Delayed Deconfinement and the Hawking-Page Transition

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Based on arXiv:2008.04950 with Christian Copetti, Alba Grassi and Zohar Komargodski

Introduction

In large-N Yang–Mills theory a criterion for confinement is whether the free energy is $\mathcal{O}(1)$, receiving contributions from color singlets, or $\mathcal{O}(N^2)$ reflecting gluons contributions.

Thanks to the AdS/CFT correspondence the deconfinement transition can be interpreted in terms of black hole formation in AdS via the Hawking–Page phase transition. [Witten]

Our goal is to revisit this idea in the context of $\mathcal{N} = 4$ supersymmetric Yang–Mills theory. More precisely, we would like to know if the superconformal index $\mathcal{I}$ can "deconfine" and account for the Hawking–Page transition.

Introduction

A central tool in the study of large-N confinement/deconfinement transition in $\mathcal{N} = 4$ SYM is a unitary matrix model [Sundborg; Aharony-Marsano-Minwalla-Papadodimas-Van Raamsdonk]

$$\int \prod_{i=1}^{N} \frac{d\theta_i}{2\pi} \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} (a_n - 1) \operatorname{Tr} U^n \operatorname{Tr} U^{-n}\right),$$

with a_n some known real couplings. The phase transition is usually described in the following way:

- If $\underline{a_n - 1 < 0}$, the most dominant contribution is $\operatorname{Tr} U^n = 0$ with $n < N$. Eigenvalues are spread on the unit circle typical of confined phase.
- If $\underline{a_n - 1 > 0}$ (for some n), eigenvalues have support only on a subset of the unit circle, center symmetry is broken $\rightarrow$ deconfined phase.

Introduction

At the special point where

$$\mathrm{sgn}(a_n - 1) < 0 \longrightarrow \mathrm{sgn}(a_n - 1) > 0\,,$$

we should discard the confined saddle and switch to a deconfined one. This is a weakly first order phase transition since the two phases never co-exist.

Here we will argue that, for complex couplings a_n, this logic has to be revised

- The large-N saddle point analysis receives additional cancellations in such a way that the phase transition does not take place when $(\mathrm{Re}\, a_n - 1)$ changes sign. This is what we call **"Delayed Deconfinement"**.
- The phase transition is **first-order** and this will be important for the application to gravity.

Complex Couplings

We describe these new features using a single complex coupling model with $a_1 = |a_1|e^{i\phi}$.

It is also convenient to introduce a Hubbard–Stratonovich complex field g to decouple the double trace interaction [Klebanov-Hashimoto; Liu]

$$Z[a_1, N] = \int_{\gamma(\phi)} dg g \exp\left(-\frac{N^2 g^2}{4a_1}\right) \int [DU] \exp\left(\frac{Ng}{2}\left(\mathrm{Tr}\, U + \mathrm{Tr}\, U^{-1}\right)\right) ,$$

where $\mathrm{Re}(a_1) > 0$ and $\gamma(\phi)$ is a line in complex plane at angle ϕ.

The second integral is also known as [Gross-Witten, Wadia] GWW matrix model. For real g its genus zero free-energy $F_0(g)$ is given by

$$F_0(g) = \begin{cases} \frac{g^2}{4}, & \text{ungapped, one-cut} \\ g - \frac{1}{2}\log g - \frac{3}{4}\,, & \text{gapped, one-cut}\,. \end{cases}$$

Multi-Cuts

We would like to study the GWW model for complex g where a complete picture is not yet available. The matrix model could have *multi-cuts* phases

$$Z[g,N] \sim \sum_{N_1+\cdots+N_s=N} Z[g,N_1,\cdots,N_s]\,,$$

where we sum over all arrangements of eigenvalues.
For a given g we can think about sub-dominant large-N saddles in the sum as instanton solutions with action

$$\exp(-N^2 A(g))\,,$$

connected to the dominant saddle via tunnelling. In the gapped one-cut phase this action is [Marino]

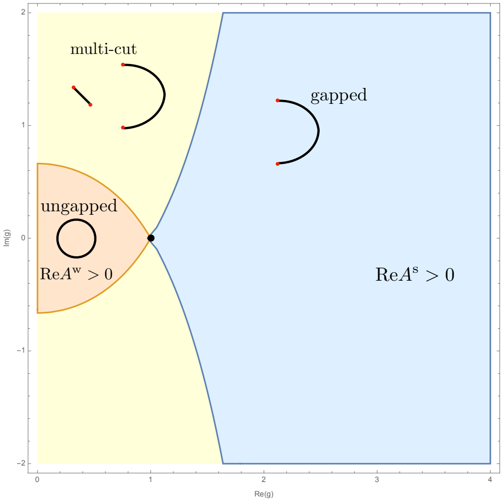
$$A^{\mathrm{s}}(g) = 2\sqrt{g(g-1)} - \cosh^{-1}(2g-1)\ .$$

We expect that there is a region in g-plane where the gapped one-cut can be analytically continued.

In order to determine such region we require that matrix model instanton tunnelling is suppressed [David]:

$$\mathrm{Re}(A^s(g)) > 0\,.$$

We can similarly determine such region also in the ungapped one-cut phase.



multi-cut
gapped
ungapped
$\mathrm{Re}A^{\mathrm{w}} > 0$
$\mathrm{Re}A^{\mathrm{s}} > 0$
Re(g)
Im(g)
0
1
2
3
4
2
1
0
−1
−2

The Deconfinement Curve

The full complex-HS integral at large-N is given by

$$Z[a_1, N] \approx \sum_{g_*} \exp\left(N^2 Q(a_1, g)\right), \quad \text{where} \quad Q(a_1, g) \equiv -\frac{g^2}{4a_1} + F_0(g)\,.$$

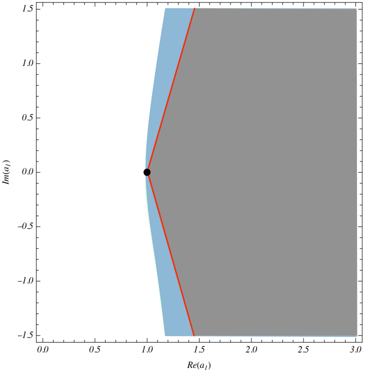
The saddle $g_* = 0$ leads to a confining behavior with $\mathcal{O}(1)$ contribution. We can find a second self-consistent, $\mathrm{Re}(A^s(g_*)) > 0$, complex saddle:

$$g_* = a_1 + \sqrt{a_1(a_1 - 1)}\,.$$

As a function of complex a_1, there is a region $\mathrm{Re}(Q(a_1, g_*)) > 0$ where the partition function has exponentially growing deconfined behavior.

The boundary of the region with exponentially growing behavior is what we call deconfinement curve $\mathcal{C}_D$

$$\mathcal{C}_D \,:\, \mathrm{Re}(Q(a_1, g_*)) = 0\,.$$



- For real a_1, the integral is over $\mathbb{R}$, and $\mathcal{C}_D$ is equivalent to

$$a_1 = 1\,.$$

  However, for complex a_1 the transition happens behind $\mathrm{sgn}(\mathrm{Re}\,a_1 - 1) > 0$ in the region bounded by $\mathcal{C}_D$ $\Rightarrow$ deconfinement is delayed.

- In the blue region the ungapped and gapped phase of the full model co-exist but only the ungapped saddle dominates. Across $\mathcal{C}_D$ the gapped saddle begins to dominate. This is a **first order** transition. For real a_1 the blue region pinches at $a_1 = 1$ and there is never phase co-existence. This is instead a weakly first order transition.

Superconformal Index

In 4D $\mathcal{N} = 1$ SCFTs a natural observable to study is $Z_{S^3 \times S^1}$

- Counts supersymmetric states on $S^3 \times \mathbb{R}_{\rm time}$,
- $\to$ BPS local operator in flat space (State-Operator Correspondence), [Römelsberger; Kinney-Maldacena-Minwalla-Raju]
- Independent of exactly marginal deformations.

Pick a supercharge Q such that $\{Q, Q^\dagger\} \sim \Delta - 2j_1 + \frac{3}{2}r = 0$, then

$$\mathcal{I} = \mathrm{Tr}_{\mathcal{H}_{\rm BPS}} (-1)^F p^{\frac{1}{3}(\Delta + j_1) + j_2} q^{\frac{1}{3}(\Delta + j_1) - j_2} .$$

For $\mathcal{N} = 4$ SYM $\Rightarrow$ 1/16-BPS local operators.

Superconformal Index

The $\mathcal{N} = 4$ superconformal index belongs to the class of unitary matrix models described here with

$$a_n(p,q) - 1 = \frac{((pq)^{n/3} - 1)^3}{(1-p^n)(1-q^n)}\,.$$

For $p, q \in \mathbb{R}$ and $0 < p, q < 1 \Rightarrow a_n(p,q) - 1 < 0$. At large-$N$

$$\log \mathcal{I} \sim \mathcal{O}(1)$$

where $\mathrm{Tr}\, U^n = 0$ is the most dominant contribution $\rightarrow$ confined phase.

This is disappointing for holography since we expect deconfinement to be described by large BPS black holes in AdS_5 whose entropy grows like $\mathcal{O}(N^2)$.

BPS Black Holes in AdS_5

Charged and rotating 1/16-BPS black holes in AdS_5 [Gutowski-Reall; Chong-Cvetic-Lu-Pope; Kunduri-Lucietti-Reall]

- Electric Charges $Q_1, Q_2, Q_3 \to U(1)^3 \subset SO(6)$
- Angular Momentum J_1, J_2
- Extremality $\to$ non-linear relation between 5 charges.

The simplest black hole has $Q_1 = Q_2 = Q_3 \equiv Q$, $J_1 = J_2 \equiv J$ and its Bekenstein–Hawking entropy is given by

$$S_{BH} = \frac{\mathrm{Area}}{4G_N} = 2\pi\sqrt{3Q^2 - N^2 J}$$

Hawking–Page Transition

S_{BH} can be obtained from a Legendre transform of the black hole free energy [Sen; Benini-Hristov-Zaffaroni; Review by Zaffaroni]

$$F_{BH} = -\frac{4N^2(\omega - \pi i)^3}{27\omega^2} \, .$$

with complex chemical potentials $\Delta \leftrightarrow Q$ and $\omega \leftrightarrow J$ subject to $3\Delta = 2\omega - 2\pi i$.

We can define a Hawking–Page line in ω-plane separating a thermal gravitons dominated phase from an AdS black hole phase

$$\mathrm{Re}(F_{BH}) = 0 \, .$$

Index Deconfinement

Recent developments indicates that to capture the $\mathcal{O}(N^2)$ behavior it is necessary to allow for general complex fugacities. [Cabo Bizet-Cassani-Martelli-Murthy; Choi-J.-Kim-S.-Kim-Nahmgoong; Benini-Milan]

In our work we adopted a general parametrization $p = q = ye^{i\psi}$ and studied the behavior of the index as a function of (y, ψ).

The couplings $a_n(p)$ become complex and we can use all our tools to compute $\mathcal{C}_D$ starting from the model with a single complex coupling

$$a_1(p) = 1 - \frac{(1 - p^{2/3})^3}{(1 - p)^2} \,.$$

The transition is first order so we cannot ignore corrections to $\mathcal{C}_D$ due to $a_{n>1}$ couplings.

However, near the HP transition region the effects of $a_{n>1}$ are numerically small and already studying a_2 leads to remarkable agreement with gravity.

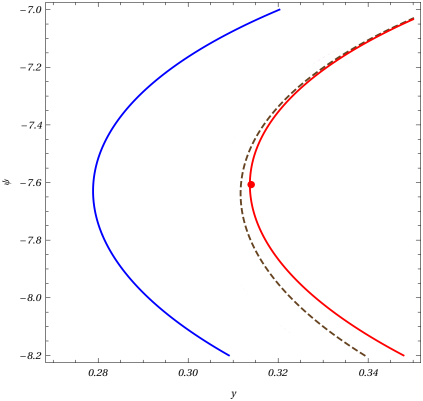
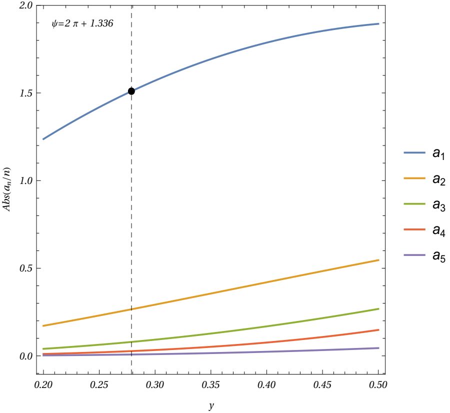


Figure 1: $\mathcal{C}_D$ for a_1 model — $\mathcal{C}_D$ for $a_1 + a_2$ model — $\mathrm{Re}(F_{BH}) = 0$

Thank You for Your Attention!

Extra


ψ=2 π + 1.336
Abs(a_n/n)
y
2.0
1.5
1.0
0.5
0.0
0.20
0.25
0.30
0.35
0.40
0.45
0.50
a_1
a_2
a_3
a_4
a_5

Two-Terms Model

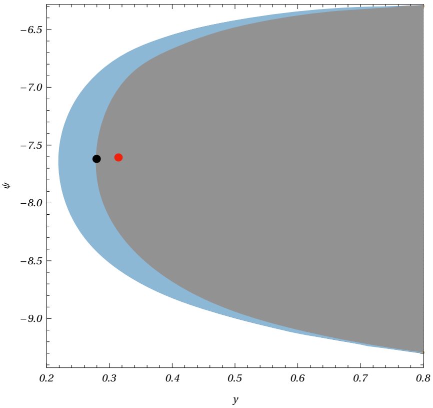
Even if there is no parametric suppression for $a_{n>1}$ these couplings are sufficiently small and one can study a systematic truncation of the model.

We can look at a model with both a_1 and a_2

$$Z[a_1, a_2, N] = \int [DU] \exp\left(a_1 \operatorname{Tr} U \operatorname{Tr} U^{-1} + \frac{a_2}{2} \operatorname{Tr} U^2 \operatorname{Tr} U^{-2}\right) ,$$

and repeat the saddle point analysis using two HS fields g, h to obtain

$$\int dg\, dh \exp\left[-\frac{N^2 g^2}{4a_1} - \frac{N^2 h^2}{2a_2} + N^2 F_0(g, h)\right] .$$



$$|p_{\mathrm{HP}}| \approx 0.314 \Rightarrow |p_\star| < |p_{\mathrm{HP}}|$$

$$|p_\star^{(2)}| \approx 0.312 \qquad \text{model with } a_1 + a_2$$