6d (2,0) correlators and Quantum M-theory

Luis Fernando Alday

University of Oxford

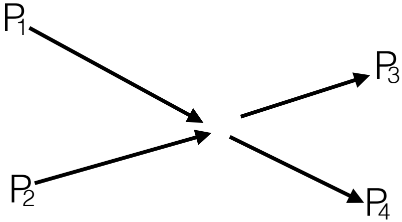
IGST - August 2020

Based on work with S. Chester and H. Raj [arXiv:2005.07175] and X. Zhou [arXiv:2006.06653,arXiv:2006.12505]

What will this talk be about?

Conformal Field Theories techniques to study Scattering Amplitudes in theories of Gravity, and in particular String and M-theory.

Target: 4pt (super) graviton amplitude in $M-$theory.



Mandelstand variables

$$= \mathcal{A}(\ell, s, t, u)$$

11D Planck length

Scattering amplitudes in M-theory

Perturbative M-theory: 11D Sugra plus higher derivative corrections

$$S_{eff}[g] = \frac{1}{2\kappa^2}\int d^{11}x\sqrt{g}\left(\mathcal{R} + \alpha\ell^6\mathcal{R}^4 + \beta\ell^{12}D^6\mathcal{R}^4 + \underbrace{\gamma\ell^{14}D^8\mathcal{R}^4 + \cdots}_{\text{unknown}}\right)$$

- Higher derivative terms carry powers of ℓ, the 11D Planck length.
- $D^2\mathcal{R}^4$ and $D^4\mathcal{R}^4$ can be shown to be absent. From $D^8\mathcal{R}^4$ nothing is known!

Scattering amplitudes in M-theory

11D 4pt graviton amplitude in a momentum expansion (in flat space!)

$$\mathcal{A}(\underset{\text{momenta}}{p_i},\underset{\text{polarisation vectors}}{\zeta_i}) = \ell^9 \hat{K}(p_i,\zeta_i)\left(\underset{SUGRA}{\frac{1}{s\,t\,u}} + \underset{\mathcal{R}^4}{\ell^6\alpha} + \underset{D^6\mathcal{R}^4}{\ell^{12}\beta\, s\,t\,u} + \cdots + \text{loops}\right)$$

- All polarisation dependence inside a universal factor $\hat{K}$.
- The coefficients of higher polynomials in s, t, u are not known.
- Loops are complicated non-analytic functions of s, t, u.

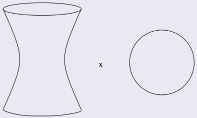
$$\mathcal{A}^{loop-sugra} = \ell^9 \times \Phi_{11\mathrm{D}}(s,t) = \ell^9 s^{5/2} t^{5/2} u^{-7/2} \log(\sqrt{u} - \sqrt{-s}) + \cdots$$

Let's extend this to curved space-time!

A CFT window into M-theory

AdS/CFT duality for M-theory

M-theory on $AdS_7 \times S^4$ $\Leftrightarrow$ 6d SCFT living in the boundary of AdS_7



- Maximally SUSY theory in $6d$.
- 6d $(2,0)$ theory
  with central charge c.

$L_{S^4} = 1/2 L_{AdS}$.

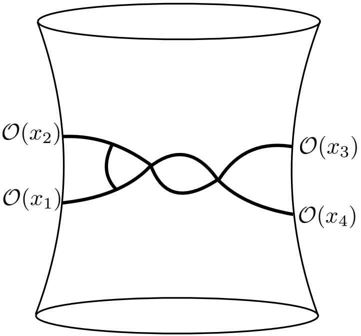
$$\left(\frac{L_{AdS}}{\ell}\right)^9 \sim c$$

$$\Downarrow$$

Momentum expansion (powers of ℓ) $\leftrightarrow$ $1/c$ expansion.

Dictionary

M-theory amplitude on $AdS_7 \times S^4$ $\leftrightarrow$ Correlator of local operators in the 6d $(2,0)$ theory



- The graviton on AdS corresponds to $\mathcal{O}_2$
- KK modes on S^4 corresponds to $\mathcal{O}_k$

$\downarrow$

1/2 BPS operators of dim. $2k$

We want $\langle \mathcal{O}_{k_1}(x_1)\mathcal{O}_{k_2}(x_2)\mathcal{O}_{k_3}(x_3)\mathcal{O}_{k_4}(x_4)\rangle$ in a $1/c$ expansion.

6d $(2,0)$ Correlators - Kinematics

- Maximally susy theory in 6d, with super-conformal algebra

$$OSp(8^*|4) \supset \underbrace{SO(6,2)}_{\text{conformal symmetry}} \oplus \underbrace{SO(5)}_{\text{R-symmetry}}$$

- $1/2-$BPS operators transform in the symmetric-traceless of $SO(5)$

$$\mathcal{O}_{I_1 \cdots I_k}(x) \quad \rightarrow \quad \mathcal{O}_k(x,y) = \mathcal{O}_{I_1 \cdots I_k}(x) y^{I_1} \cdots y^{I_k}, \quad y^2 = 0$$

- We consider 4pt-functions of these guys

$$\langle \mathcal{O}_k(x_1,t_1) \cdots \mathcal{O}_k(x_4,t_4) \rangle = \left(\frac{y_{12} y_{34}}{x_{12}^4 x_{34}^4} \right)^k \underbrace{\mathcal{G}(U,V,\sigma,\tau)}_{\text{Polynomial in } \sigma,\tau}$$

where $U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}, \sigma = \frac{y_{13} y_{24}}{y_{12} y_{32}}, \tau = \frac{y_{14} y_{23}}{y_{12} y_{34}}$.

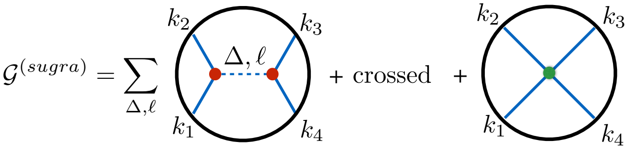
6d $(2,0)$ Correlators $1/c$ expansion

Leading non-trivial order (the supergravity approximation)

$$\mathcal{G}(U,V;\sigma,\tau) = \underbrace{\mathcal{G}^{(0)}(U,V;\sigma,\tau)}_{disconnected} + \boxed{\frac{1}{c}\mathcal{G}^{(sugra)}(U,V;\sigma,\tau)} + \cdots$$

Standard recipe

- Perform a KK reduction of the 11D Sugra effective action on S^4.
- Read off cubic and quartic vertices from the AdS_7 effective action.
- Write down & compute all exchange and contact Witten diagrams.



Not possible in practise! Rather, use consistency conditions.

The right language: Mellin space

$\mathcal{G}(U,V;\sigma,\tau) \to \mathcal{M}(s,t,u;\sigma,\tau) \equiv \mathcal{M}(s,t;\sigma,\tau)$, with $s+t+u=8k$.

$$\boxed{\mathcal{G}(U,V;\sigma,\tau) = \int_{-i\infty}^{i\infty} ds dt U^s V^t \underbrace{\Gamma_{\{k_i\}}(s,t,u)}_{\text{A prefactor}} \quad \underbrace{\mathcal{M}(s,t;\sigma,\tau)}_{M-\text{theory amplitude in } AdS_7\times S^4}}$$

$\mathcal{M}(s,t;\sigma,\tau)$ is a meromorphic function with very nice properties!

1. Crossing symmetry. ✓
2. Exchanged operators lead to simple poles:

$$\mathcal{M}_{exch}(s,t) = \sum_{m=0}^{\infty} \lambda^2_{\Delta,\ell} \frac{Q_{\ell,m}(u,t)}{s-(\Delta-\ell)-2m} + \text{regular} \quad \checkmark$$

3. Superconformal Ward identities:

$$(\text{Shift operator}) \circ \mathcal{M}(s,t;\sigma,\tau) = 0 \quad \checkmark$$

Rather than the effective action use 1+2+3! [Rastelli-Zhou]

MRV amplitudes

With this method we can produce results case by case, but this is still very inefficient and its hard to see any structure...we need a new idea!

Maximally R-symmetry violating amplitudes

- Each operator depends on a 6d point x and a null 5d vector y:

$$\langle \mathcal{O}_k(x_1, y_1)\mathcal{O}_k(x_2, y_2)\mathcal{O}_k(x_3, y_3)\mathcal{O}_k(x_4, y_4)\rangle$$

Point in $\mathbb{R}^{1,5}$ Null 5d vector

- Choose a configuration where y_1, y_3 are aligned $\to \sigma = 0, \tau = 1$.

$$\boxed{MRV(s,t) = \mathcal{M}(s,t;0,1)}$$

This suppresses all sugra exchanges in the u-channel (as $y_1 \cdot y_3 = 0$) and the amplitude simplifies drastically!

MRV amplitudes

Stress tensor multiplet four-point function in $AdS_7 \times S^4$

$$\mathcal{M}_{2222}(s,t;\sigma,\tau) = \frac{P(s,t;\sigma,\tau)}{(s-4)(s-6)(t-4)(t-6)(u-4)(u-6)}$$

$$\begin{aligned}
&-5\,160\,960 + 2\,512\,896\, s - 445\,440\, s^2 + 34\,176\, s^3 - 960\, s^4 + 4\,386\,816\, t - 1\,967\,872\, s\, t + 315\,008\, s^2\, t - \\
&21\,392\, s^3\, t + 520\, s^4\, t - 1\,507\,072\, t^2 + 609\,152\, s\, t^2 - 84\,576\, s^2\, t^2 + 4764\, s^3\, t^2 - 90\, s^4\, t^2 + \\
&268\,800\, t^3 - 95\,264\, s\, t^3 + 10\,792\, s^2\, t^3 - 448\, s^3\, t^3 + 5\, s^4\, t^3 - 26\,320\, t^4 + 7892\, s\, t^4 - 654\, s^2\, t^4 + \\
&15\, s^3\, t^4 + 1344\, t^5 - 324\, s\, t^5 + 15\, s^2\, t^5 - 28\, t^6 + 5\, s\, t^6 + 3\,022\,848\, \sigma - 2\,629\,632\, s\, \sigma + 655\,872\, s^2\, \sigma - \\
&61\,440\, s^3\, \sigma + 1920\, s^4\, \sigma - 2\,055\,168\, t\, \sigma + 1\,797\,632\, s\, t\, \sigma - 429\,760\, s^2\, t\, \sigma + 37\,120\, s^3\, t\, \sigma - 1040\, s^4\, t\, \sigma + \\
&521\,984\, t^2\, \sigma - 461\,760\, s\, t^2\, \sigma + 103\,760\, s^2\, t^2\, \sigma - 7840\, s^3\, t^2\, \sigma + 180\, s^4\, t^2\, \sigma - 61\,568\, t^3\, \sigma + \\
&55\,760\, s\, t^3\, \sigma - 11\,440\, s^2\, t^3\, \sigma + 680\, s^3\, t^3\, \sigma - 10\, s^4\, t^3\, \sigma + 3328\, t^4\, \sigma - 3168\, s\, t^4\, \sigma + 568\, s^2\, t^4\, \sigma - \\
&20\, s^3\, t^4\, \sigma - 64\, t^5\, \sigma + 68\, s\, t^5\, \sigma - 10\, s^2\, t^5\, \sigma - 1\,916\,928\, \sigma^2 + 1\,222\,656\, s\, \sigma^2 - 279\,552\, s^2\, \sigma^2 + \\
&27\,264\, s^3\, \sigma^2 - 960\, s^4\, \sigma^2 + 1\,222\,656\, t\, \sigma^2 - 762\,112\, s\, t\, \sigma^2 + 168\,704\, s^2\, t\, \sigma^2 - 15\,728\, s^3\, t\, \sigma^2 + \\
&520\, s^4\, t\, \sigma^2 - 279\,552\, t^2\, \sigma^2 + 168\,704\, s\, t^2\, \sigma^2 - 35\,568\, s^2\, t^2\, \sigma^2 + 3076\, s^3\, t^2\, \sigma^2 - 90\, s^4\, t^2\, \sigma^2 + \\
&27\,264\, t^3\, \sigma^2 - 15\,728\, s\, t^3\, \sigma^2 + 3076\, s^2\, t^3\, \sigma^2 - 232\, s^3\, t^3\, \sigma^2 + 5\, s^4\, t^3\, \sigma^2 - 960\, t^4\, \sigma^2 + 520\, s\, t^4\, \sigma^2 - \\
&90\, s^2\, t^4\, \sigma^2 + 5\, s^3\, t^4\, \sigma^2 + 2\,580\,480\, \tau - 918\,528\, s\, \tau + 57\,088\, s^2\, \tau + 12\,416\, s^3\, \tau - 1792\, s^4\, \tau + 64\, s^5\, \tau - \\
&918\,528\, t\, \tau - 108\,544\, s\, t\, \tau + 171\,200\, s^2\, t\, \tau - 34\,256\, s^3\, t\, \tau + 2592\, s^4\, t\, \tau - 68\, s^5\, t\, \tau + 57\,088\, t^2\, \tau + \\
&171\,200\, s\, t^2\, \tau - 74\,528\, s^2\, t^2\, \tau + 10\,416\, s^3\, t^2\, \tau - 572\, s^4\, t^2\, \tau + 10\, s^5\, t^2\, \tau + 12\,416\, t^3\, \tau - 34\,256\, s\, t^3\, \tau + \\
&10\,416\, s^2\, t^3\, \tau - 1008\, s^3\, t^3\, \tau + 30\, s^4\, t^3\, \tau - 1792\, t^4\, \tau + 2592\, s\, t^4\, \tau - 572\, s^2\, t^4\, \tau + 30\, s^3\, t^4\, \tau + \\
&64\, t^5\, \tau - 68\, s\, t^5\, \tau + 10\, s^2\, t^5\, \tau + 3\,022\,848\, \sigma\tau - 2\,055\,168\, s\, \sigma\tau + 521\,984\, s^2\, \sigma\tau - 61\,568\, s^3\, \sigma\tau + \\
&3328\, s^4\, \sigma\tau - 64\, s^5\, \sigma\tau - 2\,629\,632\, t\, \sigma\tau + 1\,797\,632\, s\, t\, \sigma\tau - 461\,760\, s^2\, t\, \sigma\tau + 55\,760\, s^3\, t\, \sigma\tau - \\
&3168\, s^4\, t\, \sigma\tau + 68\, s^5\, t\, \sigma\tau + 655\,872\, t^2\, \sigma\tau - 429\,760\, s\, t^2\, \sigma\tau + 103\,760\, s^2\, t^2\, \sigma\tau - 11\,440\, s^3\, t^2\, \sigma\tau + \\
&568\, s^4\, t^2\, \sigma\tau - 10\, s^5\, t^2\, \sigma\tau - 61\,440\, t^3\, \sigma\tau + 37\,120\, s\, t^3\, \sigma\tau - 7840\, s^2\, t^3\, \sigma\tau + 680\, s^3\, t^3\, \sigma\tau - \\
&20\, s^4\, t^3\, \sigma\tau + 1920\, t^4\, \sigma\tau - 1040\, s\, t^4\, \sigma\tau + 180\, s^2\, t^4\, \sigma\tau - 10\, s^3\, t^4\, \sigma\tau - 5\,160\,960\, \tau^2 + 4\,386\,816\, s\, \tau^2 - \\
&1\,507\,072\, s^2\, \tau^2 + 268\,800\, s^3\, \tau^2 - 26\,320\, s^4\, \tau^2 + 1344\, s^5\, \tau^2 - 28\, s^6\, \tau^2 + 2\,512\,896\, t\, \tau^2 - 1\,967\,872\, s\, t\, \tau^2 + \\
&609\,152\, s^2\, t\, \tau^2 - 95\,264\, s^3\, t\, \tau^2 + 7892\, s^4\, t\, \tau^2 - 324\, s^5\, t\, \tau^2 + 5\, s^6\, t\, \tau^2 - 445\,440\, t^2\, \tau^2 + 315\,008\, s\, t^2\, \tau^2 - \\
&84\,576\, s^2\, t^2\, \tau^2 + 10\,792\, s^3\, t^2\, \tau^2 - 654\, s^4\, t^2\, \tau^2 + 15\, s^5\, t^2\, \tau^2 + 34\,176\, t^3\, \tau^2 - 21\,392\, s\, t^3\, \tau^2 + \\
&4764\, s^2\, t^3\, \tau^2 - 448\, s^3\, t^3\, \tau^2 + 15\, s^4\, t^3\, \tau^2 - 960\, t^4\, \tau^2 + 520\, s\, t^4\, \tau^2 - 90\, s^2\, t^4\, \tau^2 + 5\, s^3\, t^4\, \tau^2
\end{aligned}$$

MRV amplitudes

Stress tensor multiplet four-point function in $AdS_7 \times S^4$

$$\mathcal{M}_{2222}(s,t;\sigma,\tau) = \frac{P(s,t;\sigma,\tau)}{(s-4)(s-6)(t-4)(t-6)(u-4)(u-6)}$$

$$\begin{aligned}
&-5\,160\,960 + 2\,512\,896\, s - 445\,440\, s^2 + 34\,176\, s^3 - 960\, s^4 + 4\,386\,816\, t - 1\,967\,872\, s\, t + 315\,008\, s^2\, t - \\
&21\,392\, s^3\, t + 520\, s^4\, t - 1\,507\,072\, t^2 + 609\,152\, s\, t^2 - 84\,576\, s^2\, t^2 + 4764\, s^3\, t^2 - 90\, s^4\, t^2 + \\
&268\,800\, t^3 - 95\,264\, s\, t^3 + 10\,792\, s^2\, t^3 - 448\, s^3\, t^3 + 5\, s^4\, t^3 - 26\,320\, t^4 + 7892\, s\, t^4 - 654\, s^2\, t^4 + \\
&15\, s^3\, t^4 + 1344\, t^5 - 324\, s\, t^5 + 15\, s^2\, t^5 - 28\, t^6 + 5\, s\, t^6 + 3\,022\,848\, \sigma - 2\,629\,632\, s\, \sigma + 655\,872\, s^2\, \sigma - \\
&61\,440\, s^3\, \sigma + 1920\, s^4\, \sigma - 2\,055\,168\, t\, \sigma + 1\,797\,632\, s\, t\, \sigma - 429\,760\, s^2\, t\, \sigma + 37\,120\, s^3\, t\, \sigma - 1040\, s^4\, t\, \sigma + \\
&521\,984\, t^2\, \sigma - 461\,760\, s\, t^2\, \sigma + 103\,760\, s^2\, t^2\, \sigma - 7840\, s^3\, t^2\, \sigma + 180\, s^4\, t^2\, \sigma - 61\,568\, t^3\, \sigma + \\
&55\,760\, s\, t^3\, \sigma - 11\,440\, s^2\, t^3\, \sigma + 680\, s^3\, t^3\, \sigma - 10\, s^4\, t^3\, \sigma + 3328\, t^4\, \sigma - 3168\, s\, t^4\, \sigma + 568\, s^2\, t^4\, \sigma - \\
&20\, s^3\, t^4\, \sigma - 64\, t^5\, \sigma + 68\, s\, t^5\, \sigma - 10\, s^2\, t^5\, \sigma - 1\,916\,928\, \sigma^2 + 1\,222\,656\, s\, \sigma^2 - 279\,552\, s^2\, \sigma^2 + \\
&27\,264\, s^3\, \sigma^2 - 960\, s^4\, \sigma^2 + 1\,222\,656\, t\, \sigma^2 - 762\,112\, s\, t\, \sigma^2 + 168\,704\, s^2\, t\, \sigma^2 - 15\,728\, s^3\, t\, \sigma^2 + \\
&520\, s^4\, t\, \sigma^2 - 279\,552\, t^2\, \sigma^2 + 168\,704\, s\, t^2\, \sigma^2 - 35\,568\, s^2\, t^2\, \sigma^2 + 3076\, s^3\, t^2\, \sigma^2 - 90\, s^4\, t^2\, \sigma^2 + \\
&27\,264\, t^3\, \sigma^2 - 15\,728\, s\, t^3\, \sigma^2 + 3076\, s^2\, t^3\, \sigma^2 - 232\, s^3\, t^3\, \sigma^2 + 5\, s^4\, t^3\, \sigma^2 - 960\, t^4\, \sigma^2 + 520\, s\, t^4\, \sigma^2 - \\
&90\, s^2\, t^4\, \sigma^2 + 5\, s^3\, t^4\, \sigma^2 + 2\,580\,480\, \tau - 918\,528\, s\, \tau + 57\,088\, s^2\, \tau + 12\,416\, s^3\, \tau - 1792\, s^4\, \tau + 64\, s^5\, \tau - \\
&918\,528\, t\, \tau - 108\,544\, s\, t\, \tau + 171\,200\, s^2\, t\, \tau - 34\,256\, s^3\, t\, \tau + 2592\, s^4\, t\, \tau - 68\, s^5\, t\, \tau + 57\,088\, t^2\, \tau + \\
&171\,200\, s\, t^2\, \tau - 74\,528\, s^2\, t^2\, \tau + 10\,416\, s^3\, t^2\, \tau - 572\, s^4\, t^2\, \tau + 10\, s^5\, t^2\, \tau + 12\,416\, t^3\, \tau - 34\,256\, s\, t^3\, \tau + \\
&10\,416\, s^2\, t^3\, \tau - 1008\, s^3\, t^3\, \tau + 30\, s^4\, t^3\, \tau - 1792\, t^4\, \tau + 2592\, s\, t^4\, \tau - 572\, s^2\, t^4\, \tau + 30\, s^3\, t^4\, \tau + \\
&64\, t^5\, \tau - 68\, s\, t^5\, \tau + 10\, s^2\, t^5\, \tau + 3\,022\,848\, \sigma\, \tau - 2\,055\,168\, s\, \sigma\, \tau + 521\,984\, s^2\, \sigma\, \tau - 61\,568\, s^3\, \sigma\, \tau + \\
&3328\, s^4\, \sigma\, \tau - 64\, s^5\, \sigma\, \tau - 2\,629\,632\, t\, \sigma\, \tau + 1\,797\,632\, s\, t\, \sigma\, \tau - 461\,760\, s^2\, t\, \sigma\, \tau + 55\,760\, s^3\, t\, \sigma\, \tau - \\
&3168\, s^4\, t\, \sigma\, \tau + 68\, s^5\, t\, \sigma\, \tau + 655\,872\, t^2\, \sigma\, \tau - 429\,760\, s\, t^2\, \sigma\, \tau + 103\,760\, s^2\, t^2\, \sigma\, \tau - 11\,440\, s^3\, t^2\, \sigma\, \tau + \\
&568\, s^4\, t^2\, \sigma\, \tau - 10\, s^5\, t^2\, \sigma\, \tau - 61\,440\, t^3\, \sigma\, \tau + 37\,120\, s\, t^3\, \sigma\, \tau - 7840\, s^2\, t^3\, \sigma\, \tau + 680\, s^3\, t^3\, \sigma\, \tau - \\
&20\, s^4\, t^3\, \sigma\, \tau + 1920\, t^4\, \sigma\, \tau - 1040\, s\, t^4\, \sigma\, \tau + 180\, s^2\, t^4\, \sigma\, \tau - 10\, s^3\, t^4\, \sigma\, \tau - 5\,160\,960\, \tau^2 + 4\,386\,816\, s\, \tau^2 - \\
&1\,507\,072\, s^2\, \tau^2 + 268\,800\, s^3\, \tau^2 - 26\,320\, s^4\, \tau^2 + 1344\, s^5\, \tau^2 - 28\, s^6\, \tau^2 + 2\,512\,896\, t\, \tau^2 - 1\,967\,872\, s\, t\, \tau^2 + \\
&609\,152\, s^2\, t\, \tau^2 - 95\,264\, s^3\, t\, \tau^2 + 7892\, s^4\, t\, \tau^2 - 324\, s^5\, t\, \tau^2 + 5\, s^6\, t\, \tau^2 - 445\,440\, t^2\, \tau^2 + 315\,008\, s\, t^2\, \tau^2 - \\
&84\,576\, s^2\, t^2\, \tau^2 + 10\,792\, s^3\, t^2\, \tau^2 - 654\, s^4\, t^2\, \tau^2 + 15\, s^5\, t^2\, \tau^2 + 34\,176\, t^3\, \tau^2 - 21\,392\, s\, t^3\, \tau^2 + \\
&4764\, s^2\, t^3\, \tau^2 - 448\, s^3\, t^3\, \tau^2 + 15\, s^4\, t^3\, \tau^2 - 960\, t^4\, \tau^2 + 520\, s\, t^4\, \tau^2 - 90\, s^2\, t^4\, \tau^2 + 5\, s^3\, t^4\, \tau^2
\end{aligned}$$

Much simpler in the MRV limit!

$$MRV(s,t) = (u-8)(u-10)\left(\frac{1}{s-4} + \frac{1}{4(s-6)} + \frac{1}{t-4} + \frac{1}{4(t-6)}\right)$$

MRV amplitudes

$$MRV(s,t) = \underbrace{(u-8)(u-10)}_{\text{double zero}} \left(\underbrace{\frac{1}{s-4} + \frac{1}{4(s-6)} + \frac{1}{t-4} + \frac{1}{4(t-6)}}_{\text{no poles in the } u\text{-channel}} \right)$$

- Highly non-trivial at the level of exchange Witten diagrams.
- All cubic couplings are fixed in terms of the scalar couplings.
- We can write down the general MRV amplitude for all k_1, k_2, k_3, k_4!
- $R-$symmetry can be used to restore the full σ, τ dependence!

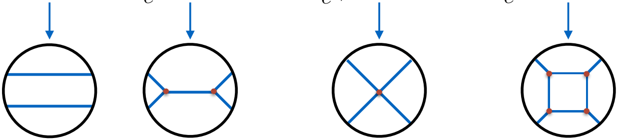
$$\mathcal{M}^{(sugra)}(s,t;\sigma,\tau) = \sum_{i,j} \sigma^i \tau^j \left(\sum_{h_{min}}^{h_{max}} \frac{R_h^{i,j}(t,u)}{s-2h} + \text{crossed} \right)$$

- Compact and explicit expression for all sugra amplitudes!

Higher orders

- Let's now consider higher $1/c$ terms...

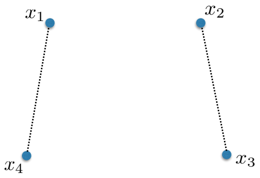
$$\mathcal{G}(U,V) = \mathcal{G}^{(0)}(U,V) + \frac{1}{c}\mathcal{G}^{(sugra)}(U,V) + \frac{1}{c^{5/3}}\mathcal{G}^{(\mathcal{R}^4)}(U,V) + \frac{1}{c^2}\mathcal{G}^{(loop-sugra)}(U,V) + \cdots$$



- The next order corresponds to the quartic vertex $\mathcal{R}^4$.
- In Mellin space the solution is simply a polynomial.
- Fixed again by consistency conditions.

What about loops? They actually follow from Trees!

Fact: As operators become null separated (only possible in Minkowski space-time) the correlator develops singularities



$$\longrightarrow \quad \mathcal{G}(U,V) \sim \frac{h(U)}{V^{\alpha}}$$

Unitarity method in *AdS*

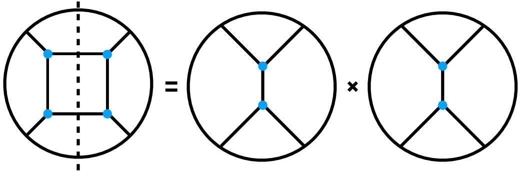
1. At a given order in $1/c$ these singularities follow from CFT-data at previous orders.
2. The whole correlator can be reconstructed from this singularities!

[L.F.A; Caron-Huot]

Tree-level correlators $\rightarrow$ loop correlators!

Loops from trees

- Each solution is the 'square' of the solutions at previous order!



- In Mellin space, meromorphic function with simultaneous-poles:

Complicated but explicit :)

$$\mathcal{M}^{(loop-sugra)}(s,t) = \sum_{m,n} \frac{c_{mn}}{(s-2m)(t-2n)} + \text{crossed}$$

- Quantum $M-$theory amplitude on AdS!
- The first quantum correction for a non-Lagrangian theory!

Flat-space limit

- All flat space results extended to *AdS*!

$$\mathcal{M}(s,t;\sigma,\tau)=\frac{1}{c}\mathcal{M}^{(sugra)}(s,t;\sigma,\tau)+\frac{1}{c^{5/3}}\mathcal{M}^{(\mathcal{R}^4)}(s,t;\sigma,\tau)+\frac{1}{c^2}\mathcal{M}^{(loop-sugra)}(s,t;\sigma,\tau)+\cdots$$

- From the amplitude in *AdS* we can recover the amplitude in flat-space (take L_{AdS} large, rescaling s,t,u accordingly)

$$\lim_{s,t\to\infty}\mathcal{M}(s,t;\sigma,\tau)=\ell^9\underbrace{(t\,u+t\,s\,\sigma+s\,u\,\tau)^2}_{\text{prefactor } \hat{K} \text{ for } p_i\cdot\zeta_i=0}\left(\underbrace{\frac{1}{s\,t\,u}+\ell^6\alpha}_{\text{tree-level results in flat space}}+\underbrace{\ell^9\Phi_{11\mathrm{D}}(s,t)}_{\text{Precisely the expected one-loop result!!}}+\cdots\right)$$

Note: We recover precisely the 11D result from a 6d CFT computation!

Conclusions

- We have developed powerful tools to compute systematically String and M-theory amplitudes in curved space-time.
- We computed all holographic correlators, dual to AdS tree amplitudes, for any maximally susy background! (A milestone in the field of holographic correlators)
- We computed $1/c^2$ correlators in the 6d $(2,0)$ theory (first loop computation in a non-lagrangian theory!)
- We can obtain a wealth of CFT-data about this theory.

$$\Delta_4 = 12 - \frac{288}{91}\frac{1}{c} - 18.91\frac{1}{c^2} + \cdots$$

For the near future

- The 6d $(2,0)$ theory gives a non-perturbative definition of $M-$theory. Use the bootstrap to answer questions about it!
- Target question: is $D^8\mathcal{R}^4$ present or not? just 'measure' Δ_4!