(Mixed BC and) Wilson lines as 1d defects

Diego H. Correa

Universidad Nacional de La Plata - CONICET - Argentina

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Plan of the Talk

- Wilson lines as 1d defects
- Excitations on the line $\leftrightarrow$ fluctuations on the dual worldsheet
- Mixed BC for fluctuations on the worldsheet
- Case study: Wilson lines in ABJM

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Motivations to study Wilson loops as conformal defects

- CFT data: Local operators + defects
- Wilson loops are examples of line defects
- WL are observables with valuable physical interpretation in any gauge theory
  - $q\bar{q}$ potential
  - Bremsstrahlung function
- There is a variety of sophisticated theoretical tools to their study
  - AdS/CFT Correspondence
  - Integrability
  - Conformal Boostrap
  - Supersymmetric Localization

Wilson lines as Conformal Defects

For a conformal field theory in d dimensions we will consider a straight Wilson line preserving 1-dimensional conformal symmetry:

$$SO(2,1) \times SO(d-1) \subset SO(2,d)$$

Example: $1/2$ BPS Maldacena Wilson line in $\mathcal{N} = 4$ SYM in 4 dimensions

$$W = \mathrm{tr}\left(\mathcal{P} e^{\int dt (iA_t + \Phi^6)}\right)$$

$$SO(2,1) \times SO(3) \times SO(5) \times (16 \text{ susy}) = OSp(4^*|4)$$

Correlators on the Wilson line

VEVs of Wilson loops with operator insertions → notion of correlation functions on the defect *(Drukker, Kawamoto 06)*

$$\langle\langle \mathcal{O}_1(t_1)\cdots\mathcal{O}_n(t_n)\rangle\rangle = \frac{\langle \mathrm{tr}\left(\mathcal{P}\mathcal{O}_1(t_1)\cdots\mathcal{O}_n(t_n)e^{\int dt(iA_t+\Phi^6)}\right)\rangle}{\langle \mathrm{tr}\left(\mathcal{P}e^{\int dt(iA_t+\Phi^6)}\right)\rangle}$$

Constrained from conformal symmetry as usual

- 2-point functions ↔ scale dimensions

$$\langle\langle \mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\rangle\rangle = \frac{1}{|t_1 - t_2|^{2\Delta_1}}$$

- 3-point functions ↔ structure constants

$$\langle\langle \mathcal{O}_1(t_1)\mathcal{O}_2(t_2)\mathcal{O}_3(t_3)\rangle\rangle = \frac{c_{123}}{|t_1-t_2|^{\Delta_1+\Delta_2-\Delta_3}|t_1-t_3|^{\Delta_1+\Delta_3-\Delta_2}|t_2-t_3|^{\Delta_2+\Delta_3-\Delta_1}}$$

- higher-point functions ↔ OPE (Operator Product Expansion)

Insertions from the Displacement multiplet

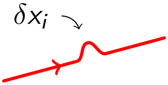
8 bosonic insertions $\oplus$ **8 fermionic insertions**

- Φ^I $\quad I = 1, \cdots 5$ $\quad \Delta = 1$
- $\mathbb{D}_i = iF_{ti} + D_i\Phi^6$ $\quad i = 1, \cdots 3$ $\quad \Delta = 2$
- ψ_α^A $\quad \Delta = 3/2$

(Giombi, Roiban, Tseytlin 17)

⊛ **Displacement operator:** characterizes small deformations of the defect in transverse directions

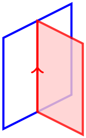
$$\delta W = \int dt \, \mathrm{tr}\left(\mathcal{P}\delta x_i(t)\mathbb{D}_i(t)e^{\int dt(iA_t+\Phi^6)}\right)$$



⊛ Coherent insertions of displacement operators can account for arbitrary Maldacena-Wilson loops

Dual description of the line defect

Example: dual of a straight Wilson line



AdS$_{d+1}$ Minimal Area: $z = \sigma \quad t = \tau \quad x_i = 0$

$$ds^2_{d+1} = \frac{dz^2 + dt^2 + dx_i^2}{z^2} \longrightarrow ds^2_2 = \frac{d\sigma^2 + d\tau^2}{\sigma^2} \qquad \text{AdS}_2$$

⊛ Fluctuations around AdS_2 world-sheet in static gauge

8 transverse fluctuations $\oplus$ **8 fermionic fluctuations**

- 5 $m_B^2 = 0$ scalars, S^5 fluctuations ($\Delta = 1$)
- 3 $m_B^2 = 2$ scalars, AdS_5 transv. fluctuations ($\Delta = 2$)
- 8 $|m_F| = 1$ fermions ($\Delta = 3/2$)

Consistent with the mass $\leftrightarrow$ scale dim AdS/CFT correspondence:

scalar fields: $m_B^2 = \Delta(\Delta - d)$
spinor fields: $|m_F| = \Delta - \frac{d}{2}$

Other interesting deformations

$$\text{tr}\left(\mathcal{P}e^{\int dt(iA_t+\zeta\Phi^6)}\right) \qquad \text{Interpolating WL} \quad \textit{(Polchinski, Sully 11)}$$

⊛ This interpolating WL can be expressed in terms of **insertions of** Φ^6 along the Maldacena-Wilson loop

$$\text{tr}\left(\mathcal{P}e^{\int dt(iA_t+\zeta\Phi^6)}\right) =$$

Other interesting deformations

$$\mathrm{tr}\left(\mathcal{P}e^{\int dt(iA_t+\zeta\Phi^6)}\right)$$

Interpolating WL *(Polchinski, Sully 11)*

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$$\mathrm{tr}\left(\mathcal{P}e^{\int dt(iA_t+\zeta\Phi^6)}\right) = \mathrm{tr}\left(\mathcal{P}e^{\int dt(iA_t+\Phi^6)}\right) + \\ -(1-\zeta)\int dt\,\mathrm{tr}\left(\mathcal{P}\Phi^6(t)e^{\int dt(iA_t+\Phi^6)}\right) + \cdots$$

Other interesting deformations

$$\mathrm{tr}\left(\mathcal{P}e^{\int dt(iA_t+\zeta\Phi^6)}\right)$$

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Computation of $\langle W^{(\zeta)}\rangle$ requires renormalization of ζ $\longrightarrow$ it runs

$$\beta_\zeta = \mu\frac{d\zeta}{d\mu} = -\frac{\lambda}{8\pi^2}\zeta(1-\zeta^2) + \mathcal{O}(\lambda^2)$$

(*Polchinski, Sully* 11; *Beccaria, Giombi, Tseytlin* 17)

Renormalization Group Flow in the defect

$$\beta_\zeta = -\frac{\lambda}{8\pi^2}\zeta(1-\zeta^2) + \mathcal{O}(\lambda^2) = 0 \quad \nearrow \zeta = 0, \quad \searrow \zeta = \pm 1$$

$$\Delta_{\Phi^6} = 1 - \frac{\lambda}{8\pi^2} + \mathcal{O}(\lambda^2) < 1$$

Φ^6 insertion is relevant (in $d = 1$) and triggers a RG flow

$\zeta = 0$ UV fixed point	$\longrightarrow$	$\zeta = 1$ IR fixed point
$\mathrm{tr}\left(\mathcal{P}e^{\int dt\ iA_t}\right)$	$\longrightarrow$	$\mathrm{tr}\left(\mathcal{P}e^{\int dt(iA_t+\Phi^6)}\right)$
Ordinary WL		Maldacena WL

In the dual description:

Maldacena WL $\leftrightarrow$ Dirichlet b.c. in the S^5 *(Maldacena 98)*

Ordinary WL $\leftrightarrow$ Neumann b.c. in the S^5 *(Alday, Maldacena 07)*

Dual description of a RG flow

RG flow from multi-trace deformations $\leftrightarrow$ mixed B.C. *(Witten 01)*

$$\phi = z^{\Delta_-}\alpha(t,x_i) + z^{\Delta_+}\beta(t,x_i) \qquad \Delta_\pm = \frac{d}{2} \pm \frac{d}{2}\sqrt{1+\frac{4m^2}{d^2}}$$

$$\text{B.C.} \quad \alpha - \frac{\delta W(\beta)}{\delta\beta} = 0 \qquad \text{for } W(\beta) \text{ some boundary term}$$

Dual description of the Wilson line RG flow

5 massless scalars fields from fluctuations along the S^5

$$\phi_a = \alpha_a(t) + z\beta_a(t) \qquad \Delta_- = 0 \qquad \Delta_+ = 1$$

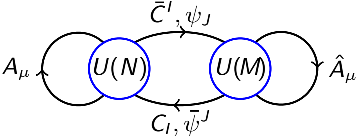
Mixed B.C. for the dual of the Wilson line RG flow *(Polchinski,Sully 11)*

$$\text{quadratic } W \quad \rightarrow \quad \chi\alpha_a + \beta_a = 0$$

- χ is dimensionful $\rightarrow$ it breaks conformal invariance
- It breaks susy when the fermions has $m_F = 1$

Wilson lines in ABJM

- $\mathcal{N} = 6$ Chern-Simons-matter theory



- There is a 1/2 BPS Wilson line as well *(Drukker,Trancanelli 09)*

$$W = \mathrm{tr}\left(\mathcal{P} e^{\int dt \;\; iL(t)}\right)$$

$$L(t) = \begin{pmatrix} A_t - \frac{2\pi i}{k}\mathcal{M}^I_J C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}}\eta\bar{\psi}^1_+ \\ -i\sqrt{\frac{2\pi}{k}}\bar{\eta}\psi_1^+ & \hat{A}_t - \frac{2\pi i}{k}\mathcal{M}^I_J \bar{C}^J C_I \end{pmatrix}$$

$$\mathcal{M} = \mathrm{diag}(-1,1,1,1) \qquad \eta\bar{\eta} = 2i$$

$$SO(2,1) \times SO(2) \times SU(3) \times (12 \text{ susy}) = SU(1,1|3)$$

Excitations in the ABJM Wilson line

- Displacement operator insertions *(Bianchi,Griguolo,Preti,Seminara 17, Bianchi,Preti,Vescovi 18, Bianchi, Bliard, Forini, Griguolo, Seminara 20)*
- Dual description: AdS_2 along $AdS_4 \times CP^3$. The mass spectrum of the fluctuations is *(Forini,Puletti, Ohlsson Sax 12, Aguilera-Damia,DC,Silva 14)*

8 transverse fluctuations $\oplus$ **8 fermionic fluctuations**

- 6 $m_B^2 = 0$ scalars, CP^3 fluctuations
- 2 $m_B^2 = 2$ scalars, AdS_4 fluctuations (transverse to AdS_2)
- 6 $|m_F| = 1$ spinor fluctuations
- 2 $m_F = 0$ spinor fluctuations

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- 2 $m_F = 0$ spinor fluctuations

⊛ Having massless fermions is crucial to find susy b.c., other than Dirichlet $\Rightarrow$ Richer family of BPS Wilson loops

1-parameter family of BPS Wilson lines in ABJM

(Drukker,Plefka,Young 08; Chen,Wu 08; Rey,Suyama,Yamaguchi 08; Ouyang,Wu,Zhang 15)

$$L(t) = \begin{pmatrix} A_t - \frac{2\pi i}{k}\mathcal{M}^I_J C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}}\eta\zeta\bar{\psi}^1_+ \\ -i\sqrt{\frac{2\pi}{k}}\bar{\eta}\zeta\psi_1^+ & \hat{A}_t - \frac{2\pi i}{k}\mathcal{M}^I_J \bar{C}^J C_I \end{pmatrix}$$

$\mathcal{M} = \text{diag}(-1, 2\zeta^2 - 1, 1, 1)$ $\quad \eta\bar{\eta} = 2i$ $\quad$ 4 susy's for any ζ

$\zeta = 0$	$\longleftrightarrow$	$\zeta = 1$
bosonic 1/6 BPS WL	$\longleftrightarrow$	1/2 BPS WL
CP^3angles: 4 Dir. + 2 Neu.	$\longleftrightarrow$	CP^3angles: 6 Dir.

- Revealing the dual string description was our main motivation
- The VEV of these WL's is independent of ζ. As $\langle W \rangle := e^{-F_{\rm 1d}}$, the interpolation between Neumann and Dirichlet should correspond to a marginal deformation rather than to an RG flow

Supersymmetric mixed boundary conditions in AdS_2

- Dirichlet/Neumann b.c. have been discussed in *(Sakai, Tanii 85)*
- We consider a complex massless scalar and massless spinor

$$\delta\phi = \bar{\varepsilon}\psi, \qquad \delta\psi = -i\gamma^\alpha\partial_\alpha\phi\varepsilon,$$

AdS_2 Killing spinor

$$\varepsilon(t,z) = z^{-1/2}\xi(t) + z^{1/2} i\gamma_0\,\dot{\xi}(t) \qquad \xi(t) = \xi_0 + t\;\xi_1$$

- Expanding near the boundary

$$\begin{aligned}
\phi(t,z) &= \big(\alpha(t)+\cdots\big) + z\big(\beta(t)+\cdots\big) \qquad \tfrac{1}{2}(\mathbb{1}_2 \pm i\gamma_1)\eta\pm = \eta_\pm \\
\psi(t,z) &= z^{1/2}\left(\eta_- + z\,\gamma_3\dot{\eta}_- + \cdots\right) + z^{1/2}\left(\eta_+ + z\,\gamma_3\dot{\eta}_+ + \cdots\right)
\end{aligned}$$

Resulting susy transformations

$$\begin{aligned}
\delta\beta &= \bar{\xi}\gamma_3\dot{\eta}_+ + \dot{\bar{\xi}}\gamma_3\eta_+ \qquad & \delta\alpha &= \bar{\xi}\eta_- \\
\delta\eta_+ &= \beta\xi & \delta\eta_- &= \dot{\alpha}\gamma_3\xi
\end{aligned}$$

Supersymmetric mixed boundary conditions in AdS_2

Dirichlet BC: $\alpha(t) = 0$ & $\eta_-(t) = 0$ 4 susy's

Neumann BC: $\beta(t) = 0$ & $\eta_+(t) = 0$ 4 susy's

Supersymmetric mixed boundary conditions in AdS_2

Dirichlet BC: $\alpha(t) = 0$ & $\eta_-(t) = 0$ 4 susy's

Neumann BC: $\beta(t) = 0$ & $\eta_+(t) = 0$ 4 susy's

- Mixed boundary conditions

If we choose: $\chi\alpha(t) + \beta(t) = 0$

$$\delta(\chi\alpha(t) + \beta(t)) = 0 \quad \text{requires} \quad \chi\eta_- + \gamma_3\dot{\eta}_+ = 0$$

$$\delta(\chi\eta_- + \gamma_3\dot{\eta}_+) = 0 \quad \text{requires} \quad \dot{\xi} = 0$$

Type I mixed BC: $\chi\alpha + \beta = 0$ & $\chi\eta_- + \gamma_3\dot{\eta}_+ = 0$ 2 susy's

- Less susy than Dirichlet/Neumann
- χ is dimensionful $\rightarrow$ scale invariance is broken

Supersymmetric and conformal mixed boundary conditions

Can we have mixed b.c. without breaking scale invariance?

Recall: $\phi(t,z) \simeq \alpha(t) + z\beta(t)$

Supersymmetric and conformal mixed boundary conditions

Can we have mixed b.c. without breaking scale invariance?

Recall: $\phi(t,z) \simeq \alpha(t) + z\beta(t)$

- **Dimensionless** interpolating parameter if we choose:

$$i\chi\dot{\alpha}(t) + \beta(t) = 0 \quad \Leftrightarrow \quad \begin{cases} \chi\dot{\alpha}_1(t) + \beta_2(t) = 0 \\ \chi\dot{\alpha}_2(t) - \beta_1(t) = 0 \end{cases}$$

- Preservation of the fermions b.c., doesn't require to restrict the Killing spinor $\rightarrow$ **as many susy's** as Dirichlet/Neumann

Type II mixed BC: $i\chi\dot{\alpha} + \beta = 0$ & $i\chi\eta_- + \gamma_3\eta_+ = 0$ 4 susy's

Correlators in the defect

- $\Box\phi = 0 \quad + \quad$ b.c. with sources $f_i(t)$
- Compute on-shell action and then

$$\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{\delta^2 S_{\text{onshell}}}{\delta f_i(t) \delta f_j(t')}$$

Type I mixed BC:

$$\begin{cases} (\chi\phi_1 + \partial_z \phi_2)\,|_{z=\epsilon} = f_1(t) \\ (\chi\phi_2 + \partial_z \phi_1)\,|_{z=\epsilon} = f_2(t) \end{cases}$$

The correlator **does not** correspond to that one of a CFT

$$\langle \tilde{\mathcal{O}}_1(\omega_1) \tilde{\mathcal{O}}_1(\omega_2) \rangle = \frac{\delta(\omega_1 + \omega_2)|\omega_1|}{\chi^2 - \omega_1^2}$$

$$\langle \mathcal{O}_1(t_1) \mathcal{O}_1(t_2) \rangle \simeq \begin{cases} \frac{1}{\pi} \log|t_1 - t_2| & \text{for } \chi \to 0 \\ -\frac{1}{\pi} \frac{1}{(t_1 - t_2)^2 \chi^2} & \text{for } \chi \to \infty \end{cases}$$

Correlators in the defect

- $\Box\phi = 0 \quad + \quad$ b.c. with sources $f_i(t)$
- Compute on-shell action and then

$$\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{\delta^2 S_{\text{onshell}}}{\delta f_i(t) \delta f_j(t')}$$

Type II mixed BC:

$$\begin{cases} (\chi \partial_t \phi_1 + \partial_z \phi_2)\,|_{z=\epsilon} = f_1(t) \\ (-\chi \partial_t \phi_2 + \partial_z \phi_1)\,|_{z=\epsilon} = f_2(t) \end{cases}$$

The correlator **does** correspond to that one of a CFT

$$\langle \tilde{\mathcal{O}}_1(\omega_1) \tilde{\mathcal{O}}_1(\omega_2) \rangle = \frac{\delta(\omega_1 + \omega_2)}{(\chi^2 - 1)|\omega_1|}$$

$$\langle \mathcal{O}_1(t_1) \mathcal{O}_1(t_2) \rangle \simeq \frac{\log|t_1 - t_2|}{\pi(1 - \chi^2)}$$

Type II plus an extra constraint

Type II interpolates between:

$$\beta = 0 \leftrightarrow \dot{\alpha} = 0 \qquad \Leftrightarrow \qquad \partial_n \phi = 0 \leftrightarrow \phi = \phi_0 \text{ (unspecified)}$$

- Type II mixed BC: Neumann BC $\leftrightarrow$ Smeared Dirichlet BC

- To interpolate between Neumann and Dirichlet we further fix ϕ_0. We can do this by integrating type II BC

Type III mixed BC: $i\chi\alpha + \int_{-\infty}^{t} \beta(t')dt' = 0 \quad \& \quad i\chi\eta_- + \gamma_3\eta_+ = 0$

Our main result

Type III mixed BC:

- preserve 4 susy's
- interpolate between Dirichlet and Neumann
- Interpolating parameter is dimensionless

Our proposal is they account for the ζ-dependent supersymmetric family of Wilson loops in ABJM

Further check: We computed the 1-loop correction to the open string partition function and, despite using χ-dependent BC, it is independent of χ (is vanishing)

Marginal operator in the line

$$\langle W^{(\zeta)} \rangle = \langle\!\langle e^{\zeta^2 \int_{-\infty}^{\infty} d\tau \ \mathcal{O}_M} \rangle\!\rangle_{bosonic}$$

Expanding for small ζ

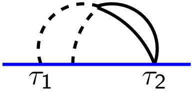
$$\langle W^{(\zeta)} \rangle = 1 + \zeta^2 \int_{-\infty}^{\infty} d\tau \ \langle\!\langle \mathcal{O}_B + \mathcal{O}_F \rangle\!\rangle + O(\zeta^4)$$

$$\begin{aligned} \mathcal{O}_B(\tau) &= \begin{pmatrix} C_2(\tau)\bar{C}^2(\tau) & 0 \\ 0 & C_2(\tau)\bar{C}^2(\tau) \end{pmatrix} \\ \mathcal{O}_F(\tau) &= \frac{1}{2}\int_{-\infty}^{\infty} d\tau' \mathcal{P} \begin{pmatrix} 0 & \bar{\psi}^1_+(\tau) \\ \psi_1^+(\tau) & 0 \end{pmatrix} \begin{pmatrix} 0 & \bar{\psi}^1_+(\tau') \\ \psi_1^+(\tau') & 0 \end{pmatrix} \end{aligned}$$

- Non-local dual BC $\leftrightarrow$ Non-local marginal insertion

- is the combination $\mathcal{O}_B + \mathcal{O}_F$ marginal?
- ABJM vertices $\mathrm{tr}(C\bar{C}\bar{\psi}\psi)$ mix them at 1-loop

$$\langle\langle \mathcal{O}_F(\tau_1)\mathcal{O}_B(\tau_2)\rangle\rangle =$$



$$+\cdots$$

One could verify that $\mathcal{O}_B + \mathcal{O}_F$ has vanishing anomalous dimension at 1-loop

- Under certain susy transformation

$$\int_{-\infty}^{\infty} d\tau \ \langle\langle \delta_{\text{susy}}(\mathcal{O}_B + \mathcal{O}_F)\rangle\rangle = 0$$

Summary

⊛ Mixed BC in AdS_2 can be used to describe either:

- RG flows in a $d = 1$ defect theory
- Marginal deformations in a $d = 1$ defect theory

⊛ Type III (mixed, non-local and supersymmetric) BC in AdS_2

can be used to account for

supersymmetric ζ-dependent family of Wilson loops in ABJM

Some open questions

- Which are the exactly marginal operator in the line?
- Is there a 1/12 BPS family of Wilson loops representing a $d = 1$ RG flow corresponding to the Type I BC?
- New domain point/cusps in the line: abrupt changes in ζ
- is there an exactly computable bremsstrahlung coefficient?
- What is the relation between ζ and χ?
- Is it possible to construct these Mixed BC with dimensionless interpolating parameter in higher dimensional AdS spaces?