Entanglement in QM and QFT

4/5 - The "Extensive Mutual Information" model

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It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

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Outline

Some references

- Some papers on the general structure of EE: https://arxiv.org/abs/1202.2070, https://arxiv.org/abs/1108.4038.
- Original paper on EE of CFTs in $d = 2$ by Calabrese and Cardy: https://arxiv.org/abs/hep-th/0405152.
- The expression for the EE universal term in the case of smooth entangling regions in $d = 4$ is due to Solodukhin: https://arxiv.org/abs/0802.3117.
- The unitarity bounds are due to Hoffman and Maldacena: https://arxiv.org/abs/0803.1467.
- The connection between the disk entanglement entropy and the three-sphere partition function is due to Casini, Huerta and Myers: https://arxiv.org/abs/1102.0440.
- A detailed account on the EE for entangling regions containing singularities and references can be found in: https://arxiv.org/abs/1904.11495.
- The relation between the corner coefficient σ and the stress tensor two point function C_T was conjectured in my paper with Myers and Witczak-Krempa: https://arxiv.org/abs/1505.04804 based on free-field and holographic calculations, and proved later for general CFTs by Faulkner, Leigh and Parrikar in: https://arxiv.org/abs/1511.05179.
- The "extensive mutual information" model was proposed in https://arxiv.org/abs/cond-mat/0505563 and https://arxiv.org/abs/hep-th/0405111 by Casini, Fosco and Huerta.

General structure of EE and universal terms

General structure of EE for CFTs

Given CFT_d and smooth entangling region V, EE takes the generic form

$$S_{\mathrm{EE}}^{(d)} = b_{d-2}\frac{H^{d-2}}{\delta^{d-2}} + b_{d-4}\frac{H^{d-4}}{\delta^{d-4}} + \cdots + \begin{cases} b_1 \frac{H}{\delta} + (-1)^{\frac{d-1}{2}} s^{\mathrm{univ}}\,, & (\text{odd } d) \\ b_2 \frac{H^2}{\delta^2} + (-1)^{\frac{d-2}{2}} s^{\mathrm{univ}} \log\left(\frac{H}{\delta}\right) + b_0\,, & (\text{even } d) \end{cases}$$

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Coefficients $b_{d-2}, \ldots, b_1$ are "non-universal": they are not well-defined in the continuum. They are "local" in the sense that they come from short-range correlations across ∂V.

Universal terms: even dimensions

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In even dimensions, the universal term is logarithmic and s^{univ} is given by a linear combination of local integrals over ∂V weighted by theory-dependent coefficients which can be shown to coincide with the trace-anomaly charges,

$$\langle T_\mu^\mu \rangle = -2(-)^{d/2} A \mathcal{X}_d + \sum_n B_n I_n \, .$$

Universal terms: even dimensions

For instance, in $d = 4$:

$$\langle T^\mu_\mu \rangle = -\frac{a}{16\pi^2}\mathcal{X}_4 + \frac{c}{16\pi^2} C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$

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For unitary CFTs, a and c constrained to the range:

$$\frac{1}{3} \leq \frac{a}{c} \leq \frac{31}{18}\,.$$

Universal terms: even dimensions

a and c can be isolated by considering entangling surfaces corresponding to spheres and cylinders, respectively,

$$S^{(4)}_{\rm EE}|_{\rm sphere} \supset -4a \log(R/\delta)\,, \quad S^{(4)}_{\rm EE}|_{\rm cylinder} \supset -\frac{c}{2}\frac{L}{R}\log(R/\delta)\,,$$

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For comparison, in $d = 6$ there are three "B-type" charges, B_1, B_2, B_3, besides the "A-type" one.

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In F, dependence on geometric details of V and dependence on the details of the CFT are no longer disentangled from each other. In $d = 5, 7, \ldots$ similar story: for s^{univ} for $\partial V = \mathbb{S}^{d-2}$ equals free energy on $\mathbb{S}^d$.

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non-universal and local; universal and local;
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No one has ever payed much attention to the constant coefficient b_0 appearing for even-dimensional theories. Just like $b_{d-2}, \ldots, b_1$, this is a non-universal piece. However, all this pollution has a local origin and b_0 also contains a universal non-local part which does not depend on the regulator details...

Singular regions

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$$S_{\rm EE}^{(3)}|_{\rm corner} = b_1 \frac{H}{\delta} - a^{(3)}(\Omega) \log\left(\frac{H}{\delta}\right) + \tilde{b}_0 \, ,$$

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The dependence of $a^{(3)}(\Omega)$ on the opening angle changes (apparently rather drastically) from one CFT to another.

Singular regions

Some general properties:

$$a^{(3)}(\pi + \Omega) = a^{(3)}(\pi - \Omega)\,, \quad a^{(3)}(\Omega) \geq 0\,,$$

$$\partial_\Omega a^{(3)}(\Omega) \leq 0\,, \quad \partial_\Omega^2 a^{(3)}(\Omega) \geq -\frac{\partial_\Omega a^{(3)}(\Omega)}{\sin\Omega} \quad \text{for} \quad \Omega \in [0, \pi]\,.$$

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In the very-sharp and almost-smooth limits, the function behaves as

$$a^{(3)}(\Omega\simeq 0)=\frac{k}{\Omega}+\mathcal{O}(\Omega)\,,\quad a^{(3)}(\Omega\simeq\pi)=\sigma\cdot(\Omega-\pi)^2+\mathcal{O}(\Omega-\pi)^4\,.$$

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k is a constant which coincides with the universal coefficient corresponding to a slab region for general theories.

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Leading coefficient in almost-smooth regime, σ, is related to the stress-energy tensor two-point function coefficient* C_T through

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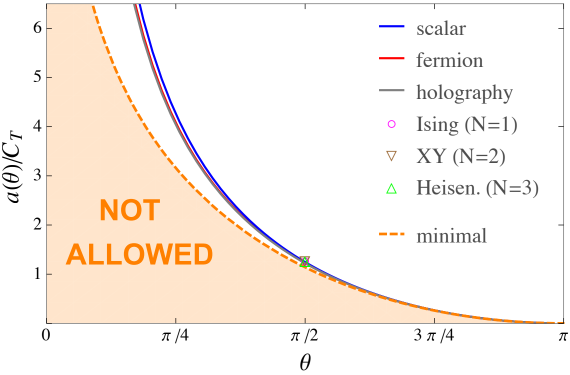
*For general CFTs in d dimensions, the stress-tensor correlator behaves as $\langle T_{ab}(x) T_{cd}(0) \rangle = C_T I_{ab,cd}(x)/|x|^{2d}$ where $I_{ab,cd}(x)$ is a fixed tensorial structure, and the only theory-dependence appears through C_T.

[In $d = 4$, C_T is proportional to the trace-anomaly coefficient c]

Singular regions



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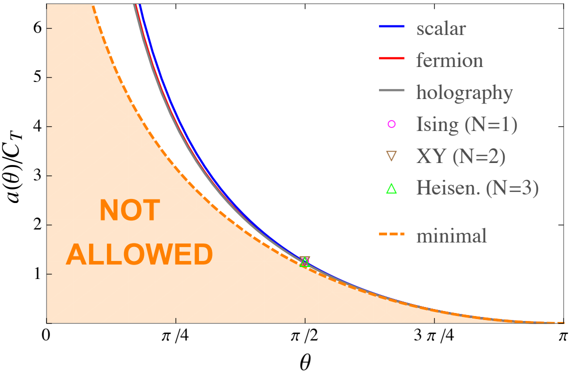

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Lower bound on $a^{(3)}(\Omega)$ valid for general CFTs

$$a^{(3)}(\Omega) \geq \mathfrak{a}_{\min}(\Omega)\,, \quad \text{where} \quad \mathfrak{a}_{\min}(\Omega) \equiv \frac{\pi^2 C_T}{3} \log[1/\sin(\Omega/2)]$$

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Similar logarithmic enhancement of universal term

$$S_{\mathrm{EE}}^{(4)}|_{\mathrm{cone}} = b_2 \frac{H^2}{\delta^2} - a^{(4)}(\Omega) \log^2 (H/\delta) + \tilde{b}_0 \log (H/\delta) + b_0 ,$$

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but now

$$a^{(4)}(\Omega) = \frac{c}{4} \cdot \frac{\cos^2 \Omega}{\sin \Omega}$$

for all CFTs.

Only dependence on the theory under consideration appears through the charge c

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$$S_{\rm EE}^{(d)(\rm even)}\big|_{\rm (hyper)cone} \supset (-1)^{\frac{d-2}{2}} a^{(d)}(\Omega) \log^2 (H/\delta)$$

where

$$a^{(d)}(\Omega) = \frac{\cos^2\Omega}{\sin\Omega} \sum_{j=0}^{\frac{d-4}{2}} \left[\gamma_j^{(d)} \cos(2j\Omega)\right]$$

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Still some degree of universality in almost-smooth limit $\Leftrightarrow$ C_T

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- Polyhedral corner of opening angles $\theta_1, \theta_2, \dots, \theta_j$

$$S_{\rm EE}^{(4)}|_{\rm polyh.} = b_2 \frac{H^2}{\delta^2} - w_1 \frac{H}{\delta} + v(\theta_1, \theta_2, \cdots, \theta_j) \log\left(\frac{H}{\delta}\right) + b_0$$

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- Infinite wedge of opening angle Ω

$$S_{\rm EE}^{(4)}|_{\rm wedge} = b_2 \frac{H^2}{\delta^2} - f(\Omega) \frac{H}{\delta} + b_0$$

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[In general, it is possible both $I_3(A;B,C) \geq 0$ and $I_3(A;B,C) \leq 0$].

Imposing $I_3(A;B,C) = 0$ + some physically reasonable requirements such as causality and Poincaré invariance, strongly restricts the form of EE and mutual information in general d.

The “extensive mutual information” model

The result defines the EMI model. Its EE is given by

$$S_{\rm EE}^{\rm EMI} = \kappa_{(d)} \int_{\partial A} {\rm d}^{d-2}\sigma_1 \int_{\partial A} {\rm d}^{d-2}\sigma_2 \, \frac{n^i(x_1) n^j(x_2) \delta_{ij}}{|x_1 - x_2|^{2(d-2)}}$$

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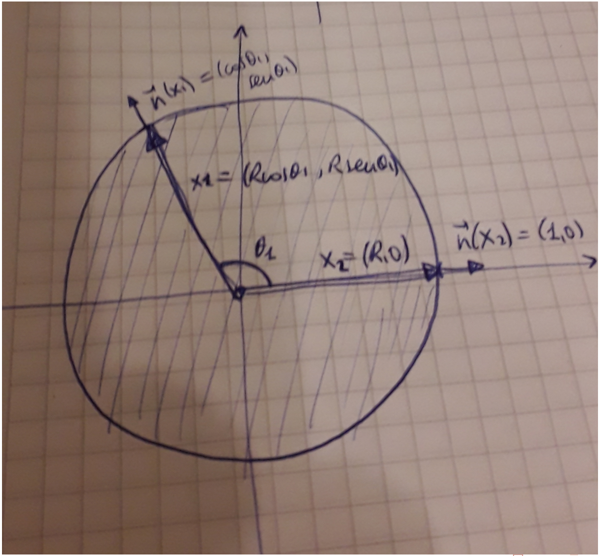
Regardless of this, the model respects all general principles of EE and is a useful tool for understanding various features. Computationally, even simpler than Ryu-Takayanagi formula.

EE of a disk in $d = 3$



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1) allow only for angles larger than δ/R;

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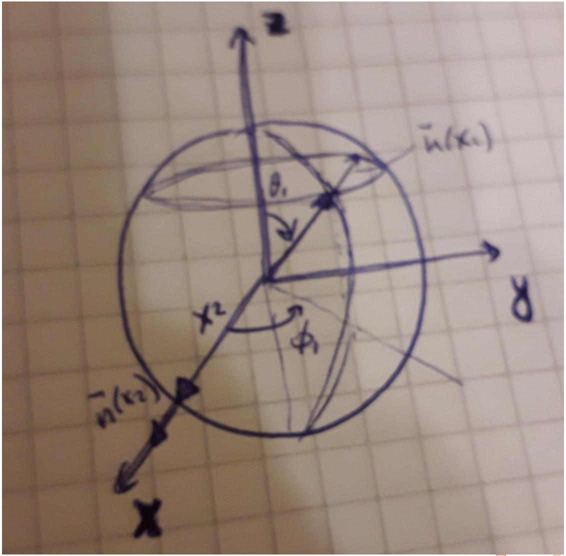
Universal piece unchanged, $F = 2\pi^2\kappa_{(3)}$, whereas b_1 depends on regulator.

EE across sphere in $d = 4$



EE across sphere in $d = 4$

Radius-R spherical entangling surface.

EE across sphere in $d = 4$

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Regulators:
1) allow only for angles larger than δ/R;
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First regulator:

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$$= 4\pi^2 \kappa_{(4)} \frac{R^2}{\delta^2} - 4\pi^2 \kappa_{(4)} \log\left(R/\delta\right) - \pi^2 \kappa_{(4)} \left[\frac{2}{3} + 4\log 2\right].$$

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Would-be trace-anomaly coefficient $a_{\mathrm{EMI}} = \pi^2 \kappa_{(4)}$. Non-universal piece b_2 depends on regulator. b_0 has a "piece" which remains the same and one which varies*.

EE across cylinder in $d = 4$

Cylinder of radius R.

EE across cylinder in $d = 4$

Cylinder of radius R. In cylindrical coordinates we can write $x_1 = (R\cos\phi_1, R\sin\phi_1, z_1)$, $\vec{n}(x_1) = (\cos\phi_1, \sin\phi_1, 0)$, and $x_2 = (R, 0, z_2)$, $\vec{n}(x_2) = (1, 0, 0)$, where we already took advantage of the circular symmetry of the surface.

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We can set $z_1 = 0$ and regulate $\int_{-L/2}^{L/2} \mathrm{d}z_1 = L$.

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We can set $z_1 = 0$ and regulate $\int_{-L/2}^{L/2} \mathrm{d}z_1 = L$. In the resulting expression, we can perform the integral over z_2, which requires no regulation,

$$S_{\mathrm{EE}}^{\mathrm{EMI}} = 2\pi R^2 \cdot L \cdot \kappa_{(4)} \cdot \int \mathrm{d}\phi_1 \frac{\pi\cos\phi_1}{16R^3\sin^3(\phi_1/2)} \,.$$

EE across cylinder in $d = 4$

First regulator:

$$S_{\mathrm{EE}}^{\mathrm{EMI}} = \pi^2 \kappa_{(4)} \frac{RL}{\delta^2} - \frac{3\pi^2 \kappa_{(4)}}{4} \frac{L}{R} \log\left(R/\delta\right) - \frac{\pi^2 \kappa_{(4)}}{2} \left[\frac{1}{12} + 3\log 2\right] \frac{L}{R}\,.$$

Second regulator:

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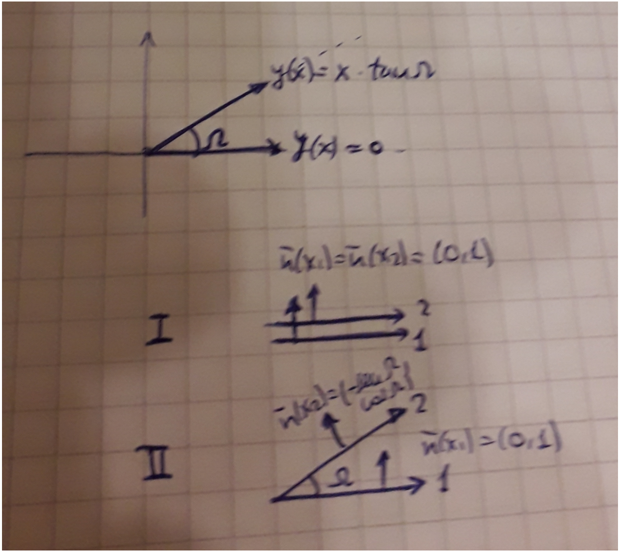
EE for a corner in $d = 3$



EE for a corner in $d = 3$

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$$s_{\rm I} = \kappa_{(3)} \int \mathrm{d}X_1 \int \mathrm{d}X_2 \frac{1}{(X_1 - X_2)^2} \,.$$

Second contribution: $\vec{n}(X_1) = (0, -1)$, $\vec{n}(X_2) = (-\sin\Omega, \cos\Omega)$, $\mathrm{d}\sigma_1 = \mathrm{d}X_1$, $\mathrm{d}\sigma_2 = \mathrm{d}X_2 / \cos\Omega$,

$$s_{\rm II} = -\kappa_{(3)} \int \mathrm{d}X_1 \int \mathrm{d}X_2 \frac{1}{(X_1 - X_2)^2 + \tan^2\Omega X_2^2} \,.$$

EE for a corner in $d = 3$

Possible regulator:

$$\int \mathrm{d}X_1 \to \int_\delta^H \mathrm{d}X_1\,, \quad \int \mathrm{d}X_2 \to \left[\int_0^{x_1-\delta} \mathrm{d}X_2 + \int_{x_1+\delta}^{\infty} \mathrm{d}X_2\right].$$

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Final result:

$$S_{\mathrm{EE}}^{\mathrm{EMI}} = \frac{4\kappa_{(3)}H}{\delta} - a_{\mathrm{EMI}}^{(3)}(\Omega)\log(H/\delta) + \mathcal{O}(\delta^0)\,,$$

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$$a_{\mathrm{EMI}}^{(3)}(\Omega) = 2\kappa_{(3)}[1 + (\pi - \Omega)\cot\Omega]\,.$$

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This satisfies all general properties for a decent EE corner function. In particular, in the very-sharp and almost-smooth limits,

$$a_{\mathrm{EMI}}^{(3)}(\Omega \simeq 0) = \frac{k}{\Omega} + \mathcal{O}(\Omega)\,, \quad a_{\mathrm{EMI}}^{(3)}(\Omega \simeq \pi) = \sigma \cdot (\Omega - \pi)^2 + \mathcal{O}(\Omega - \pi)^4\,.$$

$k_{\mathrm{EMI}} = 2\pi\kappa_{(3)}$ and $\sigma_{\mathrm{EMI}} = 2\kappa_{(3)}/3$.

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$k_{\mathrm{EMI}} = 2\pi\kappa_{(3)}$ and $\sigma_{\mathrm{EMI}} = 2\kappa_{(3)}/3$.Value of the would-be stress-tensor two-point function charge $C_T^{\mathrm{EMI}} = 16\kappa_{(3)}/\pi^2$.

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$$S_{\mathrm{EE}}^{\mathrm{EMI}} = \frac{2\pi\kappa_{(4)}}{\sin^2\Omega}\int\rho_1\mathrm{d}\rho_1\int\rho_2\mathrm{d}\rho_2\int_0^{2\pi}\mathrm{d}\phi_1\frac{[\cos^2\Omega\cos\phi_1+\sin^2\Omega]}{[a-b\cos\phi_1]^2},$$

where $a \equiv \rho_1^2 + {\rho_2}^2 + (\rho_1-\rho_2)^2/\tan^2\Omega$, $b \equiv 2\rho_1\rho_2$.

EE across a cone in $d = 4$

Performing the angular integrals, we are left with

$$S_{\text{EE}}^{\text{EMI}} = \frac{4\pi^2 \kappa_{(4)}}{\sin^2 \Omega} \left[\cos^2 \Omega \, s_{\text{I}} + \sin^2 \Omega \, s_{\text{II}} \right] ,$$

where

$$s_{\text{I}} = \int \int \frac{b \, \rho_1 \rho_2}{(a^2 - b^2)^{3/2}} \mathrm{d}\rho_1 \mathrm{d}\rho_2 \, , \quad s_{\text{II}} = \int \int \frac{a \, \rho_1 \rho_2}{(a^2 - b^2)^{3/2}} \mathrm{d}\rho_1 \mathrm{d}\rho_2 \, .$$

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Putting pieces together, we are left with

$$S_{\mathrm{EE}}^{\mathrm{EMI}} \supset -\frac{3\pi^2 \kappa_{(4)}}{8} \cdot \frac{\cos^2 \Omega}{\sin \Omega} \log^2 (H/\delta) \, .$$

Angular dependence is the expected one.

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Angular dependence is the expected one. Also, $c_{\rm EMI} = 3\pi^2\kappa_{(4)}/2$, which matches the cylinder result.

Bonus track: EE across a deformed sphere

Parametrizing the deformed sphere as

$$r(\Omega_{d-2}) = 1 + \varepsilon \sum_{\ell,m_1,\ldots,m_{d-3}} a_{\ell,m_1,\ldots,m_{d-3}} Y_{\ell,m_1,\ldots,m_{d-3}}(\Omega_{d-2}) \,,$$

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where the $Y_{\ell,m_1,\dots,m_{d-3}}(\Omega_{d-2})$ are real (hyper)spherical harmonics, for general CFTs, the universal contribution s^{univ} takes the form

$$s^{\text{univ}} = s_0^{\text{univ}} + \varepsilon^2 s_2^{\text{univ}} + \mathcal{O}(\varepsilon^3) \,,$$

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Sphere is a local extremum of the EE. Leading correction controlled by stress-tensor two-point function charge C_T.

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No one has verified this result in the EMI...

It from Qubit
Simons Collaboration on
Quantum Fields, Gravity and Information

Instituto
Balseiro