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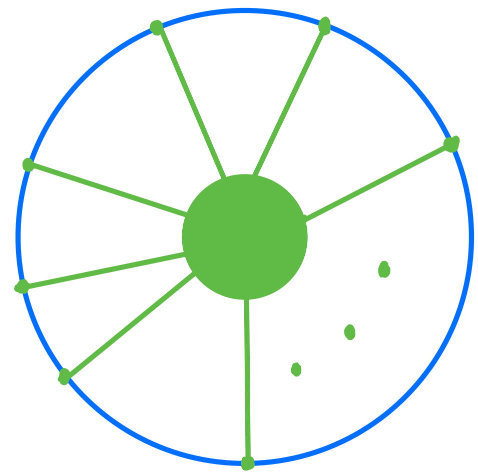
Ambitwistor Strings on $AdS_3 \times S^3$

based on : 2007.07234 K. Roehrig + D.S.
ongoing work G. Korchemsky, K. Roehrig + D.S.

see also : 2007.06574 L. Eberhardt, S. Komatsu + S. Mizera

Goal (unachieved!):

To obtain compact expressions for AdS amplitudes / boundary correlators with arbitrary multiplicity in supergravity limit



Inspiration from flat space:

Cachazo - He - Yuan

$$M^{(n)}_{sugra}(k_i, \epsilon_i, \tilde{\epsilon}_i) = \sum_{z_i \,:\, S_i(\underline{z})=0} \frac{Pf'(k_i, \epsilon_i, z_i)\; Pf'(k_i, \tilde{\epsilon}_i, z_i)}{\det'(\partial S_i / \partial z_j)}$$

$$S_i(\underline{z}) = \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j}$$

scattering equations

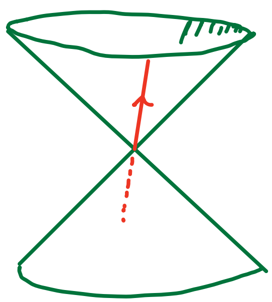
Ambitwistor Strings

$$S = \int_\Sigma P_\mu \bar{\partial} X^\mu - \frac{1}{2} e P_\mu P^\mu$$

$$P_\mu \in \Omega^{1,0}(\Sigma)$$
$$e \in \Omega^{0,1}(\Sigma, T_\Sigma)$$

- chiral worldsheet theory / complexification of worldline

- $P^2 = 0$ constraint generates $\delta X^\mu = \alpha P^\mu$, $\delta P_\mu = 0$, $\delta e = \bar{\partial}\alpha$



- $X \sim X + \alpha P$ so target is really space of null r s "ambitwistor space"

- Vertex ops $\sim e^{ik\cdot X}$ BRST invariant iff $k^2=0$.

Since $X(z)X(0)\sim 0$ no massive states in spectrum

- With n vertex operators, integrate out X to find $\sum k_i=0$ and

$$\bar\partial P = \sum_i k_i\,\bar\delta(z-z_i) \Rightarrow P(z)=\sum_i \frac{k_i}{z-z_i} \Rightarrow P^2=\sum_{i,j}\frac{k_i\cdot k_j}{(z-z_i)(z-z_j)}$$

- Usual basis of Beltrami differentials $e\in H^{0,1}(\Sigma, T_\Sigma(-z_1\cdots -z_n))$ gives

$$\frac{1}{2}\int_\Sigma e\,P^2 = \frac{1}{2}\sum_i \overset{\text{moduli}}{e_i}\,\mathrm{Res}_i(P^2) = \sum_i e_i\Big(\underbrace{\sum_{j\neq i}\frac{k_i\cdot k_j}{z_i-z_j}}_{\text{scattering eq}^{\text{n}}\text{s}}\Big)$$

and integrating over moduli e_i imposes scattering eq$^{\text{n}}$s.

- Anomaly free in $d=10$ if also include $\int \frac{1}{2}\psi_\mu \bar\partial \psi^\mu + \frac{1}{2}\tilde\psi_\mu \bar\partial \tilde\psi^\mu$ and also gauging $\mathcal{G} \equiv p \cdot \psi$ and $\tilde{\mathcal{G}} \equiv p \cdot \tilde\psi$ along with $H \equiv \frac{1}{2}p^2$

SL(1|1) worldsheet algebra

$$\mathcal{G}(z)\,\mathcal{G}(\omega) \sim \frac{2H}{z-\omega} \qquad \tilde{\mathcal{G}}(z)\,\tilde{\mathcal{G}}(\omega) \sim \frac{2H}{z-\omega} \qquad \mathcal{G}(z)\,\tilde{\mathcal{G}}(\omega) \sim 0$$

- Complete (GSO+) spectrum $\Rightarrow$ Type II supergravity
- Pfaffians in CHY come from ψ, $\tilde\psi$

Since we obtain correct, n-particle sugra tree amplitudes, will be a consistent theory on any background solving the (super-) Einstein eq^ns

Ambitwistor Strings on $AdS_3 \times S^3$ $(\times M_4)$

- $AdS_3 \times S^3 \cong SL(2,\mathbb{R}) \times SU(2)$ (we need complexification)
- Pure NS flux $H_{abc} = -f_{abc}$ background solves Einstein eq^{n}s

$$S = \int_\Sigma J_a (g^{-1}\bar\partial g)^a + \frac{1}{2} m(\Psi, \bar D \Psi) + \frac{1}{2} m(\tilde\Psi, \bar\partial \tilde\Psi)$$
$$- eH + \chi G + \tilde\chi \tilde G$$

- $J \in \Omega^{1,0}(\Sigma, g^*)$ independent field (~~not~~ $g^{-1}\partial g$). No WZ term, as would break chirality here
- m a bi-invariant metric on $SL(2) \times SL(2)$, with $\kappa^{ab} m_{ab} = 0$

$\bar D \Psi = \bar\partial \Psi^a + f^a{}_{bc} (g^{-1}\bar\partial g)^b \Psi^c$ covariant, but $\tilde\Psi$ non-cov

- Currents become

$$H = \frac{1}{2} m(J,J), \quad G = J\cdot\psi + \frac{1}{6} m(\psi,[\psi,\psi]), \quad \tilde{G} = J\cdot\tilde{\psi} - \frac{1}{6} m(\tilde{\psi},[\tilde{\psi},\tilde{\psi}])$$

and obey same $SL(1|1)$ algebra as in flat space

$$G(z)\,G(w) \sim \tilde{G}(z)\,\tilde{G}(w) \sim \frac{2H}{z-w}$$

anomaly free iff $m_{ab}\,\kappa^{ab} = 0 \Leftrightarrow$ Einstein eq^{n}s

- $$Q = \oint cT + \tilde{c}H + \gamma G + \tilde{\gamma}\tilde{G} + \text{ghosts}$$

hol. stress tensor on Σ

$$T = J_a (g^{-1}\partial g)^a + \text{fermions}$$

$Q^2 = 0$ provided $d = 10$ and $m_{ab}\,\kappa^{ab} = 0$

- Spectrum is Type II supergravity on $AdS_3 \times S^3 (\times M_4)$

$$U = c\tilde{c}\, \delta^2(\gamma)\, \psi^a \tilde{\psi}^b V_{ab}(g)$$ no normal ordering since gg OPE trivial

- For bulk-boundary propagators, choose

$$V^+_{ab}(g) = \frac{\langle\lambda| t_a |\lambda\rangle \langle\lambda| t_b |\lambda\rangle}{[\tilde{\lambda}| g(z) |\lambda\rangle^4} \qquad V^-_{ab} = \frac{[\tilde{\lambda}| g^{-1} t_a g |\tilde{\lambda}][\tilde{\lambda}| g^{-1} t_b g |\tilde{\lambda}]}{[\tilde{\lambda}| g(z) |\lambda\rangle^4}$$

where $|\tilde{\lambda}]\langle\lambda| = \begin{pmatrix} \bar{x} \\ 1 \end{pmatrix} \begin{pmatrix} x & 1 \end{pmatrix} = \begin{pmatrix} x\bar{x} & \bar{x} \\ x & 1 \end{pmatrix}$ describes the boundary point

- Worldsheet correlator (cf flat space Pfaffians) can be computed explicitly using JJ and $J\psi$ OPEs, even at n points (see paper)

The Gaudin Model

- AdS scattering eq^{n}s come from moduli of $\int e\, H = \int e\, \frac{1}{2} J^2$

- OPE of J with vertex operator shows H acts on bulk-boundary propagators as

$$H(z) = \frac{1}{2} \sum_i \frac{\Delta_i(\Delta_i - 2)}{(z - z_i)^2} + \sum_i \frac{1}{z - z_i} \overbrace{\left(\sum_{j \neq i} \frac{e_i f_j + e_j f_i + \frac{1}{2} h_i h_j}{z_i - z_j} \right)}^{H_i}$$

vanishes since massless / BRST closed

$e_i = -x_i^2 \partial/\partial x_i - \Delta_i x_i$,

$f_i = \partial/\partial x_i$, $h_i = 2 x_i \partial/\partial x_i + \Delta_i$

This is sl_2 Gaudin Hamiltonian acting on boundary coords

$z_i \Rightarrow$ worldsheet co-ord / inhomogeneity parameter

rep^{n}s "spins" determined by dual boundary operators

- Also appears in KZ connection for standard WZW model

$$\left(\frac{\partial}{\partial z_i} - \frac{1}{k+h^\vee} \sum_{j \neq i} \frac{t_i \cdot t_j}{z_i - z_j} \right) \Psi(\underline{x}, \underline{z}) = 0$$

- Here instead we evolve correlator $\Psi(\underline{x}, \underline{z})$ through 'times' e_i:

$$\exp\left(-\int e H\right) |\Psi\rangle = \int \exp\left(-\sum_i{}' \overset{\text{moduli}}{e_i}\, E_i(\underline{z})\right) |\underline{E}\rangle\langle \underline{E} | \Psi\rangle \, d\underline{E}$$

eigenvalue of $H_i = \mathrm{Res}_i\, H(z)$

Integrating over these $n-3$ moduli e_i again imposes scattering eq^{n}s

$$\int d^{n-3} e_i \; e^{-\int eH} |\Psi\rangle = \int \prod_i{}' \bar{\delta}\left(E_i(z_i)\right) |\underline{E}\rangle\langle \underline{E} | \Psi\rangle \, d\underline{E}$$

- Analogous relation to "AdS Gross-Mende limit" as $k \to -h^\vee$ "critical" level in WZW model [Feigin, Frenkel, Reshetikhin]

- Since $H(z) \in \Omega^0[\Sigma, K_\Sigma^2(z_1 + \cdots + z_n)]$ and $H_i = \mathrm{Res}_i H \in \Omega^0(\Sigma_i, K_i(\sum_{j\neq i} z_j))$ the eigenvalues $E_i(z)$ must take the form

$$\mathcal{E}(z) = \sum_{i,j} \frac{c_{ij}}{(z-z_i)(z-z_j)} \quad , \quad E_i(z) = \mathrm{Res}_i\, \mathcal{E} = \sum_{j\neq i} \frac{c_{ij}}{z_i - z_j}$$

where $c_{ij} = c_{ji} \in \mathbb{C}$ and

$c_{ii} = 0$ (no double poles - massless particles on AdS)

$\sum_j c_{ij} = 0$ (no poles at $z=\infty$ - well-defined CFT on Σ_i invariant under global $SL(2)$ on boundary)

- AdS scattering eq$^{\text{n}}$s $E_i = 0$ same form as in flat space, replacing $s_{ij} \to c_{ij}$. Related to (but $\neq$) Mellin parameters

- What sort of eigenstates should we consider?

$[H, \sum S_i] = 0$ • invariant under <u>worldsheet</u> $SL(2)$

global bdry sl_2

Bethe states not enough (Bethe vacuum breaks global $SL(2)$)

- Use SoV [Sklyanin]: look for zeros (in z) of operator

$$f(z) = \sum_i \frac{\partial/\partial x_i}{z - z_i}$$

$\Rightarrow$ diagonalize by <u>Fourier transform</u>

- <u>Problem</u>: This FT does not sit well with global $SL(2)$ transformations of boundary. Perhaps resolve by choosing 1 particle to live at $z = \infty$ on Σ [ongoing w/ Korchemsky, Roehrig, DS]

- **Simplification:** only need to know eigenstates on support of scattering eq$^{\text{n}}$s, since

$$\int \frac{dz_1 \cdots dz_n}{\mathrm{vol}\, SL(2)}\, d\underline{E} \prod_i{}' \bar{\delta}(E_i(\underline{z}))\, |E(\underline{z},\underline{x})\rangle\langle E(\underline{z},\underline{x})|\Psi(\underline{z},\underline{x})\rangle$$

$$= \sum_{\underline{z}_* :\, E_i(\underline{z}_*)=0} \int d\underline{E}\, \frac{1}{\det{}'(\partial E_i/\partial z_j)}\, |E(\underline{z}_*,\underline{x})\rangle\langle E(\underline{z}_*,\underline{x})|\Psi(\underline{z}_*,\underline{x})\rangle$$

eg $n=4$ In general, eigenstate $\phi(\underline{x},\underline{z})$ obeys ode

$$\frac{d^2\phi}{dX^2} + \left[\frac{1}{X} + \frac{1}{X-1} + \frac{3}{X-Z}\right]\frac{d\phi}{dX} + \left[\frac{4X-2-E(Z)}{X(X-1)(X-Z)}\right]\phi = 0$$

$X = \dfrac{x_{12}\,x_{34}}{x_{23}\,x_{41}}$, $Z = \dfrac{z_{12}\,z_{34}}{z_{23}\,z_{41}}$. General sol$^{\text{n}}$ involves Heun f$^{\text{n}}$s

but when $E(Z_*)=0$ reduces to $\displaystyle \frac{1}{(X-Z)^2}\Big[a + b\ln\big(X^Z(1-X)^{1-Z}\big)\Big]_{Z=Z_*}$

Conclusions

- There exist compact expressions for n-particle AdS amplitudes in supergravity limit (also for YM, bi-adjoint scalar...)
- In AdS, scattering eq^s take same form as in flat space

$$E_i(\underline{z}) = \sum_{j\neq i} \frac{c_{ij}}{z_i - z_j} \overset{!}{=} 0 \qquad c_{ij} = c_{ji}\,, \quad c_{ii} = 0\,, \quad \sum_j c_{ij} = 0$$

where $E_i(\underline{z})$ are eigenvalues of $sl_2 \times su_2$ Gaudin Hamiltonian

- Amplitude formula $\displaystyle\int \sum_{\underline{z}\,:\,E_i(\underline{z})=0} \frac{1}{\det'\left(\frac{\partial E_i}{\partial z_j}\right)} \left| E(\underline{z},\underline{x})\right\rangle \left\langle E(\underline{z},\underline{x})\right| \Psi(\underline{z},\underline{x})\rangle \, d\underline{c}$

somewhat reminiscent of scattering wavepackets in flat space

- Bi-adjoint scalar formula proved [Eberhardt, Komatsu, Mizera] by unpacking back into Witten diagrams

Thank You!