Strong coupling expansion of circular Wilson loops and string theories in $\mathrm{AdS}_5 \times S^5$ and $\mathrm{AdS}_4 \times CP^3$

Simone Giombi, Arkady Tseytlin

arXiv:2007.08512

$\frac{1}{2}$ BPS circular WL's in $\mathcal{N}=4$ SYM and ABJM:

[Erickson, Semenoff, Zarembo; Drukker, Gross; Pestun] [Drukker, Marino, Putrov]

SYM: $\mathcal{W} = \text{Tr}\, P e^{\int (iA+\Phi)}$, in planar limit $\langle \mathcal{W} \rangle = N \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$

$\lambda \gg 1$: $\langle \mathcal{W} \rangle = \frac{N}{\lambda^{3/4}} \sqrt{\frac{2}{\pi}}\, e^{\sqrt{\lambda}} + ...$

compare to string theory: Tr $(...) \to Z_{\text{str}}$ (no $\frac{1}{N}$ in $\mathcal{W}$ [Lewkowycz, Maldacena])

disk partition function near AdS_2 minimal surface

$$\langle \mathcal{W} \rangle = Z_{\text{str}} = \frac{1}{g_s} Z_1 + \mathcal{O}(g_s)\,, \qquad Z_1 = \int [dx] ...\, e^{-T \int d^2\sigma\, L}$$

$$\text{SYM}: \ \langle \mathcal{W} \rangle = \sqrt{\frac{2}{\pi}} \frac{N}{\lambda^{3/4}}\, e^{\sqrt{\lambda}} + ...\,, \qquad g_s = \frac{\lambda}{4\pi N}\,, \quad T = \frac{\sqrt{\lambda}}{2\pi}\,, \qquad \lambda = g_{\text{YM}}^2 N$$

$$\text{ABJM}: \ \langle \mathcal{W} \rangle = \frac{N}{4\pi\lambda}\, e^{\pi\sqrt{2\lambda}} + ...\,, \qquad g_s = \frac{\sqrt{\pi}\,(2\lambda)^{5/4}}{N}\,, \qquad T = \frac{\sqrt{2\lambda}}{2}\,, \quad \lambda = \frac{N}{k}$$

both $\langle \mathcal{W} \rangle$ have remarkably universal form at strong coupling:

reason – dual string theories have similar structure

$$\langle \mathcal{W} \rangle = W_1 \left[1 + \mathcal{O}(T^{-1}) \right] + \mathcal{O}(g_s)$$

$$W_1 = c_1 \frac{\sqrt{T}}{g_s}\ e^{2\pi T}\,, \qquad c_1 = \frac{1}{(\sqrt{2\pi})^{n-3}} = \frac{1}{\sqrt{2\pi}}\,\bar{c}_1\,, \qquad n = 5,\ 4$$

dual string theory in $\mathrm{AdS}_n \times M^{10-n}$: $\mathrm{AdS}_5 \times S^5$ and $\mathrm{AdS}_4 \times CP^3$

- $e^{2\pi T} = e^{-TV_{\mathrm{AdS}_2}}$ – area of AdS_2 minimal surface [Berenstein, Corrado, Fischler, Maldacena]

- $\bar{c}_1 = \frac{1}{(\sqrt{2\pi})^{n-4}}$ – numerical constant in Z_1 [Drukker, Gross, AT; Kruczenski, Tirziu; Buchbinder, AT; ...]

- $\sqrt{T}$ prefactor – from universal dependence of Z_1 on AdS radius

- extra $\frac{1}{\sqrt{2\pi}}$ remains to be explained

sensitive to defn of string path integral measure; implicitly checked by computing ratio of $\frac{1}{2}$ and $\frac{1}{4}$ BPS WL's [Medina-Rincon, Zarembo, AT]

1-loop $AdS_n \times M^{10-n}$ superstring partition function

near AdS_2 minimal surface (with AdS scale R) in static gauge

$$\log Z_1 = -\tfrac{1}{2} \log \frac{[\det(-\nabla^2+2)]^{n-2}\,[\det(-\nabla^2)]^{10-n}}{[\det(-\nabla^2+\tfrac{1}{2})]^{2n-2}\,[\det(-\nabla^2-\tfrac{1}{2})]^{10-2n}}$$

$$\log Z_1 = B_2 \log(\mathrm{R}\,\Lambda) + \log \bar{c}_1\,, \qquad B_2 = \tfrac{1}{4\pi}\int d^2\sigma \sqrt{g}\, R^{(2)} = \chi$$

- $B_2 = \zeta_{\rm tot}(0) = \chi$ is universal and n-independent

$$Z_1 \sim (\sqrt{T})^{\chi}\,, \qquad (Z_1)_{\rm disk} \sim \sqrt{T}\,, \qquad T = \frac{\mathrm{R}^2}{2\pi\alpha'}$$

- computing determinants on the disk ($V_{AdS_2} = -2\pi$)

$$\bar{c}_1 = \exp\Big[-\tfrac{1}{2}\int_0^\infty dv\, v\Big(\tanh(\pi v)\big[(n-2)\ln(v^2+\tfrac{9}{4}) + (10-n)\ln(v^2+\tfrac{1}{4})\big]$$
$$-\coth(\pi v)\big[(2n-2)\ln(v^2+1) + (10-2n)\ln(v^2)\big]\Big)\Big] = \frac{1}{(\sqrt{2\pi})^{n-4}}$$

- disk with h handles: $g_s^{-1} \to g_s^{\chi}$, $\sqrt{T} \to (\sqrt{T})^{\chi}$, $\chi = 1 - 2h$ =Euler

$$\langle \mathcal{W} \rangle = e^{2\pi T} \sum_{h=0}^{\infty} c_{h+1} \left(\frac{g_s}{\sqrt{T}} \right)^{2h-1} \left[1 + \mathcal{O}(T^{-1}) \right]$$

- remarkably consistent with structure of $\frac{1}{N}$ corrections in SYM for $N \gg 1, \lambda \gg 1$ [Drukker, Gross; Pestun]

$$\langle \mathcal{W} \rangle = e^{\frac{\lambda}{8N}} L_{N-1}^{1}(-\tfrac{\lambda}{4N}) = e^{\sqrt{\lambda}} \sum_{h=0}^{\infty} \frac{\sqrt{2}}{96^h \sqrt{\pi}\, h!} \frac{\lambda^{\frac{6h-3}{4}}}{N^{2h-1}} \left[1 + \mathcal{O}(\tfrac{1}{\sqrt{\lambda}}) \right]$$

$$\to \quad c_{h+1} = \tfrac{1}{2\pi\, h!} \left(\tfrac{\pi}{12} \right)^h , \qquad\qquad c_1 = \tfrac{1}{2\pi}$$

- $\frac{g_s}{\sqrt{T}}$ is natural expansion parameter; for above c_{p+1} powers exponentiate:

$$\langle \mathcal{W} \rangle = W_1\, e^{H} \left[1 + \mathcal{O}(T^{-1}) \right] , \qquad W_1 = \frac{1}{2\pi} \frac{\sqrt{T}}{g_s} e^{2\pi T} , \qquad H \equiv \frac{\pi}{12} \frac{g_s^2}{T}$$

- H= "handle insertion operator": exp H expected in "dilute handle gas" approximation (thin far-separated handles) relevant for large T

- another interpretation of exponentiation [Drukker, Fiol]:

start with circular WL in k-symmetric $SU(N)$ representation

for large k, N and λ with $\kappa = \frac{k\sqrt{\lambda}}{4N}$ =fixed

$\langle \mathcal{W} \rangle \sim \exp(-S_{\rm D3})$ – determined by action of classical D3-brane solution

for $1 \ll k \ll N$ should apply also to WL in k-fundamental rep

described by minimal string surface ending on k-wrapped circle

$S_{\rm D3} = N f(\kappa) = -k\sqrt{\lambda} - \frac{k^3 \lambda^{3/2}}{96N^2} + ... = \ -2\pi k T - k^3 \frac{\pi}{12} \frac{g_{\rm s}^2}{T} + \mathcal{O}(\frac{g_{\rm s}^4}{T^3})$

extrapolating to $k = 1$ gives: $\quad \langle \mathcal{W} \rangle \sim \exp(-S_{\rm D3}) \to \exp(2\pi T + H)$

- similar structure of topological expansion for $\frac{1}{2}$ BPS circular WL in ABJM?

$\langle \mathcal{W} \rangle$ should be series in $\frac{\sqrt{T}}{g_{\rm s}} = \frac{N}{\sqrt{8\pi}\,\lambda} = \frac{k}{\sqrt{8\pi}} \sim$ CS level

leading and first subleading $\frac{1}{N}$ corrections found explicitly in [Drukker, Marino, Putrov]

$$\langle \mathcal{W} \rangle = \ e^{\pi\sqrt{2\lambda}} \Big(\frac{N}{4\pi\lambda} + \frac{\pi\lambda}{6N} + ... \Big) = W_1 \Big(1 + \frac{\pi}{12} \frac{g_{\rm s}^2}{T} + ... \Big)\,, \qquad W_1 = \frac{1}{\sqrt{2\pi}} \frac{\sqrt{T}}{g_{\rm s}} e^{2\pi T}$$

same (!) $H = \frac{\pi}{12} \frac{g_{\rm s}^2}{T}$ term at $\frac{1}{N^2}$ ("disk with one handle") order as in SYM

• H exponentiates as in SYM case?

D2-brane description of exponentiation? (cf. [Drukker, Plefka, Young; Cookmeyer, Liu, Pando Zayas])

so far does not seem so ... [Beccaria, Giombi, AT]

• hope for progress in detailed understanding $\frac{1}{N}$ corrections on string side