# THE HIERARCHY PROBLEM

The Higgs potential in the SM takes the form

$$V(H) = -m^2 |H|^2 + \lambda |H|^4$$

After minimizing, in unitary gauge we write

$$H = \begin{pmatrix} 0 \\ v + f \end{pmatrix} \frac{1}{\sqrt{2}}$$

where $v$ is the Higgs vev and $f$ is the observed Higgs boson. Minimizing the potential we have

$$v^2 = m^2/\lambda$$

The W boson mass ~~g~~ is given by

$$m_W = \frac{gv}{2}$$

where $g$ is the $SU(2)$ gauge coupling. The mass of the Higgs boson is given by

$$m_f^2 = 2m^2 = 2\lambda v^2$$

Then, for $m_f = 125$ GeV and $v = 246$ GeV we have $m \approx 90$ GeV and $\lambda \approx 0.13$.

The hierarchy problem arises because the Higgs mass parameter receives quantum corrections from high scales.

$$m^2 = m^2\Big|_{\text{bare}} + \delta m^2\Big|_{\text{quantum corrections}}$$

The quantum corrections are quadratically divergent,

top quark

$$= -\frac{3\lambda_t^2}{8\pi^2}\Lambda^2$$


$$= \frac{9g^2}{64\pi^2}\Lambda^2$$

$$= \frac{3\lambda}{8\pi^2}\Lambda^2$$

Then, for $\Lambda_{top} > 1.5$ TeV, $\delta m^2 > 10\, m^2$.

For $\Lambda_{gauge} > 3.5$ TeV, $\delta m^2 > 10\, m^2$.

For $\Lambda_{Higgs} > 4$ TeV, $\delta m^2 > 10\, m^2$

We see that if the cutoff $\Lambda$ is greater than a few TeV, we have $\delta m^2 \gg m^2$. Then the bare Higgs mass parameter must be finely tuned against the quantum corrections to reproduce the observed value of the Higgs mass. This seems extremely contrived and unnatural. This is the "hierarchy problem" or "naturalness problem" of the SM.

In framing the hierarchy problem we have used a crude cut-off regulator. One may worry that the hierarchy problem is simply an artifact of this choice of regulator. For example, there are no quadratic divergences when using dimensional regularization. However, the hierarchy problem can be seen to be independent of regulator when $\Lambda$ is interpreted as a physical mass threshold where new degrees of freedom appear. The hierarchy problem then represents the extreme sensitivity of the Higgs mass to new physics that appears at energies above the weak scale. Most physicists believe that we will need new physics at the Planck scale $M_{Pl}$ to regulate quantum gravity. The fact that the hypercharge gauge coupling exhibits a Landau pole at high energies also requires new physics.

The sensitivity to new physics at scales $\Lambda$ of order $M_{Pl}$ is the "big hierarchy problem". However, our calculation of previous page shows that there is a "little hierarchy problem" even $\Lambda \sim 5$ TeV.

## Solutions to the Hierarchy Problem

### Supersymmetry

This is a symmetry that relates bosons to fermions. For every particle in the SM we introduce a partner with the opposite spin. The chiral symmetry that protects the mass ~~fr~~ of the fermionic superpartner of the Higgs from high scales will now also protect the mass of the Higgs.

### Composite Higgs ($\equiv$ Warped Extra Dimensions)

The Higgs is not an elementary scalar but a composite of more fundamental constituents. Then there are no quadratically divergent contributions to the Higgs mass parameter from scales above the compositeness scale $\Lambda$. If $\Lambda$ is close to the weak scale, the hierarchy problem is solved. The AdS/CFT correspondence relates composite Higgs models to solutions to the hierarchy problem based on warped extra dimensions (Randall Sundrum models).

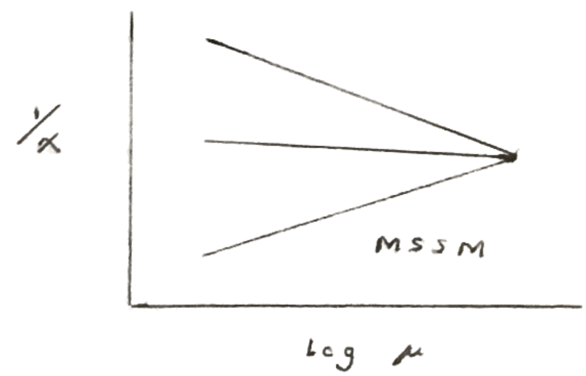
These are the two traditional approaches to the hierarchy problem. Other proposals include Large Extra Dimensions, Anthropic solution based on the Multiverse and the Relaxion.

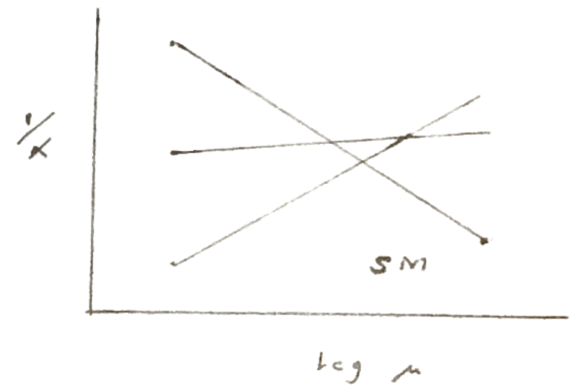
## Weak Scale Supersymmetry

Weak scale supersymmetry is an attractive solution to the hierarchy problem. Here are some of its most attractive features.

① Solves hierarchy problem. But all simple models suffer from a large residual fine tuning of order one part in a hundred.

② Unification of strong, weak and hypercharge interactions.




③ Has natural dark matter candidates. In simple models, the relic abundance does not come out easily. Much of parameter space excluded by direct and indirect detection, and by direct searches.

④ Top Yukawa in MSSM flows to a fixed point. Close to the observed value.

$$\mu \frac{d y_t}{d\mu} \simeq \frac{y_t}{16\pi^2}\left( 6 y_t^2 - \frac{16}{3} g_3^2 - \dots \right)$$

(5) MSSM predicts a light Higgs, in agreement with observation. Higgs is still somewhat heavier than expected. This is a major source of the residual tuning.

(6) The new supersymmetric particles only contribute to precision electroweak observables at one loop. The precision electroweak constraints are naturally satisfied.

Supersymmetry is a symmetry that "rotates" bosons into fermions and vice versa. Schematically,

$$\hat{Q} \, | \text{spin } 0 \rangle \longrightarrow | \text{spin } 1/2 \rangle$$

$$\hat{Q} \, | \text{spin } 1/2 \rangle \longrightarrow | \text{spin } 0 \rangle$$

How does this help with hierarchy problem? If supersymmetry is a symmetry of the theory,

$$[ \hat{H} , \hat{Q} ] = 0$$

Consider,

$$\hat{H} ( \hat{Q} \, | \text{spin } 0 \rangle ) = \hat{H} \, | \text{spin } 1/2 \rangle = m_F \, | \text{spin } 1/2 \rangle$$
$$\hat{Q} ( \hat{H} \, | \text{spin } 0 \rangle ) = \hat{Q} \, m_B \, | \text{spin } 0 \rangle = m_B \, | \text{spin } 1/2 \rangle$$

Since these operations commute, $m_B = m_F$. In supersymmetric theories, for every boson in nature there is a fermion with the same mass.

The supersymmetric partner of the Higgs boson is the Higgsino. Since the Higgsino is a fermion, its mass is protected by chiral symmetry. Since supersymmetry equates the mass of the Higgs and Higgsino, the mass of the Higgs is also protected.

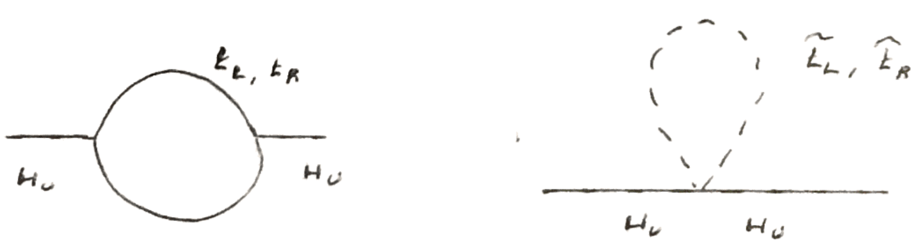
The minimal supersymmetric extension of the SM is called the "Minimal Supersymmetric Standard Model" (MSSM). The particle content of the MSSM is shown below.

| SM Particle | | Superpartner | | |
|---|---|---|---|---|
| $g$ (gluon) | | $\tilde{g}$ (gluino) | | |
| $W^{\pm}, W^{3}$ | | $\tilde{W}^{\pm}, \tilde{W}^{3}$ (wino) | | |
| $B$ | | $\tilde{B}$ (bino) | | |
| $L_\alpha$ | leptons | $\tilde{L}_\alpha$ | sleptons | |
| $e^c_\alpha$ | | $\tilde{e}^c_\alpha$ | | |
| $Q_\alpha$ | ~~s~~quarks | $\tilde{Q}_\alpha$ | squarks | |
| $u^c_\alpha$ | | $\tilde{u}^c_\alpha$ | | |
| $d^c_\alpha$ | | $\tilde{d}^c_\alpha$ | | |
| $H_u$ | Higgs fields | $\tilde{H}_u$ | Higgsinos | $Y/2 = +1/2$ |
| $H_d$ | | $\tilde{H}_d$ | | $Y/2 = -1/2$ |
| $G$ (graviton) | | $\tilde{G}$ (gravitino) | | |

Two Higgs fields are required to cancel anomalies. $H_u$ gives mass to the up-type quarks and $H_d$ to the down-type quarks and leptons.

Once electroweak symmetry is broken, $\tilde{B}$, $\tilde{W}^3$ and the neutral components of the Higgsinos all mix. The mass eigenstates are called "neutralinos". Similarly $\tilde{W}^{\pm}$ and the charged components of the Higgsinos mix after electroweak symmetry breaking. The mass eigenstates are called charginos.

In the MSSM, the solution to the hierarchy problem can be understood as arising from cancellations between the SM contributions to the Higgs mass and new contributions from loops involving the superpartners.




Since the superpartners have not been observed, their masses must be greater than that of the corresponding SM particles. Supersymmetry cannot be an exact symmetry, only an approximate one. The quadratic divergences are then cut off at the masses of the superpartners, $\Lambda \to \tilde{m}$. Then the superpartners must have masses below a TeV or so, and may be within the reach of the LHC.

In the SM baryon number and lepton number are accidental symmetries. However, they are not accidental symmetries of the MSSM. To avoid large baryon and lepton number violation we usually impose an additional symmetry known as "R-parity".

$$R = (-1)^{\overbrace{3(\underset{\substack{\uparrow\\ \text{baryon number}}}{B} - \underset{\substack{\uparrow\\ \text{lepton number}}}{L}) + 2\underset{\text{spin}}{S}}}$$

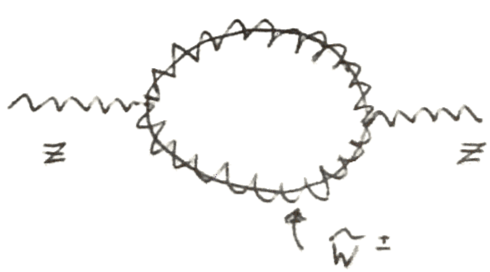
Under R-parity all SM particles, including both $H_u$ and $H_d$ have even parity, while all superpartners are odd. This immediately implies that the lightest supersymmetric particle ("LSP") is stable.

If the LSP is stable, it could constitute the observed dark matter. The candidate particles in the MSSM are the sneutrino, the gravitino and the neutralino.

An important consequence of R-parity is that all supersymmetric particles must be pair produced in colliders. There can be no single production since the initial state has R-parity $+1$ and any final state with just one supersymmetric particle would have R-parity $-1$.

Once produced, each superpartner will eventually decay down to the SM particles and the LSP. If the LSP constitutes dark matter, it will be invisible at colliders. Since supersymmetric particles are pair produced, we have 2 LSP's in every signal event. Therefore the characteristic collider signals of supersymmetry with R-parity involve missing energy. Well-known signals include "jets + MET" and "dileptons + MET".

R-parity also explains why the corrections from supersymmetric particles to precision electroweak observables are small. The supersymmetric particles are odd under R-parity and can only contribute at loop level.

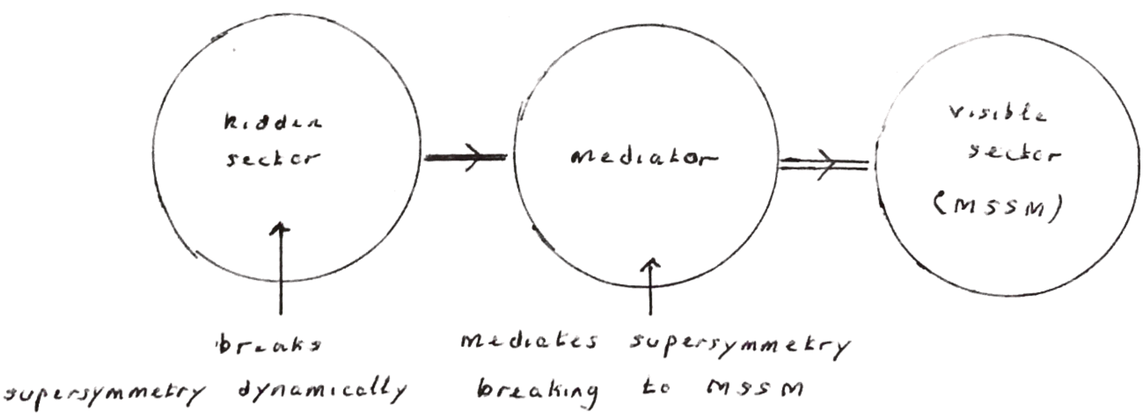



Tree level exchange of super-symmetric particles is forbidden.

In general the MSSM can lead to large flavor changing neutral currents, Although these effects are only generated at loop level, they are too large unless there is alignment between the masses of the scalar superpartners and the corresponding SM fermions. This is known as the "SUSY Flavor Problem".

Also, in general the MSSM can lead to large new sources of CP violation. This is the "SUSY CP Problem".

A realistic SUSY model must address these features. A typical SUSY model looks like this.




The masses of the supersymmetric particles depend on how supersymmetry is mediated to the visible sector. Examples include gauge mediation, gaugino mediation and anomaly mediation. These can resolve the "SUSY Flavor Problem" and "SUSY CP Problem".

Most simple supersymmetric models are severely constrained by the data, and are fine tuned at the per cent level or worse. There are two reasons for this. The first has to do with the value of the Higgs mass, 125 GeV, which is well above what is expected in most simple supersymmetric models. At tree level in the MSSM there is an upper bound on the Higgs mass, $m_H \leqslant m_Z$. To raise the Higgs mass to 125 GeV requires large radiative corrections. These generally need the stop masses to lie in the few TeV range, or even heavier, leading to large fine tuning. This can be avoided by going beyond the MSSM, but the models are more complicated.

A second source of fine tuning arises from the fact that the LHC bounds on the stops are now of order 800 GeV or higher. The quadratically divergent contributions to the Higgs mass parameter are cut off at the stop mass, but are softened to a logarithmic divergence,

$$\delta m_H^2 = \frac{3 \lambda_t^2}{8\pi^2} m_{\tilde{t}}^2 \log \left( \frac{\Lambda^2}{m_{\tilde{t}}^2} \right)$$

Here $\Lambda$ is the scale at which supersymmetry breaking is mediated to the visible sector, and is of order 100 TeV or higher in simple models. This corresponds to fine tuning at the few per cent level.

## Composite Higgs Models

In this class of theories, the Higgs is not an elementary particle, but a composite made up of more fundamental constituents. In this scenario, the quadratically divergent contributions to the Higgs mass parameter are cut off at the compositeness scale, which we denote by $\Lambda$. For sufficiently low $\Lambda$, the hierarchy problem is solved.

In this framework, the Higgs boson would be expected to have large self-couplings at the compositeness scale $\Lambda$. At this scale operators such as

$$K_S \frac{(D_\mu D_\nu H)^\dagger D^\mu D^\nu H}{\Lambda^2}$$

are expected to be present. Using the methods of "naive dimensional analysis" we can estimate $K_S$ to be of order one. Then precision electroweak constraints require the compositeness scale $\Lambda$ to be above 5 TeV. This is far above the Higgs mass of 125 GeV. To explain this, we usually take the Higgs to be a pseudo-Nambu-Goldstone Boson (pNGB).

In composite Higgs models the SM quarks and leptons acquire masses through "partial compositeness". We illustrate this for the case of the up-type quarks. We include in the Lagrangian terms of the form,

$$\mathcal{L} \sim Q \, O_{Q^c} + u^c \, O_u$$

Here $Q$ is the doublet of $SU(2)_L$ containing SM quarks and $u^c$ is the $SU(2)_L$ singlet quark. $O_{Q^c}$ and $O_u$ are operaters in the strongly coupled sector of which the Higgs is a composite. At the compositeness scale this leads to terms of the form,

$$Q \, \tilde{Q}^c + u^c \, \tilde{U}$$

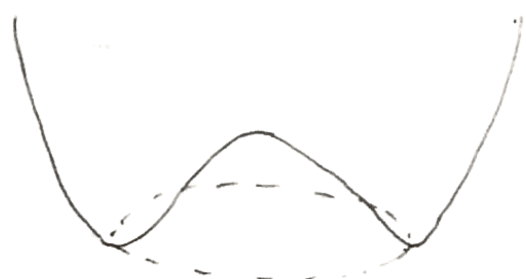
Where $\tilde{Q}^c$ and $\tilde{U}$ are resonances of the strongly coupled sector with masses of order the compositeness scale. The strongly coupled sector is expected to contain terms of the form

$$\tilde{Q} \, \tilde{Q}^c + \tilde{U}^c \, \tilde{U} + \tilde{Q} \, H \, \tilde{U}$$

Here $\tilde{Q}$ and $\tilde{U}^c$ are strongly coupled resonances and $H$ is the Higgs. Through these interactions the SM quarks $Q$ and $U^c$ mix with $\tilde{Q}$ and $\tilde{U}^c$, thereby becoming partially composite. Through the mixing they acquire Yukawa couplings to the Higgs.

In realistic composite Higgs models the Higgs is usually identified as the pNGB of a global symmetry that is spontaneously broken by the strong dynamics that generates the Higgs. It is therefore important to have a detailed understanding of the properties of Goldstone bosons in general. We now discuss how to write down the low energy effective Lagrangian for Goldstone bosons based on symmetry considerations.

Consider a scalar field $\phi$ with a $U(1)$ symmetry, $\phi \to e^{i\alpha}\phi$. The potential for $\phi$ is assumed to be of the "Mexican hat" form.



$$V(\phi) = -m^2 \phi^+\phi + \lambda(\phi^+\phi)^2$$

We can expand $\phi$ about the minimum of the potential,

NGB

$$\phi = \frac{1}{\sqrt{2}}\left(f + \varphi(x)\right) e^{i\theta(x)/f}$$

radial mode

Under $\phi \to e^{i\alpha}\phi$, $\theta \longrightarrow \theta + \alpha$. The global symmetry is realized nonlinearly through the shift of $\theta$. Only the massless NGB $\theta(x)$ will be present in the low energy effective theory. The Lagrangian for $\theta(x)$ in the low energy theory must respect the shift symmetry

$$\theta(x) \to \theta(x) + \alpha$$

This implies that only terms with derivatives are allowed. Then the Lagrangian takes the form,

$$L(\theta) = \frac{1}{2} \partial_\mu \theta \partial^\mu \theta + c_1 \frac{(\partial_\mu \theta \partial^\mu \theta)(\partial_\nu \theta \partial^\nu \theta)}{f^4}$$

$$+ \dots$$

The coefficient $c_1$ can be determined from the model with the scalar field $\phi(x)$ after integrating out the radial mode $\rho(x)$.

The crucial observation is that any theory with a global $U(1)$ symmetry that is spontaneously broken will have a Goldstone boson $\theta(x)$ satisfying the shift symmetry $\theta(x) \to \theta(x) + \alpha$. Therefore the form of the effective Lagrangian is completely dictated by symmetry. Different ultraviolet theories will lead to different values of the coefficients in the Lagrangian like $c_1$, but the form of the Lagrangian itself is completely fixed.

For $c_1$ of order 1, this low energy theory becomes strongly coupled at $\Lambda \sim 4\pi f$. New physics must enter at or below this scale. This could be as simple as the radial mode of the linear model.

There is an alternative way to obtain the low energy effective theory for Goldstone bosons that is often more convenient. We illustrate it for the case of $U(1)$ breaking that we have been considering.

Consider the object

$$\hat{\phi} = \frac{1}{\sqrt{2}} f \, e^{i\theta(x)/f}$$

We see that $\hat{\phi}$ is almost exactly like $\phi$, but they are different because the radial mode from $\phi$ is not present in $\hat{\phi}$. The advantage of working with $\hat{\phi}$ is that it transforms linearly under $U(1)$, just like $\phi$. Under $U(1)$, $\hat{\phi} \to e^{i\alpha} \hat{\phi}$.

To obtain the low energy Lagrangian for $\theta(x)$, simply write down the most general Lagrangian for $\hat{\phi}(x)$ consistent with the rephasing symmetry,

$$\hat{\phi}(x) \rightarrow e^{i\alpha} \hat{\phi}(x)$$

Notice that terms with no derivatives such as $\hat{\phi}^{+}\hat{\phi}$ and $(\hat{\phi}^{+}\hat{\phi})^{2}$ are completely independent of $\theta(x)$ and do not contribute to the Lagrangian. We are left with,

$$\partial_\mu \hat{\phi}^{+} \partial^\mu \hat{\phi} + c_1' \frac{\partial_\mu \hat{\phi}^{+} \partial^\mu \hat{\phi} \, \partial_\nu \hat{\phi}^{+} \partial^\nu \hat{\phi}}{f^{4}}$$

$$+ \; . \; . \; .$$

Expanded out in terms of $\theta(x)$, this will give a Lagrangian of the same form as the one directly obtained using the shift symmetry of $\theta(x)$.