Five Lectures on Dark Matter

Third Lecture

Università degli Studi di Padova

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Sezione di Padova

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II Joint ICTP-Trieste/ICTP-SAIFR School on Particle Physics (June 22 — July 3, 2020)

Thermal Freeze-Out Revisited

In the first part of this lecture,
we will revisit thermal freeze-out
with the help of a new tool:
<u>the Boltzmann equation</u>

Thermal Freeze-Out Revisited

FREEZE-OUT

$$e^{x_{FO}}\, x_{FO}^{-1/2} \simeq \frac{3\sqrt{5}}{2\,\pi^{5/2}}\,\frac{g_\chi}{g_*^{1/2}(x_{\rm FO})}\, m_\chi\, M_{\rm Pl}\,\langle \sigma v_{\rm rel}\rangle$$

Thermal Freeze-Out Revisited

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NUMBER DENSITY

$$Y_\chi(T_{FO}) = \frac{n_\chi(T_{FO})}{s(T_{FO})} \simeq \frac{H(T_{FO})/\langle \sigma v_{\rm rel} \rangle}{\frac{2\pi^2}{45}\, g_{*s}(T_{FO}) T_{FO}^3} \simeq \frac{3\sqrt{5}}{2\sqrt{2}\pi}\, \frac{g_*^{1/2}(T_{\rm FO})}{g_{*s}(T_{FO})}\, \frac{x_{FO}}{m_\chi\, M_{\rm Pl}\, \langle \sigma v_{\rm rel} \rangle}$$

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MASS DENSITY

$$\Omega_\chi h^2 = \frac{\rho_\chi}{\rho_{\rm cr}/h^2} = \frac{m_\chi Y_\chi(T_{FO})\, s_0}{\rho_{\rm cr}/h^2} \simeq 2.07 \times 10^8\, \frac{g_*^{1/2}(T_{\rm FO})}{g_{*s}(T_{FO})}\, \frac{x_{\rm FO}\, {\rm GeV}^{-1}}{M_{\rm Pl}\langle \sigma v_{\rm rel}\rangle}$$

Thermal Freeze-Out Revisited

OUR GOAL

Differential equation describing time evolution of the dark matter number density

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$$\frac{dn_\chi}{dt} = ?$$

Thermal Freeze-Out Revisited

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$$\frac{dn_\chi}{dt} = ?$$

Reasons why number density changes?

Thermal Freeze-Out Revisited

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$$\frac{dn_\chi}{dt} = \text{Hubble Expansion} + \text{Collisions}$$

Hubble Expansion

The term in the Boltzmann equation accounting for Hubble expansion is universal

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In the absence of collisions: $n_\chi \propto a^{-3}$

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$$0 = \frac{d(n_\chi a^3)}{dt} = \frac{dn_\chi}{dt} a^3 + 3a^2 \frac{da}{dt} n_\chi =$$

$$a^3 \left[\frac{dn_\chi}{dt} + 3Hn_\chi \right]$$

Hubble Expansion

The term in the Boltzmann equation accounting for Hubble expansion is universal

$$\frac{dn_\chi}{dt} + 3Hn_\chi = \text{Collisions}$$

Particle Collisions

We will consider all possible collisions
changing the number of dark matter particles

$$\chi\, b \;\rightarrow\; c\, d$$

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$$\chi\, b \;\rightarrow\; c\, d$$

Boltzmann equation: $$\frac{dn_\chi}{dt} + 3Hn_\chi = \sum_{b,c,d} \mathcal{C}_{\chi b \rightarrow cd}$$

Each collision accounted for by a collision operator

Collision Operator

$$\mathcal{C}_{\chi b\to cd} = -\int (2\pi)^4 \delta^4 \left(p_\chi + p_b - p_c - p_d\right) \, d\Pi_\chi d\Pi_b d\Pi_c d\Pi_d \left|\mathcal{M}_{\chi b\to cd}\right|^2 \left[f_\chi f_b - f_c f_d\right]$$

Under "reasonable" assumption such as absence of CP violation

Collision Operator

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Lorentz-invariant phase-space

$$d\Pi_i = g_i \frac{d^3 p_i}{2E_i (2\pi)^3}$$

Under "reasonable" assumption such as absence of CP violation

Collision Operator

$$\mathcal{C}_{\chi b\to cd} = -\int (2\pi)^4 \delta^4 \left(p_\chi + p_b - p_c - p_d\right) \, d\Pi_\chi d\Pi_b d\Pi_c d\Pi_d \left|\mathcal{M}_{\chi b\to cd}\right|^2 \left[f_\chi f_b - f_c f_d\right]$$

$$f_i = \frac{n_i}{n_i^{\rm eq}} f_i^{eq} \quad \text{and} \quad f_i(E) = e^{-E_i/T}$$

$$\mathcal{C}_{\chi b\to cd} = -\langle\sigma v_{\rm rel}\rangle_{\chi b\to cd}\left[n_\chi n_b - \frac{n_\chi^{eq} n_b^{eq}}{n_c^{eq} n_d^{eq}} n_c n_d\right] = -\langle\sigma v_{\rm rel}\rangle_{cd\to\chi b}\left[\frac{n_c^{eq} n_d^{eq}}{n_\chi^{eq} n_b^{eq}} n_\chi n_b - n_c n_d\right]$$

PROBLEM 1

Under "reasonable" assumption such as absence of CP violation

The Lee-Weinberg Scenario

Cold relic annihilating
to SM final states
(in thermal equilibrium)

$$\chi\chi \rightarrow \mathrm{SM}\,\mathrm{SM}$$

Lee, Weinberg, Phys.Rev.Lett. 39 (1977)

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GOAL:

time evolution of dark matter number density

Lee, Weinberg, Phys.Rev.Lett. 39 (1977)

Dimensionless Variables

Convenient to employ dimensionless variables for number density and time

$$Y_\chi \equiv \frac{n_\chi}{s}$$

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$$\frac{dY_\chi}{dx} = -\langle \sigma v_{\rm rel} \rangle \frac{s}{Hx} \left[Y_\chi^2 - Y_\chi^{\rm eq\,2} \right]$$

PROBLEM 2

Dimensionless Variables

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PROBLEM 2

We will give a simple semi-analytical solution and then visualize the result of numerical solutions

Semi-analytical Solution

We have the differential equation

$$\frac{dY_\chi}{dx} = -\frac{\lambda(x)}{x^2}\left[Y_\chi^2 - Y_\chi^{\mathrm{eq}\,2}\right]$$

Kolb, Turner, "The Early Universe", Front.Phys. 69 (1990)

Semi-analytical Solution

We have the differential equation

$$\frac{dY_\chi}{dx} = -\frac{\lambda(x)}{x^2}\left[Y_\chi^2 - Y_\chi^{\mathrm{eq}\,2}\right]$$

Valid for s-wave, p-wave, ….

$$\lambda(x) \equiv \frac{s(x=1)}{H(x=1)}\langle\sigma v_{\mathrm{rel}}\rangle(x)$$

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Semi-analytical Solution

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Semi-analytical solutions in two opposite regimes,
before and after decoupling,
then match these two solutions at the freeze-out point

$$e^{x_{FO}}\, x_{FO}^{-1/2} \simeq \frac{3\sqrt{5}}{2\,\pi^{5/2}}\,\frac{g_\chi}{g_*^{1/2}(x_{\mathrm{FO}})}\, m_\chi\, M_{\mathrm{Pl}}\, \langle \sigma v_{\mathrm{rel}} \rangle$$

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Early Time Solution

Convenient to use deviation from equilibrium as the density variable

$$\frac{d\Delta_\chi}{dx} = -\frac{dY_\chi^{\rm eq}}{dx} - \frac{\lambda(x)}{x^2}\left[\Delta_\chi^2 + 2\Delta_\chi Y_\chi^{\rm eq}\right]$$

$$\Delta_\chi = Y_\chi - Y_\chi^{\rm eq}$$

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$$\Delta_\chi = Y_\chi - Y_\chi^{\rm eq}$$

Before freeze-out we are very close to equilibrium:

$$\frac{d\Delta_\chi}{dx} = 0$$

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Early Time Solution

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BEFORE FREEZE-OUT

$$\Delta_X = -\frac{x^2}{\lambda(x)\left[\Delta_X + 2Y_X^{\rm eq}\right]}\frac{dY_X^{\rm eq}}{dx} \qquad (x \leq x_{FO})$$

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Late Time Solution

After freeze-out, the term proportional to equilibrium density is exponentially suppressed

$$\frac{d\Delta_\chi}{dx} \simeq -\frac{\lambda(x)}{x^2}\Delta_\chi^2$$

$$Y_X \simeq \Delta_X$$

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AFTER FREEZE-OUT

$$Y_\chi^{-1}(x) \simeq \Delta_\chi^{-1} = \int_{x_{FO}}^{x} dx\,\frac{\lambda(x)}{x^2} \qquad (x \geq x_{FO})$$

Kolb, Turner, "The Early Universe", Front.Phys. 69 (1990)

Relic Density

We can evaluate the comoving density at large x, where it reaches an asymptotic value, and compute the resulting relic density

$$Y_\chi^\infty = \sqrt{\frac{45}{\pi}} \frac{g_*^{1/2}}{g_{*s}} \frac{1}{m_\chi M_{\rm Pl}} \frac{1}{J(x_f)}$$

$$J(x_f) = \int_{x_{FO}}^\infty dx \, \frac{\langle \sigma v_{\rm rel} \rangle (x)}{x^2}$$

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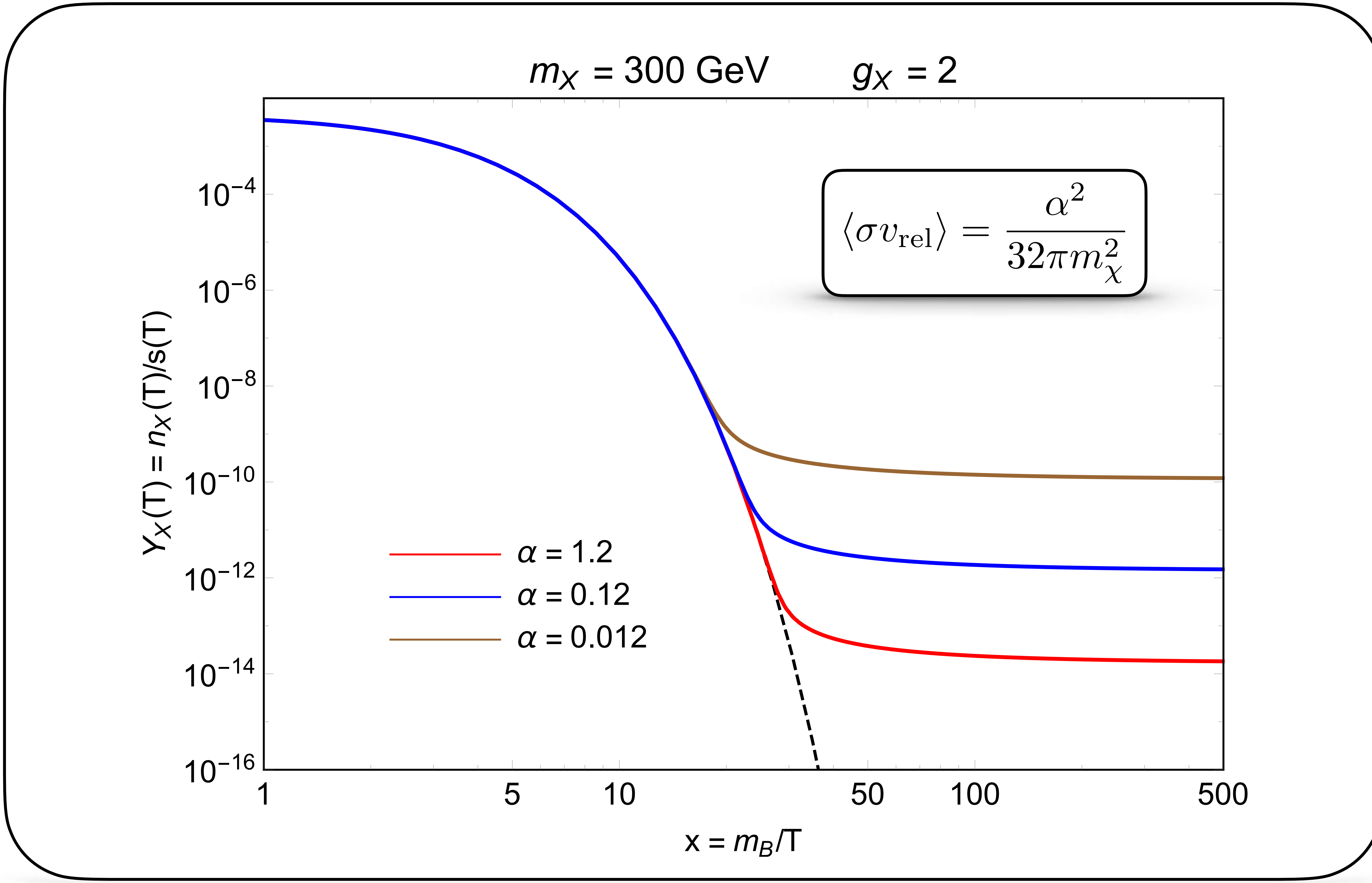
$$\Omega_\chi h^2 = \frac{1.07\times 10^9\,{\rm GeV}^{-1}}{g_*^{1/2}\,M_{\rm Pl}\,J(x_{FO})}$$

Numerical Solutions



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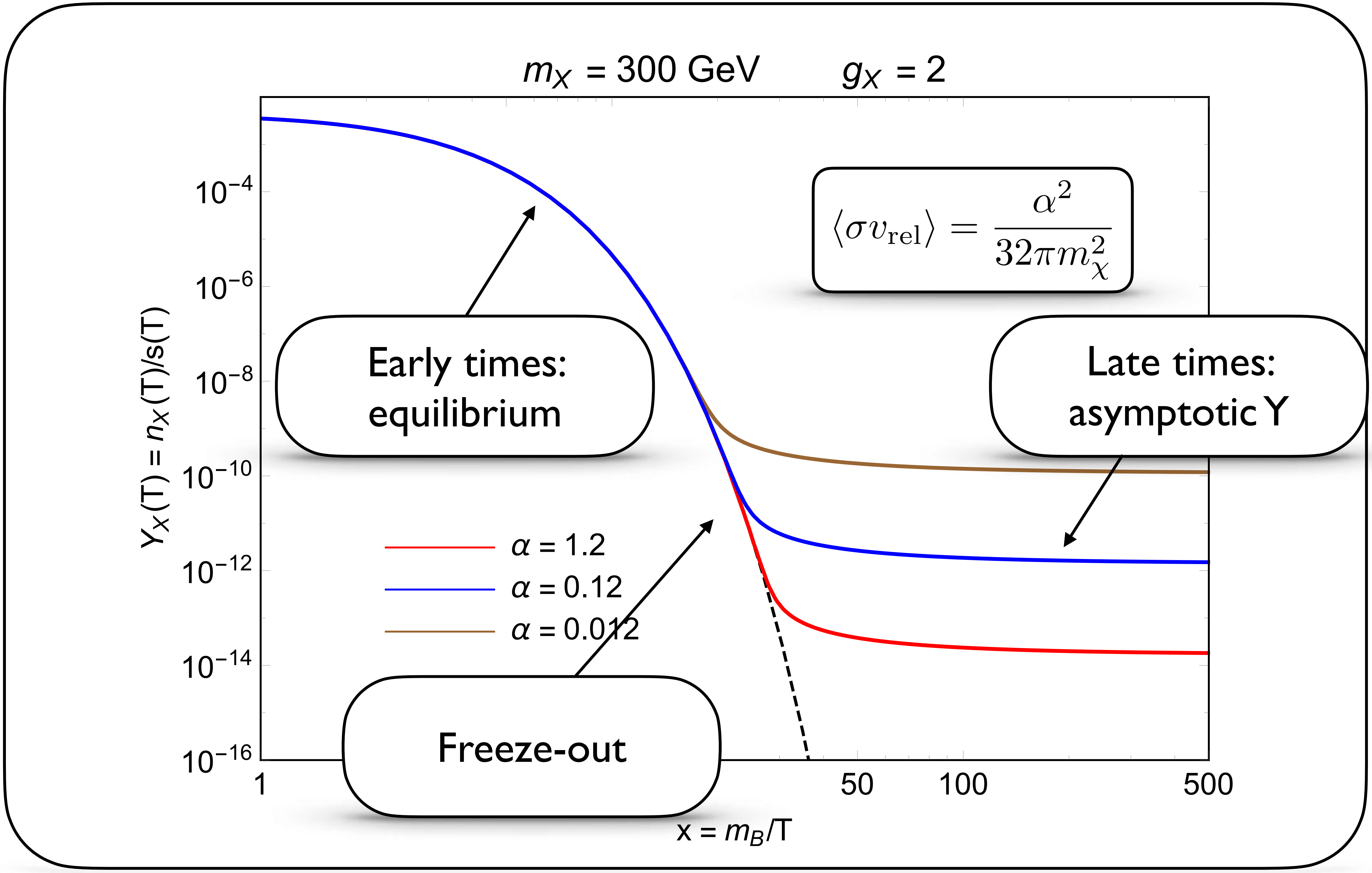

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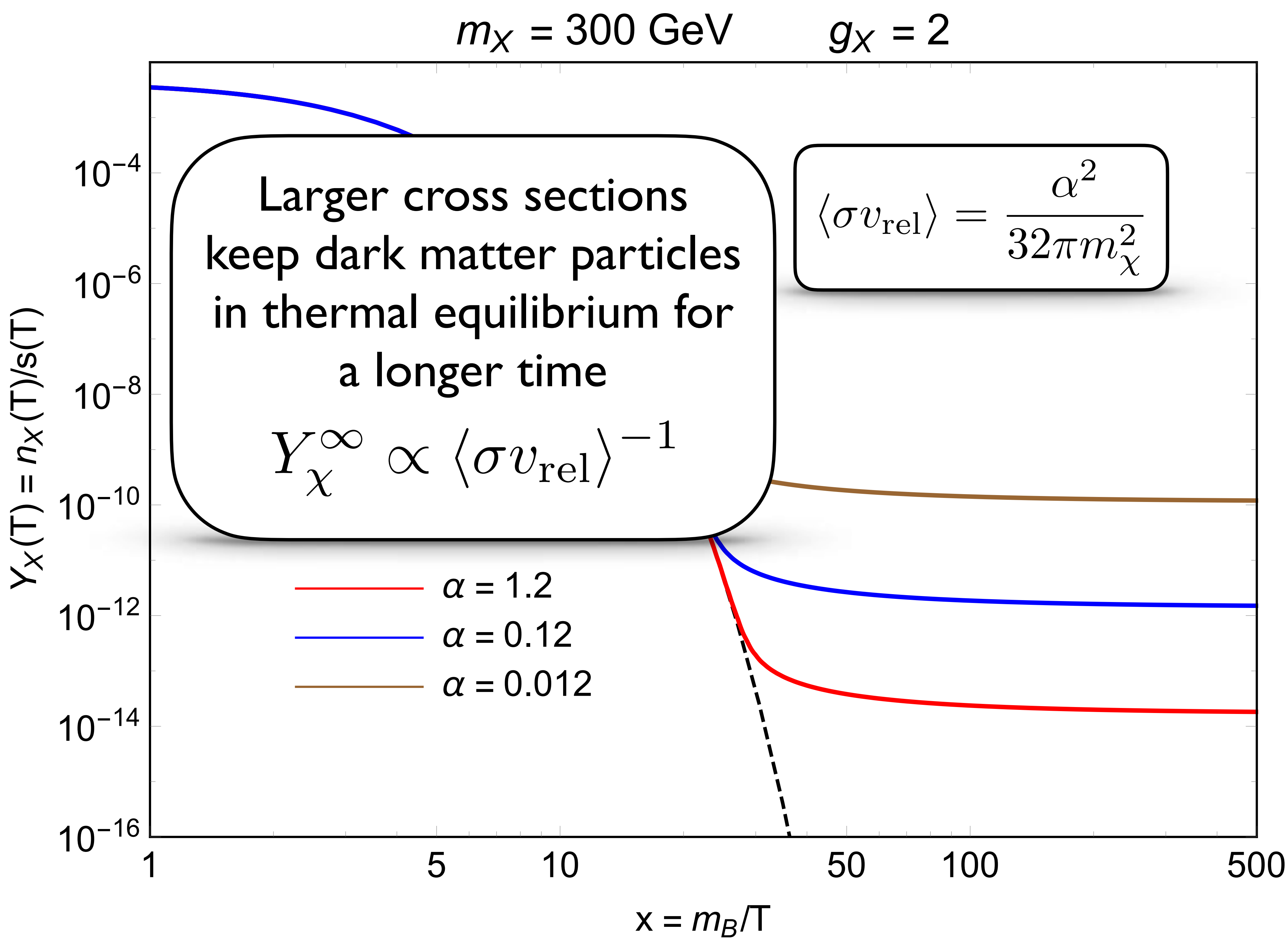

Larger cross sections keep dark matter particles in thermal equilibrium for a longer time

$$Y_\chi^\infty \propto \langle \sigma v_{\rm rel} \rangle^{-1}$$

$$\langle \sigma v_{\rm rel} \rangle = \frac{\alpha^2}{32\pi m_\chi^2}$$

Variation 1: Co-annihilations

Dark sector made of N particles all charged under a discrete simmetry (e.g., Z_2) that makes the dark matter particle stable

Griest, Seckel, Phys.Rev.D 43 (1991)

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Dark sector made of N particles all charged under a discrete simmetry (e.g., Z_2) that makes the dark matter particle stable

$$
\begin{aligned}
\psi_i \psi_j &\to \mathrm{SM\,SM} \\
\psi_i\,\mathrm{SM} &\to \psi_j\,\mathrm{SM} \\
\psi_j &\to \psi_i\,\mathrm{SM}
\end{aligned}
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Annihilations (i=j)
Co-annihilations (i≠j)

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$$\psi_i\, \mathrm{SM} \rightarrow \psi_j\, \mathrm{SM}$$

Scattering:
kinetic equilibrium

$$\psi_j \rightarrow \psi_i\, \mathrm{SM}$$

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Dark sector decays
(j >i)

Griest, Seckel, Phys.Rev.D 43 (1991)

Variation 1: Co-annihilations

Relic density can be computed by solving one single Boltzmann equation exactly as in the Lee-Weinberg scenario

$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\rm eff} v_{\rm rel} \rangle \left(n^2 - n^{{\rm eq}\,2} \right)$$

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$$\sigma_{\rm eff} \equiv \sum_{i,j} \sigma_{ij} \frac{g_i g_j}{g_{\rm eff}^2} \left(1+\Delta_i\right)^{3/2} \left(1+\Delta_j\right)^{3/2} e^{-x(\Delta_i+\Delta_j)}$$

$$n = \sum_i n_i$$

$$\Delta_i = \frac{m_i - m_1}{m_1}$$

$$g_{\rm eff} = \sum_i g_i \left(1+\Delta_i\right)^{3/2} e^{-x\Delta_i}$$

Griest, Seckel, Phys.Rev.D 43 (1991)

Variation 1: Co-annihilations

Special Requests!

Relic density can be computed by solving one single Boltzmann equation exactly as in the Lee-Weinberg scenario

$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\rm eff} v_{\rm rel} \rangle \left(n^2 - n^{\rm eq\,2} \right)$$

Price to pay for heavier states

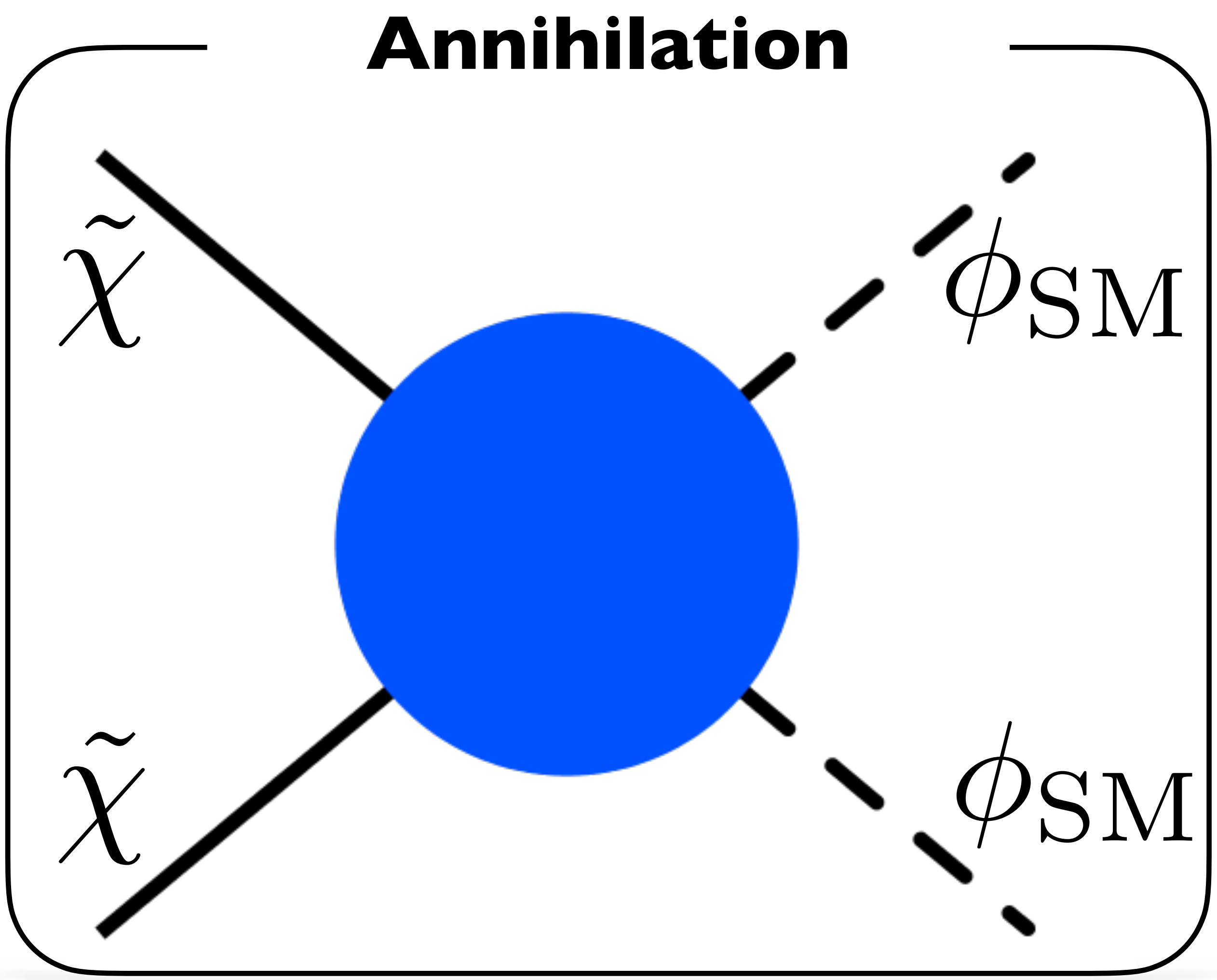
$$\sigma_{\rm eff} \equiv \sum_{i,j} \sigma_{ij} \frac{g_i g_j}{g_{\rm eff}^2} (1+\Delta_i)^{3/2} (1+\Delta_j)^{3/2} e^{-x(\Delta_i + \Delta_j)}$$

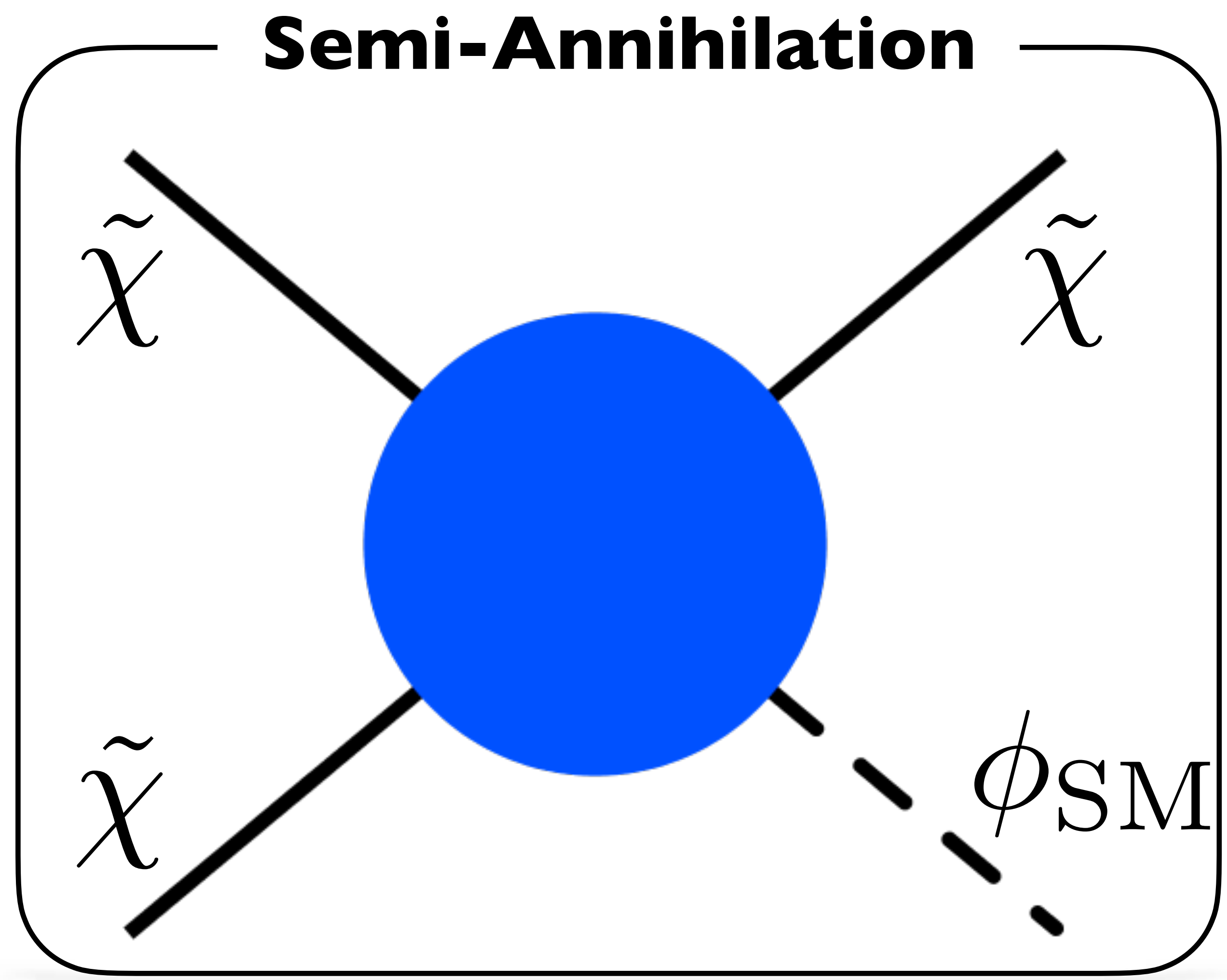
$$\Delta_i = \frac{m_i - m_1}{m_1}$$

$$g_{\rm eff} = \sum_i g_i (1+\Delta_i)^{3/2} e^{-x\Delta_i}$$

Griest, Seckel, Phys.Rev.D 43 (1991)

Variation 2: Semi-annihilations





D'Eramo, Thaler, JHEP 06 (2010)

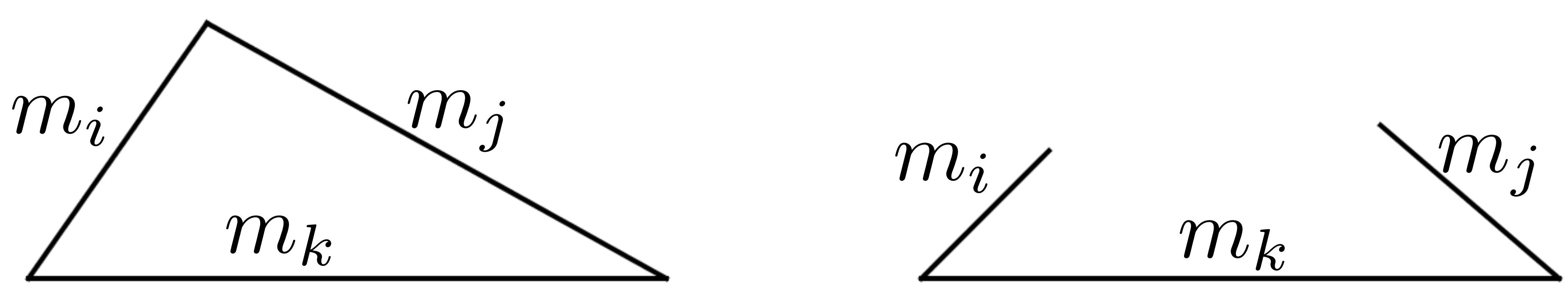
Variation 2: Semi-annihilations

$$\tilde{\chi}_i \, \tilde{\chi}_j \;\to\; \tilde{\chi}_k \, \phi_{\rm SM}$$

Stabilization symmetry must allow semi-annihilations (no Z_2)

$$\tilde{\chi}_i \tilde{\chi}_j \to \tilde{\chi}_k \phi_{\rm SM} \quad \stackrel{?}{\Rightarrow} \quad \tilde{\chi}_k \to \tilde{\chi}_i \tilde{\chi}_j \phi_{\rm SM}$$

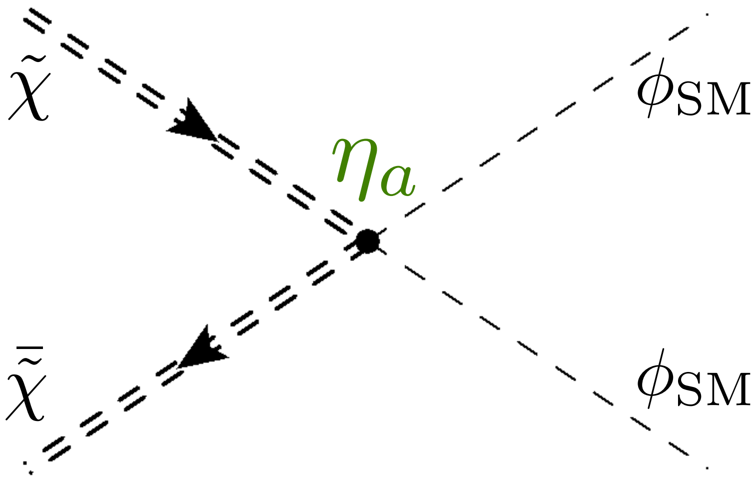
Dark matter particles stable as long as their masses satisfy a triangle inequality

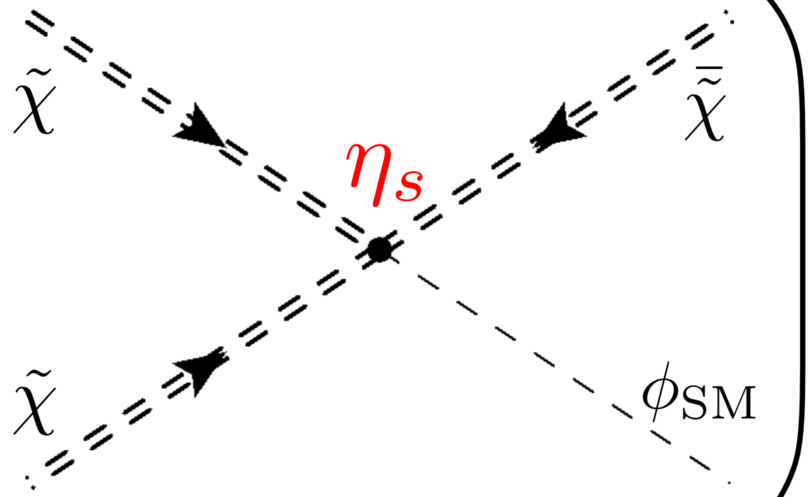


D'Eramo, Thaler, JHEP 06 (2010)

Variation 2: Semi-annihilations

	scalar	Z_3
χ	complex	$\exp\left[i\frac{2\pi}{3}\right]$
ϕ	real	0





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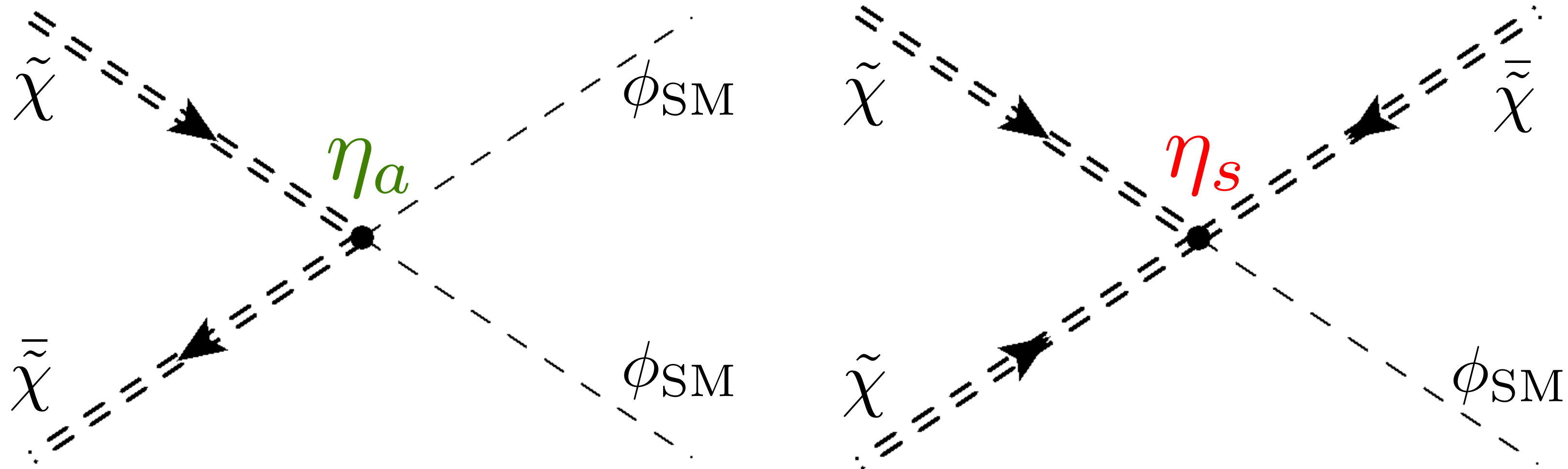
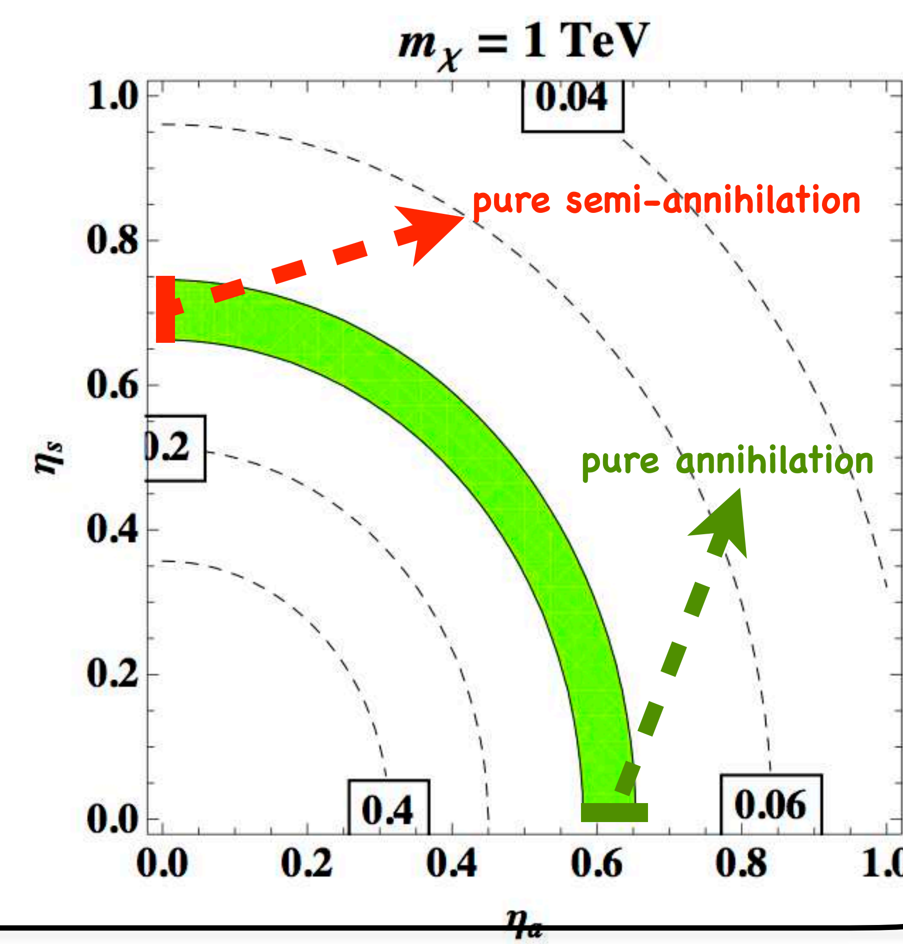


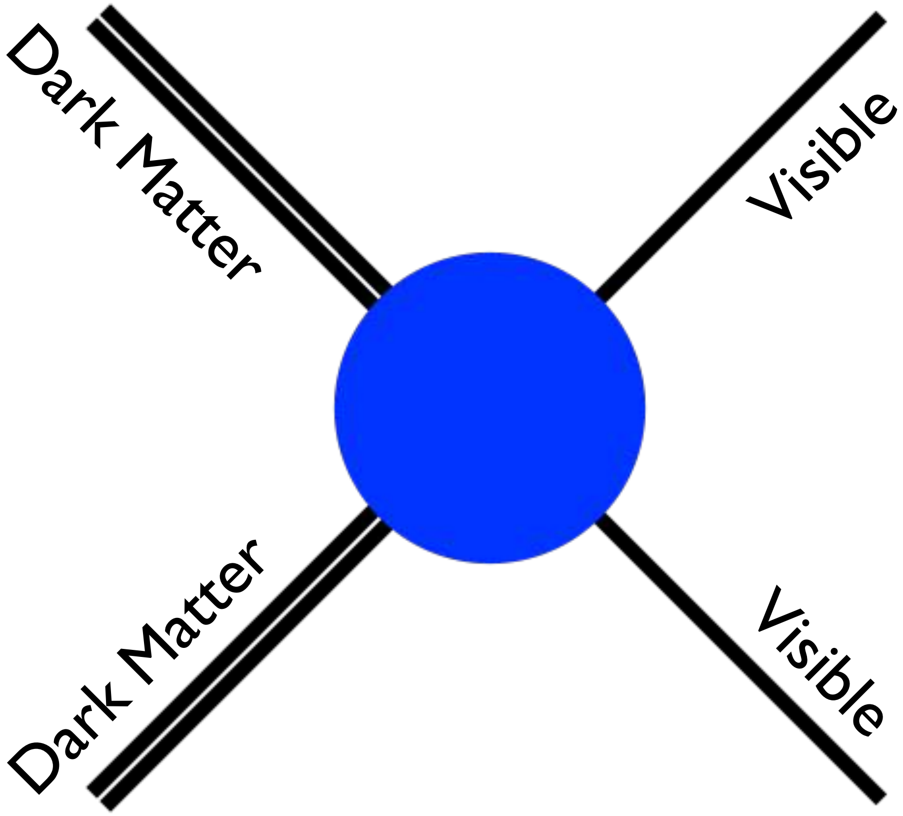
$$\frac{dn_{\tilde{\chi}}}{dt} = -3Hn_{\tilde{\chi}} - \langle\sigma v\rangle_{\tilde{\chi}\bar{\tilde{\chi}}\to\phi\phi}\left[n_{\tilde{\chi}}^2 - n_{\tilde{\chi}}^{\rm eq\,2}\right] - \frac{1}{2}\langle\sigma v\rangle_{\tilde{\chi}\tilde{\chi}\to\bar{\tilde{\chi}}\phi}\left[n_{\tilde{\chi}}^2 - n_{\tilde{\chi}}n_{\tilde{\chi}}^{\rm eq}\right]$$

expansion annihilations semi-annihilations



D'Eramo, Thaler, JHEP 06 (2010)

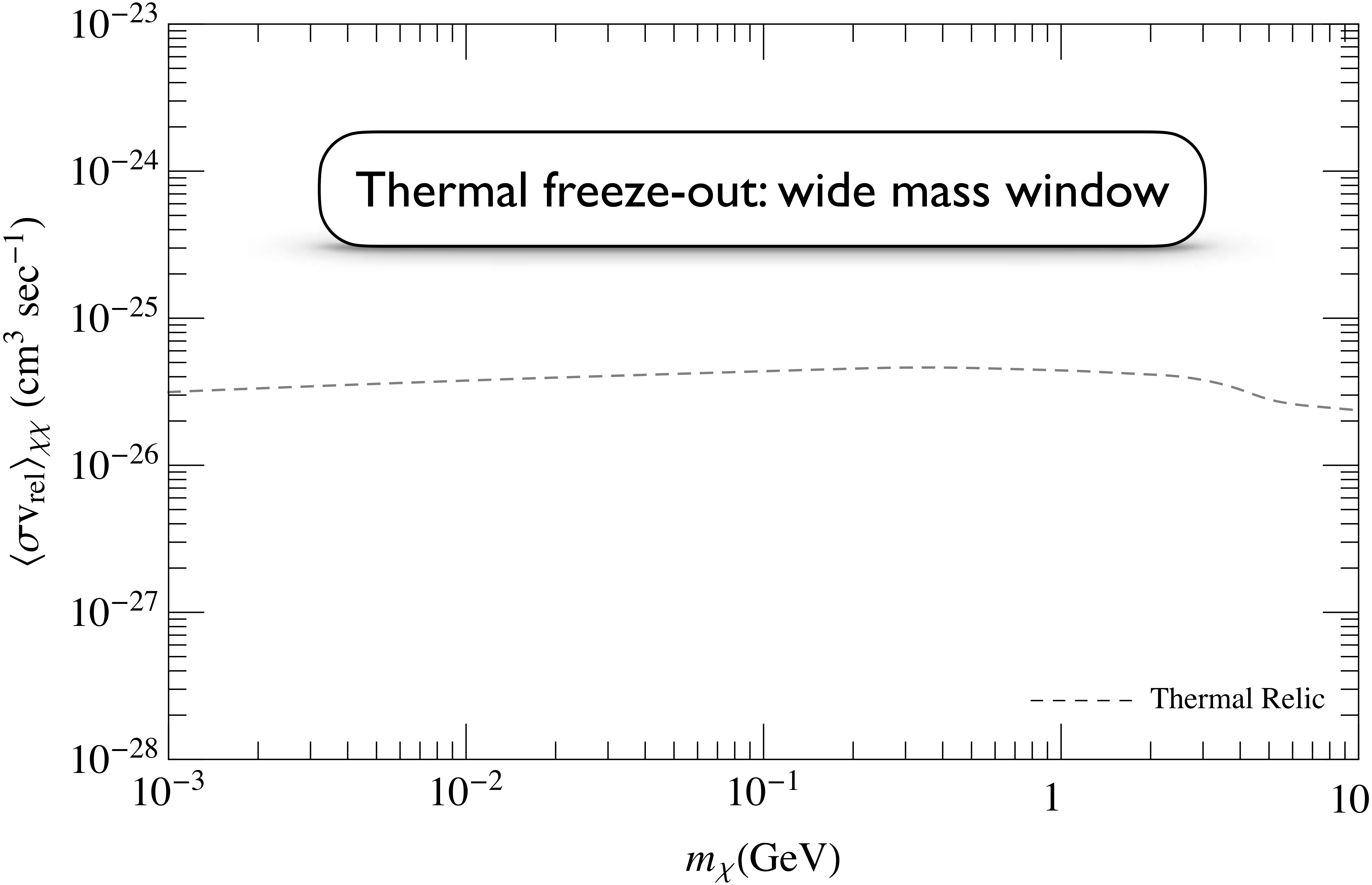
What Masses for Cold Relics?



Relic density bounds annihilation rate at freeze-out

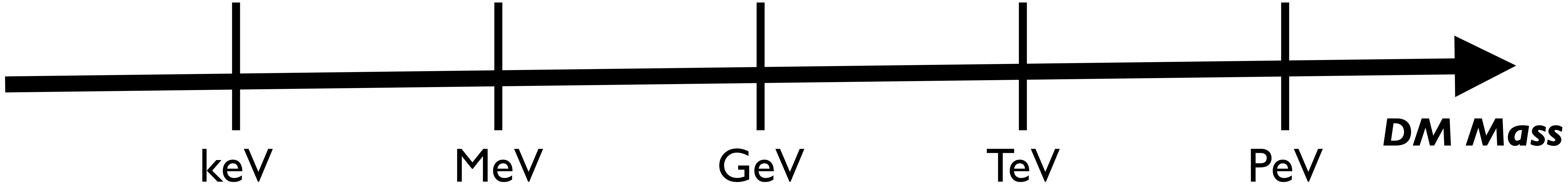
$$\langle \sigma v_{\rm rel} \rangle \simeq 3 \times 10^{-26}\,{\rm cm}^3{\rm s}^{-1}$$

What Masses for Cold Relics?



What Masses for Cold Relics?

Can we really produce a cold thermal relic for any dark matter mass?

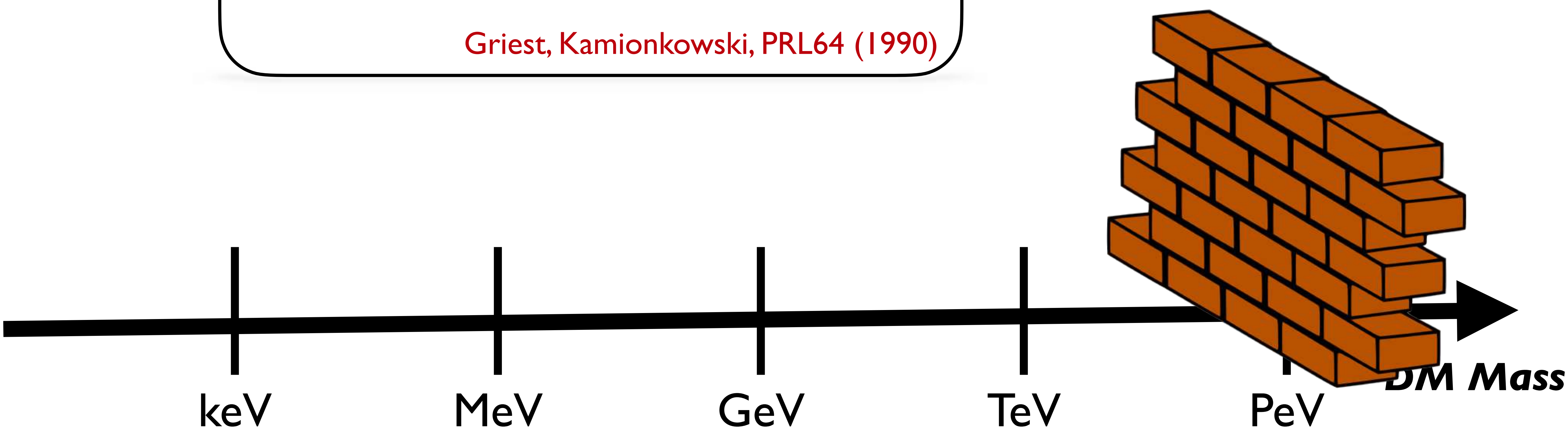


What Masses for Cold Relics?

Unitarity

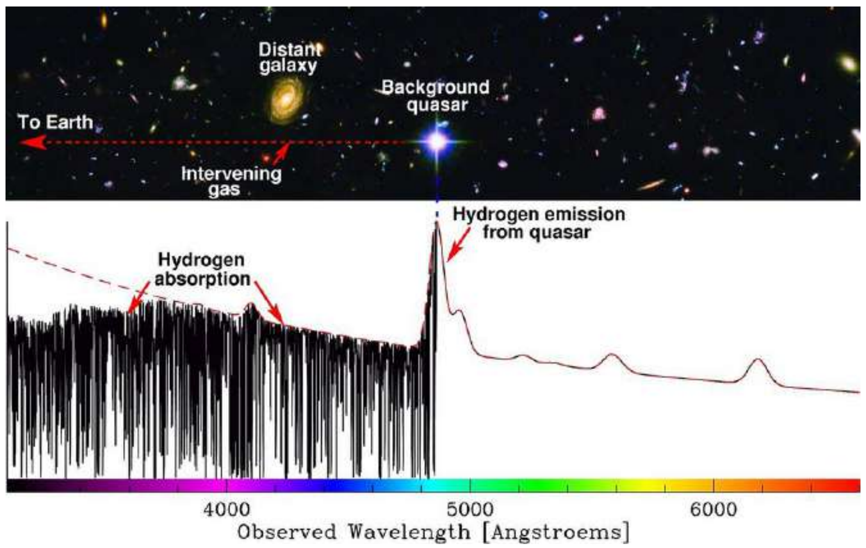
$$\sigma v \lesssim \frac{4\pi}{m_{\rm DM}^2 v}$$

Griest, Kamionkowski, PRL64 (1990)



What Masses for Cold Relics?

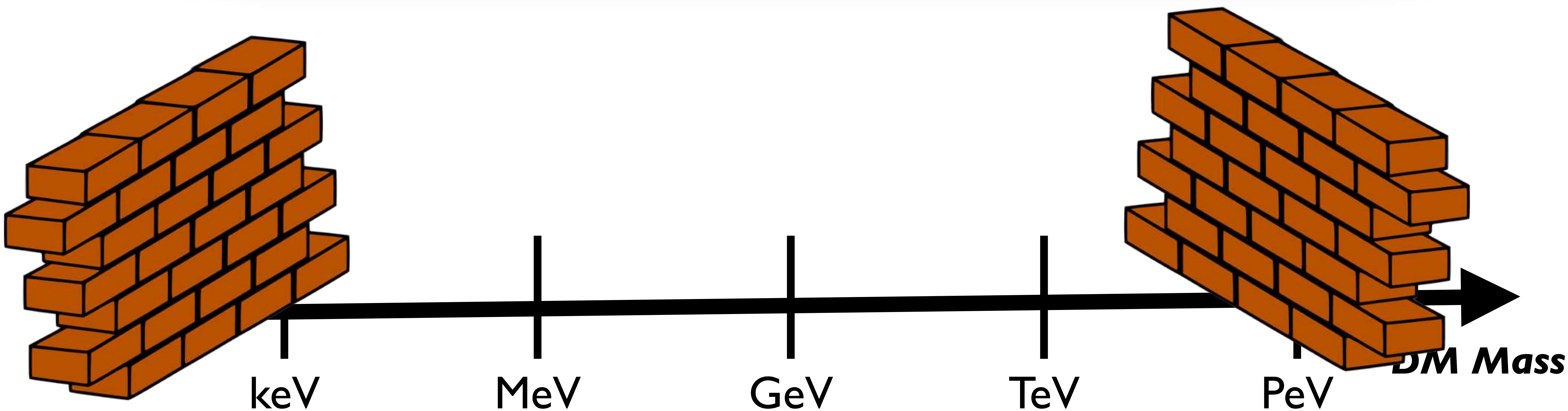
Cannot be "too hot"



Bounds on the dark matter free-streaming length

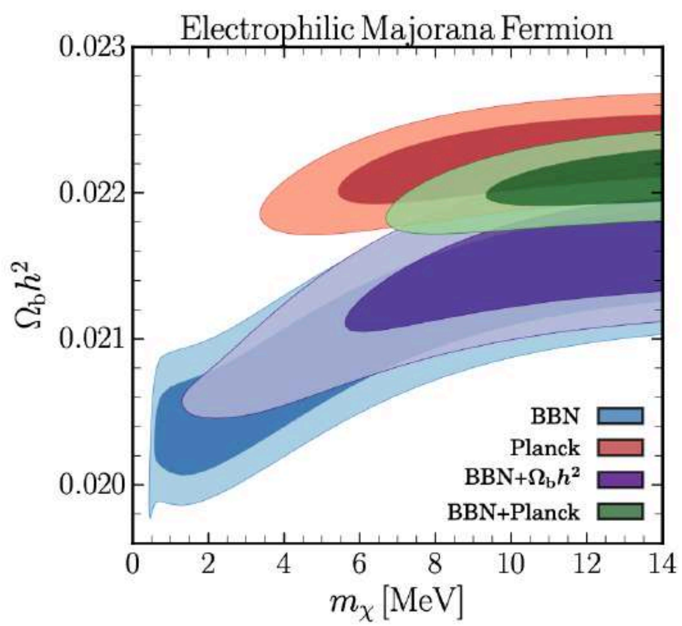
$$m_{\rm warm} \gtrsim {\rm keV}$$

Viel, Becker, Bolton, Haehnelt, PRD88 (2013)



Bounds on the dark matter mass

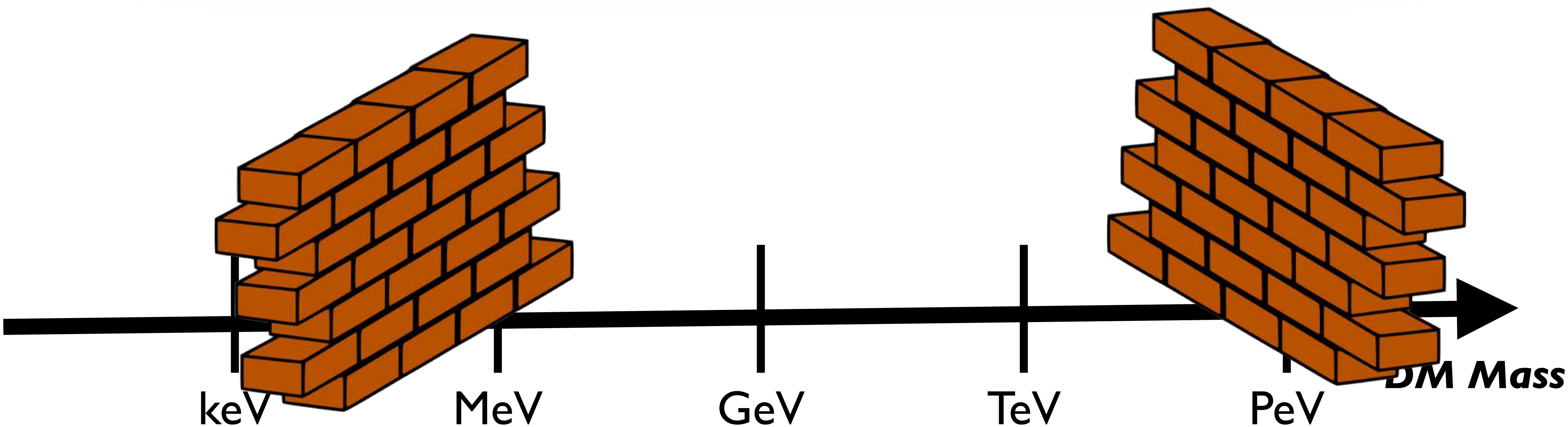
The BBN Wall



Thermal relics with mass below MeV spoil BBN and CMB

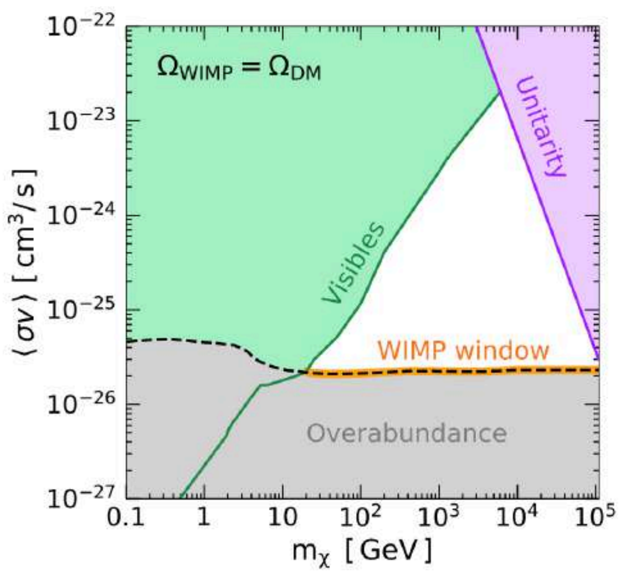
$$\mathrm{MeV} \lesssim m_{\mathrm{thermal}} \lesssim 100\,\mathrm{TeV}$$

Sabti, Alvey, Escudero, Fairbairn, Blas, arXiv:1910.01649



What Masses for Cold Relics?

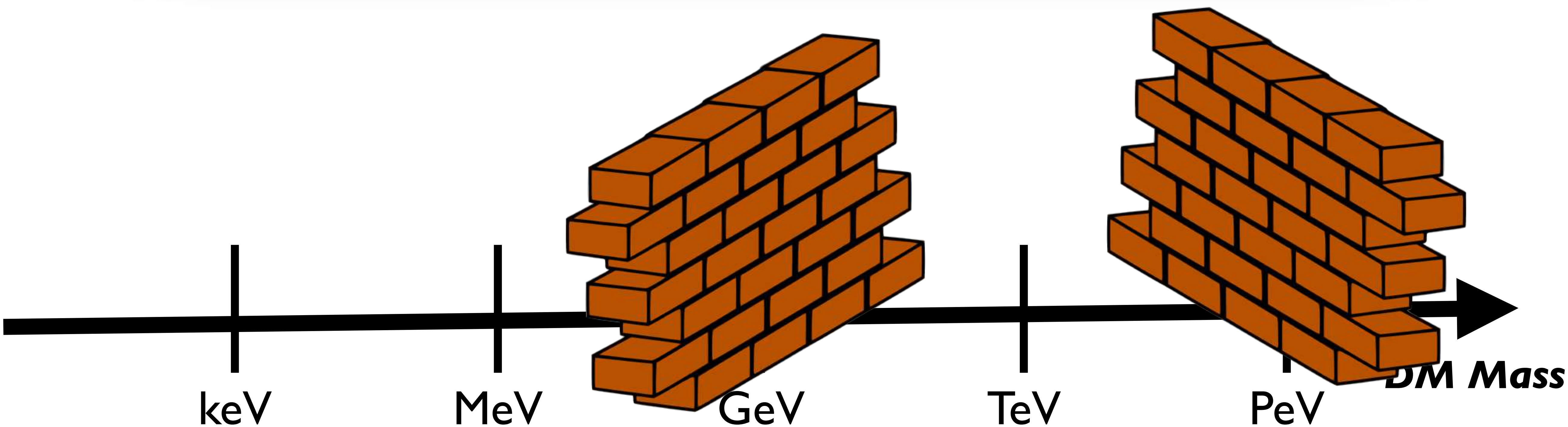
The WIMP Window



Dark matter with s-wave annihilation to visible states

$$10\,\mathrm{GeV} \lesssim m_{\mathrm{WIMP}} \lesssim 100\,\mathrm{TeV}$$

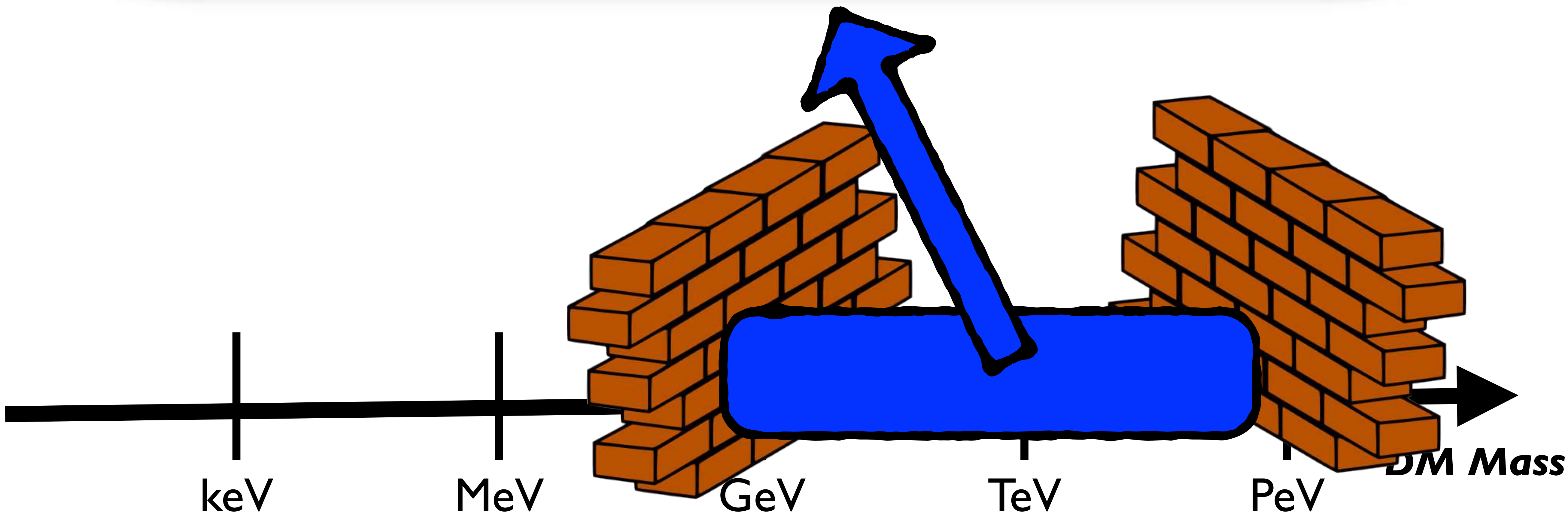
Leane, Slatyer, Beacom, Ng, PR D98 (2018)



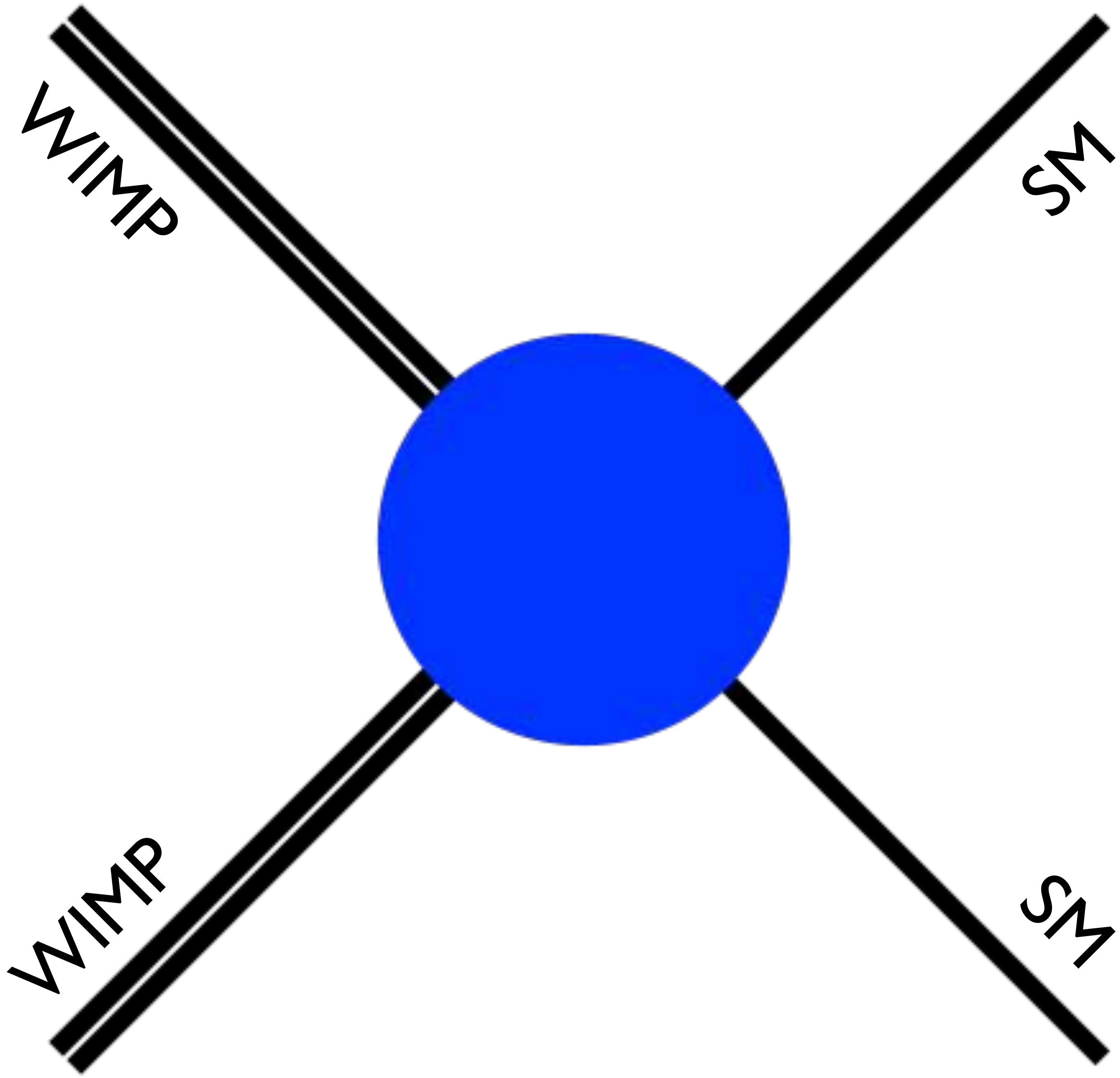
What Masses for Cold Relics?

Tomorrow we will discuss dark matter searches in this mass window

We end this lecture with a preview



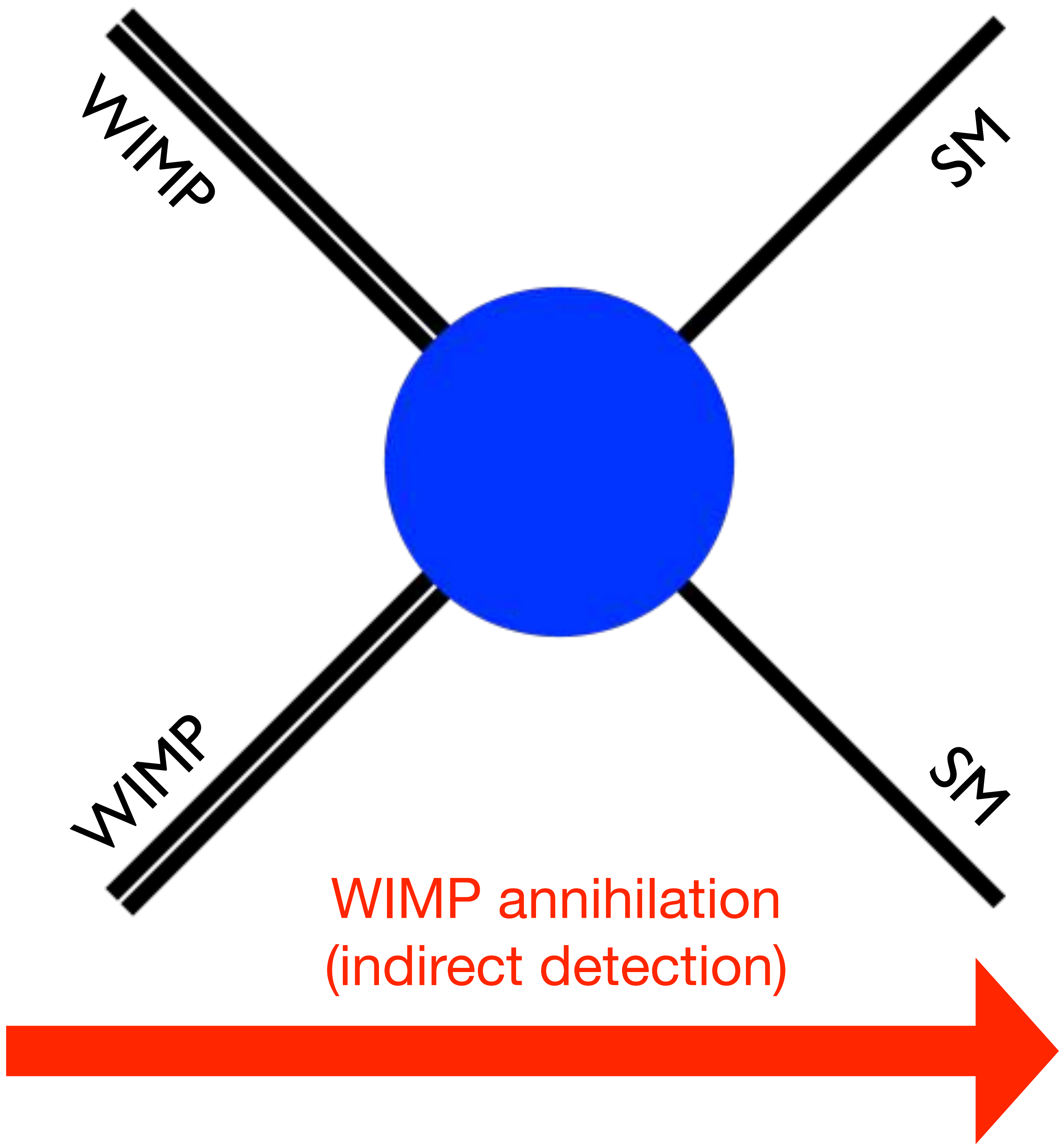
A Very Testable Paradigm



$$\Omega_\chi h^2 \simeq 0.12 \left(\frac{106.75}{g_*(T_{FO})}\right)^{1/2} \left(\frac{0.7\,\text{pb}}{\langle \sigma v_{\text{rel}} \rangle}\right)$$

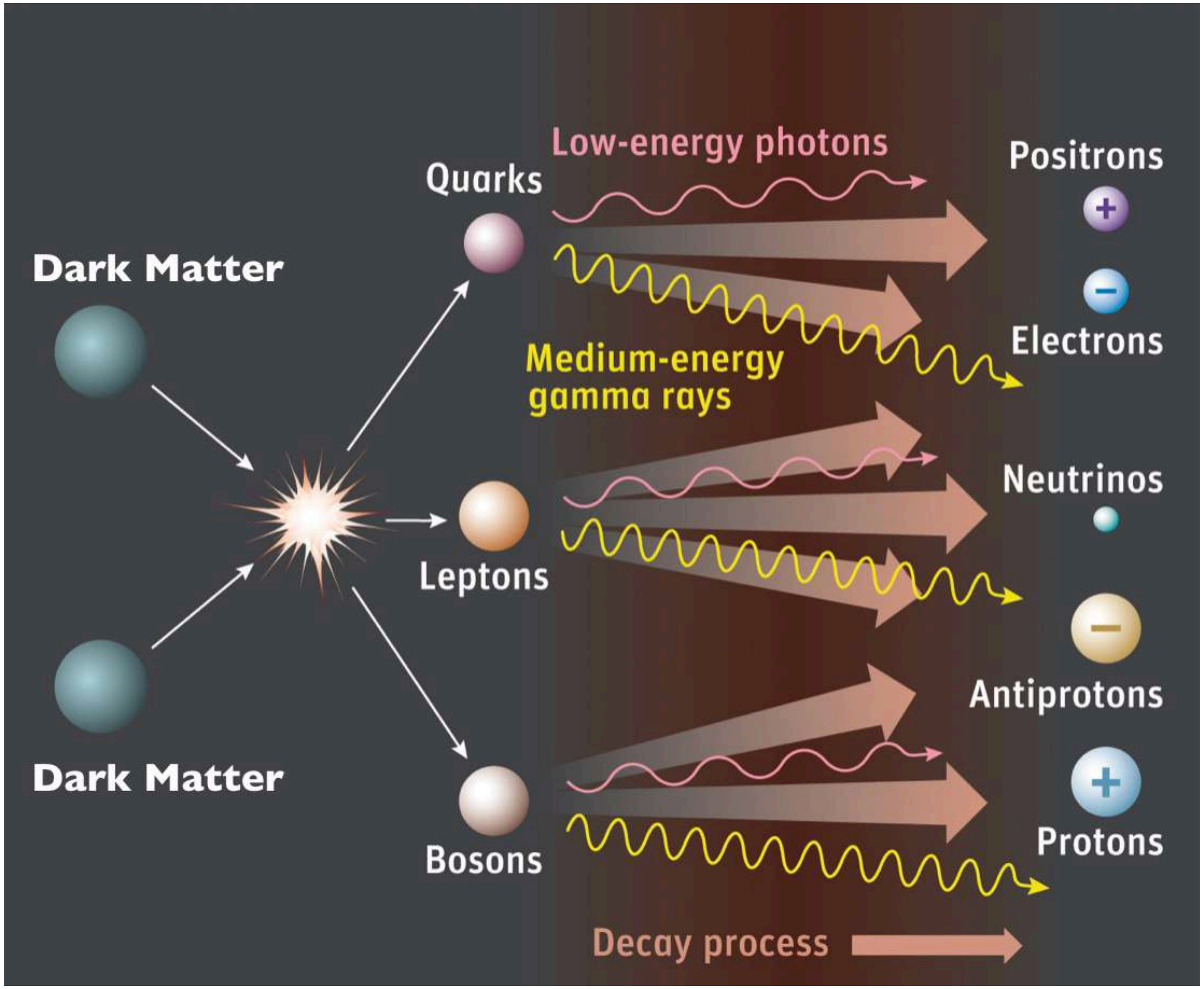
Need sizeable interactions:
can we detect them today?

A Very Testable Paradigm



A Very Testable Paradigm

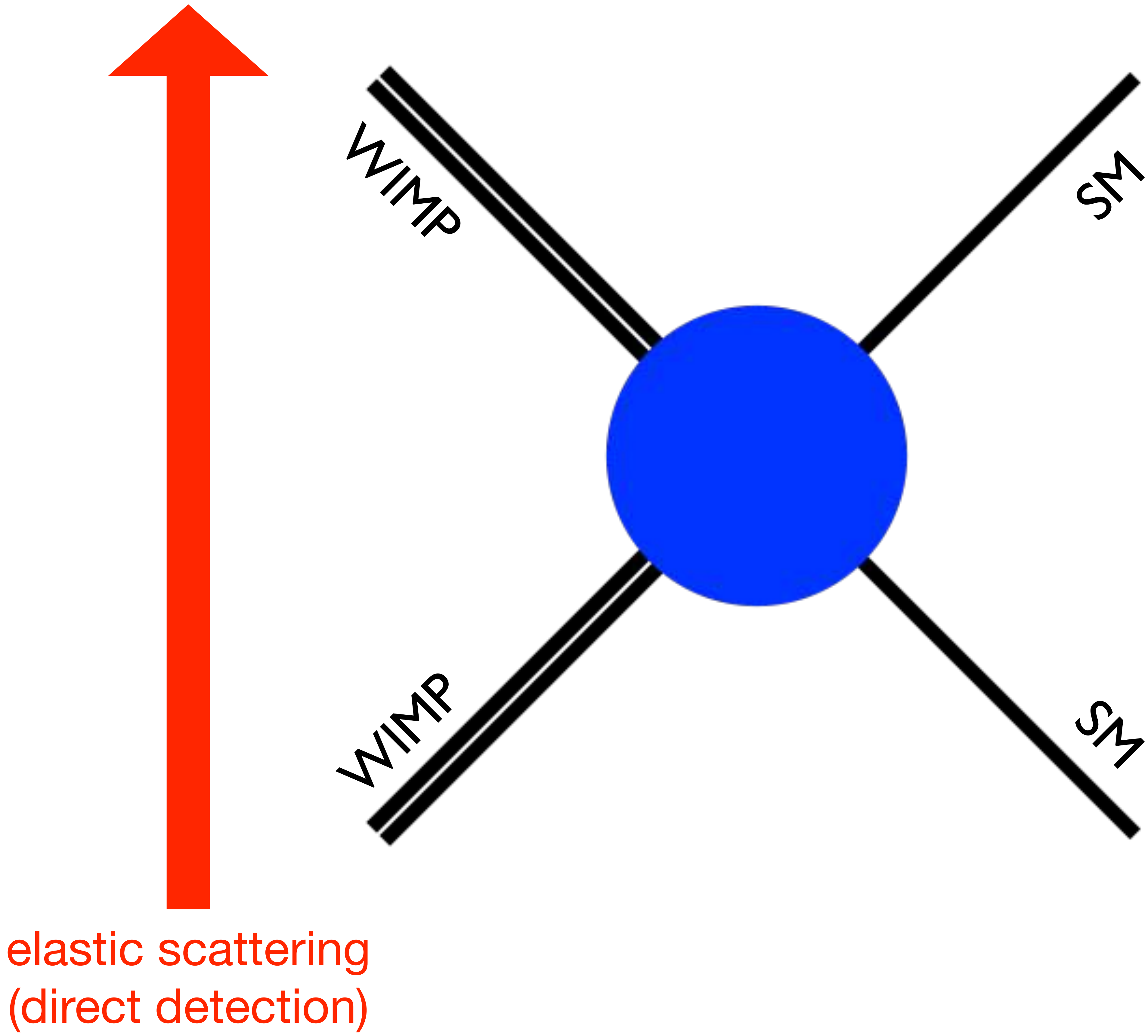
$$\chi\chi \rightarrow \mathrm{SM\,SM} \quad \text{or} \quad \chi \rightarrow \mathrm{SM\,SM}$$



Non-relativistic Dark Matter

Annihilations or decays produce cosmic rays whose characteristic energy scale is set by the DM mass

A Very Testable Paradigm

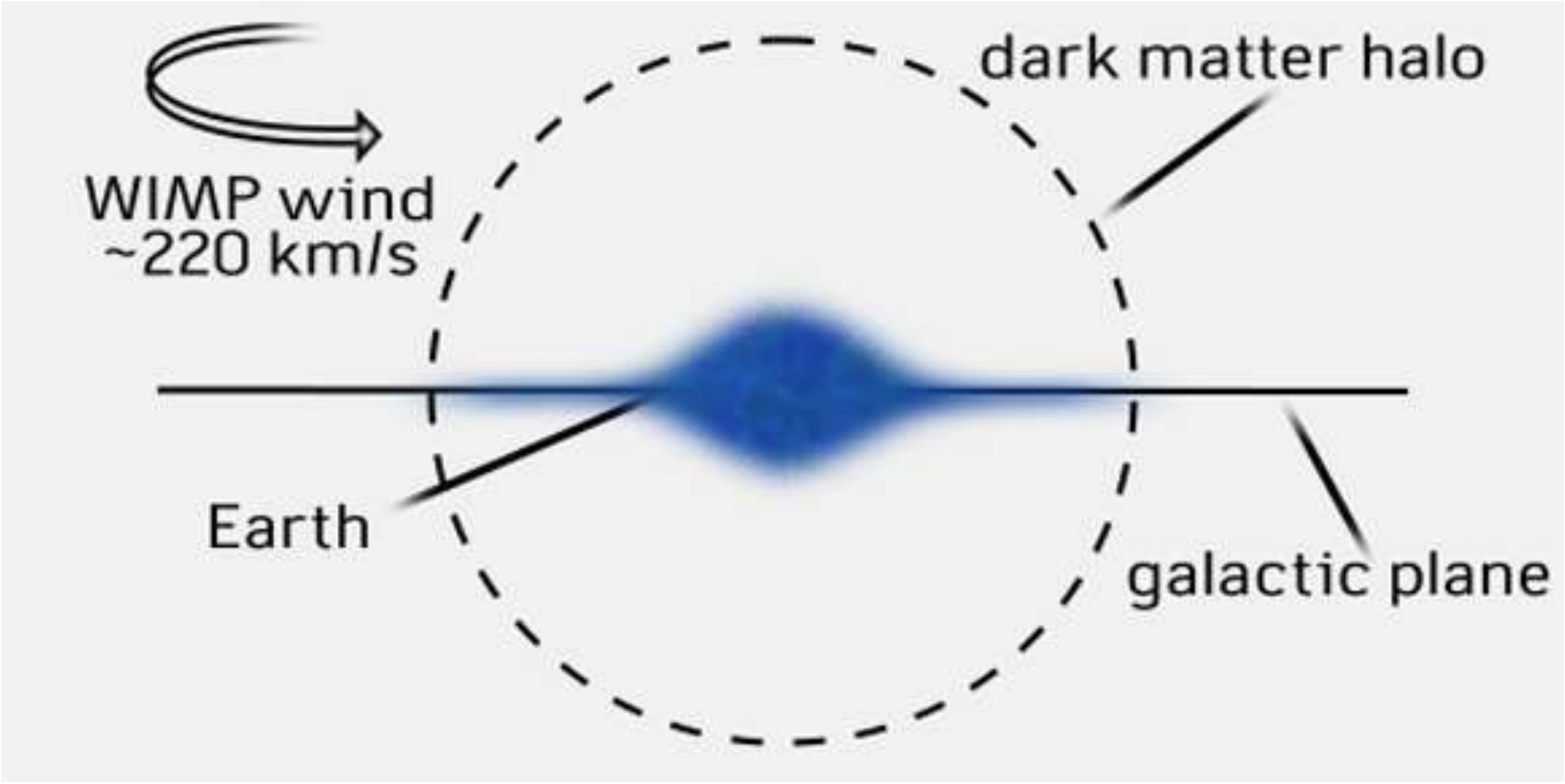


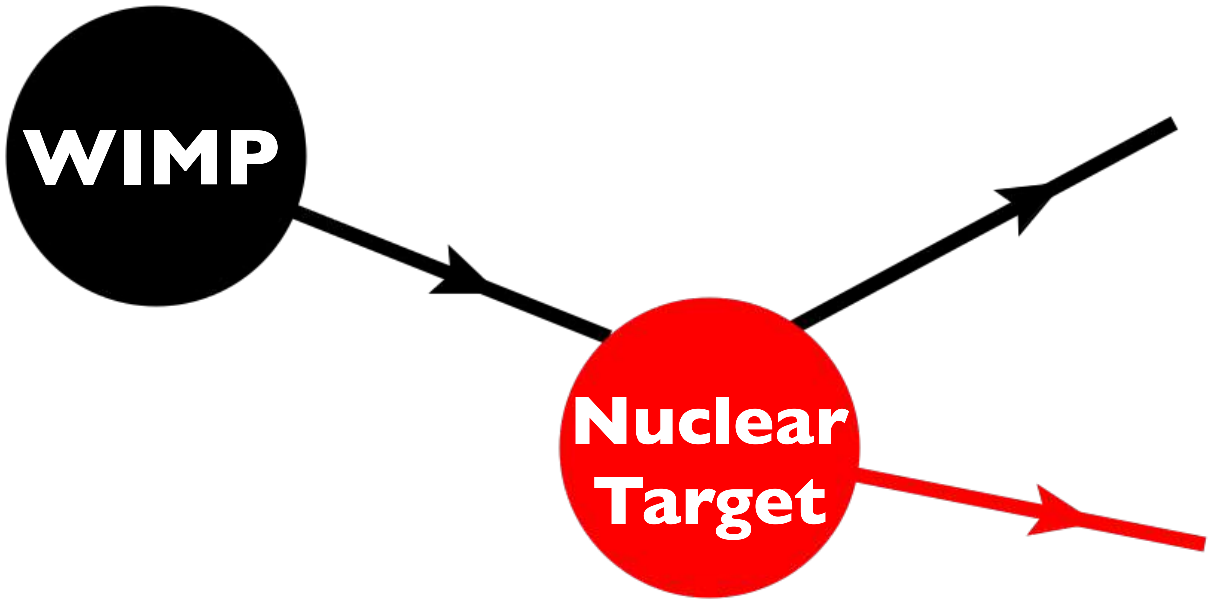
A Very Testable Paradigm

Flux of WIMPs:

10^5 cm^{-2} s^{-1}

(m_{WIMP} = 100 GeV)



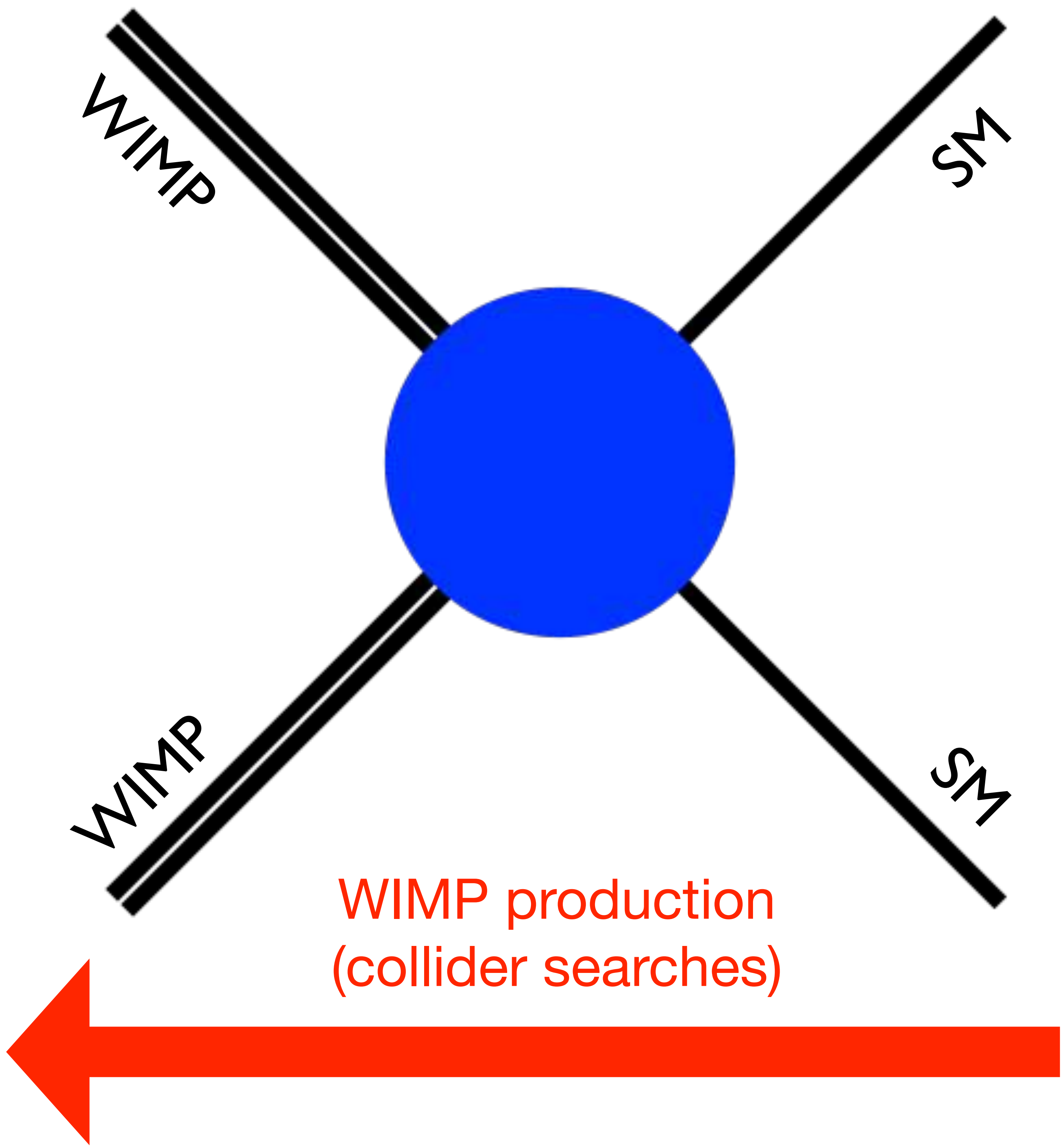


Typical recoil energy:

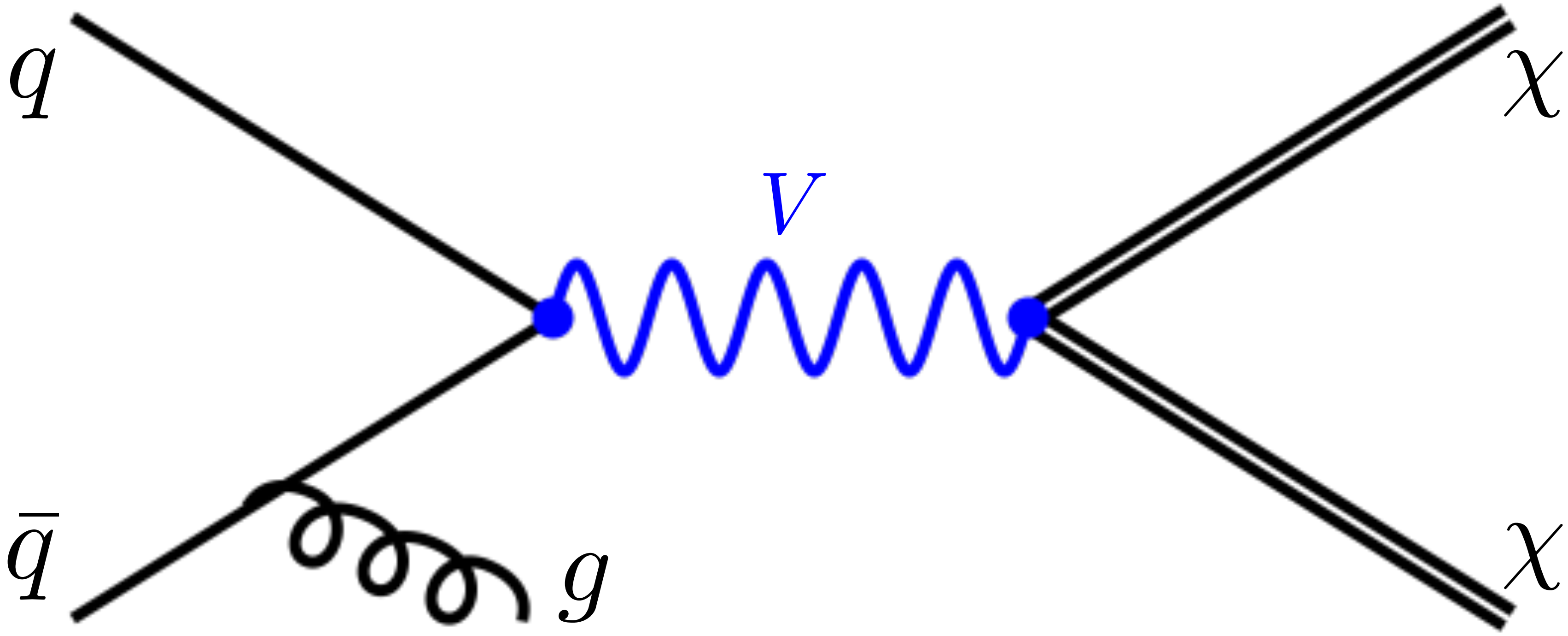
1-100 keV

Goodman and Witten, PRD31 (1985)

A Very Testable Paradigm



A Very Testable Paradigm



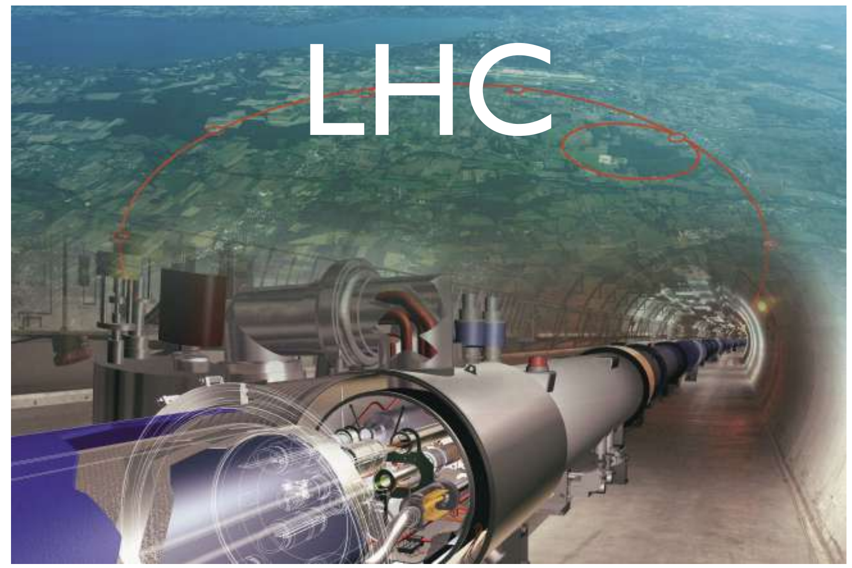
"Mono-jet" event

A Very Testable Paradigm

Multiple and complementary search strategies

A Very Testable Paradigm

Impressive Results by Current Experiments



We will learn much more soon (next 5-10 years)