In realistic MTH models the discrete Z_2 twin symmetry is extended to all the particles and interactions of the SM. Then we have a mirror copy of the entire SM, with exactly the same field content and interactions. Although the mirror particles are light, they have never been observed because they carry no charge under the SM gauge groups. Very difficult to test at colliders.

After electroweak symmetry breaking the Higgs and its twin partner mix. The Higgs production cross section is suppressed by the mixing angle. The mixing allows the Higgs to decay into invisible mirror states. Both these effects contribute to a uniform suppression of Higgs events into all SM final states. The LHC can place limits on the mixing angle.

The cosmology of MTH is complicated because of all the extra light states, but consistent frameworks exist. The MTH only stabilizes the hierarchy up to about 5 TeV. Above 5 TeV an ultraviolet completion, either in the form of supersymmetry or composite Higgs, is required.

The Axion Solution to the Strong CP Problem

The Strong CP Problem

The lightest hadrons of QCD are the 3 pions, which have masses of order 140 MeV. The lightness of the pions is a consequence of the fact that they are the pNGBs arising from the spontaneous breaking of an approximate symmetry of the QCD Lagrangian. Let us understand this.

Consider the QCD Lagrangian for two light quark flavors,

$$\mathcal{L} \supset i\bar{q}\bar{\sigma}^\mu D_\mu q + i\bar{q}^c\bar{\sigma}^\mu D_\mu q^c - \left(q M q^c + h.c.\right)$$

Here $q \equiv (u, d)$ and $q^c \equiv (u^c, d^c)$, while

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}.$$

This Lagrangian has an approximate $SU(2)_L \times SU(2)_R$ global symmetry under which q transforms under $SU(2)_L$ and q^c under $SU(2)_R$. This is violated by the mass terms, and also by electromagnetism. When QCD gets strong it develops a condensate,

$$\langle q q^c \rangle \neq 0$$

This breaks $SU(2)_L \times SU(2)_R$ down to the diagonal $SU(2)_{L+R}$. The three pions are the pNGBs arising from this symmetry breaking pattern.

However, there is a puzzle. The Lagrangian appears to have two additional $U(1)$ symmetries; a $U(1)_B$ under which $q \to e^{i\alpha} q$ and $q^c \to e^{-i\alpha} q^c$ and an axial $U(1)_A$ under which $q \to e^{i\alpha} q$ and $q^c \to e^{i\alpha} q^c$. The $U(1)_B$ remains unbroken when $\langle q q^c \rangle$ condenses. However, the $U(1)_A$ is broken, and it appears that there should be a fourth light pion.

The reason why there is no fourth light pion is that $U(1)_A$ is not actually a symmetry of the theory because of a quantum anomaly. Even though the Lagrangian is invariant under $U(1)_A$ transformations, the measure in the path integral is not invariant. Consequently, under

$$u \to e^{i\alpha} u$$

$$d \to e^{i\alpha} d$$

with u^c, d^c invariant, the Lagrangian changes as

$$\mathcal{L} \to \mathcal{L} + \alpha \frac{g^2}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

where

$$\tilde{G}^{a\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\lambda} G^a_{\rho\lambda}$$

Since $U(1)_A$ is not a symmetry, no fourth light pion.

Consider adding a term of the form,

$$\theta \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

to the QCD Lagrangian. This term is a total derivative, and does not contribute to any order in perturbation theory. However, it does not vanish non-perturbatively. Physical quantities like the vacuum energy and the electric dipole moment of the neutron depend on θ.

We have just seen that under an axial rotation $q \to e^{i\alpha} q$ with q^c invariant,

$$\theta \to \theta + 2\alpha$$

This means that there is more than one way to parametrize the same theory. Only the combination

$$\bar{\theta} \equiv \theta + \arg(\det M)$$

is physical.

If any of the quark masses is zero, θ can be set to zero and is unphysical. But otherwise, $\bar{\theta}$ is expected to be non-vanishing and has physical consequences.

A term of the form $\theta G\hat{G}$ violates CP. Therefore, if $\bar{\theta}$ is non-vanishing, there is a new source of CP violation in the theory.

It can be shown that $\bar{\theta}$ feeds into the electric dipole moment of the neutron. The non-observation of any such electric dipole moment constrains $\bar{\theta} \lesssim 10^{-9}$. Now $\bar{\theta}$ is a combination of a strong phase θ and a phase from the quark mass matrix. Since the phases in the quark mass matrix generate the large phase in the CKM, we expect that $\bar{\theta}$ should be of order 1. The strong CP problem is the question of why $\bar{\theta}$ is so small.

The simplest solution to the strong CP problem is if the up-quark mass is zero. Then θ is unphysical and can be rotated away. Unfortunately, this seems to be ruled out by lattice data.

Another class of solutions involves having CP as an exact symmetry in the ultraviolet, so that $\bar{\theta} = 0$ at high energies. The challenge is then to break CP spontaneously and induce a large CKM phase without also inducing a large $\bar{\theta}$. Models of this type can be constructed that solve the problem. They make use of the fact that in the SM, RG running of $\bar{\theta}$ only arises at 7 loops.

A third class of solutions to the strong CP problem involve the axion. These theories work by making the value of $\bar{\theta}$ depend on the vev of a dynamical field, the axion. The minimum of the potential for the axion corresponds to $\bar{\theta} = 0$, thereby solving the strong CP problem.

The Axion Solution

Before introducing the axion, let us first determine how the vacuum energy depends on the value of $\bar{\theta}$. It is convenient to begin in a basis in which the θ-term has been rotated to zero so that the entire dependence on $\bar{\theta}$ is contained in the phase of the quark mass term.

Then the Lagrangian for the two light quark flavors takes the form,

$$\mathcal{L} \supset \bar{q}_L i \gamma^\mu D_\mu q_L + \bar{q}_R i \gamma^\mu D_\mu q_R - \left(\bar{q}_R M q_L + h.c. \right)$$

Here $q = (u, d)$ and $M = e^{i\bar{\theta}/2} \hat{M}$ with

$$\hat{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

where m_u and m_d are real.

When QCD gets strong the $SU(2)_L \times SU(2)_R$ symmetry is broken to the diagonal $SU(2)$. We parametrize the Goldstone bosons as $\pi^a(x)$, where a runs from 1 to 3. These are the three pion fields.

The object U defined as

$$U = e^{\frac{i \pi^a \sigma^a}{\sqrt{2} f_\pi}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

transforms linearly under $SU(2)_L \times SU(2)_R$, and can be used to write down the low energy Lagrangian for the light pions. Under $SU(2)_L \times SU(2)_R$, U transforms as

$$U \rightarrow L U R^\dagger$$

The $SU(2)_L \times SU(2)_R$ symmetry is explicitly broken by the quark mass term. We can determine how the mass parameter M appears in the low energy theory by promoting it to a spurion that transforms as

$$M \rightarrow R M L^\dagger$$

Then, ignoring electromagnetic effects which are small, we can write down the low energy effective theory for the pNGB fields as

$$\mathcal{L} = f_\pi^2 \, \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + a f_\pi^3 \, \mathrm{Tr}\left(M U\right) + h.c.$$

The next step is to determine the vacuum of the theory. With $\bar{\theta} \neq 0$, the vacuum is no longer at $\pi^a = 0$. In particular, we shall see that π^3 acquires a vev.

The potential that has to be minimized is

$$V = -a f_\pi^3 \, Tr(MU) + h.c.$$

$$= -a f_\pi^3 \, e^{i\bar{\theta}/2} \, Tr(\hat{M} U) + h.c.$$

Here the parameter a is real, because in the limit that $\bar{\theta}$ is zero the theory respects CP.

Define $\hat{\pi}_a = \dfrac{\sqrt{2}\,\pi^a}{f_\pi}$. Then

$$U = e^{i\hat{\pi}_a \tau^a/2} = \cos\left(\frac{|\hat{\pi}|}{2}\right) + i \frac{\hat{\pi}\cdot\tau}{|\hat{\pi}|} \sin\left(\frac{|\hat{\pi}|}{2}\right)$$

We have that

$$Tr(\hat{M} U) = m_u \left\{ \cos\left(\frac{|\hat{\pi}|}{2}\right) + i \frac{\hat{\pi}_3}{|\hat{\pi}|} \sin\left(\frac{|\hat{\pi}|}{2}\right) \right\}$$

$$+ m_d \left\{ \cos\left(\frac{|\hat{\pi}|}{2}\right) - i \frac{\hat{\pi}_3}{|\hat{\pi}|} \sin\left(\frac{|\hat{\pi}|}{2}\right) \right\}$$

It is clear from this that only $\hat{\pi}_3$ can acquire an expectation value. We define $\phi = \langle \hat{\pi}_3 \rangle$. The potential that needs to be minimized to determine ϕ is,

$$V(\phi) = -a f_\pi^3 \left\{ e^{i\bar{\theta}/2} \left(m_u e^{i\phi/2} + m_d e^{-i\phi/2} \right) + h.c. \right\} =$$

$$-2 a f_\pi^3 \left\{ m_u \cos\left(\bar{\theta}/2 + \phi/2\right) + m_d \cos\left(\phi/2 - \bar{\theta}/2\right) \right\}$$

The minimum of this potential is at

$$\tan\left(\frac{\phi}{2}\right) = \left(\frac{m_d - m_u}{m_d + m_u}\right) \tan\left(\frac{\bar{\theta}}{2}\right)$$

Then the vacuum energy at the minimum is given by

$$V(\bar{\theta}) = -2a f_\pi^3 (m_u + m_d) \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\bar{\theta}}{2}\right)}$$

We see that the value of the vacuum energy at the minimum does depend on $\bar{\theta}$, as advertised.

We now introduce the axion. The axion $a(x)$ is a field that couples as

$$\frac{g^2}{32\pi^2} \left(\frac{a(x)}{f_a} + \theta\right) G^{a\mu\nu} \tilde{G}^a_{\mu\nu}$$

Notice that the theory has a spurious symmetry,

$$\theta \to \theta - \alpha$$
$$a(x) \to a(x) + \alpha f_a$$

This can be used to determine how $a(x)$ appears in the low energy Lagrangian.

$$\mathcal{L} = f_\pi^2 \, \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + a f_\pi^3 e^{i/2\left(\bar{\theta} + a(x)/f_a\right)} \mathrm{Tr}(\hat{M} U) + h.c.$$

Following exactly the same steps as earlier, we find for the vacuum energy as a function of $a(x)$,

$$V = -2a f_\pi^3 (m_u + m_d) \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2} \sin^2\left(\frac{a}{2f_a} + \frac{\bar{\theta}}{2}\right)}$$

This a is a constant, not the axion

This is the axion

This is minimized when

$$a/f_a + \bar{\theta} = 0$$

Since the effective value of $\bar{\theta}$ is exactly $\langle a \rangle / f_a + \bar{\theta}$, we have solved the strong CP problem! At the minimum of the axion potential, the strong CP phase vanishes!

The parameter f_a is called the "axion decay constant". It parametrizes the strength of axion interactions. The mass of the axion is order

$$m_a \sim \frac{\Lambda_{QCD}^2}{f_a}$$

Bounds from meson decays and from stellar cooling constrain $f_a \gtrsim 10^9$ GeV. Then the axion is expected to be very light, $m_a \lesssim 10^{-3}$ eV.