BLACK HOLE THERMODYNAMICS

Lecture 4

Euclidean Methods

BH Mechanics	BH Thermodynamics
κ : surface gravity constant on $\mathcal{H}^+$	$\kappa_+ \sim$ temp canonical ensemble (temp constant in thermal eq.)
M, J, Q conserved quantities.	$E \sim M$ energy system
$dM = \frac{\kappa}{8\pi} dA_H + \Omega_H dJ + \Phi_H dQ$	$dE = T dS + \sum_i \mu_i dN_i$
$\delta A_H \geq 0$ area increase	$dS \geq 0$ entropy increase

We will find that

$$T_H = \frac{\hbar}{2\pi} \kappa \rightarrow \text{Hawking Temperature}$$

$$\underbrace{S_{BH} = \frac{A_H}{4 l_p^2}}_{\text{Einstein-Hilbert } \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}\right)} \rightarrow \text{Bekenstein-Hawking entropy formula.}$$

Euc Schw BH:

Schw:

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$\downarrow$ Euc Schw: $\quad t = -i t_E \quad$ Wick Rotation

$$ds_E^2 = \left(1-\frac{2M}{r}\right)dt_E^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Zoom near $r = 2M$, define new coords

$$r = \frac{\rho^2}{8M} + 2M \qquad dr = \frac{\rho}{4M}\,d\rho$$

look at behaviour as $\rho \to 0$ $\quad (r \to 2M)$

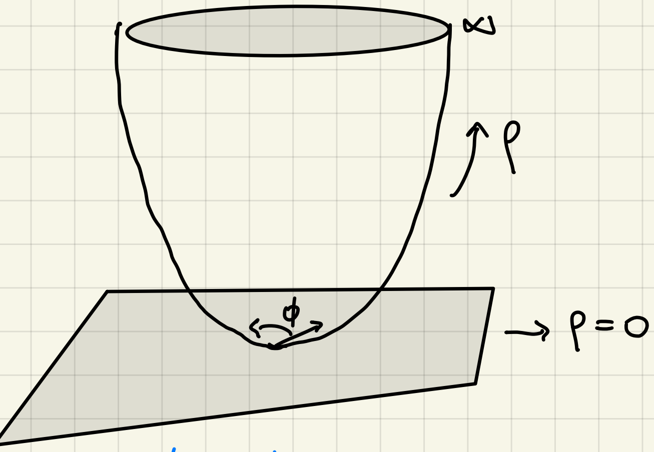
$$1 - \frac{2M}{r} = 1 - \frac{1}{1+\frac{\rho^2}{16M^2}} = \frac{\rho^2}{16M^2} + O(\rho^4)$$

$$ds_E^2 \underset{\rho\to 0}{=} \frac{\rho^2}{16M^2}dt_E^2 + \frac{1}{\cancel{\rho^2/16M^2}}\,\cancel{\frac{\rho^2}{16M^2}}\,d\rho^2 + (2M)^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \dots$$

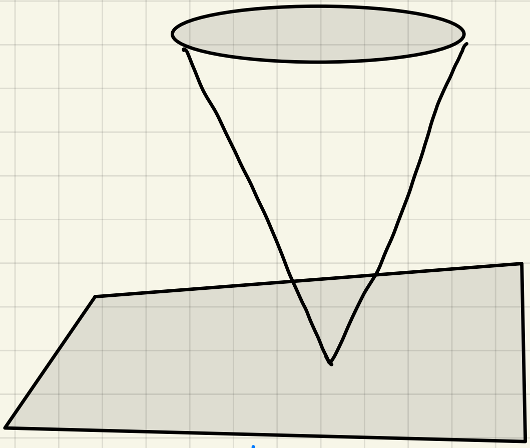
$$= \underbrace{\frac{\rho^2}{16M^2}dt_E^2 + d\rho^2}_{\mathbb{R}^2 \text{ in polar coords}} + \underbrace{(2M)^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{S^2} + \dots$$

$$\rho^2\, d\phi^2 + d\rho^2 \Rightarrow \phi = \frac{t_E}{4M}$$

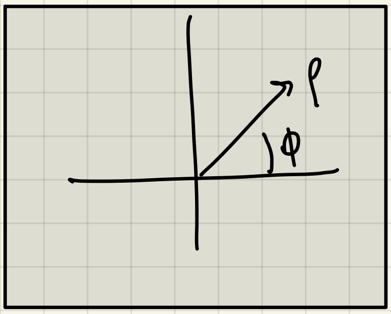
$$\rho \in [0, \infty) \qquad \phi \in [0, 2\pi)$$



$\phi \sim \phi + 2\pi$



if $\phi \not\sim \phi + 2\pi$



If we want a regular Euclidean geometry for Schw

$$\phi \sim \phi + 2\pi$$

$$\Rightarrow t_E \sim t_E + 2\pi\,(4M)$$

QFT interpretation

What does it means for Euc time to be periodic ?

$$t_E \sim t_E + \beta$$

From a path integral we would have

$$Z[\Phi_i, \Phi_f] = \int_{\Phi(0)=\Phi_i}^{\Phi(t_f)=\Phi_f} D\Phi \; e^{\frac{i S[\Phi]}{\hbar}} = \langle \Phi_i | e^{-\frac{itH}{\hbar}} | \Phi_f \rangle$$

$$i S[\Phi] = i \int dt\, d^3x\, L \underset{t = i t_E}{=} - S_E = - \int dt_E\, d^3x\, L$$

$$e^{-itH/\hbar} \rightarrow e^{-t_E H/\hbar}$$

$$\Rightarrow t_E \sim t_E + \beta$$

$$Z[\beta] = \mathrm{Tr}\left(e^{-\beta H}\right)$$: partition function @ finite temp

for ang momentum $$Z[\beta,\Omega] = \mathrm{Tr}\left(e^{-\beta(H-\Omega J)}\right) \qquad T = 1/\beta$$

Interpretation implemented for Euc Schw

$$t_E \sim t_E + 2\pi(4M)$$

$$\Rightarrow \beta = 2\pi \cdot \underbrace{4M}_{\kappa^{-1}} \Rightarrow T_H = \frac{1}{8\pi M}$$

$$\boxed{T_H = \frac{\kappa}{2\pi}}$$

Rindler spacetime: back to Lorentzian

$$ds_E = \underbrace{\frac{\rho^2}{16M^2}\,dt_E^2 + d\rho^2}_{\mathbb{R}^2 \text{ in polar coords}} + \underbrace{(2M)^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{S^2} + \dots$$

$\downarrow$ back to Lorentzian

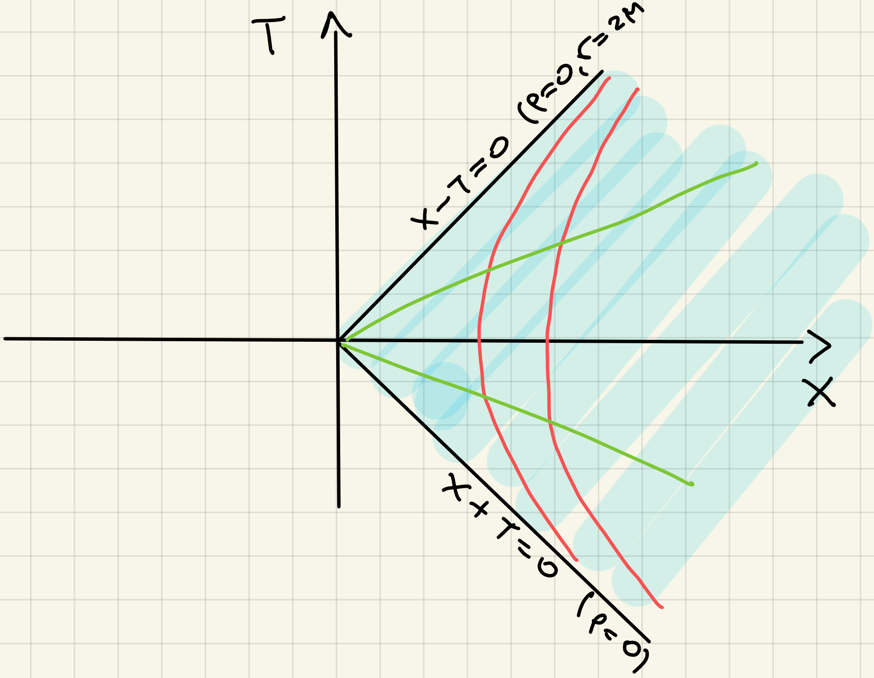
$$= \underbrace{-\frac{\rho^2}{16M^2}\,dt^2 + d\rho^2}_{\text{Rindler}} + (2M)^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$T = \rho\sinh\left(\frac{t}{4M}\right) \qquad X = \rho\cosh\left(\frac{t}{4M}\right)$$

$$X^2 - T^2 = \rho^2$$

$$= -dT^2 + dX^2 + \dots$$

$$\frac{T}{X} = \tanh\left(t/4M\right)$$



Rindler: acc obs
in Minkowski

acc : $\frac{1}{4M} = \kappa$