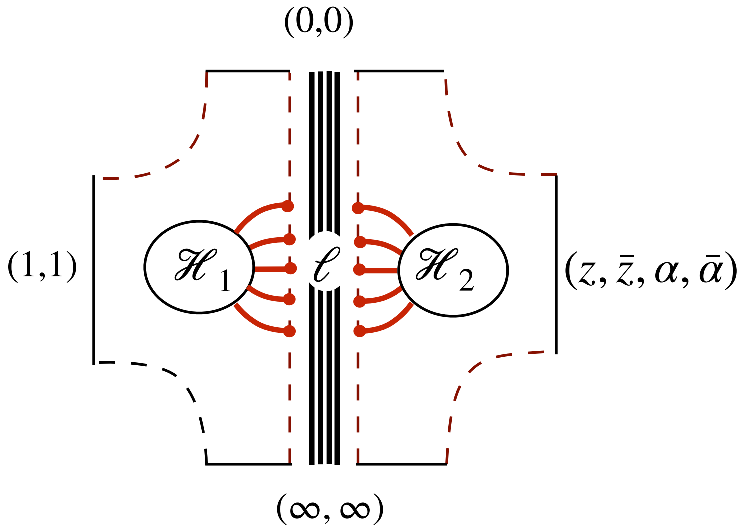
Hexagonalisation: a generalisation of the TBA setup to world-sheet of higher genus

— gives a prescription for summing up the virtual particles wrapping the spacial cycles.

— weights= [symmetric bi-local] x [antisymmetric bi-local] x [matrix factor]

Octagon: the simplest non-trivial object

weights = [symmetric bi-local] x 1 x 1



Is there hidden integrability in the Octagon?

Octagon = Free fermions $\Rightarrow$ (Fredholm) determinant $\Rightarrow$ integrability?

$$\mathbb{O}_\ell = \sum_{\pm} \langle \ell \,|\exp(\psi^* \mathbf{C} \psi^*) \;\; \exp(\psi \mathbf{K} \psi)\,|\, \ell \rangle$$

$$\psi^* \mathbf{C} \psi^* = \sum_{m,n=0}^{\infty} \psi^*_m \mathrm{C}_{nm} \psi^*_n$$

Bridge length ℓ = charge of Fermi vacuum

$$\psi \mathbf{K} \psi = \sum_{m,n=0}^{\infty} \psi_m \mathrm{K}_{nm} \psi_n$$

— In the null square limit $\log(z/\bar{z}) = e^{-s/g}, \; g \to 0$

the octagon is a tau-function for the semi-infinite Toda lattice $\tau_\ell(s), \; \ell \geq 0.$

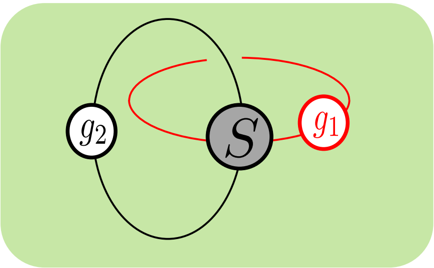
$$\mathbb{O}_\ell = \mathrm{e}^{-s^2/4}\, \tau_\ell(s) \qquad\qquad s^2 \frac{\tau_\ell\, \tau_{\ell+2}}{\tau^2_{\ell+1}} = (s\partial_s)^2 \log \tau_{\ell+1}$$

Q. Is it possible to write Hirota equations in the general case?
Physical excitations by integrable deformation?

Is it possible to rephrase the hexagonalization in terms of the Quantum Spectral Curve?

The weights of the virtual particles in the deformed octagon and the decagon have nontrivial matrix part.

e.g. for the Decagon



The strong coupling expressions can be interpreted as functional determinant of the QSC

$$\mathbb{O}_0 \simeq \exp 2g \int_{-\infty}^{\infty} \frac{d\vartheta}{2\pi}\, \varphi \cosh\vartheta \log(1 + Y(\vartheta))$$

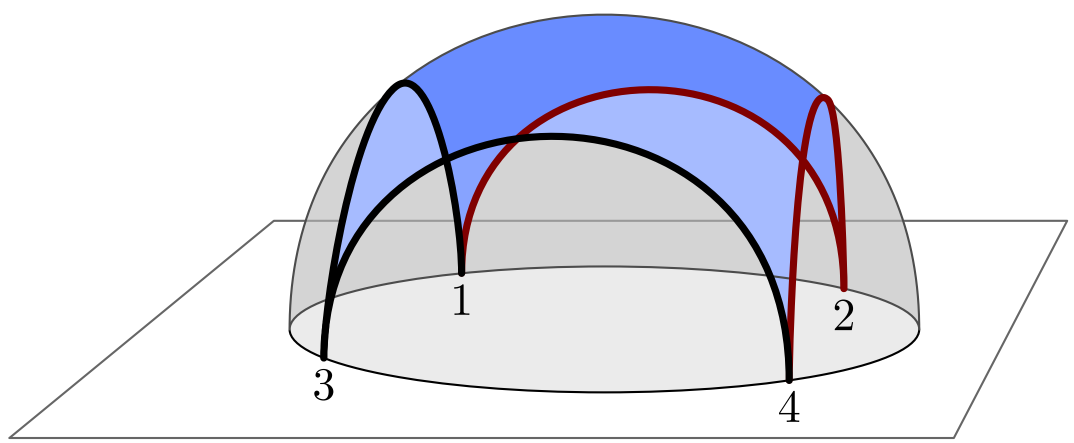
SU(2|2) character

What about finite coupling?

Possible approach: diagonalise the matrix part as in the nested TBA using the (wrong) string hypothesis and then interpret the result in terms of functional equations as in TBA. Is that realistic?

Gauge-string correspondence in the semi-classical limit

interpretation of the leading strong coupling expansion of the octagon in terms of minimal area attached to four BMN geodesics. The problem is well posed but the solution is still missing.



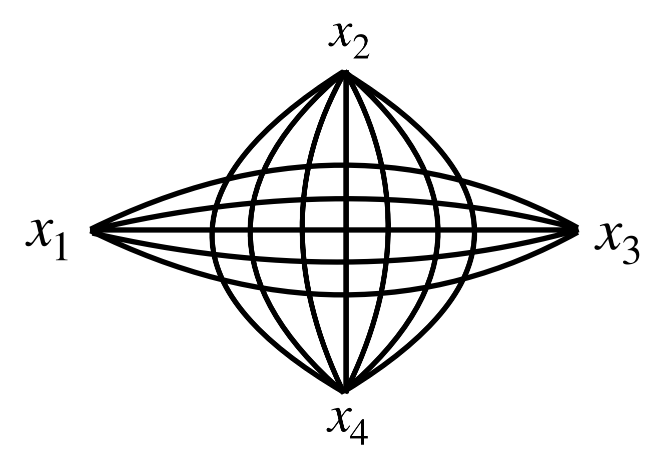
How to extend the map between integrability and Feynman graphs?

Weak coupling: interpretation of the terms in the expansion of the determinant as Feynman graphs

In the determinant representation of the octagon

$\mathbb{O}_\ell = \det(1 + \mathbf{R})$

Diagonal minors = $n \times (n + \ell)$ Basso-Dixon fishnets



In the fishnet limit (strong twists ~1/g in both channels and g small) the octagon is the generating function for the Basso-Dixon fishnets

Non-diagonal minors = fishnets with defects?

Some papers on the octagon:

The original papers: F. Coronado, arXiv:1811.00467, 1811.033282

The octagon as a determinant:
V. Petkova, I. Kostov, D. Serban. arXiv:1903.05038, arXiv:1905.11467

Non-planar correlators from octagons: T. Bargheer, F. Coronado, P. Vieira, 1909.04077.

Description in terms of a Modified Bessel kernel,

weak+strong coupling expansions,

Toda tau function in the null-square limit:

A. Belitsky, G. Korchemsky, 2003.01121, 2006.01831

Relation to Basso-Dixon fishnets:

F. Coronado, arXiv:1811.00467, 1811.033282, B. Basso, L. Dixon 1705.03545;

In the null square limit [A.Belitsky, G. Korchemsky 1907.13131]

- relation to the 6-gluon MHV amplitude and the BES kernel:

B. Basso, L. Dixon, G. Papathanasiou 1705.03545