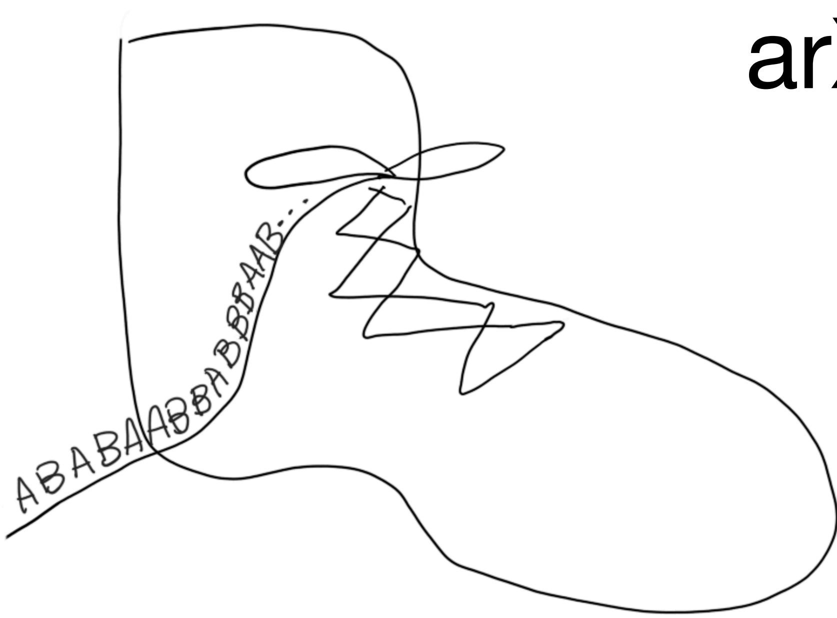
Bootstrapping Matrices

Henry Lin (Princeton)

arXiv:2002.08387



IGST Conference 2020

Bootstrap

- Inequalities are (almost) as good as equalities when we have many of them

- Bootstrap

  1. Restricted set of “data”

  2. Some positive norm

  3. Another relation (e.g. crossing) which does not obviously satisfy (2)

see also Anderson Kruczenski 2017
Jevicki Rodrigues 1983

(Multi-)Matrix Model

only single traces

$$Z = \lim_{N\to\infty} \int dA\, dB\, e^{-N\,\mathrm{Tr}\,V(A,B)}$$

$$\langle \mathrm{Tr}\,\mathcal{O}(A,B)\rangle = \lim_{N\to\infty} \frac{1}{Z} \int dA\, dB\, e^{-N\,\mathrm{Tr}\,V(A,B)}\, \mathrm{Tr}\,\mathcal{O}(A,B)$$

0) Does it exist?
1) Determine values as a function of couplings.

Positivity constraints

Let ϕ be an arbitrary superposition of matrices.

$$\left\langle \operatorname{tr} \phi^\dagger \phi \right\rangle \geq 0$$

For single matrix model,

$$\phi = \sum \alpha_i A^i \quad \Rightarrow \quad \mathcal{M} \succeq 0$$

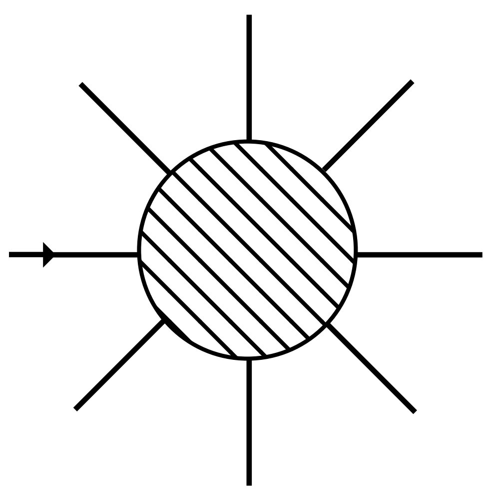
$$\mathcal{M}_{ij} = \langle \operatorname{tr} A^{i+j} \rangle$$

For multi-matrix models, take all possible 'strings,' e.g.,

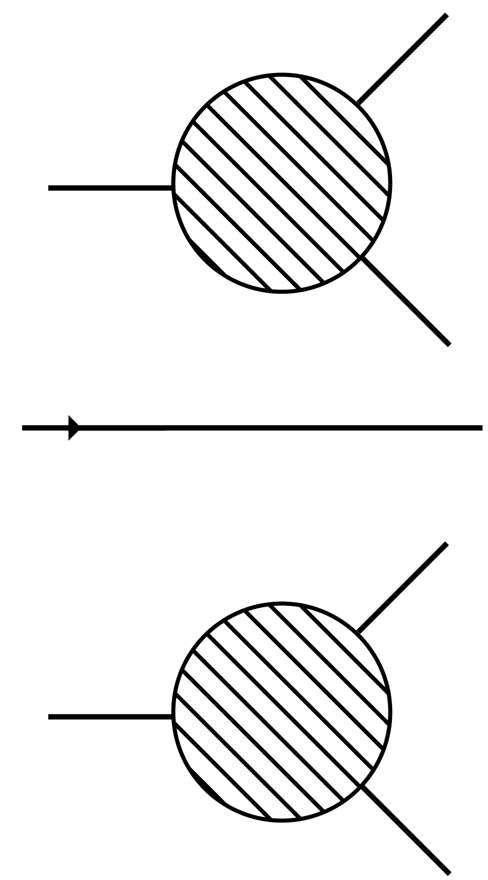
$$\phi = A + B + AB + \cdots + AB^2 ABAB + \cdots$$

Loop equations

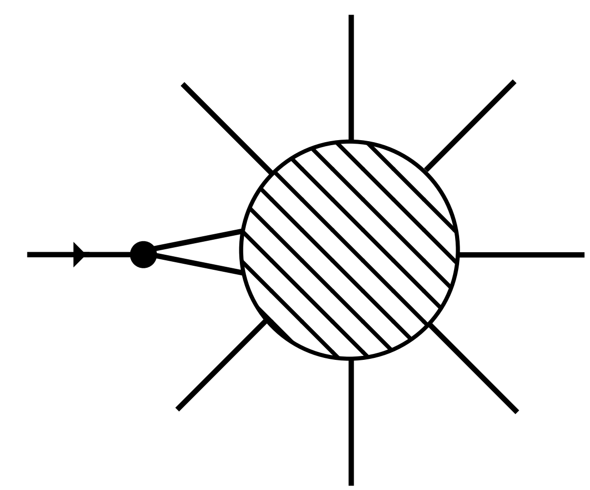
$$0 = \int_{A,B} \frac{\partial}{\partial A_{ij}} \left[\mathcal{O}(A,B)_{ij} e^{-S(A,B)} \right] = \int_{A,B} \frac{\partial}{\partial B_{ij}} \left[\mathcal{O}'(A,B)_{ij} e^{-S(A,B)} \right]$$



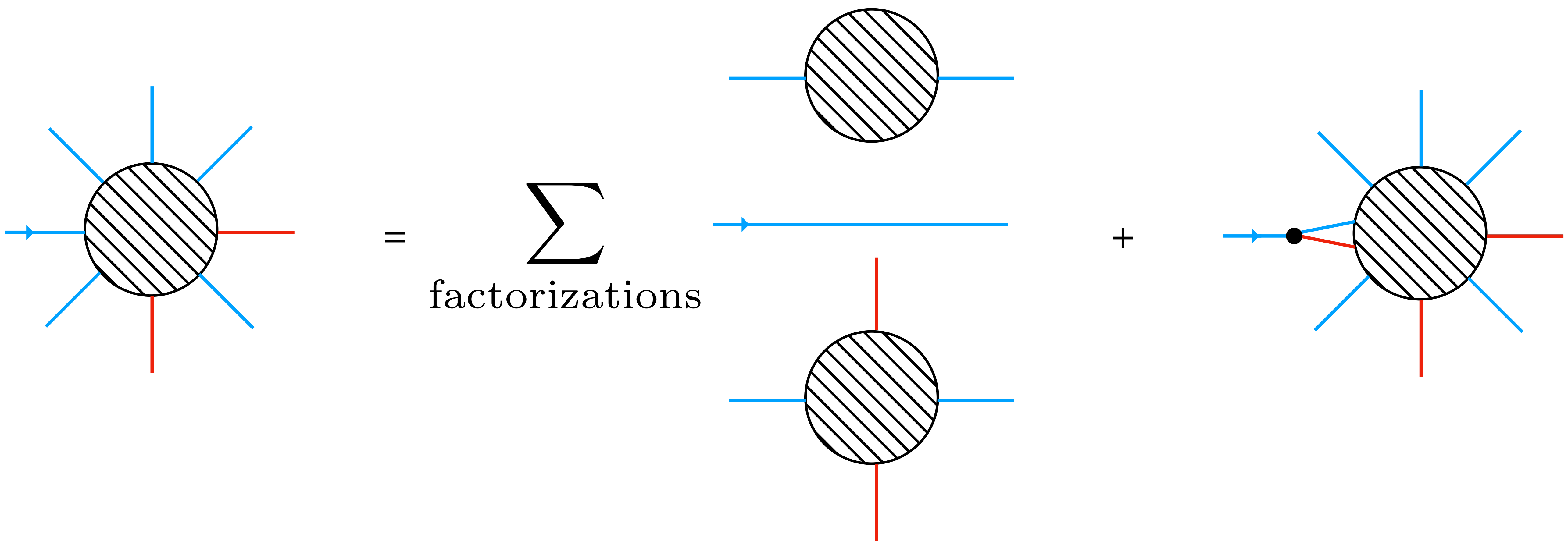
$$= \sum_{\text{factorizations}}$$



$$+$$



Loop equations



Search space

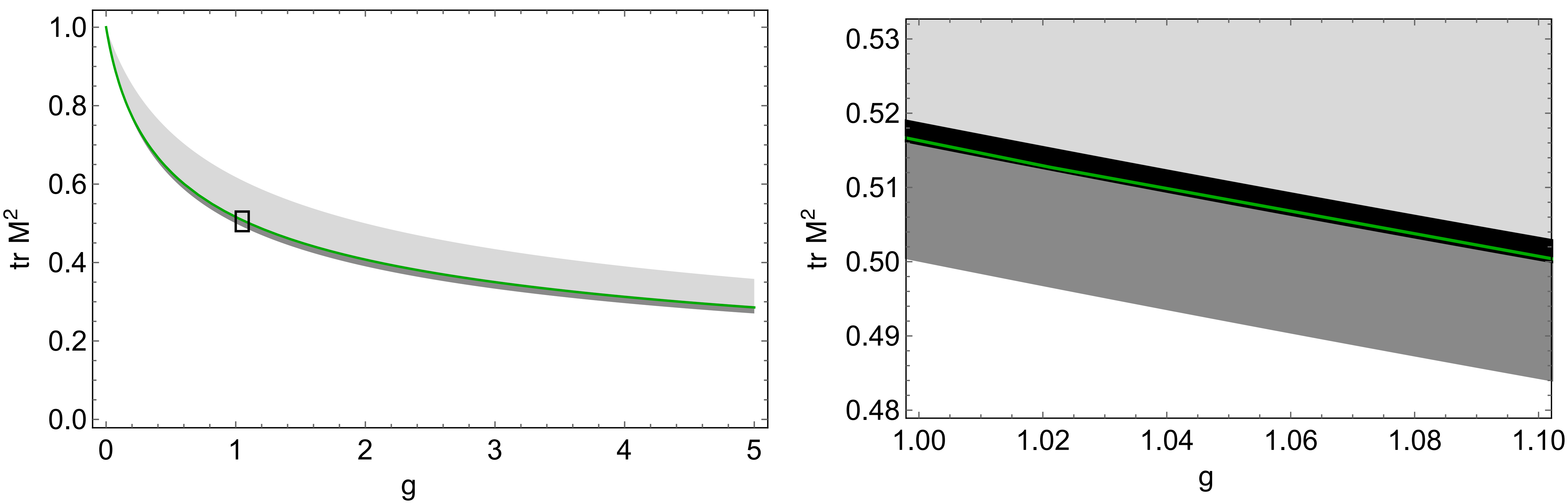
Loop equations relate low-pt functions to higher-pt functions. “Initial conditions”

search space *S* = min set of correlators such that the loop equations fully determine the rest

- 1-matrix model: potential of degree *D*, dim *S* = *D* - 2
- *m*-matrix model:

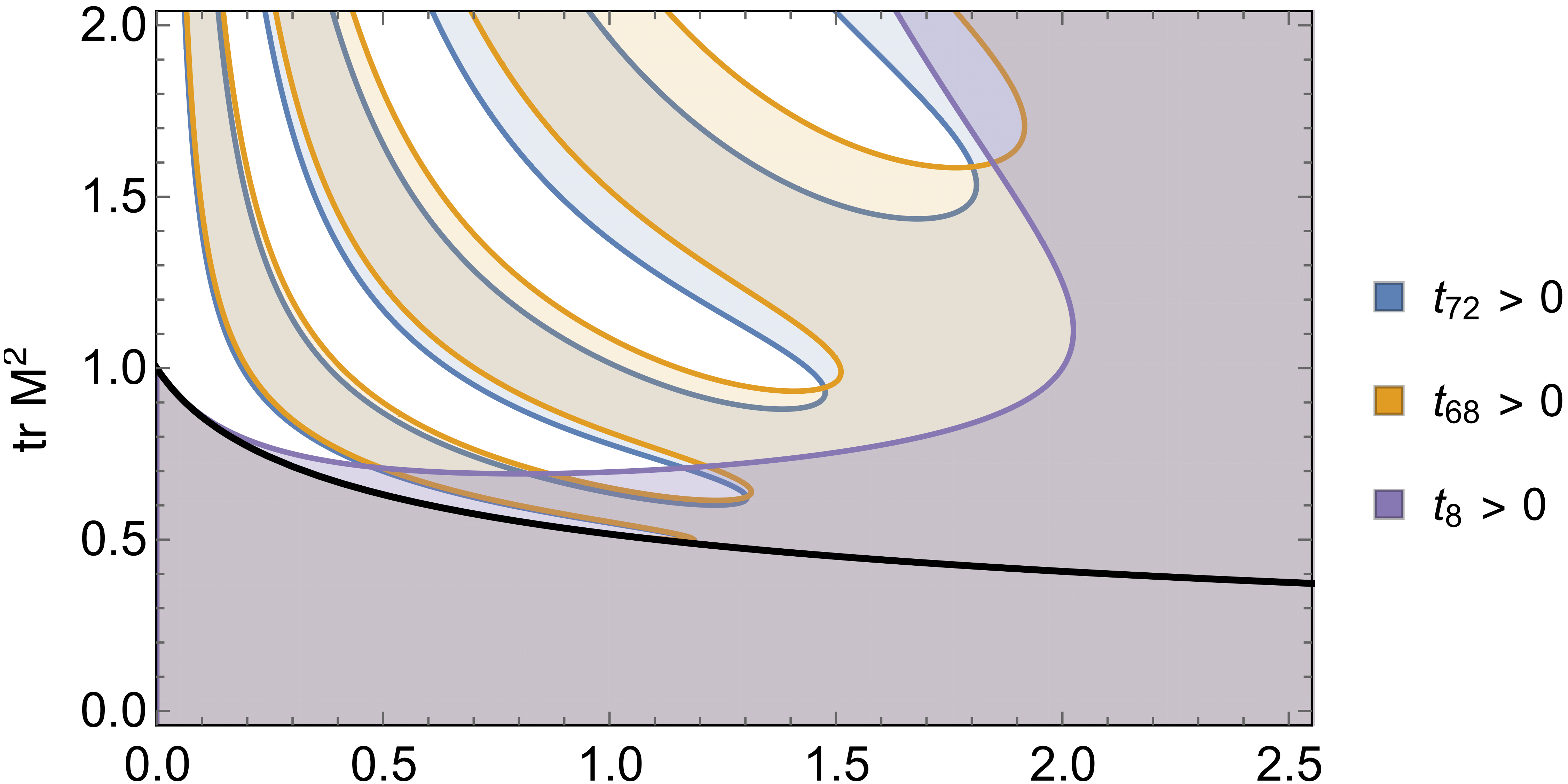
$$k\,m^k/k \sim m^{k+D}/(k+D) \quad \Rightarrow k \sim m^D$$

1-matrix

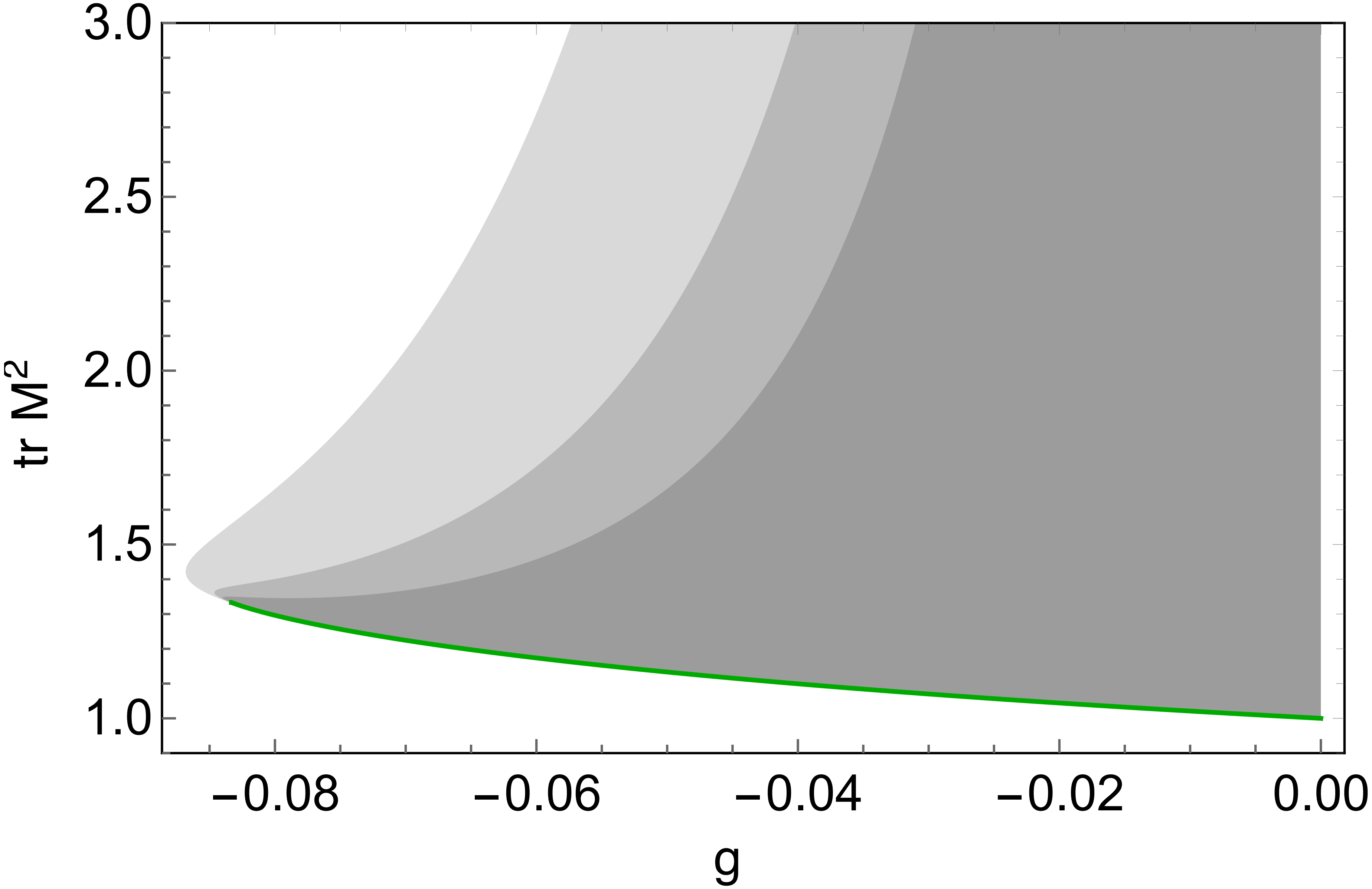


$$V = \frac{1}{2}M^2 + \frac{g}{4}M^4$$

1-matrix

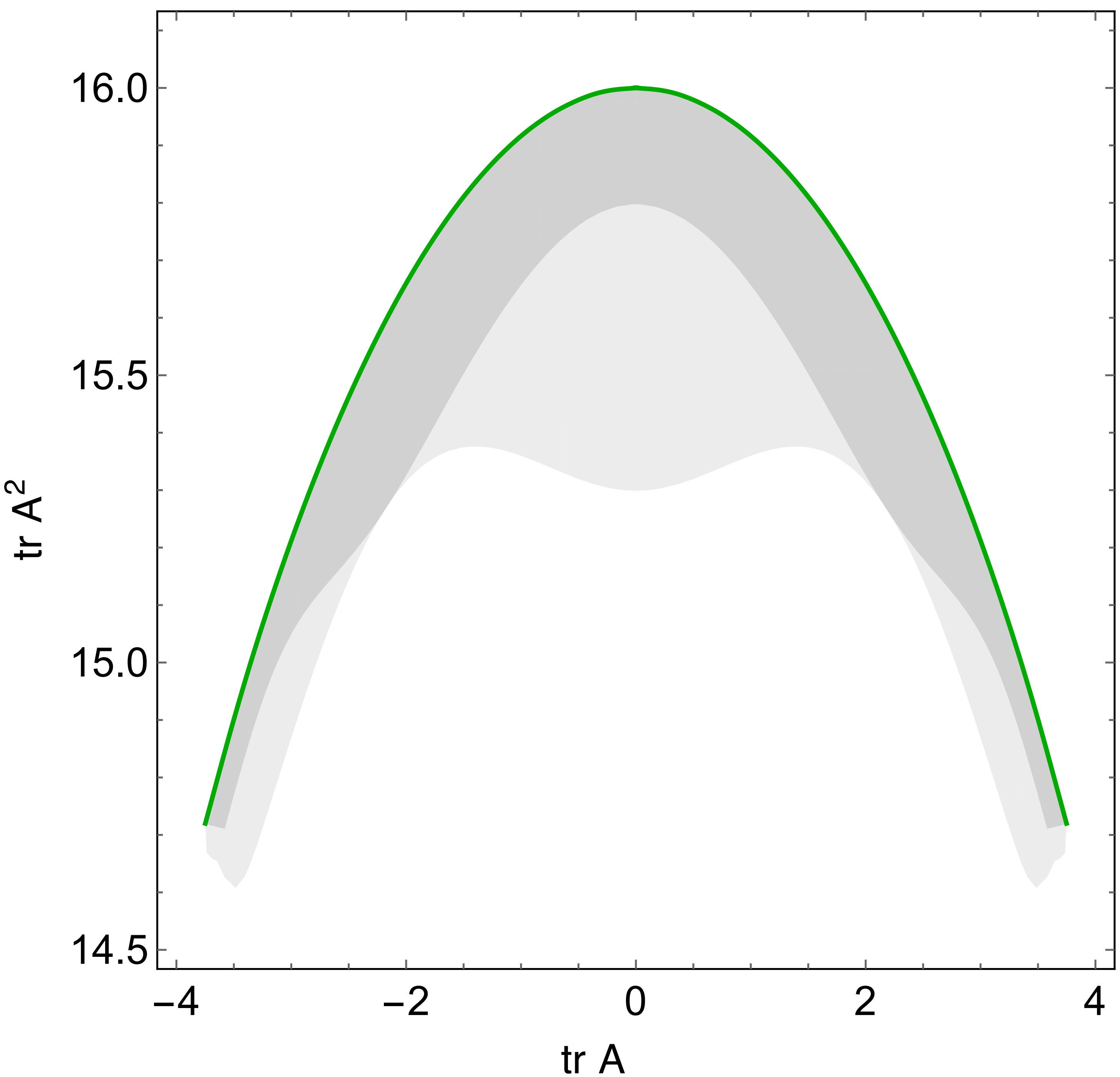


Peninsula

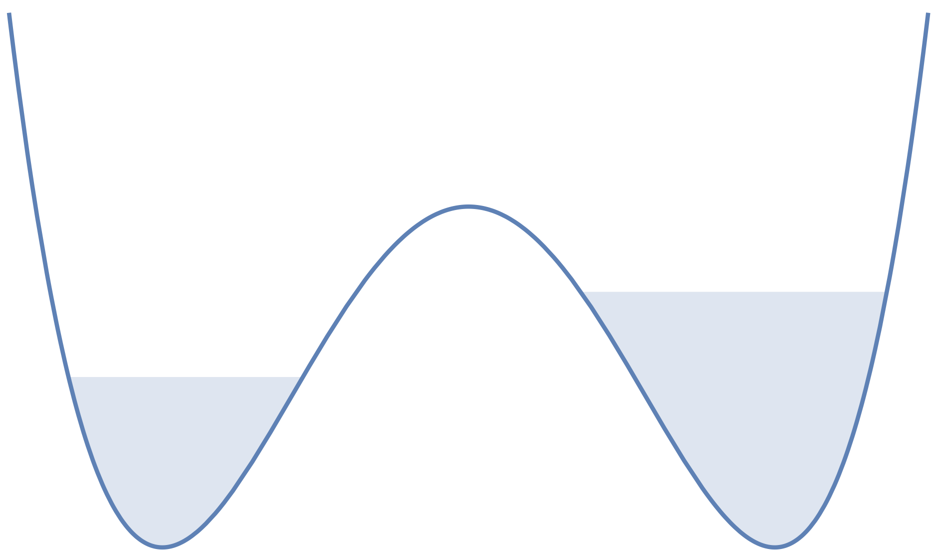


$$V = \frac{1}{2}M^2 + \frac{g}{4}M^4$$

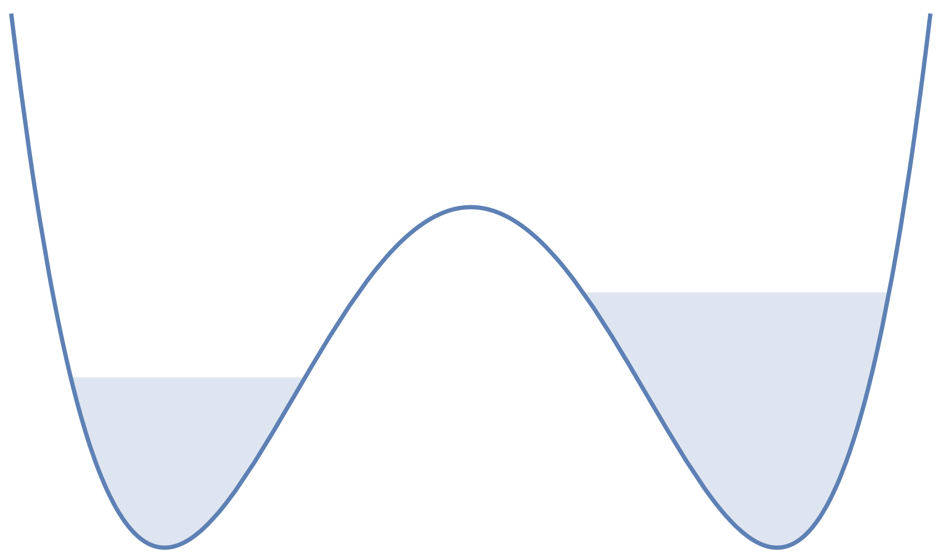
2-cut model



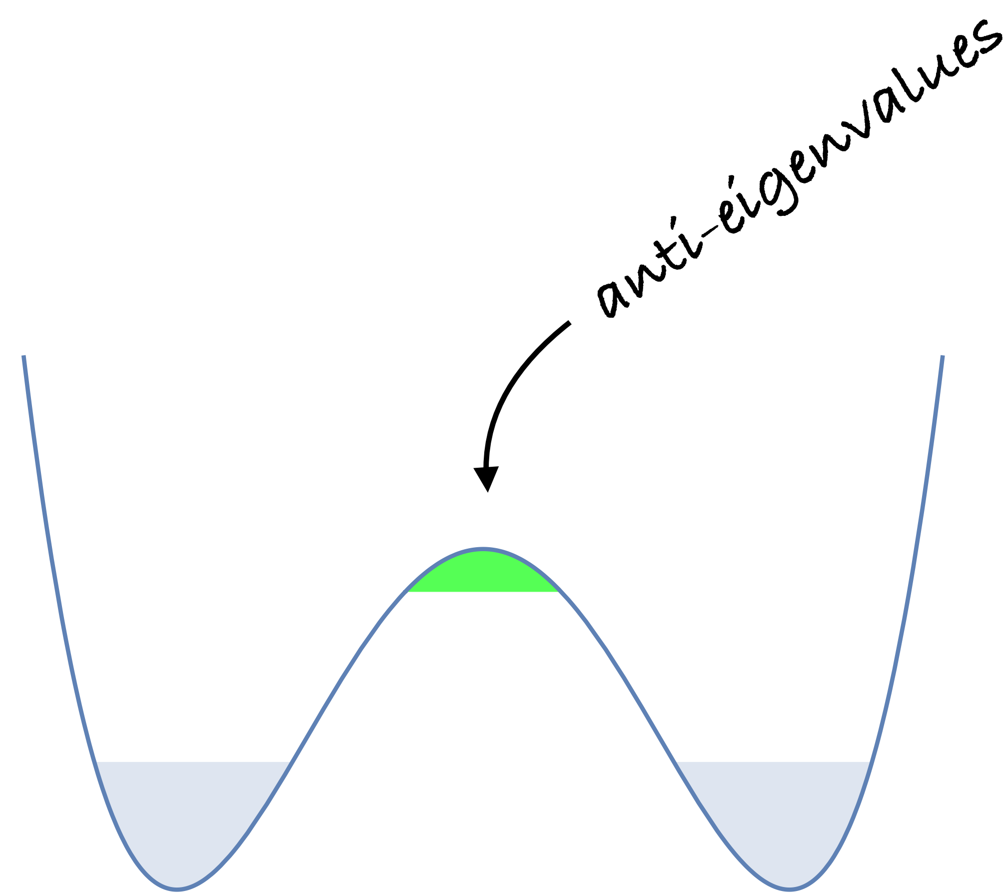
$$V = -\frac{1}{2}M^2 + \frac{g}{4}M^4$$



[aside] filling fractions



dim S ≥ # filling fractions



dim S = # unphysical filling fractions

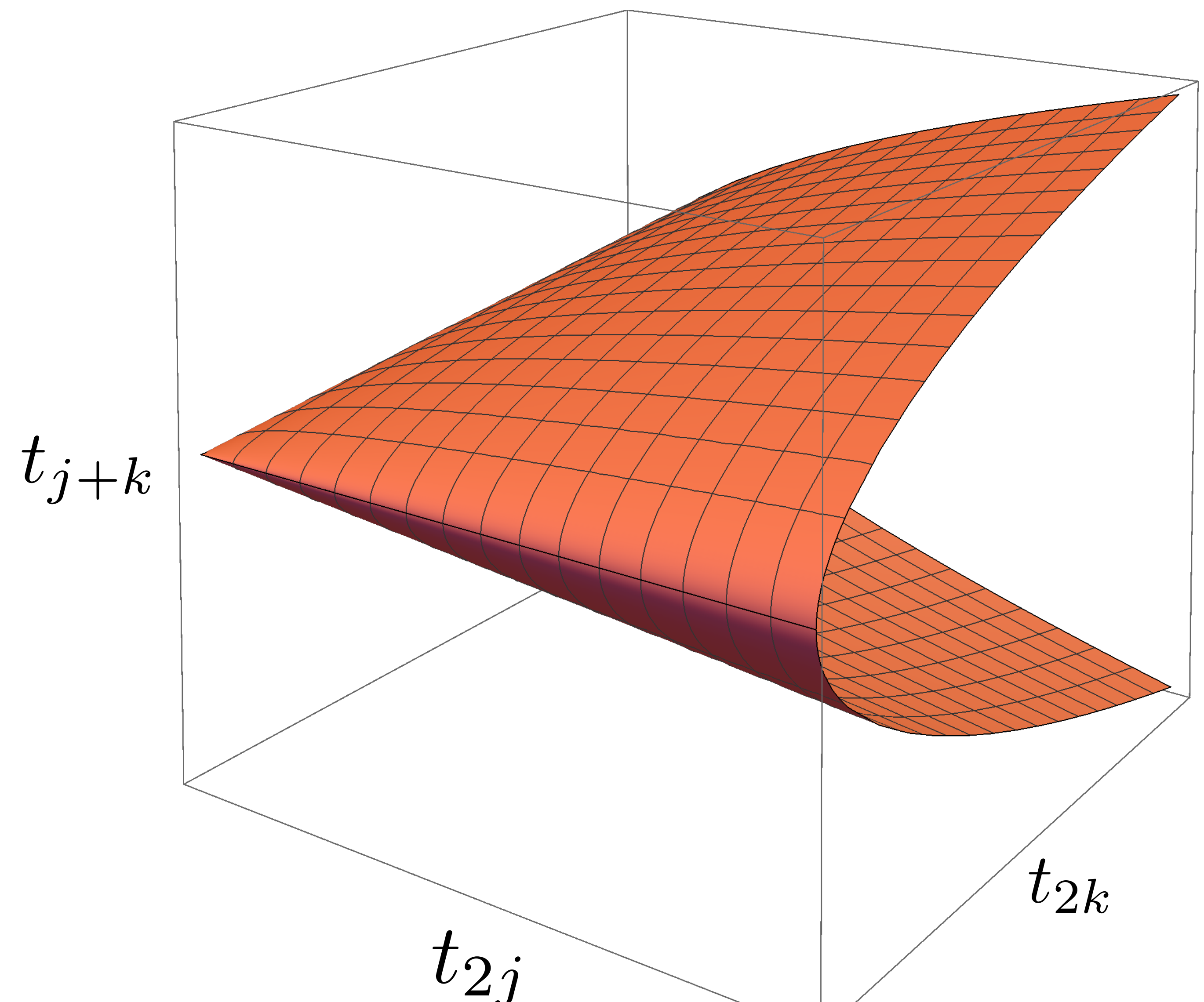
$$S_{\text{eff}}[\rho]/N=\int dx\rho(x)V(x)-\int dxdx'\rho(x)\rho\left(x'\right)\log\left|x-x'\right|$$

Why does the method work well?

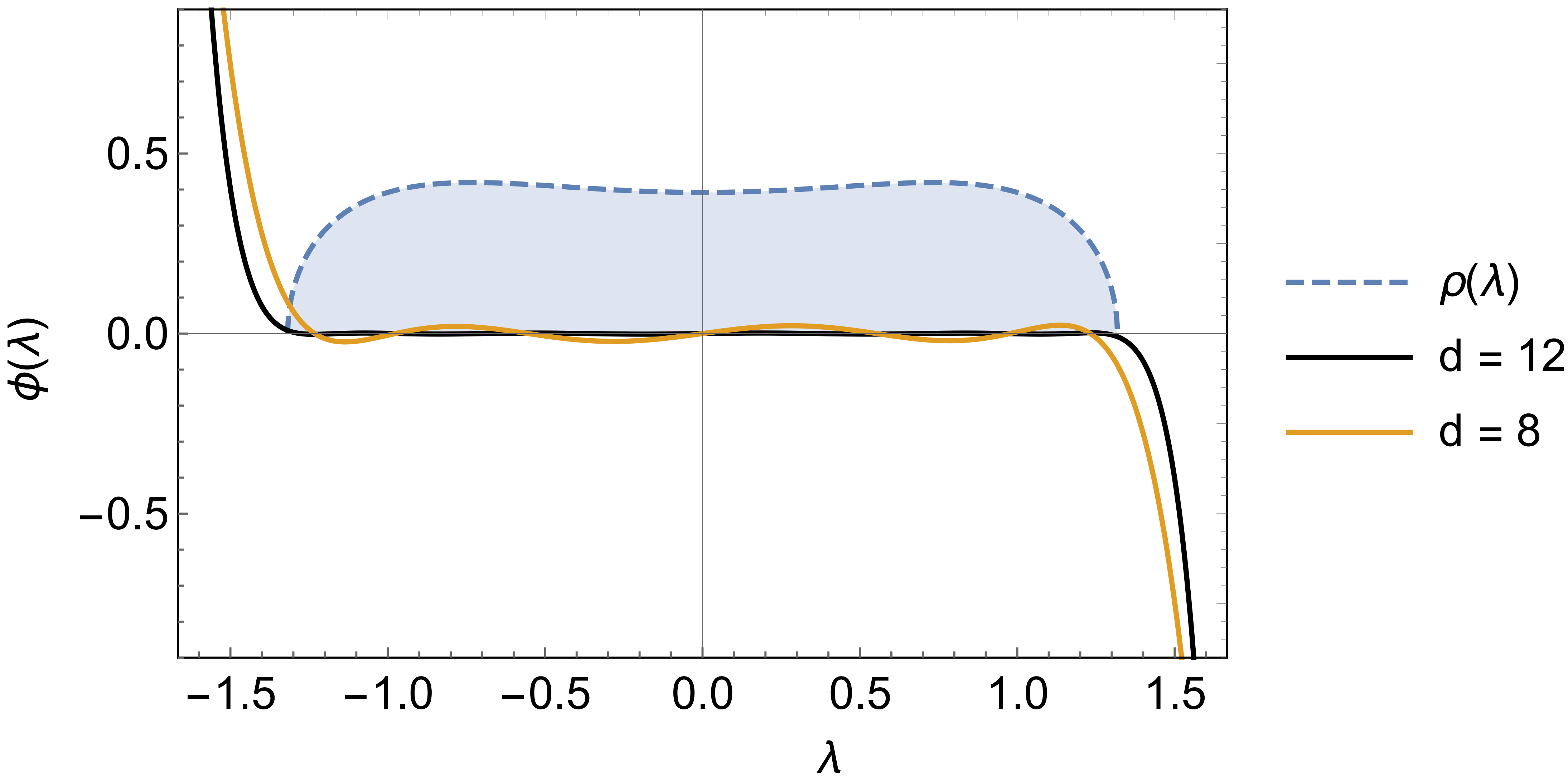
The space of positive semi-definite matrices forms a high-dim convex cone.

We expect the method to work well if the exact solution lies near the bd of the cone

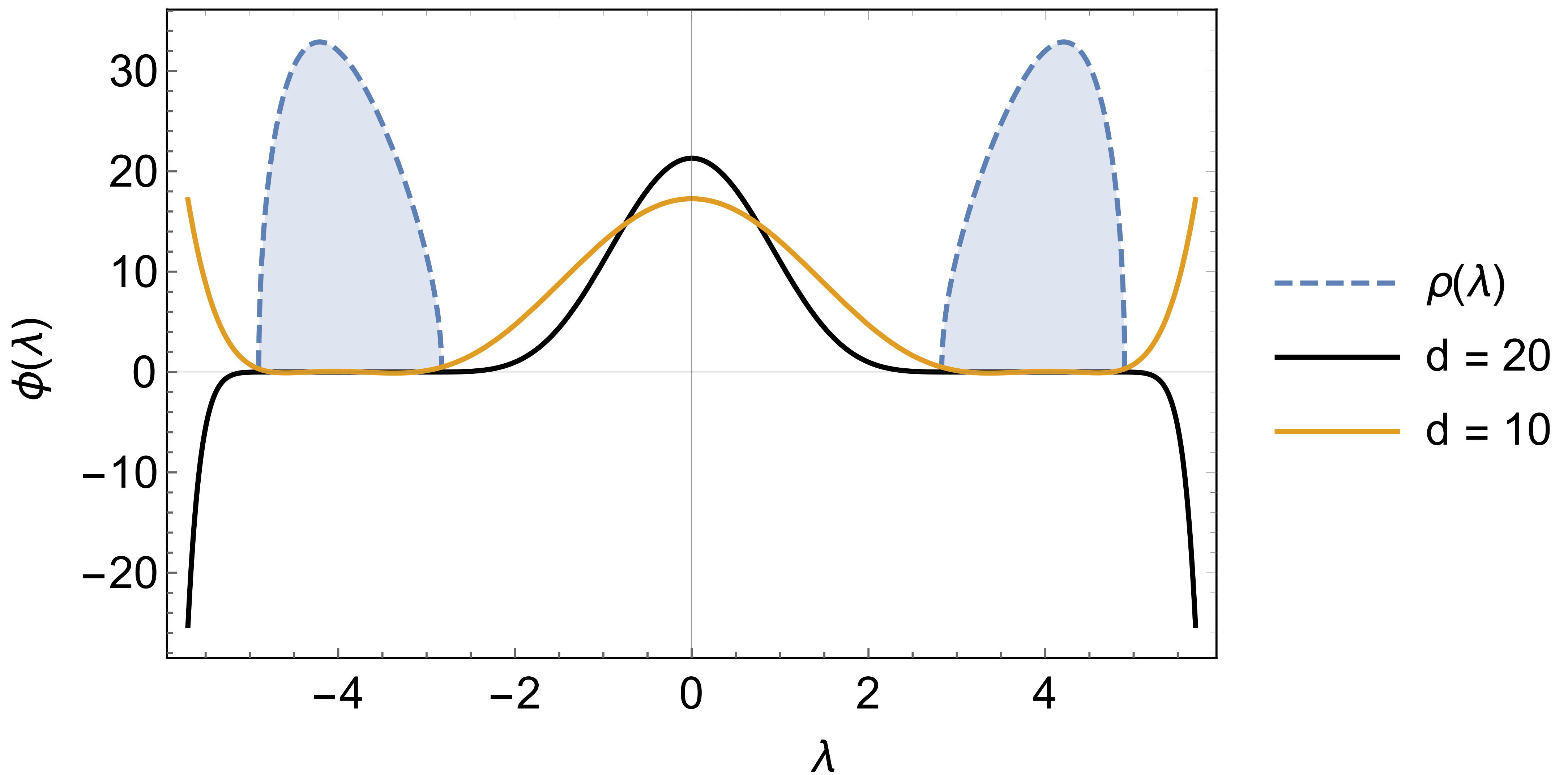
$$\mathcal{M}_{jk} = \left[\begin{array}{cc} t_{2j} & t_{j+k} \\ t_{j+k} & t_{2k} \end{array} \right] \succeq 0$$



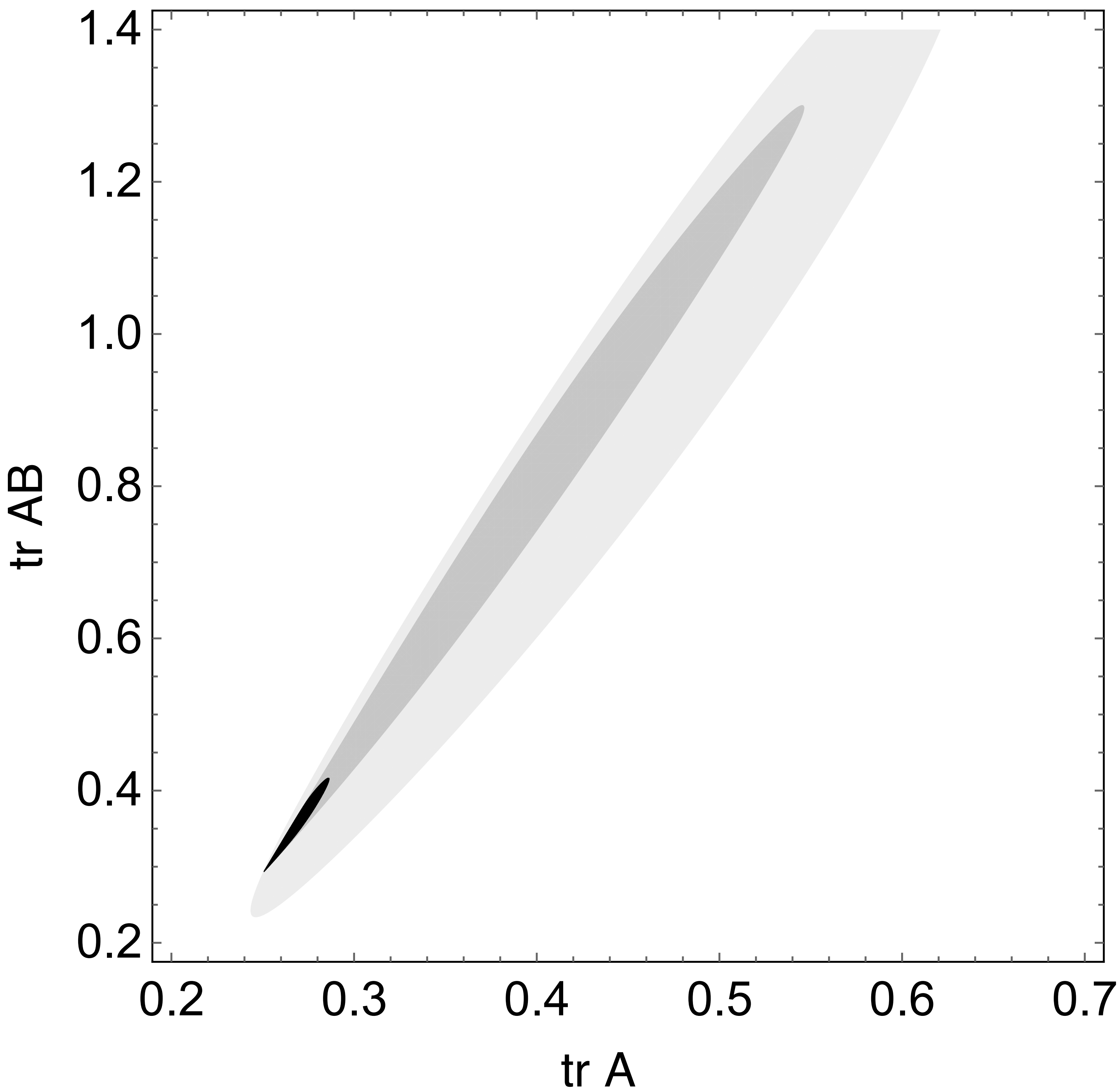
Nearly null vector



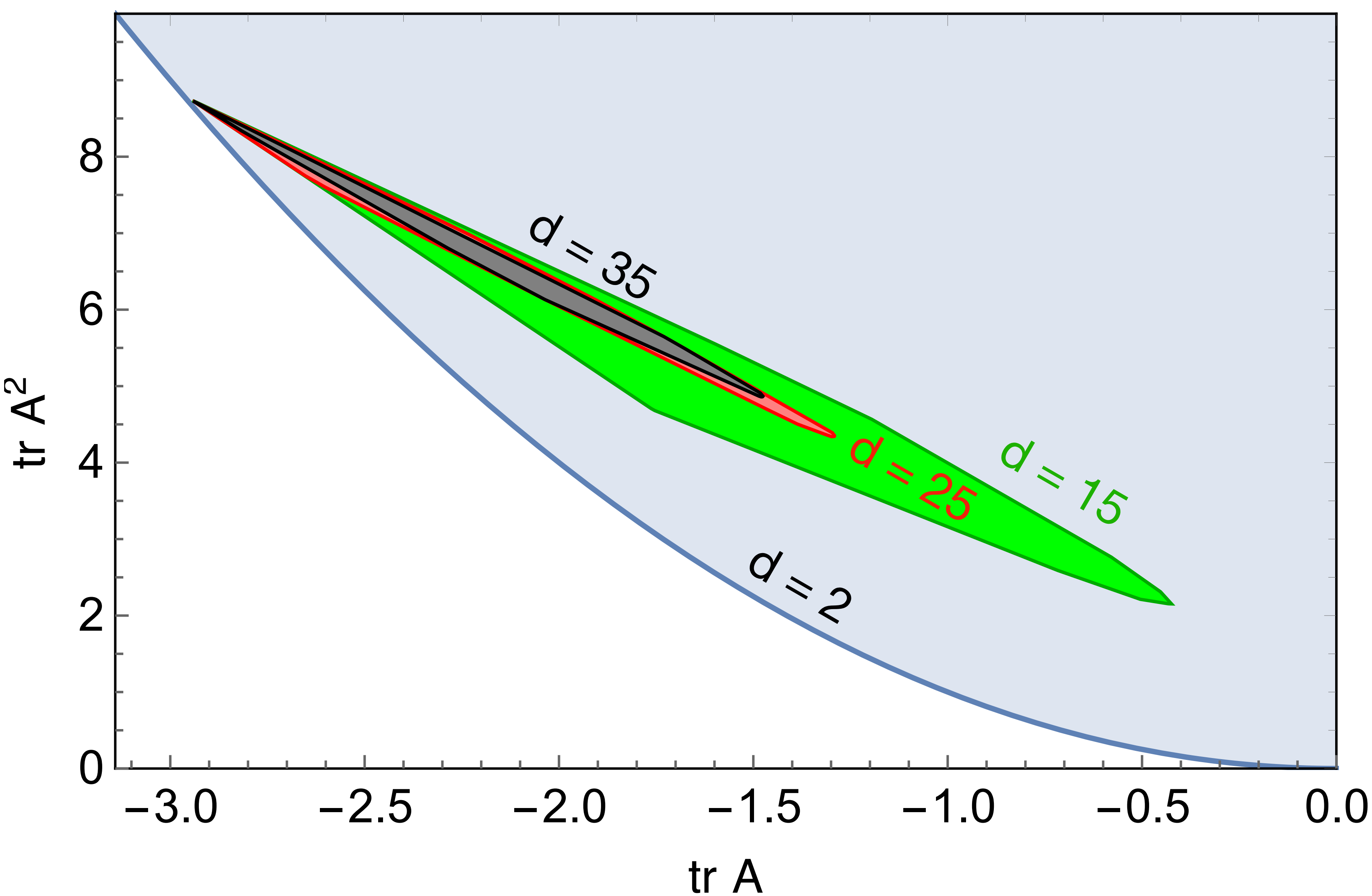
Nearly null vector



Ising model



Multi-matrix, W quartic



$$V = \frac{1}{4}A^4 + gAB^2 + (A \leftrightarrow B)$$

Future directions

Search for new string worldsheet theories?

Fermions/complex couplings

1/N corrections

see Anderson Kruczenski 2017

Higher dimensions (c=1, BFSS/BMN, …, Large *N* QCD?)

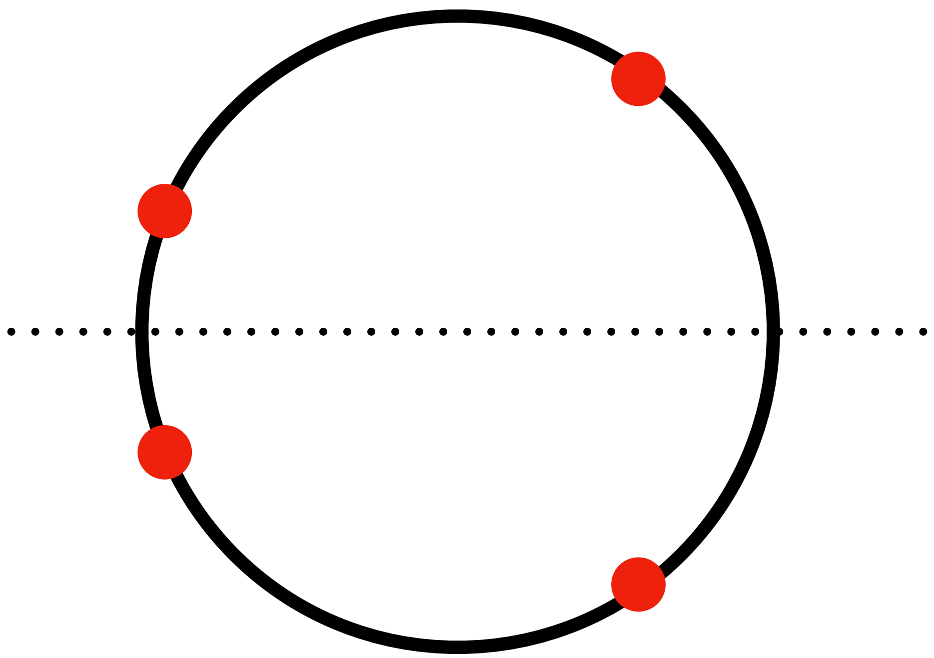
QM discussion

Han, Hartnoll, Kruthoff; 2004.10212 + WIP

$$\int dM e^{-S}(\cdots)$$

We have only been using positivity of the measure

Alternatively, *reflection positivity*



No problem with fermions

Time translation symmetry

$$0 = \langle \partial_t O \rangle = \langle [H, O] \rangle$$

BFSS matrix model

$$H = \frac{1}{2}\,\mathrm{Tr}\left(P_I^2 - \frac{1}{2}\left[X^I, X^J\right]^2 - \frac{1}{2}\psi\gamma^I\left[\psi, X^I\right]\right)$$

SUSY-invariant ground states:

$$\langle 0|\{Q_\alpha, \mathcal{O}_F\}|0\rangle = \langle 0|[Q_\alpha, \mathcal{O}_B]|0\rangle = 0$$

Bound on size of matrices

Take $\mathcal{O}_F \sim Q_\alpha \Rightarrow K + V + F = 0$

$$\mathcal{O}_F \sim \mathrm{Tr}\, P^I \psi\gamma^I \Rightarrow K + F/2 = 0$$

$$\mathcal{O}_F \sim \mathrm{Tr}\, X^I \psi\gamma^I$$

BFSS matrix model

Apply gauge invariance + Large N + positivity

$$\left\langle 0 \left| \mathrm{Tr}\left(X^1 X^1\right) \right| 0 \right\rangle \left\langle 0 \left| \mathrm{Tr}\left(X^1 X^1 X^1 X^1\right) \right| 0 \right\rangle \geq \frac{N^4 \pi^2}{2}$$

Polchinski ‘99

Size of matrices ~ size of gravity region ~ $N^{1/3}$

Can we derive an upper bound?

Higher moments?