3. The SYK model

Today we will study the QM side. We will focus on the SYK model:
- Understand the model
- Compute the two-point functions
- See the emergence of the Schwarzian in the IR and the linear-in-T specific heat.

(3.1) SYK model is a simple, finite, quantum mechanical model.

The protagonists will be N Majorana fermions:

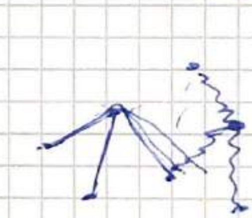
$$\boxed{\psi_i} \quad i=1,\dots,N \;.\quad \psi_i = \psi_i^\dagger \;.\quad \{\psi_i,\psi_j\} = \delta_{i,j} \quad i,j=1,\dots,N$$

One can explicitly build this recursively $\Rightarrow$ ψ_i are $2^{N/2}\times 2^{N/2}$ matrices.

The SYK Hamiltonian is given by:

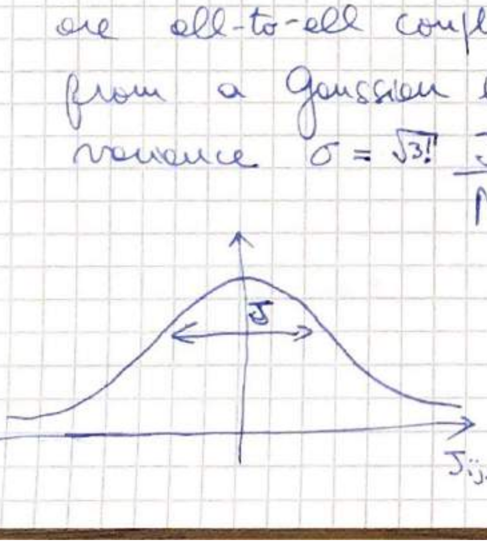
$\frac{N!}{4!(N-4)!}$ J's

$$H = \sum_{ijkl=1}^{N} J_{ijkl}\, \psi_i \psi_j \psi_k \psi_l$$



So it is also a $2^{N/2}\times 2^{N/2}$ matrix. The "couplings" J_{ijkl} are all-to-all couplings that are chosen randomly from a Gaussian ensemble with mean $\mu = 0$ and variance $\sigma = \sqrt{3!}\,\frac{J}{N^{3/2}}$.

(DISORDERED QUIVERS & COLD HORIZONS Anninos, Anous, Denef)



J_{ijkl}

* J is fixed
* The scaling w/ N is important for the large-N limit.
* We will study averages over couplings

• Fermions in QM

We want to find representations of the clifford algebra

$$\{\psi_i, \psi_j\} = \delta_{ij} \qquad i,j = 1,\dots,N$$

Majorana fermions so $\psi_i = \psi_i^\dagger$.

Now let's define $c_i = \frac{1}{\sqrt{2}}(\psi_{2i} - i\,\psi_{2i+1})$
(Assuming $N = 2K$, even) $c_i^\dagger = \frac{1}{\sqrt{2}}(\psi_{2i} + i\,\psi_{2i+1})$ $\Big\}$ ~~Assuming N=2K is even~~ $i = 1,\dots,k$

$\Rightarrow$ It is easy to probe that $\{c_i, c_j\} = \{c_i^\dagger, c_j^\dagger\} = 0$

$$\{c_i, c_j^\dagger\} = \delta_{ij}$$

$\Rightarrow$ This are the canonical anti-commutation relations for fermions.

$\Rightarrow$ Now we know how to build the representation

Take $|0\rangle$ such that $c_i|0\rangle = 0$ and then the states of the basis are $(c_1^\dagger)^{n_1} \dots (c_k^\dagger)^{n_k}|0\rangle$ w/ $n_i = 0, 1$

$\Rightarrow$ It is clear now that the Hilbert state has dimension $2^k = 2^{N/2}$.

• How do we build the representation? We can do it recursively:

* For $N = 2 \Rightarrow \psi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \psi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

* For any $k \Rightarrow \psi_i^{(k)} = \psi_i^{(k-1)} \otimes \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad i = 1,\dots,N-2$

$$\psi_{N-1}^{(k)} = \frac{1}{\sqrt{2}} I_{2^{k-1}} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\Rightarrow \boxed{\psi_i = 2^{N/2} \times 2^{N/2} \text{ matrices}}$ $\quad \psi_N^{(k)} = \frac{1}{\sqrt{2}} I_{2^{k-1}} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

- At operational level:
  - Choose the random couplings
  - Compute everything
  - Choose another set of couplings.
  - Repeat
  - Average over all results.

- Sometimes, we will also talk about the q-SYK,

$$\boxed{H_q = i^{q/2} \sum J_{i_1 \dots i_q} \psi_{i_1} \dots \psi_{i_q}}$$

$$\sigma \simeq \frac{J}{N^{\frac{q-1}{2}}}$$

3.2 Solving the model in the large-N limit

- Two ways: a. Path integral.
  b. Diagrams.

a. We are more familiar w/ actions than H, so:

(Euclidean)

$$S_{SYK} = \int d\tau \left[\frac{1}{2} \sum_i \psi_i \partial_\tau \psi_i + \sum_{1 \le i<j<k<l \le N} J_{ijkl} \psi_i \psi_j \psi_k \psi_l \right]$$

If I want the partition function then

$$Z(J_{ijkl}) = \int D\psi_i \, e^{-S_{SYK}}$$

Now we need to introduce the average over J's.

$\boxed{\text{CAVEAT: How to do it}}$

$$\langle Z \rangle_J \sim \int dJ_{ijkl} \, e^{-\sum \frac{J_{ijkl}^2}{2\left(\frac{3! J^2}{N^3}\right)}} Z(J_{ijkl})$$

The point is to compute the two point fn. $\boxed{\text{See section 4.3 G. Sarosi}}$

$$G_{ij}(\tau,\tau') = \langle T \, \psi_i(\tau), \psi_j(\tau') \rangle$$

$$\Rightarrow \boxed{G(\tau,\tau') = \frac{1}{N} \sum_{i=1}^{N} G_{ii}(\tau,\tau')} \qquad 1 = \int DG \, \delta\left(NG - \sum_i \psi_i \psi_i\right)$$

Lagrange multiplier Σ

b. Large-N diagrammatics

Ok, now it's time to treat SYK as a 0+1 QFT. We have free fermions when $J=0$ and we will do perturbation theory in J.

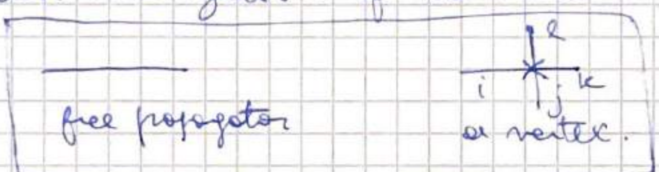
Let's remember some things: $\psi_i(\tau) = e^{\tau H}\psi_i e^{-\tau H}$

* $J=0 \Rightarrow H=0 \Rightarrow \psi_i(\tau) \equiv \psi_i$ and $\{\psi_i, \psi_j\} = \delta_{ij}$

$$G^{free}_{ij}(\tau) = \frac{1}{2}\delta_{ij}\,\mathrm{sgn}(\tau)$$

$$\boxed{G^{free}(\tau) = \frac{1}{N}\sum_{i=1}^{N} G^{free}_{ii} = \frac{1}{2}\,\mathrm{sgn}\,\tau.}$$

* For each realization of the model we have

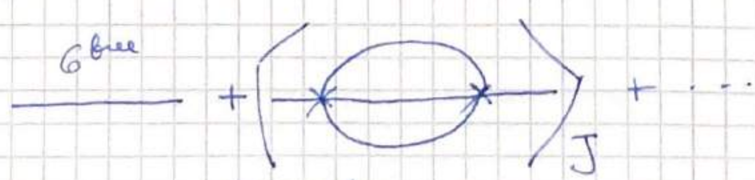


* We need to do a diagram and then average over disorder $\Rightarrow$ this is gaussian $\Rightarrow$ we will treat J_{ijkl} as a "field" with

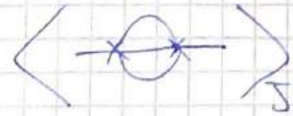
$$- - - - - \quad \langle J_{i_1 j_1 k_1 l_1} J_{i_2 j_2 k_2 l_2}\rangle_J = \frac{3!\,J^2}{N^3}\delta_{i_1 i_2}\delta_{j_1 j_2}\delta_{k_1 k_2}\delta_{l_1 l_2}$$

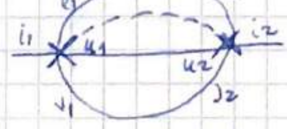
* So let's draw some diagrams.

- Note that we always need even vertices because $\langle J_{ijkl}\rangle = 0$

* $G(\tau) =$ 

$+ \dots$



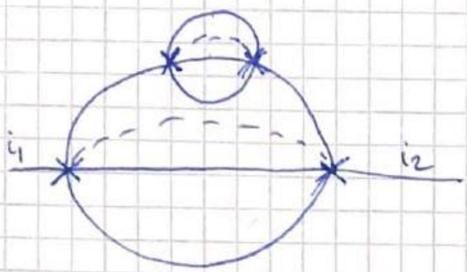
$=$ 

$$= \frac{3!\,J^2}{N^3}\delta_{i_1 i_2}\delta_{j_1 j_2}\delta_{k_1 k_2}\delta_{l_1 l_2}$$

$$G_{l_1 l_2}\,G_{k_1 k_2}\,G_{j_1 j_2}$$

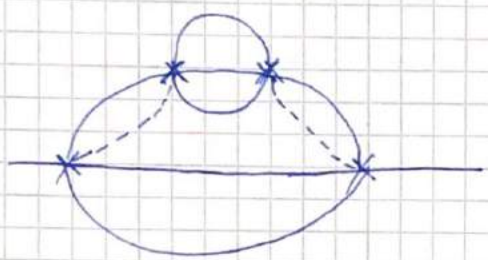
$$= J^2 \cdot N^0 (G^{free})^3 \qquad = \frac{3!\,J^2}{N^3} G_{ll}\,G_{kk}\,G_{jj}\,\delta_{i_1 i_2}$$

* Now J^4



$$= \frac{J^4}{N^6}\left(G^{free}_{kk}\right)^5 \underbrace{G_{l_1 l_3}\, G_{l_1 l_3}}_{\delta_{l_1 l_3} \to G_{ll}}\, \delta_{i_1, i_2}$$

$$= \frac{J^4}{N^6}\left(G^{free}_{kk}\right)^6 = \boxed{J^4 \cdot N^0}$$



$$= \frac{J^4}{N^6}\left(G^{free}_{kk}\right)^2$$

$$\begin{cases} G_{i_1 i_2} & \begin{cases} G_{j_1 j_2} & G_{k_1 k_2} \\ G_{j_1 j_2} & \end{cases} \\ G_{i_1 i_2} & \end{cases}$$

$$= \frac{J^4}{N^6}\left(G^{free}_{kk}\right)^2 G^{free}_{ii}\, G^{free}_{jj}\, G_{k_1 k_2}$$

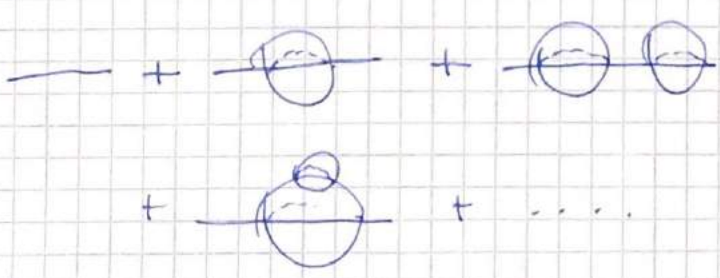
$$= \boxed{\frac{J^4}{N^2}}$$

$\Rightarrow$ in the large-N limit this diagram is suppressed

(like planar diagrams for matrices)

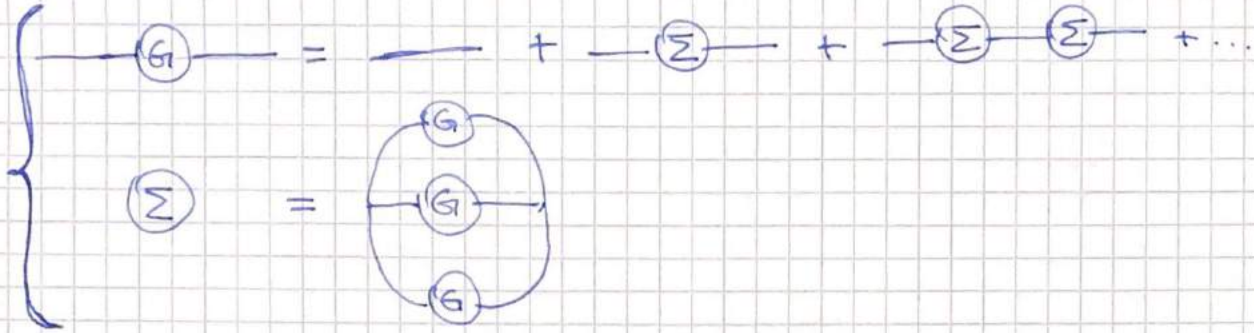
Show diagrams in Trunin

$\Rightarrow G(\tau) =$



$+ \ldots$

We can write this as:



1° line: $G^{free} + G^{free}\,\Sigma\,G^{free} + G^{free}\,\Sigma\,G^{free}\,\Sigma\,G^{free} + \dots$

$$G^{free}\underbrace{\left[1 + \Sigma G^{free} + \Sigma G^{free}\,\Sigma G^{free} + \dots\right]}_{\text{geometric sum } \sum_{k=0}^{\infty} (\Sigma G)^k = \frac{1}{1-\Sigma G}}$$

$$= G^{free}\left(1 - \Sigma G^{free}\right)^{-1}$$

$$\boxed{G = \left((G^{free})^{-1} - \Sigma\right)^{-1}}$$ ⇝ But we know $(G^{free})^{-1} = \delta(\tau-\tau')\,\partial_{\tau'}$

$$\Longrightarrow \quad \boxed{\begin{aligned} G &= (\partial_\tau - \Sigma)^{-1} \\ \Sigma &= J^2 G^3 \end{aligned}} \quad \underline{\text{or}} \quad \boxed{\begin{aligned} G &= (\partial_\tau - \Sigma)^{-1} \\ \Sigma &= J^2 G^{q-1} \end{aligned}}$$

Dyson-Schwinger equations

We managed to "solve" the theory at large-N at all orders in J !!

Well we haven't solved it yet.

3.3 Solutions to SD eqs

$$\boxed{\begin{aligned} G &= (\partial_\tau - \Sigma)^{-1} \\ \Sigma &= J^2 G^{3 \to q-1} \end{aligned}}$$

Free fermions $\longrightarrow$??

UV ($J=0$), $J(\tau-\tau') \ll 1$ $\qquad$ IR, $J(\tau-\tau') \gg 1$

∂_τ is a very UV term. In fact $\partial_\tau = \delta(\tau-\tau')\partial_\tau$. So let's neglect it for now and see what happens.

Crucial observation: these eqs. have an extra symmetry

$$\tau \to \phi(\tau) \quad \& \quad G(\tau,\tau') \to [\phi'(\tau)\phi'(\tau')]^{\Delta}\, G(\phi(\tau),\phi(\tau'))$$

$$\Sigma(\tau,\tau') \to [\phi'(\tau)\phi'(\tau')]^{\Delta(q-1)}\, \Sigma(\phi(\tau),\phi(\tau'))$$

$\Rightarrow$ So both are conformal two point functions. It turns out that $\Delta = 1/q$. $\Rightarrow$ EMERGENT CONFORMAL SYMMETRY in the IR, that is EXPLICITLY BROKEN by the ∂_τ term.

$\Rightarrow$ If it has conformal symmetry $\Rightarrow$ what's the 2-pt fcn?

$$G_c(\tau) = \frac{\#\, \mathrm{sgn}(\tau)}{|\tau|^{2\Delta}}$$

One can check that

$$\Sigma_c = J^2 \#^{q-1} \frac{\mathrm{sgn}(\tau)}{|\tau|^{\Delta(q-1)}}$$

solves the SD eqs in the IR.

- Reparametrizations are also solutions. But some reparametrizations also leave the solution unchanged

$$\tau \to \phi(\tau) = \frac{a\tau+b}{c\tau+d} \qquad ad-bc=1$$

The solution spontaneously breaks the reparametrization sym to $SL(2,\mathbb{R})$.

Emergence of the Schwarzian   Rosenhaus 1807.03334

We found an entire space of solutions

$$G(\tau,\tau') = b\,\frac{\mathrm{sgn}(\tau_{12})}{J^{2\Delta}}\,\frac{\phi'(\tau)^{\Delta}\,\phi'(\tau')^{\Delta}}{|\phi(\tau)-\phi(\tau')|^{2\Delta}}.$$

Now we want to move a bit away from the conformal point. Then we need to include the effect of the $\delta(\tau-\tau')$. The idea would be to use the conformal solution but expand it for short times $J|\tau-\tau'| \ll 1$.

$$\left.\begin{aligned}\tau-\tau' &= \tau_-\\ \tau+\tau' &= \tau_+\end{aligned}\right\}\quad G \Rightarrow b\,\frac{\mathrm{sgn}(\tau_-)}{|J\tau_-|^{2\Delta}}\left(1+\frac{\Delta}{6}\tau_-^2\,\mathrm{Sch}(f(\tau_+),\tau_+) + \dots\right)$$

Plugging this into the action we get

$$S_{eff} = \frac{N}{J}\int d\tau_+\,\mathrm{Sch}(f(\tau),\tau) + \dots$$

Thermodynamics at large q

So far we know $\underset{UV}{\text{free fermions}}$?? $\underset{IR}{\text{Near conformal}}$

At large q, we can solve all the way. Remember q is the number of fermions interacting.

$$G = \frac{\mathrm{sgn}\,\tau}{2}\left(1+\frac{g(\tau)}{q}+\dots\right) \Rightarrow \begin{cases} G = (\partial_\tau - \Sigma)^{-1}\\ \Sigma = J^2 G^{q-1}\end{cases}$$

$$\Rightarrow \boxed{\partial_\tau^2 g(\tau) = J^2 e^{2g(\tau)}} \qquad g(0)=g(\beta)=0$$

$$e^{g(\tau)} = \frac{2v^2}{\sqrt{(\beta J)^2 v^2}\,\cos\left(v\left(\frac{2\pi\tau}{\beta}-1\right)\right)}\ ; \quad \cos v = \frac{2v}{\beta J}$$

But for $\beta J \gg 1$, $v = \frac{\pi}{2} - \frac{\pi}{\beta J} + \frac{2\pi}{(\beta J)^2} + \cdots$

$$\beta F = -\frac{N\beta}{8q^2} \int_0^\beta d\tau \left[(\partial_\tau g)^2 + \frac{J^2}{2} e^{2g} \right]$$

$$\Rightarrow \beta F = -\frac{N}{q^2} \left[\beta J - \frac{\pi^2}{4} + \frac{\pi^2}{2\beta J} \right] + \frac{N}{2} \log 2$$

$$S = S_0 + \frac{N}{q^2} \left(-\frac{\pi^2}{4} + \frac{\pi^2}{\beta J} + \cdots \right)$$

$$\boxed{C = \frac{N}{q^2} \frac{\pi^2}{\beta J}}$$

$\rightarrow$ As promised, SYK gives linear-in-T specific heat.

$\boxed{\text{SUMMARY}}$: We studied the SYK model

- QM fermions in the large N w/ disorder
- We found that in the IR, there is an emergent reparametrization symmetry that is broken explicitly and spontaneously.
- The "soft mode" action is Schwarzian and in the low temperature limit, the entropy is linear in T.

• Resumen:

Clase 2: $S_{JT} = -\frac{\phi_0}{16\pi G_N}\left[\int_{\mathcal{M}} d^2x\sqrt{h}\,R + 2\int_{\partial M} K\right]$

$\boxed{\text{2d gravity}}$ $-\frac{1}{16\pi G_N}\left[\int_{\mathcal{M}} d^2x\sqrt{h}\,\phi(R+2)\right\} + \int_{\partial M}\phi_b K\Big]$

Clase 3: $H_{SYK} = \frac{1}{4!}\sum_{ijkl=1}^{N} J_{ijkl}\psi_i\psi_j\psi_k\psi_l$

$\boxed{\text{QM c/desorden}}$ $\langle J_{ijkl} J_{ijkl}\rangle = \frac{J^2}{N^3}$

Parametros en la teoría de gravedad: $G_N, \phi_0, \tilde{\phi}$

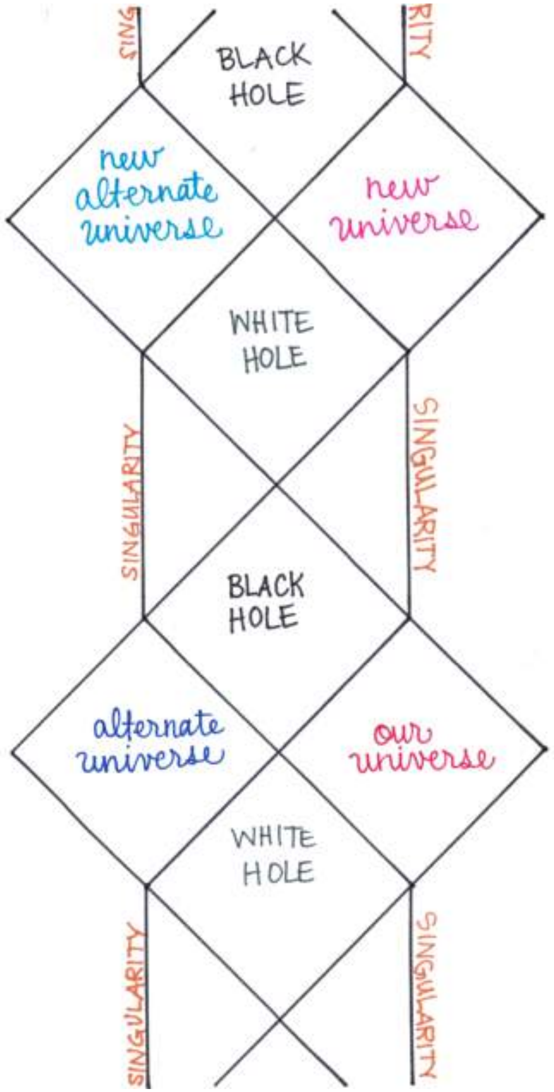
Parametros en la teoría de SYK: N, S_0, J

$$\left.\begin{aligned} \Rightarrow S_{JT} &= \frac{\phi_0}{4G} + \frac{\tilde{\phi}\pi}{4G}\cdot\frac{1}{\beta} + \dots \\ S_{SYK} &= \underbrace{N\left(\frac{1}{2}\log 2 - \frac{\pi^2}{q^2 4}\right)}_{S_0} + \frac{N}{q^2}\frac{\pi^2}{\beta J} + \dots \end{aligned}\right\}$$

Entonces podemos fácilmente identificar: $\boxed{G_N \sim \frac{1}{N} \quad \phi_0 \sim S_0 \quad \tilde{\phi} \sim \frac{1}{J}}$

* Obviamente no sabemos cual es (ni si existe) una teoría dual en el bulk a SYK (una teoría de cuerdas?)
  - String theory black hole solvable as SYK??

Sin embargo, sigue siendo un buen "toy model" para estudiar gravedad cuántica y la emergencia del espacio-tiempo.



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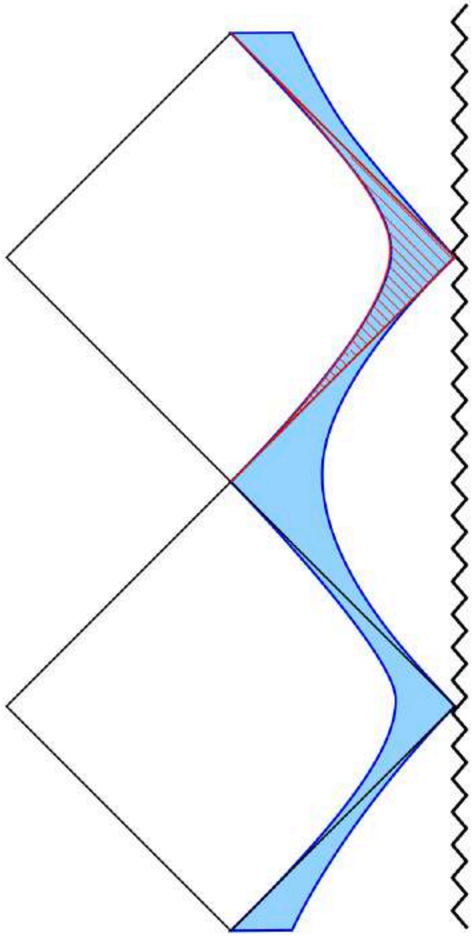

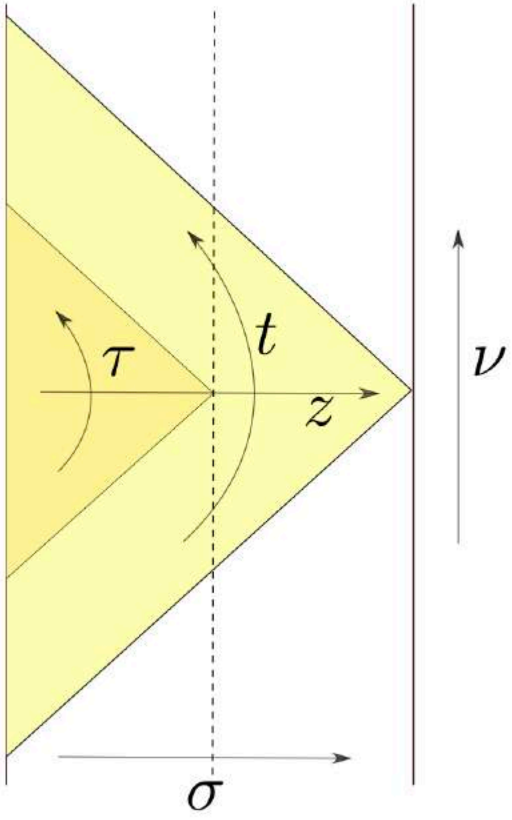

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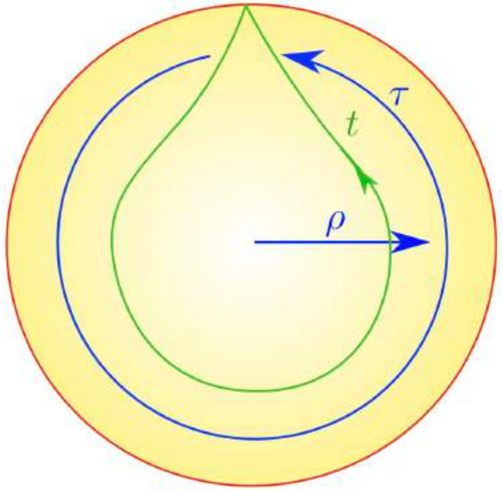

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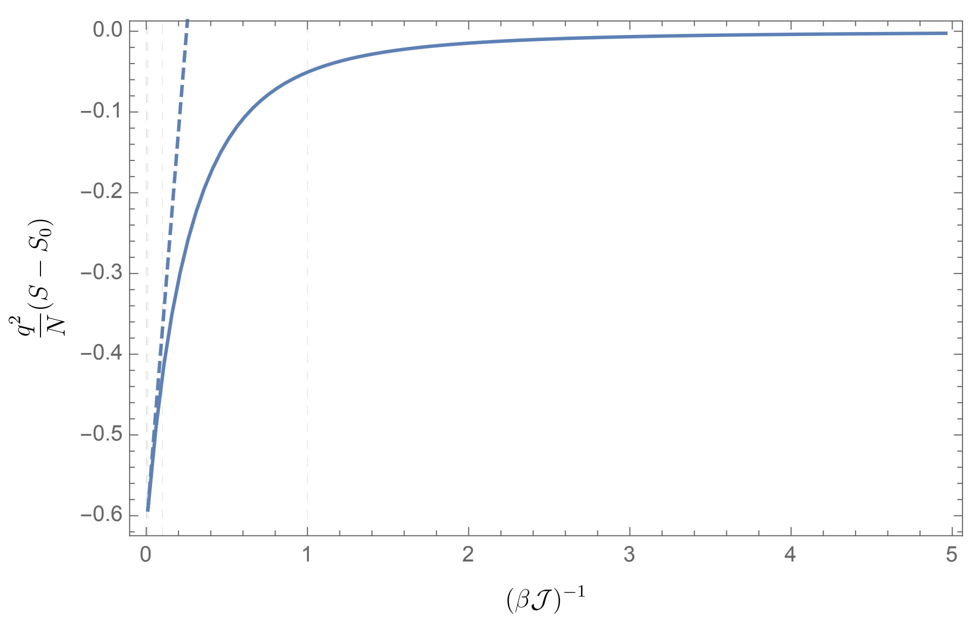
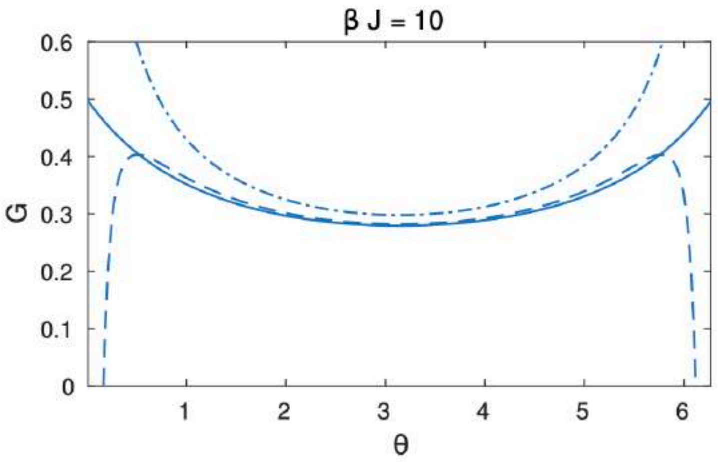
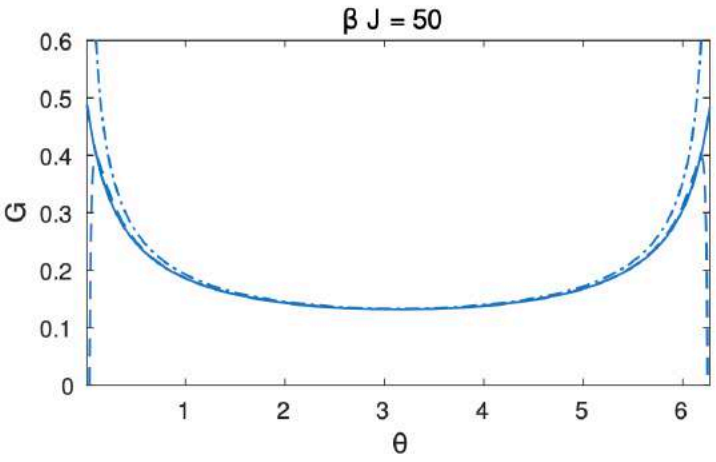
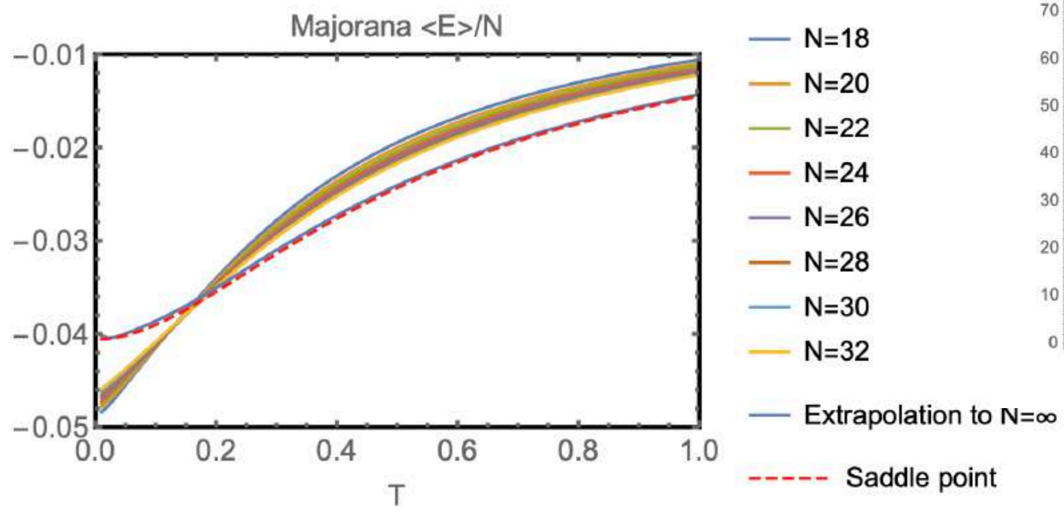
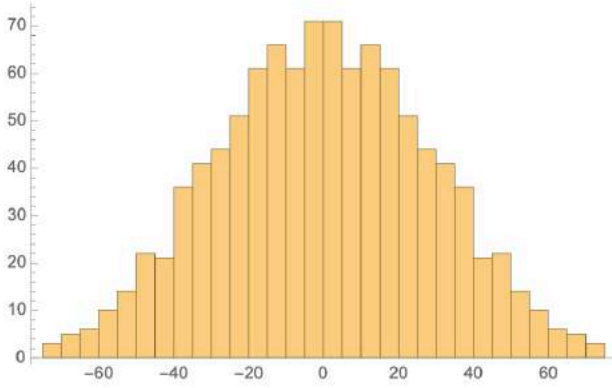
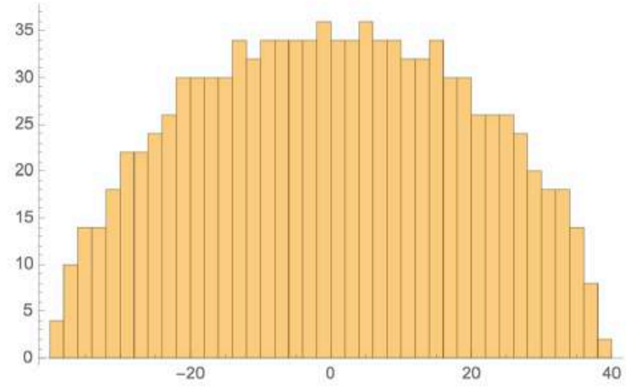




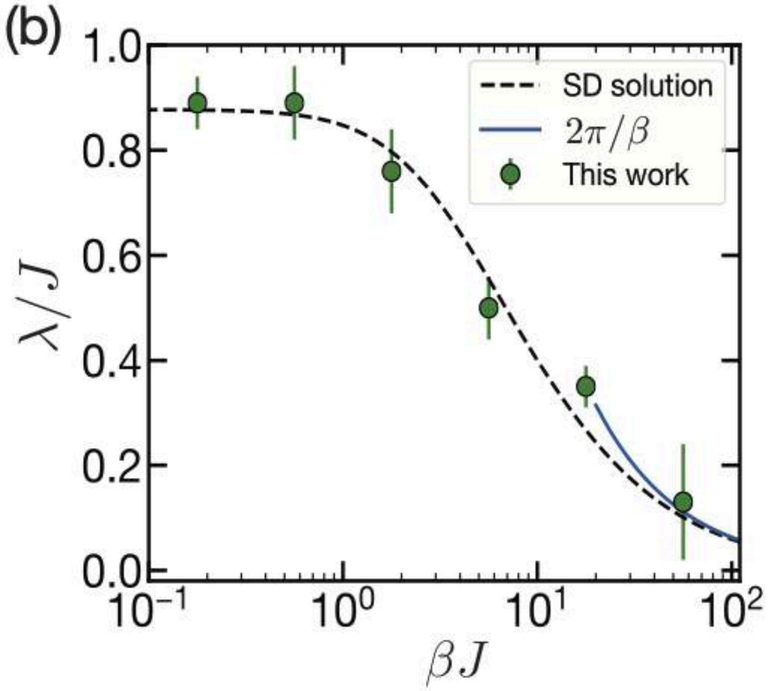


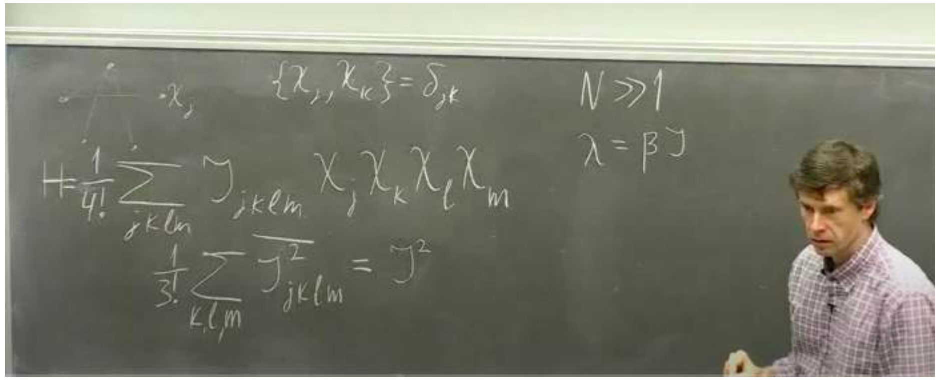






[Black Holes and Random Matrices, '16]


(b)
1.0
0.8
0.6
0.4
0.2
0.0
λ/J
10^{-1}
10^{0}
10^{1}
10^{2}
βJ
SD solution
$2\pi/\beta$
This work



December 1992

Gapless spin-fluid ground state in a random, quantum Heisenberg magnet

Subir Sachdev and Jinwu Ye

Departments of Physics and Applied Physics, P.O. Box 2157, Yale University, New Haven, CT 06520

In[105]:= `fermions = Table[{1}, {a, 1, 2}];`

$$\texttt{fermions[[1]]} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & -I\\ I & 0\end{pmatrix};$$

$$\texttt{fermions[[2]]} = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix};$$

In[108]:=
```
Do[n = 2 k;
  f = Table[{1}, {i, 1, n}];
  Do[f[[i]] = KroneckerProduct[fermions[[i]], PauliMatrix[3]], {i, 1, Length[fermions]}];
  f[[1 + Length[fermions]]] = 1/√2 KroneckerProduct[IdentityMatrix[2^(k-1)], PauliMatrix[1]];
  f[[2 + Length[fermions]]] = 1/√2 KroneckerProduct[IdentityMatrix[2^(k-1)], PauliMatrix[2]];
  fermions = Table[{1}, {i, 1, n}];
  Do[fermions[[i]] = f[[i]], {i, 1, n}], {k, 2, kmax}]
```

In[109]:= `Length[f]`

Out[109]= 6

In[110]:= `Length[f[[1]]]`

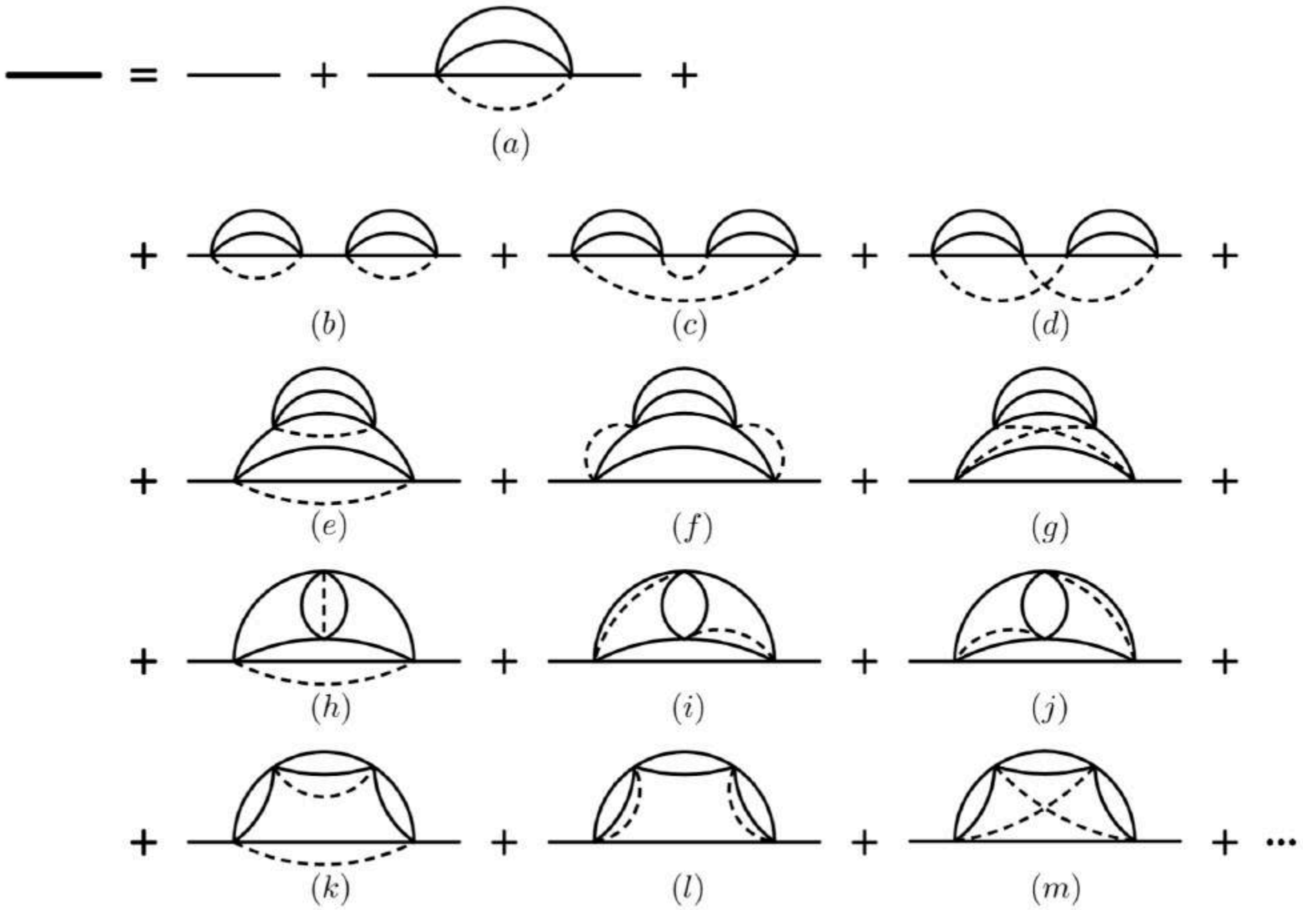
Out[110]= 8

In[111]:= `Table[f[[i]] // MatrixForm, {i, 1, 6}]`

Out[111]=

$$\left\{\begin{pmatrix}
0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}}\\
\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0
\end{pmatrix},
\begin{pmatrix}
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}\\
\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0
\end{pmatrix},
\begin{pmatrix}
0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0\\
\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}}\\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0
\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}
0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0\\
\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}}\\
0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0
\end{pmatrix},
\begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\
\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0\\
0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0\\
0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0
\end{pmatrix},
\begin{pmatrix}
0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\
\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0\\
0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0\\
0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0
\end{pmatrix}\right\}$$


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)