Domain walls as integrable boundary states in N=4 SYM

Charlotte Kristjansen
Niels Bohr Institute

Based on:

- A. Gimenez-Grau, C.K., M. Volk & M. Wilhelm, arXiv:1912.02468[hep-th],
- JHEP 04 (2020) 132
- M. de Leeuw, T. Gombor, C.K., G. Linardopoulos & B. Pozsgay ArXiv:1912.09338[hep-th], JHEP 01, (2020) 176
- C.K., D. Müller & K. Zarembo, ArXiv:2005.01392[hep-th], to appear in JHEP

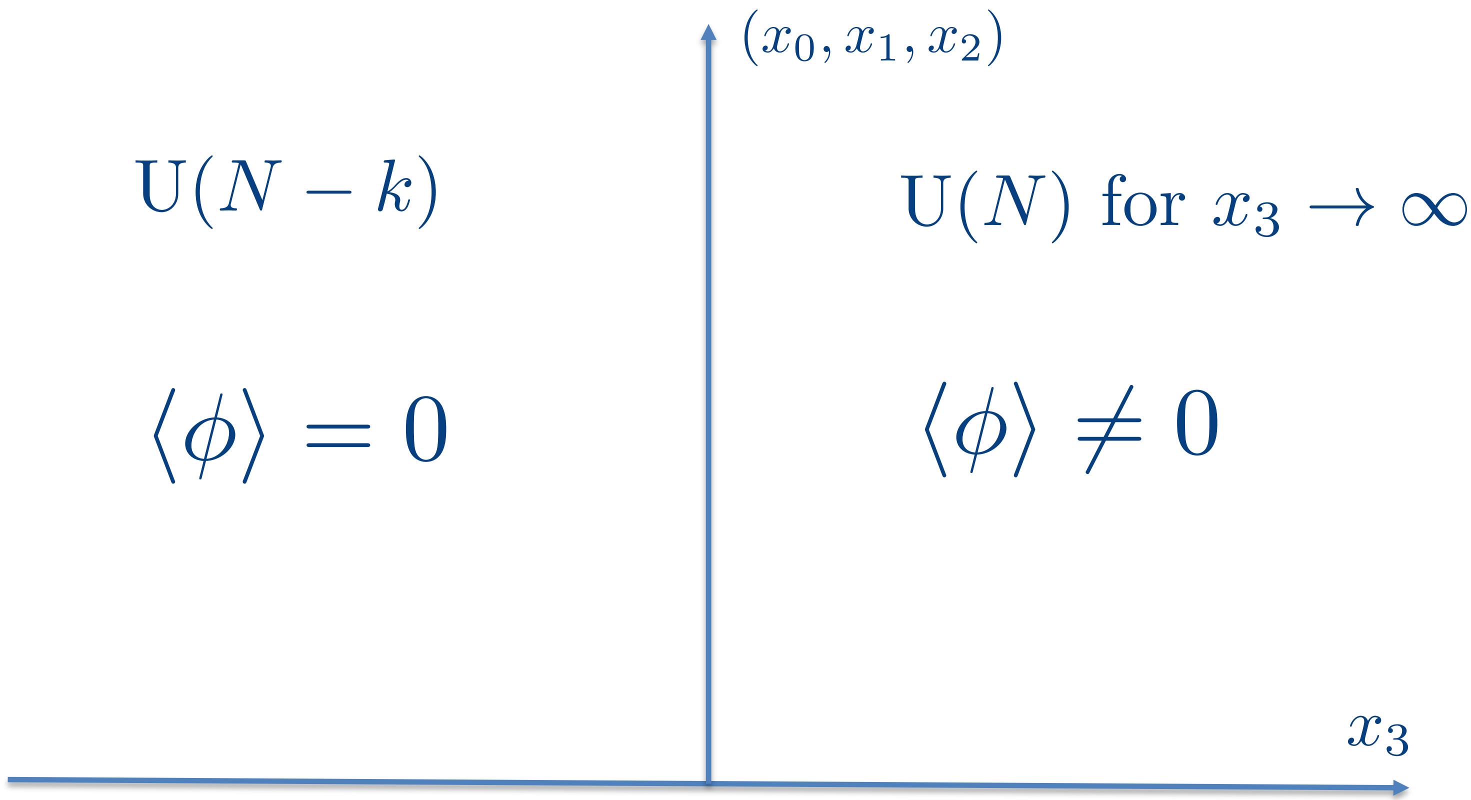
IGST2020
Sao Paolo, Brazil
August 27^{th}, 2020

Motivation

- Insights on the interplay between conformal symmetry, supersymmetry and integrability
- Exact results for novel types of observables such as one-point functions
- Positive tests of AdS/dCFT dictionary for set-ups with and without supersymmetry
- Interesting connections to statistical physics: matrix product states and quantum quenhes.
- Possible cross-fertilization with the boundary conformal bootstrap program.

The defect set-up

$\mathcal{N} = 4$ SYM



Classical Fields (simplest case)

Assume only x_3 -dependence and $x_3 > 0$, $A_\mu^{\rm cl} = 0, \qquad \Psi_A^{\rm cl} = 0$

Classical e.o.m.:

(x_3 is distance to defect)

$$\frac{d^2\phi_i^{\rm cl}}{dx_3^2} = \left[\phi_j^{\rm cl}, \left[\phi_j^{\rm cl}, \phi_i^{\rm cl}\right]\right].$$

Solution:

$$\phi_i^{\rm cl} = \frac{1}{x_3}\begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \ i = 1,2,3$$

Constable, Myers & Tafjord '99

$$\phi_4^{\rm cl} = \phi_5^{\rm cl} = \phi_6^{\rm cl} = 0$$

where t_i, i=1,2,3, constitute a k-dimensional irreducible repr. of SU(2). (Nahm eqns. also fulfilled.)

Set-up ½ BPS (Gaiotto & Witten '08)

The quantum fields

For $x_3 > 0$:

$$\Phi_i, \Psi, A_\mu = \begin{array}{c} \begin{array}{cc} k & N-k \end{array} \\ \left[\begin{array}{c|ccc} x & y & y & y \\ \hline y & z & z & z \\ y & z & z & z \\ y & z & z & z \end{array}\right] \end{array} \begin{array}{c} k \\ N-k \end{array}$$

For k>1: x and y fields are massive, $m^2 \propto 1/x_3^2$, emergent AdS space.
z fields are massless

Buhl-Mortensen, de Leeuw, Ipsen, C.K, Wilhelm '16

For k=1: No classical fields, specific b.c. at the defect

Ipsen, & Vardinghus '19

C.K, Müller, Zarembo '20

	$\Phi_{4,5,6}, A_{0,1,2}, c$	$\Phi_{1,2,3}, A_3$
x, y	Dirichlet	Neumann
z	no BCs	no BCs

AdS/dCFT set-ups

	D3-D5	D3-D7	D3-D7
Symmetry of vevs	SU(2)	SU(2) × SU(2)	SO(5)
Dim. of rep. / Flux	k	k_1, k_2	$d = \frac{(n+1)(n+2)(n+3)}{6}$
Gauge Groups	$\mathrm{U}(N)$, $\mathrm{U}(N-k)$	$\mathrm{U}(N)$, $\mathrm{U}(N-k_1k_2)$	$\mathrm{U}(N)$, $\mathrm{U}(N-d)$
Supersymmetry	1/2 BPS	None	None
Brane geometry	$\mathrm{AdS}_4 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^2 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^4$

One-point functions in dCFT's

$$\langle \mathcal{O}_\Delta^{\text{bulk}}(x)\rangle = \frac{C}{|x_3|^\Delta}$$

Cardy ´84
McAvity & Osborn '95

Normalization given by: $$\lim_{x_3\to\infty}\langle \mathcal{O}_\Delta^{\text{bulk}}(y+x)\mathcal{O}_{\Delta'}^{\text{bulk}}(z+x)\rangle = \frac{\delta_{\Delta\Delta'}}{|y-z|^{2\Delta}}$$

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_\Delta(x)\rangle = \left(\text{Tr}(\phi_{i_1}\ldots\phi_{i_\Delta}) + \ldots\right)\Big|_{\phi_i\to\phi_i^{\text{cl}}=\frac{t_i}{x_3}}$$

Matrix Product State associated with the defect:

$$|\text{MPS}_\text{k}\rangle = \sum_{\vec{i}} \text{tr}[t_{i_i}\ldots t_{i_L}]|\phi_{i_1}\ldots\phi_{i_L}\rangle,$$

deLeeuw, C.K.
& Zarembo '15,

Object to calculate: $$C_k\left(\mathbf{u}\right) = \frac{\langle \text{MPS}_k\,|\mathbf{u}\,\rangle_L}{\langle \mathbf{u}|\mathbf{u}\rangle^{\frac{1}{2}}}$$

Integrability criterion

When can $\langle \text{MPS}_k | \mathbf{u} \rangle_L$ be calculated in closed form?

Integrability criterion: $\hat{Q}_{2m+1}|MPS_k\rangle = 0, \quad m \geq 1$

Ghoshal & Zamolodchikov '94

Piroli, Pozsgay Vernier '17

$|MPS_k\rangle$ only involves excitation pairs with momenta $(+p, -p)$

$|MPS_k\rangle$ boundary state which only allows pure reflection (BYB also required)

Also inspired by
Korepin '82, Izgerzin '87,Tsuchiya '98, Pozsgay '13, Brockmann et al '14, Buhl-Mortensen, de Leeuw, CK & Zarembo '15, Foda and Zarembo '15

Integrability of MPS

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
Dim. of rep./ Flux	k	k_1, k_2	$d = \frac{(n+1)(n+2)(n+3)}{6}$
$\vert MPS \rangle$	Integrable	Non-integrable	Integrable
Overlaps	Exact formula derived	—	Exact formula derived

Reflection matrix which fulfills BYB of SO(6) spin chain and has the appropriate symmetries can be found for the two cases with $Q_{2m+1}|MPS\rangle = 0$

de Leeuw, Gombor C.K & Linardopoulos, Pozsgay '19.

Solution SO(5) symmetric D3-D7case

de Leeuw, C.K & Linardopoulos,'18.

de Leeuw, Gombor C.K & Linardopoulos, Pozsgay '19.

Result for C_n:

- Exact formula valid for any L, M, N^+, N^- and n

$$\frac{\langle \mathbf{u} | \mathrm{MPS}_n \rangle}{\langle \mathbf{u} | \mathbf{u} \rangle^{1/2}} = \Lambda_n \cdot \sqrt{\frac{Q_0(0)\, Q_0\left(\frac{1}{2}\right)}{\bar{Q}_+(0)\, \bar{Q}_+\left(\frac{1}{2}\right) \bar{Q}_-(0)\, \bar{Q}_-\left(\frac{1}{2}\right)}} \cdot \sqrt{\frac{\det G_+}{\det G_-}}$$

$$\Lambda_n = 2^L \sum_{q=-\frac{n}{2}}^{\frac{n}{2}} q^L \left[\sum_{p=-\frac{n}{2}}^{q} \frac{Q_0(p-\frac{1}{2})}{Q_0(q-\frac{1}{2})} \frac{Q_-(q) Q_-(\frac{n}{2}+1)}{Q_-(p) Q_-(p-1)} \right] \left[\sum_{r=q}^{\frac{n}{2}} \frac{Q_0(r+\frac{1}{2})}{Q_0(q+\frac{1}{2})} \frac{Q_+(q) Q_+(\frac{n}{2}+1)}{Q_+(r) Q_+(r+1)} \right].$$

Q's: Baxter polynomials, G Gaudin matrix:

$$\langle \mathbf{u} | \mathbf{u} \rangle \propto \det G = \det G_+ \det G_-,$$

Higher loops: D3-D5 case (1/2 BPS)

Tree level Formula works upon modification by a flux factor (su(2) sector)

Buhl-Mortensen, de Leeuw, Ipsen, C.K, Wilhelm '17

$$C_k = i^L \tilde{T}_{k-1}(0) \sqrt{\frac{Q(\frac{i}{2})Q(0)}{Q^2(\frac{ik}{2})}} \sqrt{\frac{\det G_+}{\det G_-}} \, \mathbb{F}_k$$

$$\mathbb{F}_k = 1 + g^2 \Big[\Psi(\tfrac{k+1}{2}) + \gamma_E - \log 2 \Big] \Delta^{(1)} + O(g^4) \,,$$

and a replacement in the Bethe equations and the transfer matrix

Beisert & Staudacher '05

$$e^{ip} = \frac{u + \frac{i}{2}}{u - \frac{i}{2}} \longrightarrow \frac{x(u + \frac{i}{2})}{x(u - \frac{i}{2})}, \qquad u(x) = x + \frac{g^2}{x}, \qquad g^2 = \frac{\lambda}{8\pi^2}$$

(plus dressing phase via bootstrap plus wrapping corrections via TBA)

Buhl-Mortensen, de Leeuw, Ipsen, C.K, Wilhelm '16

NB: A non-trivial field theory calculation is needed for this statement (involving diagonalizing the mass matrix using fuzzy spherical harmonics, supersymmetric regularization and renormalization).

Higher loops: D3-D5 case (1/2 BPS)

Recently reproduced by a bootstrap argument (assuming string integrability) — Komatsu & Wang '20

$\mathbb{F}_k$: Originates from boundary dressing phase

$\sum_{-\frac{k-1}{2}}^{\frac{k-1}{2}}$ in $T_{k-1}(u)$ originates from sum over boundary bound states

Extended to the full theory — Gombor & Bajnok '20

One-point function of chiral primaries calculated via localization
(No Bethe roots, no flux factor) — Komatsu & Wang '20

Higher loops D3-D7 cases (No susy)

Perturbative program set up:

$SU(2) \times SU(2)$ symmetric case (non-integrable) Gimenez-Grau, C.K, Volk, Wilhelm '18

$SO(5)$ symmetric case Gimenez-Grau, C.K, Volk, Wilhelm '19

Match with string theory in d.s.l. to two leading orders for

- One-point functions of chiral primaries Gimenez-Grau, C.K, Volk, Wilhelm '18, '19
- Expectation values of Maldacena-Wilson lines Bonansea, Idiab, C.K, Volk '20

Challenges of the SO(5) case

- One-point functions only non-vanishing for full $SO(6)$ sector
- Localization techniques do not work
- Argument against higher loop integrability in Gombor & Bajnok '20

Other sectors & higher loops from k=1

C.K, Müller, Zarembo '20

$k = 1$ formula is the analytical continuation of the $k > 1$ formula

Classical fields vanishing, specific b.c. at the defect

Feynman diagrammatics is completely different

Formulas start out at a higher order in g

$$\text{For } x_3 > 0: \quad A_\mu, \Phi_i, \Psi = \overset{\qquad 1 \qquad\quad N-1}{\left[\begin{array}{c|ccc} x & y & y & y \\ \hline y & z & z & z \\ y & z & z & z \\ y & z & z & z \end{array}\right]} \begin{array}{c} 1 \\ \\ N-1 \\ \end{array}$$

Boundary conditions

	$\Phi_{4,5,6}, A_{0,1,2}, c$	$\Phi_{1,2,3}, A_3$
x, y	Dirichlet	Neumann
z	no BCs	no BCs

Leading order contribution

Propagators for scalars:

$$D_\kappa(x,y) = \frac{1}{4\pi^2}\left(\frac{1}{|x-y|^2} + \frac{\kappa}{|\bar{x}-y|^2}\right), \quad \kappa = \begin{cases} 1 & \text{Neumann} \\ -1 & \text{Dirichlet} \\ 0 & \text{no BCs.} \end{cases}$$

$$\bar{x} = (x_0, x_1, x_2, -x_3)$$

$$\langle X^{1a}(x) X^{b1}(y)\rangle = \frac{g_{\rm YM}^2 \delta^{ab}}{2}\Big(D_1(x,y) - D_{-1}(x,y)\Big) = \frac{g_{\rm YM}^2 \delta^{ab}}{4\pi^2 |\bar{x}-y|^2},$$

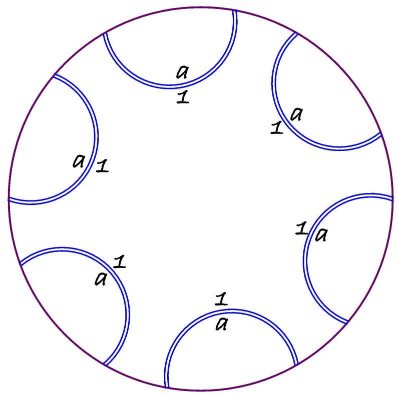
$$X = \phi_1 + i\phi_4, \text{ etc.}$$

Propagators for fermions in the SU(2|3) sector

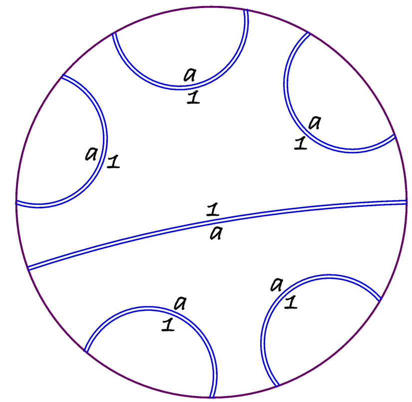
$$\langle \Psi^{1a}_\alpha(x) \Psi^{b1}_\beta(y)\rangle = \frac{g_{\rm YM}^2}{8\pi^2}\,\epsilon_{\alpha\beta}\,\delta^{ab}\;\cdot\;\frac{\bar{x}_3 - y_3}{|\bar{x}-y|^4}.$$

OBS: No divergences as $x \to y$

Feynman diagrams



Leading for large-N



Sub-leading for large-N

$$C_{k=1} = 2\left(\frac{\lambda}{16\pi^2}\right)^{L/2}\frac{\langle \mathrm{VBS}|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}}$$

C.K., Müller, Zarembo '20

$$\langle \mathrm{VBS}| = (\langle XX| + \langle YY|)^{\otimes L/2}, \qquad SU(2) \text{ sector}$$

$$\langle \mathrm{VBS}| = (\langle XX| + \langle YY| + \langle ZZ| + \langle \uparrow\downarrow| - \langle \downarrow\uparrow|)^{\otimes L/2}, \qquad SU(2|3) \text{ sector}$$

Closed expression of factorized determinant form
Result agrees with $k \to 1$ limit of formula with flux factor
(the higher order in g is encoded in the Zhukovsky map).

Summary

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
$\vert MPS \rangle$	Integrable	Non-integrable	Integrable
One-point functions	Exact formula derived at tree level and one-loop. Bootstrapped to all orders.	—	Exact formula derived at tree level.
Match with string theory: Local obs. (1-pt. fcts) Non-local obs. (Wilson lines)	 yes yes	 yes yes	 yes yes

Future directions

- Understanding the integrability/non-integrability from the string theory side
- Higher loop integrability for D3-D7?
- Derive the TBA for D3-D5
- Wilson loops by localization
- Connections to the boundary analytic bootstrap program

Thank you