In composite Higgs models, precision electroweak constraints require that the compositeness scale be larger than about 5 TeV. On the other hand, the mass of the Higgs boson is only 125 GeV. Even in theories in which the Higgs emerges as a composite pNGB, the mass of the Higgs tends to be of order a TeV rather than 100 GeV. This is because the gauge couplings, Yukawa couplings and Higgs self-coupling all violate the shift symmetries that protect the mass of the pNGB Higgs.

To understand the problem, let us consider a toy model in which an SU(3) global symmetry is broken down to SU(2) subgroup. We gauge the unbroken SU(2) subgroup so that it is similar to the SM. We parametrize the Goldstone bosons as $\pi^A(x)$, where A runs over the 5 broken generators. The object

$$\hat{H} = e^{i\frac{\pi^A T^A}{f}} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

transforms linearly under SU(3), and can be used to write down the low energy Lagrangian for the pNGBs. Define $\pi^A T^A \equiv \pi$.

We can parametrize π as

$$\pi = \left(\begin{array}{cc|c} -\eta/2 & 0 & h \\ 0 & -\eta/2 & \\ \hline h^{+} & & \eta \end{array}\right)$$

We will focus on the doublet h of $SU(2)$, so I will ignore η from now on. Then

$$\hat{H} = \begin{pmatrix} ih + \ldots \\ f - \frac{|h|^2}{2f} \end{pmatrix}$$

The only $SU(3)$ invariant term with 2 derivatives is

$$\partial_\mu \hat{H}^+ \partial^\mu \hat{H} = \partial_\mu h^+ \partial^\mu h + \frac{1}{4f^2} \partial_\mu (h^+h) \partial^\mu (h^+h) + \ldots$$

This theory becomes strongly coupled at momentum scales $\Lambda \sim 4\pi f$.

Now let us write down the top Yukawa coupling. This explicitly violates the $SU(3)$ symmetry.

$$\lambda_t \hat{H}^+ \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} Q t^c = \lambda_t (ih + \ldots)^+ Q t^c$$

Quantum loops will generate a quadratically divergent term, which we expect will be cutoff at the compositeness scale $\Lambda \sim 4\pi f$.

$$\frac{3 \lambda_t^2 \Lambda^2}{8\pi^2} \hat{H}^+ \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \hat{H}$$

$$= \frac{3 \lambda_t^2 \Lambda^2}{8\pi^2} |h|^2$$

But this is exactly what we would get in the SM if we identify the cutoff with the compositeness scale. Since the compositeness scale is constrained to be greater than 5 TeV, we are left a fine tuning problem at the level of one part in a hundred.

Although this is a huge improvement over the big hierarchy problem, it would be nice if we could do better. The main obstacle we are facing is that the top Yukawa coupling explicitly breaks the SU(3) global symmetry. Is there a way to make this coupling invariant under the global SU(3)?

Try promoting Q from a doublet of the $SU(2)$ subgroup to a fundamental of $SU(3)$ labelled Ψ

$$\Psi \equiv \begin{pmatrix} Q \\ T \end{pmatrix}$$

SM doublet of $SU(2)$ containing top and bottom quarks

$SU(2)$ singlet quark

Ψ now contains a new quark T that is not present in the SM.

We can write an $SU(3)$ invariant top Yukawa coupling,

$$\lambda_t \hat{H}^\dagger \Psi t^c = \lambda_t f T t^c + \lambda_t (ih + \cdots)^\dagger Q t^c$$

But now t^c acquires a mass $\lambda_t f$ with the new state T, and is not present in the low energy theory.

Then there is no top Yukawa coupling at all in the low energy theory. Complete failure.

This should have been expected because in this limit h is an exact Goldstone boson (up to gauge interactions) and has to be derivatively coupled.

However, there is a simple modification of this idea which solves the problem. Break $SU(3)$ to $SU(2)$ twice. We then have two sets of Goldstone fields,

$$\hat{H}_1 = e^{i\Pi_1/f_1} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} \qquad \hat{H}_2 = e^{i\Pi_2/f_2} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix}$$

We will take $f_1 = f_2 = f$ for simplicity, though this is not required.

We now gauge the full $SU(3)$ symmetry, rather than just the $SU(2)$ subgroup. Then one linear combination of Π_1 and Π_2 is eaten. The other linear combination denoted by Π, where $\Pi = \text{const.}\,(\Pi_1 - \Pi_2)$ contains the Higgs field, $h(x)$.

We also add to the theory an extra particle with the quantum numbers of t^c, so we have $t_1{}^c$ and $t_2{}^c$. We are also keeping the additional fermion T that we added earlier.

In unitary gauge,

$$\hat{H}_1 = e^{i\Pi/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \qquad \hat{H}_2 = e^{-i\Pi/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

We now write the terms that will generate the top Yukawa coupling,

$$\lambda_1 \hat{H}_1^\dagger \psi t_1^c + \lambda_2 \hat{H}_2^\dagger \psi t_2^c$$

We forbid a Yukawa coupling of $\hat{H}_1$ with t_2^c or $\hat{H}_2$ with t_1^c by imposing a discrete symmetry under which $\hat{H}_2 \to -\hat{H}_2$ and $t_2^c \to -t_2^c$. For convenience, we also set $\lambda_1 = \lambda_2 \equiv \frac{1}{\sqrt{2}} \lambda_t$, although this is not required. Expanding these terms out in terms of h, we obtain

$$\frac{\lambda_t}{\sqrt{2}} \left(ih, f - \frac{|h|^2}{2f} \right)^\dagger \begin{pmatrix} Q \\ T \end{pmatrix} t_1^c + \frac{\lambda_t}{\sqrt{2}} \left(-ih, f - \frac{|h|^2}{2f} \right)^\dagger \begin{pmatrix} Q \\ T \end{pmatrix} t_2^c$$

$$= \frac{\lambda_t}{\sqrt{2}} \left\{ f (t_1^c + t_2^c) T + ih Q (t_2^c - t_1^c) - \frac{|h|^2}{2f} T (t_1^c + t_2^c) \right\}$$

gives mass to t_1^c and t_2^c

top Yukawa coupling

Define

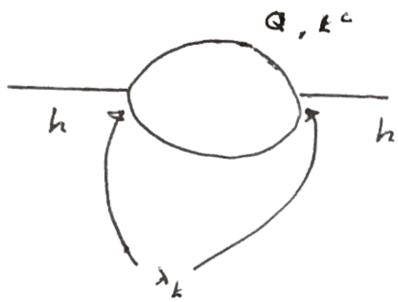
$$t^c = \frac{i}{\sqrt{2}} (t_2^c - t_1^c)$$

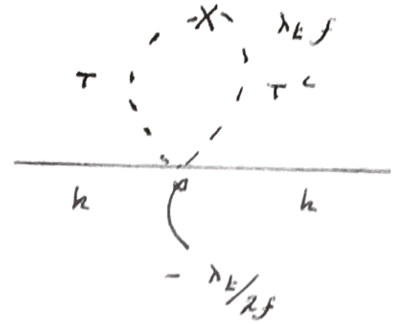
$$T^c = \frac{1}{\sqrt{2}} (t_1^c + t_2^c)$$

Then these terms become,

$$\lambda_t \left[h Q t^c + f T T^c - \frac{|h|^2}{2f} T T^c \right]$$

Let us compute corrections to the mass of h in this theory. There are, at one loop, 2 diagrams



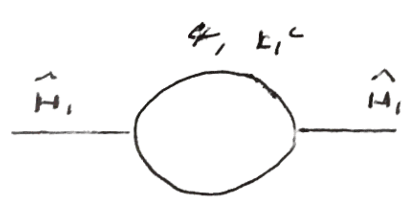


Remarkably, the quadratic divergences in these diagrams cancel exactly!

$$\frac{3\lambda_t^2}{8\pi^2}\Lambda^2 h^\dagger h - \frac{3\lambda_t^2}{8\pi^2}\Lambda^2 h^\dagger h = 0$$

Seems like a miracle! What is going on?

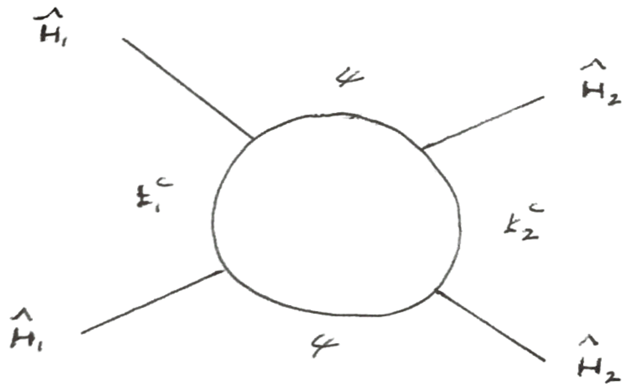
This cancellation is a consequence of "collective symmetry breaking". Go back to the $\hat{H}_1$, $\hat{H}_2$ basis. The quadratic divergences can now arise from the two diagrams



But these generate terms which are invariant under $[SU(3)]^2$

$$\frac{3\lambda_1^2}{8\pi^2}\Lambda^2 \underbrace{|\hat{H}_1|^2}_{\substack{\text{independent of} \\ \pi}} + \frac{3\lambda_2^2}{8\pi^2}\Lambda^2 \underbrace{|\hat{H}_2|^2}_{\substack{\text{independent of} \\ \pi}}$$

However, this contribution to the potential is completely independent of the pNGB field $\pi(x)$. The first diagram which contributes to the potential is of the form,



This leads to a term in the Lagrangian of the schematic form,

$$\sim \frac{\lambda_1^2 \lambda_2^2}{16\pi^2} \left| H_1^\dagger H_2 \right|^2 \log \Lambda^2$$

This contributes to the mass of the Goldstones at the level of

$$\delta m^2 \sim \frac{\lambda_t^4}{16\pi^2} f^2 \log \frac{\Lambda^2}{\lambda_t^2 f^2}$$

Since $\Lambda \sim 4\pi f$, (in composite Higgs models), $\lambda_t \sim 1$

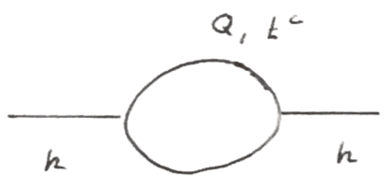
$$\delta m^2 \sim \left(\frac{\lambda_t^2}{16\pi^2}\right)^{2} \Lambda^2 \log (16\pi^2)$$

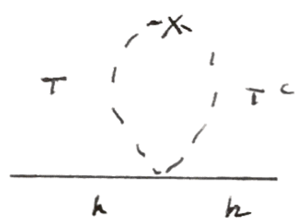
extra suppression!

The cancellation of quadratic divergences can be understood as follows. If λ_1 was zero, the theory has an ~~unbroken~~ $SU(3) \times SU(3)$ symmetry that is exact except for the gauge interactions. When this is broken to $SU(2) \times SU(2)$, we get 2 sets of Goldstone. One set of 5 Goldstones is eaten, while the other set is exactly massless. Similarly, if λ_2 was zero, the theory would again have an enhanced $SU(3) \times SU(3)$ symmetry, resulting in 5 exactly massless Goldstones. We see that to obtain a non-vanishing potential for the Goldstones, both λ_1 and λ_2 must be non-zero. But there is no quadratically divergent diagram for the Goldstones ~~that~~ at one loop that involves both λ_1 and λ_2. Therefore the leading one loop divergence can only arise at $\log \Lambda$. Since the breaking of the global symmetry that protects the masses of the Goldstones requires both λ_1 and λ_2 to be non-vanishing, we say that λ_1 and λ_2 "collectively break" the global symmetry.

The mass of the pNGB Higgs is protected because any contribution to its potential requires collective symmetry breaking.

From the point of view of the low energy effective theory there is a new fermion with mass $\lambda_t f$ that is responsible for the cancellation. This is the Dirac fermion composed of T and T^c. We refer to T and T^c as the "top partners".



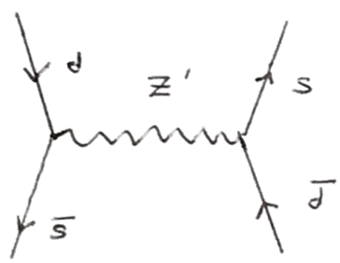


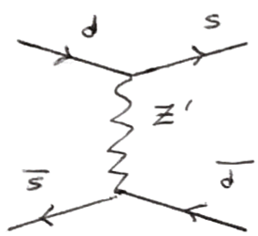
This construction predicts a new colored fermion with mass $m_T \sim \lambda_t f \sim f \sim \Lambda/4\pi$. For Λ below about 10 TeV, this should be accessible to the LHC.

Unlike the case of supersymmetry, here the cancellation arises from particles of the same spin.

These ideas can be applied to the quadratic divergences from gauge interactions and Higgs self-couplings as well. In fact, in the model we have just studied, quadratic divergences from gauge loops are also absent. However, in most simple composite Higgs models, collective symmetry breaking is employed only to cancel quadratic divergences from the top loop. The gauge and Higgs loops are cutoff at Λ.

In general, a composite Higgs model is expected to include vector resonances with masses of order the compositeness scale Λ. Since the fermions of the SM are now partially composite, they couple to these vector resonances. Then the exchange of vector resonances can lead to large flavor changing neutral currents.





The strongest bounds on flavor violation arise from processes involving the light fermions. Since the lighter fermions mix less with the composite states, their couplings to the vector resonances are smaller, and so the bounds are more easily satisfied. Nevertheless, in the absence of additional flavor symmetries, the bound on the compositeness scale Λ is of order 20 TeV.

Precision electroweak measurements also place bounds on the compositeness scale Λ. The severity of these bounds depends in part on whether the symmetry breaking pattern that generates the pNGB Higgs admits a custodial SU(2) symmetry. In the absence of such a symmetry, the operator

$$\kappa \, \frac{|(\hat{H}^{+} D_\mu \hat{H})|^2}{\Lambda^2}$$

is expected to be present at the compositeness scale, with the coefficient κ of order $16\pi^2$. The bound on the compositeness scale Λ is then of order 50 TeV, leading to severe fine tuning.

If the symmetry breaking pattern admits a custodial SU(2) symmetry, this operator is still generated. However, it is now loop suppressed, so that κ is of order 1. The bound on the compositeness scale is then of order 5 TeV, similar to the bound from the operator $(D_\mu D_\mu H)^\dagger (D_\mu D_\mu H)$.

Examples of symmetry breaking patterns that have a custodial SU(2) symmetry are SO(5)/SO(4), SO(6)/SO(5) and SO(8)/SO(7). Some simple symmetry breaking patterns such as SU(3) x U(1) / SU(2) x U(1) and SU(4) x U(1)/ SU(3) x U(1) do not possess a custodial SU(2) symmetry.

Most simple composite Higgs models today are severely constrained by LHC data, and are consequently tuned at the per cent level or worse. This is because the LHC bounds on the top partners constrain their masses to lie above a TeV. While the quadratically divergent contributions to the Higgs mass are cut off at the top partner mass, they are usually only softened to a logarithmic divergence, so that

$$\delta m_H^2 \gtrsim \frac{3 \lambda_t^2}{8 \pi^2} m_T^2 \log \left(\frac{\Lambda^2}{m_T^2} \right)$$

these group theory factors may depend on the model

Setting m_T to a TeV, we find that this class of models is tuned at the level of a few parts in a hundred.