Comments on Non-integrable Boundaries.
• Integrable Boundaries
  → Boundary conditions that preserve a "half" of higher conserved charges.
  \[ \sum \frac{1}{p_i} \]
  \[ \sum p_i \text{ even.} \]
  → Dynamical consequences
  • \( \{ p_i \rightarrow -p_i \} \rightarrow \{-p_1, -p_2, \ldots, -p_n\} \)

• Application to AdS/CFT integrability.

1. Spectrum of open string/spin-chain

Q. • Correlation Functions of open strings?
  • Other open string spectrum?
  ~ Domain wall, Non-SUSY WL WL in higher-rank rep.
2. (excited-state) $g$-function.

$\langle B \rangle$

- 3-pt (2 GG & 1 single-trace)
- 1-pt in the presence of domain wall
- Wilson loop + single trace of
  - D-instantons

$\Rightarrow$ Beyond spectrum!

TBA: $\langle B \rangle$

$\Rightarrow 1 \rightarrow \sum_{\text{open}} e^{-LE} \sim \langle B \Lambda \rangle e^{-\text{Efford} \mathbb{R}} \langle \Omega \mid B \rangle$

$\Rightarrow$ Excited-state $g$-function
  - from analytic continuation.
  - (Fredholm det) · (finite dim det)
  - (Product of phases)

Q. Some technical difficulties for theories
  - with non-diagonal scatterings, [Vu, Kostov, Serban]
  - $Q$-functions?
  - At weak coupling, we always get MPS.
    - Is it always true?
    - Can we express generic $\mid B \rangle$ in 2d QFT as MPS?
  - Rank of MPS = # of boundary bound states.
• Non-integrable boundaries in integrable theories
  (cf. a series of works by Patrick Dorey)

Integrable IB >

$\sim \left( \delta(p+g_1) \delta(p+g_2) \cdots \delta(p+g_n) \right)$

Non-integrable IB >

$\sim \delta(p+\cdots+p+g_1+\cdots+g_n)$

Single $\delta$-function

• Is this what we expect? Or is it more like
  $\delta(\cdots) + \delta(\cdots) \delta(\cdots) + \delta(\cdots) \delta(\cdots) + \cdots + \frac{\delta}{n} \delta(\cdots)$?

• Study examples:
  [de Leeuw, Kristjansen, Vardaghuis] (D3-D7 domain wall)

$\langle MPS|u_1u_2,-u_1,-u_2\rangle \sim \prod_{i=1,2} \int \sinh \frac{1}{2} \sqrt{\frac{\det G^+}{\det G^-}}$ + (More Complicated)

\[ \downarrow \]

1 \@ Large L

suppressed at large L

• $\delta(p+q) = \int dx \ e^{i(p+q)} \sim L \delta p q$

• Terms w/ more $\delta$'s dominant \@ large L
Integrable piece:

\[ f(u) : \text{"Integrable piece"} \quad \begin{array}{c} \text{2-particle} \\ \text{form factor} \end{array} \]

\[ S(u_1 + u_2) S(u_3 + u_4) \]

(More complicated) should come from 4-particle F.F.:

\[ S(u_1 + u_2) S(u_3 + u_4) + \cdots \]

Correct structure seems to be:

\[ S(u_1 + u_2) S(u_3 + u_4) + \cdots \]

non-integrable pieces

integration

\[ \text{How do we constrain all these F.F.?} \]

Form-factor bootstrap!

\[ \text{Watson} \]

\[ \text{Unitarity} \]

Different S-sectors talk to each other.

Interesting & intricate relations

Would be nice to see that AdS/CFT examples satisfy these axioms!
• Extract 4- and higher-particle form factors from [de Leeuw et al.]

• Bootstrapping non-integrable bdy.

• "Le Clair- Mussardo" formula for non-integrable g-functions?

• Numerical Bootstrap?
  - what is the most "chaotic" reflection amplitudes for free fermions?

• Nice to understand Landau diagrams for non-integrable bdy.
  - Are there some nice structures to be discovered (Hopf algebra etc...)?

(An structure in some susy... )

• They are also relevant for N4 SYM
  general WL → Non-integrable bdy.