BLACK HOLE THERMODYNAMICS

LECTURE 1

Outline

I. BH Laws: Traditional view

- ☆ Geometry: Kerr Black hole in 4D
- ☆ Komar integrals: Mass & angular momentum in GR
- ☆ Killing horizons
- ☆ Smarr relation & 1st law

II. BH Thermo:

- ☆ Euclidean methods

III. BH Entropy. Modern View

- ☆ Noether Charges in GR
- ☆ Iyer-Wald.

Overview

⇝ Our lab will Black Holes, quantify & identify repercussions of GR.

Two important lessons will be:

- ☆ GRAVITY KNOWS ABOUT THERMODYNAMICS
  (upgrade: GRAVITY KNOWS QUANTUM info)

- ☆ GRAVITY IS HOLOGRAPHIC
  (more evident & precise in AdS, expected to be true general)

PRIOR KNOWLEDGE

1) Basic Riemannian Geometry: ∇_μ, $\Gamma^\alpha_{\mu\nu}$, $R^\alpha{}_{\mu\nu\rho}$, geodesic, ...

2) Lie derivatives, Killing symmetries.

3) Integration on manifolds

4) Einstein–Hilbert action $\rightarrow$ e.o.m (equations of motion)

5) Basic Properties of Schw solution

Conventions:

$$c = 1 \qquad G = 1 \qquad \hbar = 1$$

$$\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$$

REFERENCES

BOOKS

Carroll, "Spacetime & Geometry" $\longrightarrow$ gr-qc/9712019

Wald, "General Relativity"

Poisson, "A relativist's toolkit"

ONLINE

gr-qc/9712019

Townsend, gr-qc/9707012

Compere & Fiorucci, 1801.07064

Also other online lectures

http://www.damtp.cam.ac.uk/user/tong/gr/gr.pdf ⇝ David Tong

http://www.damtp.cam.ac.uk/user/hsr1000/black_holes_lectures_2020.pdf ⇝ Harvey Reall

http://www.blau.itp.unibe.ch/Lecturenotes.html ⇝ Mathias Blau

I. BLACK HOLE LAWS

The LAWS OF BH MECHANICS

BARDEEN, CARTER, HAWKING (1973)

0th Law: the surface gravity κ is constant over the event horizon of a stationary black hole

1st Law: $$dM = \frac{\kappa}{8\pi} dA_H + \Omega_H dJ + \Phi_H dQ$$

2nd Law: $$dA_H \geq 0$$

3rd Law: κ cannot be reduced to zero by a finite number of operations

Goal: BH mechanics $\Longrightarrow$ BH thermodynamics

$$T_H = \frac{\kappa}{2\pi}$$: Hawking Temp

$$S_{BH} = \frac{c^3}{\hbar G} \frac{A_H}{4}$$: Bekenstein-Hawking entropy

$$l_P = \sqrt{\frac{\hbar G}{c^3}}$$: Planck length

Short Preamble: Schwarzschild Black Hole
Kerr Black Hole

Recommended reading: * Townsend lectures
* Caroll ch 5 & 6

Schw Solution

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

M: constant

A few facts

1) Birkhoff's Thm: Schw metric is the unique solution with spherical symmetry of (3+1)-D Einstein's eqn in the vacuum
(a spherical mass distribution cannot emit gravitational waves)

This unique position has placed Schw at the core of several test of General Relativity

2) Singularity: At $r=0$ the Schw geometry breaks down. It contaunts a curvature singularity evident from evaluating curvature squared

Question
$R_{\mu\nu}R^{\mu\nu} \overset{?}{=} 0$

$$\underbrace{R_{\mu\nu\alpha\rho}R^{\mu\nu\alpha\rho}}_{\text{Kretschmann invariant}} = \frac{48M^2}{r^6}$$

↳ lost of predictability of GR. Spacetime breaks down.

3) Horizon: surface $r=2M$ defines a null hypersurface that "divides" the spacetime into two.
Regular portion of the geometry

⇒ Coordinates systems for Schw

a) Eddington – Finkelstein coordinates

$(v, r, \theta, \phi) \longrightarrow$ In going

$(u, r, \theta, \phi) \longrightarrow$ outgoing

$$v = t + r^*$$
$$u = t - r^*$$

$$dr^* = \frac{dr}{1-\frac{2M}{r}} \Rightarrow r^* = r + 2M\ln\left(\frac{r}{2M} - 1\right)$$

b) Kruskal – Szekres coordinates

$$V = e^{v/4M}$$
$$U = -e^{-u/4M}$$

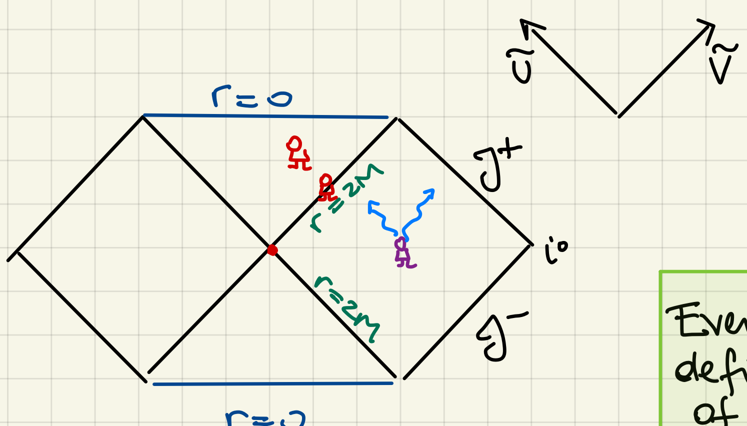
$$UV = \left(1 - \frac{r}{2M}\right) e^{r/2M}$$

$$ds^2 = -\frac{32M^3}{r} e^{-r/2M}\, dU\, dV + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

c) Penrose (conformal) diagram

$$V = \tan \tilde{U}$$
$$U = \tan \tilde{V}$$

$$-\pi/2 < \tilde{U}, \tilde{V} < \pi/2$$



•: Bifurcation 2-sphere $(U = 0 = V)$

Event Horizon: future Horizon H^+ is defined as the bndy of the causal past of future null infinity, $\partial J^-(\mathcal{J}^+)$

H^+ is a null surface.

Kerr solution

- Discovered in 1963 (schw 1916)
- Solution to vacuum Einstein Eqn which is stationary & axially symmetric

Metric in Boyer-Lindquist coordinates

$$ds^2 = -\frac{\rho^2\Delta}{\Sigma}dt^2 + \frac{\Sigma}{\rho^2}\sin^2\theta\,(d\phi - \omega dt)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Delta = r^2 - 2Mr + a^2$$

$$\Sigma = (r^2+a^2)^2 - a^2\Delta\sin^2\theta$$

$$\omega = \frac{2Mar}{\Sigma}$$ ⇝ a: constant $J = aM$: angular momentum

Properties:

1) $k = \partial_t$ $m = \partial_\phi$ are Killing vectors

k ⇝ Stationary. m ⇝ axial symm

2) Singularity: $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} = \frac{4M^2(r^2-a^2\cos^2\theta)(\rho^4 - 16a^2r^2\cos^2\theta)}{\rho^{12}}$

$$\Rightarrow \rho = 0 = r^2 + a^2\cos^2\theta$$

3) ZAMO: zero-angular-momentum observer.

U^α: four velocity

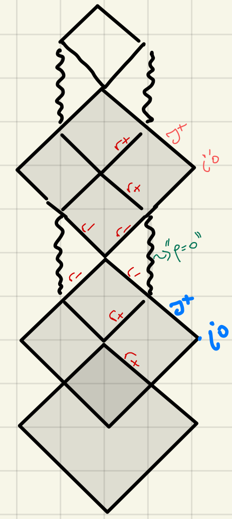
L : angular mom observer Ω: angular velocity

$L \equiv U_\alpha m^\alpha$ Zamo: $L = 0 \Rightarrow g_{\phi t}\dot{t} + g_{\phi\phi}\dot{\phi} = 0$

$\Omega \equiv d\phi/dt$

For a Zamo

$$\Omega = \frac{d\phi}{dt} = \omega = -\frac{g_{\phi t}}{g_{\phi\phi}} \quad : \text{angular velocity}$$



4) Horizons:

$$g_{rr} = \frac{\rho^2}{\Delta} \qquad \Delta = r^2 - 2Mr + a^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$g^{rr} = \frac{\Delta}{\rho^2} \longrightarrow \text{It goes to zero when } \Delta = 0$$

$$r^2 - 2Mr + a^2 = 0$$

$$r_+ = M + \sqrt{M^2 - a^2} \quad : \text{outer (Event) Horizon}$$

$$r_- = M - \sqrt{M^2 - a^2} \quad : \text{inner Horizon.}$$

Cauchy.

null vector at outer horizon

$$\chi = \partial_t + \Omega_H \partial_\phi \qquad \Omega_H = \omega(r_+) = \frac{a}{r_+^2 + a^2}$$

angular velocity of BH

$$\chi^2\Big|_{r=r_+} = 0$$

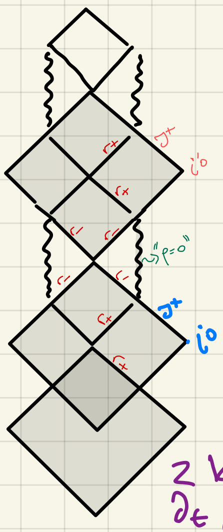
Bounds:

$M \geqslant |a|$: singularity at $\rho = 0$ behind horizon.

$(M^2 \geqslant |J|)$

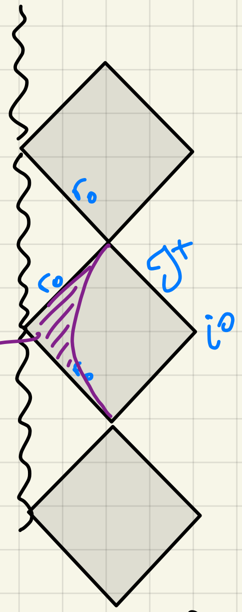
$M = a$: Extremal, $r_+ = r_-$ saturation of bound

↳ degenerate.



2 KV
$\partial_t , \partial_\phi$

non-extremal



$AdS_2 \times S^2$

$\partial_t \to SO(2,1)$

$1 \to 3$

Extremal

$r_0 = r_+ = r_-$