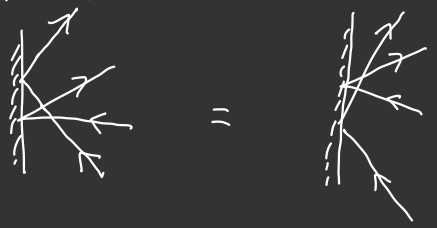
Comments on Non-integrable Boundaries

- Integrable Boundaries

⇝ Boundary conditions that preserve a "half" of higher conserved charges.

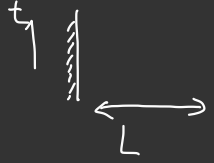
$\sum_i P_i^{\text{even}}$.

⇝ Dynamical consequences

- $\{P_1 \cdots P_n\} \longrightarrow \{-P_1, -P_2, \cdots, -P_n\}$
-  Boundary Y.B. equation. cf. Shankar-Witten.

- Application to AdS/CFT integrability.

1. Spectrum of open string / spin-chain



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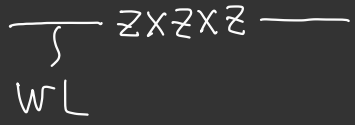

, • Open string on Giant Graviton

• $\mathcal{N}=4$ SYM w/ flavor D7-branes

BTBA:

$R \gg 1$ ⇝ BTBA eq.

Q. • Correlation Functions of open strings?

• Other open string spectrum?

⇝ Domain wall, Non-SUSY WL

WL in higher-rank rep.

- New integrable boundaries from 4d CS ? [Bittleston, Skinner]

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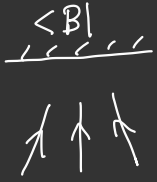
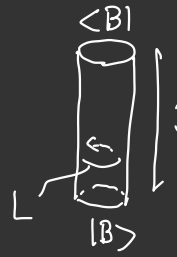
2. (excited-state) g-function.



- 3-pt (2 GG & 1 single-trace)
- 1-pt in the presence of domain wall
- Wilson loop + single trace op.
- D-instantons ----

⇝ Beyond spectrum!

TBA:



$$\rightsquigarrow \sum_{\text{open}} e^{-LE} \sim \langle B|\Omega\rangle\, e^{-E_{\text{closed}} R}\, \langle \Omega|B\rangle$$

open → density → TBA

saddle

1-loop fluctuation

⇝ Excited-state g-function from analytic continuation.

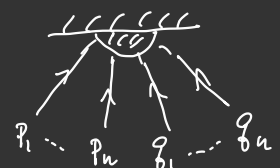
⇝ (Fredholm det) · (finite dim det) · (Product of phases)

Q.
- Some technical difficulties for theories w/ non-diagonal scatterings. [Vu, Kostov, Soban]
- Q-functions ?
- At weak coupling, we always get MPS.
  - Is it always true ?
  - Can we express generic |B> in 2d QFT as MPS ?
  - Rank of MPS = # of boundary bound states.

- Non-Integrable boundaries in Integrable theories,

(cf. a series of works by Patrick Dorey)

Integrable $|B\rangle$



$$\sim \underbrace{\delta(p_1+q_1)\,\delta(p_2+q_2)\cdots\delta(p_n+q_n)}_{n\ \delta\text{-functions}}$$

Non-Integrable $|B\rangle$ $\sim \underbrace{\delta(p_1+\cdots+p_n+q_1+\cdots+q_n)}_{\text{Single } \delta\text{-function}}$

⊛ Is this what we expect? Or is it more like

$$\delta(\cdots)+\delta(\cdots)\delta(\cdots)+\delta(\cdots)\delta(\cdots)\delta(\cdots)+\cdots+\underbrace{\delta\cdots\delta}_{n}\ ?$$

⇒ Study examples!

[de Leeuw, Kristjansen, Vardinghus] (D3-D7 domain wall)

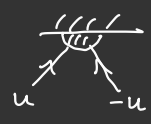
$$\langle MPS|\,u_1,u_2,-u_1,-u_2\rangle_L \sim \prod_{i=1,2} f(u_i)\sqrt{\frac{\det G^+}{\det G^-}} + (\text{More Complicated})$$

$\sqrt{\frac{\det G^+}{\det G^-}}$ → 1 @ Large L

(More Complicated) → suppressed at large L

- $\delta(p+q) = \int dx\, e^{i(p+q)} \underset{\text{finite } L}{\sim} L\,\delta_{p,-q}$
- Terms w/ more δ's dominant @ large L

- $f(u)$: "Integrable piece" $\delta(u_1+u_1')\,\delta(u_2+u_2')$



2-particle form factor

- (More complicated) should come from 4-particle F.F. :

⇝ Correct structure seems to be

$$\underbrace{\delta(\cdots) + \delta(\cdots)\,\delta(\cdots) + \cdots}_{\text{non-integrable pieces}} + \underbrace{\delta\delta\cdots\delta}_{n} \leftarrow \text{integrable}$$

⓪ How do we constrain all these F.F. ?

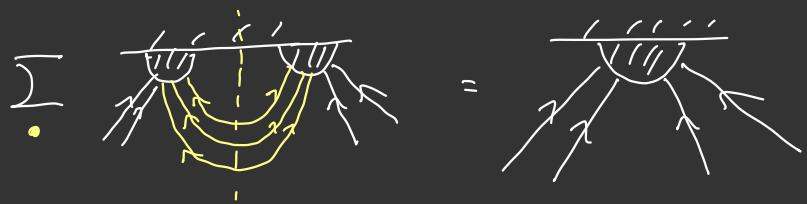
⇝ Form-factor bootstrap !

- Watson

• = S-matrix,

(closed for each "δ-sector")

- Unitarity



Different δ-sectors talk to each other.

- Interesting & Intricate relations

⇝ Would be nice to see that AdS/CFT examples satisfy these axioms !

Q
- Extract 4- and higher-particle form factors from [de Leeuw et al]
- Bootstrapping non-integrable bdy
  ↝ Bootstrapping form factors
- "Le Clair - Mussardo" formula for non-integrable g-functions ?
  * formula that expresses finite-size $\langle \mathcal{O} \rangle_L$ in terms of 2, 3, ... particle F.F. in the ∞ volume.
- Numerical Bootstrap ?
  - What is the most "chaotic" reflection amplitudes for free fermions
- Nice to understand Landau diagrams for non-integrable bdy.
  - Are there some nice structures to be discovered (Hopf algebra etc...)?

  (Ask structure in some susTic example [GMW])
- They are also relevant for N=4 SYM general WL ⟶ Non-integrable bdy.