An introduction to AdS_2 holography

Bibliography: $nAdS_2$: Maldacena, Stanford, Yang 1606.01857

SYK: Maldacena, Stanford 1604.07818

Lecture notes by G. Sarosi: 1711.08482

OUTLINE:

1. Introduction & Motivation
2. JT-gravity and AdS_2
3. The SYK model
4. Applications

1. INTRODUCTION: "Realising Borges's dream"

What is holography? $\quad S_{BH} = \frac{A}{4} \frac{k_B c^3}{\hbar G_N}$

'70's $\quad$ $\boxed{\text{Gravity in } d+1 \text{ dim}} \Longleftrightarrow \boxed{\text{Quantum system in } d \text{ dim}}$

late-'90's $\quad$ gravity in AdS_{d+1} $\Longleftrightarrow$ CFT in d-dimensions

But already in '98, Maldacena, Michelson & Strominger noted that $d=1$ was a special case. We will review this later.

From the quantum side, the theory is a 0+1 dimensional theory $\Rightarrow$ $\boxed{\text{IT HAS NO SPATIAL DIMENSIONS}}$ $\Rightarrow$ everything happens at a single point that evolves in time.

So maybe, this is the best example to understand the emergence of space-time. It is also the "easiest": the theory has finite "N" degrees of freedom, as opposed to QFT where we have to deal w/ ∞ from the beginning. I-2

$\Rightarrow$ ~'15 | gravity in n-AdS$_2$ | $\overset{\sim}{\longleftrightarrow}$ | QM w/disorder (SYK model)

Quite new - still lots to understand

SY: '92

K: '15 but no paper.

1.1. MOTIVATION: Apart from this, the physics in AdS$_2$ is important to study higher-dimensional "extremal" black holes.

So let's start our discussion in 4d w/ the following action:

$$S_L = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{g}\,R - \frac{1}{4}\int d^4x\,\sqrt{g}\,F_{\mu\nu}F^{\mu\nu} + S_{matter}$$

(classical GR coupled to electromagnetic field and matter).

One solution to this action (without matter) is the Reissner-Nordstrom BH:

$$\boxed{ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2 \qquad f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \qquad A = \frac{Q}{r}\,dt}$$

$$f(r_h) = 0 \Rightarrow r_{h\pm} = m \pm \sqrt{m^2 - Q^2} \rightsquigarrow m \geq Q$$

$$S_{BH} = \frac{\pi r_{h+}^2}{G_N} \qquad \boxed{T_H = \frac{1}{4\pi} f'(r_h) = \frac{r_{h+} - r_{h-}}{4\pi\, r_{h+}^2}}$$

Note that when $m \sim Q \Rightarrow r_{h+} \sim r_{h-} \Rightarrow T_H \simeq 0$

This is called extremal limit

Lets analyse the geometry close to that limit.

We define a small parameter $\boxed{\varepsilon = \frac{M-Q}{Q}}$ at fixed Q.

$\Rightarrow r_h = Q\left(1+\sqrt{2\varepsilon}+\dots\right)$ Now we can expand for r close to r_h.

The near-horizon geometry that we get is the Bertotti-Robinson solution:

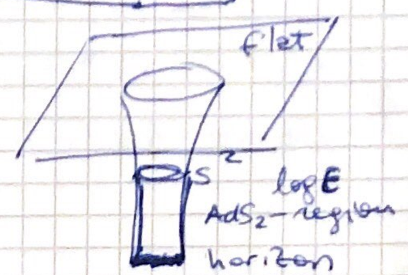
~~$ds^2 = -(\rho^2-1)d\tau^2 + d\rho^2$~~

$r = Q(1+\sqrt{2\varepsilon}\,\rho)$

$t = \frac{Q\tau}{\sqrt{2\varepsilon}}$

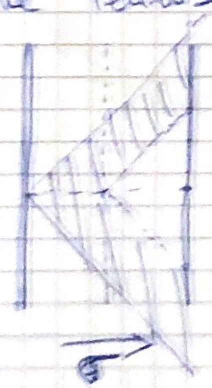
$$\boxed{\frac{ds^2}{Q^2} = -(\rho^2-1)\,d\tau^2 + \frac{d\rho^2}{(\rho^2-1)} + d\Omega_2^2}$$

$$\boxed{ds^2_{ext} = AdS_2 \times S^2}$$



1.2 A FEW WORDS ABOUT AdS_2

The Penrose diagram for AdS_2 is a strip and the coordinates that cover the full diagram are called "Global coordinates":



$$ds^2 = \frac{-dv^2 + d\sigma^2}{\sin^2\sigma}$$

$\Rightarrow$ It has two conformal boundaries at $\sigma = 0, \pi$ (In contrast to higher dimensional AdS)

The Poincaré patch is given by $ds^2 = \frac{-dt^2+dz^2}{z^2}$

The Rindler patch $ds^2 = d\rho^2 - \sinh^2\rho\, d\tau^2$

And we can also go to Euclidean space by Wick rotating t and making it periodical.

$\rightsquigarrow$ hyperbolic disk $\quad ds^2 = \frac{dt^2+dz^2}{z^2} = \frac{d\rho^2+d\tau^2}{\sinh^2\rho}$

$= d\rho^2 + \sinh^2\rho\, d\tau^2$

1.3 SO THEN, WHAT'S THE PROBLEM WITH ADS$_2$?

- Pure gravity in d=2 is topological

$$S_{EH} = -\frac{\phi_0}{16\pi G}\underbrace{\left[\int_M d^2x\,\sqrt{g}\,R + 2\int_{\partial M} K\right]}_{2\pi\chi}$$

The E.O.M. give the Einstein Equation: $R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = 0$
But for any metric in 2d, it is true that $R_{\mu\nu} = \frac{1}{2}R\,g_{\mu\nu}$,
So EE vanishes identically for any metric.

Even more, if we add matter, then we are restricted to $T^{\text{matter}}_{\mu\nu} = 0 \rightsquigarrow$ "gravity is over-constrained in 2d".
$\Rightarrow$ If we just care about the ground state $\Rightarrow S_{EH} = \frac{\phi_0}{4G_N}$ counts the "extremal" entropy but it is not possible to study excitations.

- The backreaction problem (Maldacena, Michelson, Strominger '98) the black hole gap is given by $E_{gap} \cong \frac{1}{\ell_p Q^3}$. At fixed Q, when we take $\ell_p \to 0$, the gap goes to ∞, so we can only see the ground states.

1.4 DILATON - GRAVITY THEORIES

- How do we make sense of holography in 2d then??
- Almheiri & Polchinski '14 understood how to start this analysis.

- From the 4d perspective:

$$S_L = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{g}\,R - \frac{1}{4}\int d^4x\,\sqrt{g}\,F_{\mu\nu}F^{\mu\nu}$$

Now consider the metric $ds^2 = h_{ij}\,dx^i dx^j + e^{2\psi} d\Omega_2^2$
(spherically symmetric configurations) $F = Q\sin\theta\, d\phi \wedge d\theta$.

We will get sth like

$$S_L \sim \frac{1}{G_N}\int d^2x \sqrt{-h}\left[e^{2\psi}\left(R_h + 2(\partial\psi)^2\right) + 2 - \frac{G_N}{2}e^{-2\psi}Q^2\right]$$

If $e^{\psi} \equiv \Phi \Rightarrow \sim \frac{1}{G_N}\int d^2x \sqrt{-h}\left[\Phi^2 R_h + 2(\partial\Phi)^2 + 2 - \frac{G_N Q^2}{2\Phi^2}\right]$

In the extremal case, $\Phi^2 = \phi_0 = \text{constant}$. But the idea is to go away from extremality by considering

$\Phi^2 = \phi_0 + \phi$ with $\phi \ll \phi_0$.

then the action becomes

$$\boxed{S_{JT} = \frac{\phi_0}{16\pi G_N}\int_M d^2x \sqrt{-h}\, R_h \left(+2\int_{\partial M} K\right) + \frac{1}{16\pi G_N}\int_M d^2x \sqrt{-h}\,\phi\,(R_h + 2) \left(+2\int_{\partial M}\phi_b K\right) + \dots}$$

SUMMARY :

* We will study holography in a setting where the quantum system is quantum mechanics (0+1 d)
* This is motivated by the near-extremal limit of charged and/or rotating black holes.
* "Traditional" holography does not work in 2 bulk dimensions.
* the way to understand the gravity side is by studying dilaton-gravity theories. In particular, JT gravity.