the minimal pure spinor string in a flat background.

Review of pure spinor in a flat background

The stress-energy tensor of the traditional strings is trivial because the existence of the reparametrization ghost b. That is, $Qb = T$. For the pure spinor string, the stress-energy tensor

$$T = -\frac{1}{2}\Pi_m\Pi^m - d_\alpha\partial\theta^\alpha - \omega_\alpha\partial\lambda^\alpha,$$

is annihilated by Q and the b ghost is given with the help of new variables. They are the conjugate pairs

$$(\hat{\omega}^\alpha, \hat{\lambda}_\alpha), \quad (s^\alpha, r_\alpha),$$

and are the non-minimal variables (they are constrained). They contribute to the BRST charge with $\oint \hat{\omega}^\alpha r_\alpha$ and

$$Q\hat{\lambda}_\alpha = -r_\alpha, \quad Qs^\alpha = \hat{\omega}^\alpha, \quad Qr_\alpha = Q\hat{\omega}^\alpha = 0.$$

They do not change the cohomology of the BRST charge (Berkovits, hep-th/0509120).

Review of pure spinor in a flat background

The non-minimal pure spinor has action

$$S = \int d^2z \; \frac{1}{2}\partial X_m \bar{\partial} X^m + p_\alpha \bar{\partial}\theta^\alpha + \omega_\alpha \bar{\partial}\lambda^\alpha + Q\left(\int d^2z \; s^\alpha \bar{\partial}\hat{\lambda}_\alpha\right)$$

The BRST charge is

$$Q = \oint \lambda^\alpha d_\alpha + \hat{\omega}^\alpha r_\alpha,$$

and the complete stress-energy tensor is

$$T = -\frac{1}{2}\Pi_m \Pi^m - d_\alpha \partial\theta^\alpha - \omega_\alpha \partial\lambda^\alpha - Q(s^\alpha \partial\hat{\lambda}_\alpha).$$

The b ghost is constructed to satisfy $Qb = T$.

Review of pure spinor in a flat background

The b ghost satisfying $Qb = T$ is

$$b = -s^{\alpha}\partial\hat{\lambda}_{\alpha} - \omega_{\alpha}\partial\theta^{\alpha} + \Pi_m\bar{\Gamma}^m - \frac{1}{4(\lambda\hat{\lambda})}(\lambda\gamma_{mn}r)\bar{\Gamma}^m\bar{\Gamma}^n$$

$$+\frac{1}{2(\lambda\hat{\lambda})}(\omega\gamma_m\hat{\lambda})(\lambda\gamma^m\partial\theta),$$

where

$$\bar{\Gamma}^m = \frac{1}{2(\lambda\hat{\lambda})}(d\gamma^m\hat{\lambda}) + \frac{1}{8(\lambda\hat{\lambda})^2}(r\gamma^{mnp}\hat{\lambda})\frac{1}{2}(\lambda\gamma_{np}\omega).$$

$Q\bar{\Gamma}^m$ is such that $Qb = T$.

Note that b is invariant under $\delta\omega_{\alpha} = (\lambda\gamma^m)_{\alpha}\Lambda_m$.

The idea is to do this construction in a curved background.

Pure spinor string in a curved background

Given the curved superspace coordinates Z^M, the variable d_α and the pure spinor variables. The world-sheet action in curved background is

$$S = \int d^2 z \, \frac{1}{2}\partial Z^M \bar{\partial} Z^N (G_{NM} + B_{NM}) - \bar{\partial} Z^M E_M{}^\alpha d_\alpha + \omega_\alpha \bar{\nabla} \lambda^\alpha$$

$$+ \xi \mathcal{D} \xi + \alpha' r^{(2)} \Phi,$$

where

$$\bar{\nabla} \lambda^\alpha = \bar{\partial} \lambda^\alpha + \lambda^\beta \bar{\partial} Z^M \Omega_{M\beta}{}^\alpha,$$

$$\Omega_{M\beta}{}^\alpha = \delta_\beta^\alpha \Omega_M + \frac{1}{4} (\gamma^{ab})_\beta{}^\alpha \Omega_{Mab}.$$

The background superfields are the supervielbein E, the superconnection Ω and the NSNS two-form B. They will be constrained by the BRST invariance.

Pure spinor string in a curved background

The BRST charge is $Q = \oint d\sigma \lambda^\alpha d_\alpha$. $Q^2 = 0$ and $\bar{\partial}(\lambda^\alpha d_\alpha) = 0$ put the background to satisfy constraints (Berkovits & Howe, hep-th/0112160).

$$Q^2 = 0 \Rightarrow \lambda^\alpha \lambda^\beta T_{\alpha\beta}{}^A = 0, \quad \lambda^\alpha \lambda^\beta H_{\alpha\beta a} = 0, \quad \lambda^\alpha \lambda^\beta \lambda^\gamma R_{\alpha\beta\gamma}{}^\delta = 0.$$

Some non-vanishing components of the background superfields are

$$T_{\alpha\beta a} = H_{\alpha\beta a} = -(\gamma_a)_{\alpha\beta}, \quad T_{\alpha ab} = 2(\gamma_{ab})_\alpha{}^\beta \Omega_\beta, \quad \Omega_\alpha = \frac{1}{4}\nabla_\alpha \Phi$$

This model is one-loop conformal invariant (Chandía, Vallilo, hep-th/0401226).

Pure spinor string in a curved background

Define the one-forms $E^A = dZ^M E_M{}^A$ and $\Omega_A{}^B = dZ^M \Omega_{MA}{}^B$, here $A = (a, \alpha)$. The torsion two-form is

$$T^A = \nabla E^A = dE^A + E^B \Omega_B{}^A = \frac{1}{2} E^B E^C T_{CB}{}^A,$$

the curvature two-form is

$$R_B{}^A = d\Omega_B{}^A + \Omega_B{}^C \Omega_C{}^A = \frac{1}{2} E^C E^D R_{DCB}{}^A$$

and

$$H = dB = \frac{1}{6} E^C E^B E^A H_{ABC}.$$

The Bianchi identities are

$$\nabla T^A = T^B R_B{}^A, \quad \nabla R_A{}^B = 0, \quad dH = 0.$$

The covariant derivative on a super p-form $\Psi_A{}^B$ is

$$\nabla \Psi_A{}^B = d\Psi_A{}^B + \Psi_A{}^C \Omega_C{}^A + (-1)^{p+1} \Omega_A{}^C \Psi_C{}^B.$$

Pure spinor string in a curved background

The transformation of the variables under Q
(Chandía, hep-th/0604115)

$$QZ^M = \lambda^\alpha E_\alpha{}^M, \quad Q\lambda^\alpha = -\lambda^\beta \Sigma_\beta{}^\alpha, \quad Q\omega_\alpha = d_\alpha + \Sigma_\alpha{}^\beta \omega_\beta,$$

$$Qd_\alpha = -(\lambda\gamma_a)_\alpha \Pi^a + \lambda^\beta \lambda^\gamma \omega_\delta R_{\alpha\beta\gamma}{}^\delta + \Sigma_\alpha{}^\beta d_\beta,$$

where $\Pi^A = \partial Z^M E_M{}^A$ and $\Sigma_\alpha{}^\beta = \lambda^\gamma \Omega_{\gamma\alpha}{}^\beta$ gives a local Lorentz rotation. As a check

$$Q\Pi^\alpha = \nabla\lambda^\alpha + \Pi^\beta \Sigma_\beta{}^\alpha, \quad Q\Pi^a = (\lambda\gamma^a\Pi) + \lambda^\alpha \Pi^b T_{\alpha b}{}^a - \Pi^b \Sigma_b{}^a,$$

where $\Sigma_b{}^a = \lambda^\gamma \Omega_{\gamma a}{}^b$.

The combination $(\lambda\hat{\lambda}) = \lambda^\alpha \hat{\lambda}_\alpha$ appears in the b ghost $\Rightarrow$ non-minimal variables are not inert under BRST transformations.

Non-minimal variables in a curved background

The BRST transformation of $(\hat{\lambda}_\alpha, \hat{\omega}^\alpha, r_\alpha, s^\alpha)$

$$Q\hat{\lambda}_\alpha = -r_\alpha + \lambda^\gamma \mathcal{X}_{\gamma\alpha}{}^\beta \hat{\lambda}_\alpha + \Sigma_\alpha{}^\beta \hat{\lambda}_\alpha, \quad Q\hat{\omega}^\alpha = -\hat{\omega}^\beta \lambda^\gamma \mathcal{X}_{\gamma\beta}{}^\alpha - \hat{\omega}^\beta \Sigma_\beta{}^\alpha$$

$$Qs^\alpha = \hat{\omega}^\alpha + s^\beta \lambda^\gamma \mathcal{X}_{\gamma\beta}{}^\alpha + s^\beta \Sigma_\beta{}^\alpha, \quad Qr_\alpha = \lambda^\gamma \mathcal{X}_{\gamma\alpha}{}^\beta r_\beta + \Sigma_\alpha{}^\beta r_\beta,$$

where $\mathcal{X}$ is constrained by $Q^2 = 0$ and it is given by

$$\mathcal{X}_{\gamma\alpha}{}^\beta = 3\delta_\alpha^\beta \Omega_\gamma - \frac{1}{4}(\gamma^{ab})_\alpha{}^\beta T_{\gamma ab}.$$

Let me call $\tilde{\Sigma} = \mathcal{X} + \Sigma$, and the BRST transformations are simplified to

$$Q\hat{\lambda}_\alpha = -r_\alpha + \tilde{\Sigma}_\alpha{}^\beta \hat{\lambda}_\alpha, \quad Q\hat{\omega}^\alpha = -\hat{\omega}^\beta \tilde{\Sigma}_\beta{}^\alpha$$

$$Qs^\alpha = \hat{\omega}^\alpha + s^\beta \tilde{\Sigma}_\beta{}^\alpha, \quad Qr_\alpha = \tilde{\Sigma}_\alpha{}^\beta r_\beta.$$

It is the transformation in flat spacetime plus a Lorentz rotation by $\tilde{\Sigma}$.

Non-minimal variables in a curved background

The non-minimal pure spinor variables contribute to the world-sheet action with

$$Q\left(\int d^2z\ s^\alpha \bar\nabla \hat\lambda_\alpha - 3(\bar\Pi^\alpha\Omega_\alpha)(s^\beta\hat\lambda_\beta) + \frac{1}{4}\bar\Pi^A T_{Aab}(s\gamma^{ab}\hat\lambda)\right)$$

The Noether charge of the BRST transformations is

$$Q = \oint \lambda^\alpha d_\alpha + \hat\omega^\alpha r_\alpha$$

And the energy-momentum tensor is

$$T = -\frac{1}{2}\Pi_a\Pi^a - d_\alpha\Pi^\alpha - \omega_\alpha\nabla\lambda^\alpha$$

$$-Q\left(s^\alpha\nabla\hat\lambda_\alpha - 3(\Pi^\alpha\Omega_\alpha)(s^\beta\hat\lambda_\beta) + \frac{1}{4}\Pi^A T_{Aab}(s\gamma^{ab}\hat\lambda)\right).$$

Next step: find b ghost.

Construction of the b ghost

In flat spacetime we had

$$\bar{\Gamma}^m = \frac{1}{2(\lambda\hat{\lambda})}(d\gamma^m\hat{\lambda}) + \frac{1}{8(\lambda\hat{\lambda})^2}(r\gamma^{mnp}\hat{\lambda})\frac{1}{2}(\lambda\gamma_{np}\omega),$$

with

$$Qd_\alpha = -(\lambda\gamma_m)_\alpha\Pi^m, \quad Q\omega_\alpha = d_\alpha,$$

$$Q\lambda^\alpha = 0, \quad Qr_\alpha = 0, \quad Q\hat{\lambda}_\alpha = -r_\alpha.$$

With this,

$$Q\bar{\Gamma}^m = -\frac{1}{2(\lambda\hat{\lambda})}\Pi^n(\hat{\lambda}\gamma^m\gamma_n\lambda) - \frac{1}{4(\lambda\hat{\lambda})^2}(\lambda\gamma^{np}r)(\hat{\lambda}\gamma_p\gamma^m\lambda)\bar{\Gamma}_n$$

Construction of the b ghost

In curved spacetime we have

$$\bar{\Gamma}^a = \frac{1}{2(\lambda\hat{\lambda})}(D\gamma^a\hat{\lambda}) + \frac{1}{8(\lambda\hat{\lambda})^2}(r\gamma^{abc}\hat{\lambda})\frac{1}{2}(\omega\gamma_{bc}\lambda),$$

where

$$D_\alpha = d_\alpha - \lambda^\gamma \mathcal{X}_{\gamma\alpha}{}^\beta\omega_\beta, \quad \text{well defined under } \delta\omega_\alpha = (\lambda\gamma^a)_\alpha\Lambda_a.$$

And

$$QD_\alpha = -(\lambda\gamma_a)_\alpha\Pi^a + \tilde{\Sigma}_\alpha{}^\beta D_\beta, \quad Q\omega_\alpha = D_\alpha + \tilde{\Sigma}_\alpha{}^\beta\omega_\beta,$$

$$Q\lambda^\alpha = 8(\lambda\Omega)\lambda^\alpha - \lambda^\beta\tilde{\Sigma}_\beta{}^\alpha.$$

It turns out that

$$Q\bar{\Gamma}^a = -\frac{1}{2(\lambda\hat{\lambda})}\Pi^b(\hat{\lambda}\gamma_a\gamma_b\lambda) - \frac{1}{4(\lambda\hat{\lambda})^2}(\lambda\gamma^{bc}r)(\hat{\lambda}\gamma_c\gamma^a\lambda)\bar{\Gamma}_b - \tilde{\Sigma}_a{}^b\bar{\Gamma}_b.$$

Construction of the b ghost

The b ghost becomes

$$b = -\left(s^\alpha \nabla \hat{\lambda}_\alpha - 3(\Pi^\alpha \Omega_\alpha)(s^\beta \hat{\lambda}_\beta) + \frac{1}{4}\Pi^A T_{Aab}(s\gamma^{ab}\hat{\lambda})\right)$$

$$-\omega_\alpha \Pi^\alpha + \Pi_a \bar{\Gamma}^a - \frac{1}{4(\lambda\hat{\lambda})}(\lambda\gamma_{ab}r)\bar{\Gamma}^a\bar{\Gamma}^b + \frac{1}{2(\lambda\hat{\lambda})}(\omega\gamma_a\hat{\lambda})(\lambda\gamma^a\Pi).$$

Using

$$Q\Pi^a = (\lambda\gamma^a\Pi) - \Pi^b \tilde{\Sigma}_b{}^a,$$

$$Q\Pi^\alpha = \nabla\lambda^\alpha - 3(\lambda\Omega)\Pi^\alpha - \frac{1}{2}(\lambda\gamma_{ab}\Omega)(\Pi\gamma^{ab})^\alpha + \Pi^\beta \tilde{\Sigma}_\beta{}^\alpha,$$

one obtains

$$Qb = T.$$

Final remarks

The background fields have an effect on the BRST transformations of the non-minimal pure spinor variables.

The construction of the b ghost in a curved background is possible for the case shown here.

The corresponding analysis for the type II superstring in curved background has been problematic (for me).

Thank you!!