Anomaly-free Abelian gauge symmetries

with radiative Dirac neutrino masses

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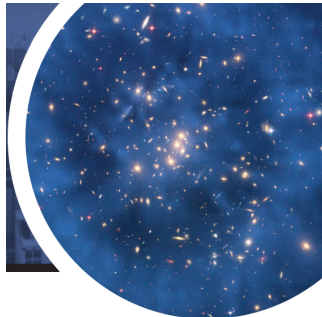

Active Symmetry

$U(1)_X$: Multi-tev : Z'_μ

SM X-Charges: $f_R \to f$

Field	u_R	d_R	$(Q)^\dagger$	e_R	$(L)^\dagger$
Symbol	u	d	Q	e	L

Dark Symmetry

$U(1)_D$: Dark photon A'_μ

N chiral Fields (CF)

$N = N' + 3$

$\oplus$ N' CF:

$$\psi_1, \psi_2 \cdots, \psi_{N'}, \psi_{N'+1}, \psi_{N'+2}, \psi_{N'+3}$$

$$\psi_1, \psi_2, \cdots, \psi_{N'}, \psi_{N'+1}\ \psi_{N'+2}\ \psi_{N'+3}$$

$m = c + 2L \rightarrow f = f(m, L)$ | $N = N' + 3$

χ-charges $(n_1, n_2 \cdots, n_{N'}, m, m, m)$

D-charges $(n_1, n_2 \cdots, n_{N'}, n_{N'+1}, n_{N'+2}, n_{N'+3})$

$$\sum_{\rho=1}^{N} n_\rho = 0 \qquad \sum_{\rho=1}^{N} n_\rho^3 = 0$$

Example: $N = 6 \rightarrow$

$\vec{n} = (-4, -4, 5, 1, 1, 1)$

$$\sum_{g=1}^{N} n_g = 0 \quad , \quad \sum_{g=1}^{N} n_g^3 = 0$$

$\vec{X} = (n_1, n_2, n_3, m, m, m)$

$U(1)_X$

Field	Symbol	$f(m,L)$	X	$B-L$
u_R	u	$\frac{4L}{3} - m$	$\frac{4L}{3} - 1$	$\frac{1}{3}$
d_R	d	$-\frac{2L}{3} + m$	$-\frac{2L}{3} + 1$	$\frac{1}{3}$
$(Q)^t$	Q	$-L/3$	$-L/3$	$-\frac{1}{3}$
e_R	e	$m - 2L$	$1 - 2L$	-1
$(L)^t$	L	L	L	1
H	h	$L - m$	$L - 1$	0
ν_{R1}	n_1	-4	-4	-4
ν_{R2}	n_2	-4	-4	-4
ν_{R3}	n_3	-5	-5	-5

$\vec{D} = (n_1, n_2, n_3, n_4, n_5, n_6)$

$U(1)_D$

Field	Symbol	X
Ψ_1	n_1	-4
Ψ_2	n_2	-4
Ψ_3	n_3	-5
Ψ_4	n_4	1
Ψ_5	n_5	1
Ψ_6	n_6	1

NEW

$N' > 3$

From: arXiv:1905.13279 [PRL] Costa, *et al*

Let a vector $\boldsymbol{z}$ with N non-zero integer entries such that

$$\sum_{i=1}^{N} z_i = 0\,, \qquad \sum_{i=1}^{N} z_i^3 = 0\,.$$

We like to build this set of N integers from two subsets $\boldsymbol{\ell}$ and $\boldsymbol{k}$ with sizes

$$\dim(\boldsymbol{\ell}) = \begin{cases} \alpha = \frac{N}{2} - 1\,, & \text{if } N \text{ even} \\ \beta = \frac{N-3}{2}\,, & \text{if } N \text{ odd} \end{cases} \quad ; \qquad \dim(\boldsymbol{k}) = \begin{cases} \alpha = \frac{N}{2} - 1\,, & \text{if } N \text{ even} \\ \beta + 1 = \frac{N-1}{2}\,, & \text{if } N \text{ odd} \end{cases}$$

- N even: Consider the following two vector-like examples of $\boldsymbol{z}$ such that

$$\boldsymbol{x} = (\ell_1, k_1, \ldots, k_\alpha, -\ell_1, -k_1, \ldots, -k_\alpha)$$
$$\boldsymbol{y} = (0, 0, \ell_1, \ldots, \ell_\alpha, -\ell_1, \ldots, -\ell_\alpha)\,.$$

- N odd:

$$\boldsymbol{x} = (0, k_1, \ldots, k_{\beta+1}, -k_1, \ldots, -k_{\beta+1})$$

$$\boldsymbol{y} = (\ell_1, \ldots, \ell_\beta, k_1, 0, -\ell_1, \ldots, -\ell_\beta, -k_1)$$

From any of this, we can build a final $\boldsymbol{z}$ which can includes *chiral* solutions

$$\boldsymbol{x} \oplus \boldsymbol{y} \equiv \left(\sum_{i=1}^{N} x_i y_i^2 \right) \boldsymbol{x} - \left(\sum_{i=1}^{N} x_i^2 y_i \right) \boldsymbol{y}\,.$$

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- N even: Consider the following two vector-like examples of $\boldsymbol{z}$ such that

$$\begin{aligned} \boldsymbol{x} &= (\ell_1, k_1, \ldots, k_\alpha, -\ell_1, -k_1, \ldots, -k_\alpha) \\ \boldsymbol{y} &= (0, 0, \ell_1, \ldots, \ell_\alpha, -\ell_1, \ldots, -\ell_\alpha)\ . \end{aligned}$$

- N odd:

$$\boldsymbol{x} = (0, k_1, \ldots, k_{\beta+1}, -k_1, \ldots, -k_{\beta+1})$$

$$\boldsymbol{y} = (\ell_1, \ldots, \ell_\beta, k_1, 0, -\ell_1, \ldots, -\ell_\beta, -k_1)$$

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https://pypi.org/project/anomalies

anomalies 0.1.4

Latest version

```
pip install anomalies
```

Released: Nov 30, 2020

Navigation

- Project description
- Release history
- Download files

Project links

- Homepage

Statistics

GitHub statistics:

- Stars: 0
- Forks: 1
- Open issues/PRs: 0

View statistics for this project via Libraries.io, or by using our public dataset on Google BigQuery

Meta

License: BSD

Author: restrepo

Maintainers

Anomalies

Implement the anomaly free solution of arXiv:1905.13729 [PRL]:

Obtain a numpy array `z` of `N` integers which satisfy the Diophantine equations

```
>>> z.sum()
0
>>> (z**3).sum()
0
```

The input is two lists `l` and `k` with any `(N-3)/2` and `(N-1)/2` integers for `N` odd, or `N/2-1` and `N/2-1` for `N` even (`N>4`). The function is implemented below under the name: `free(l,k)`

Install

```
$ pip install anomalies
```

USAGE

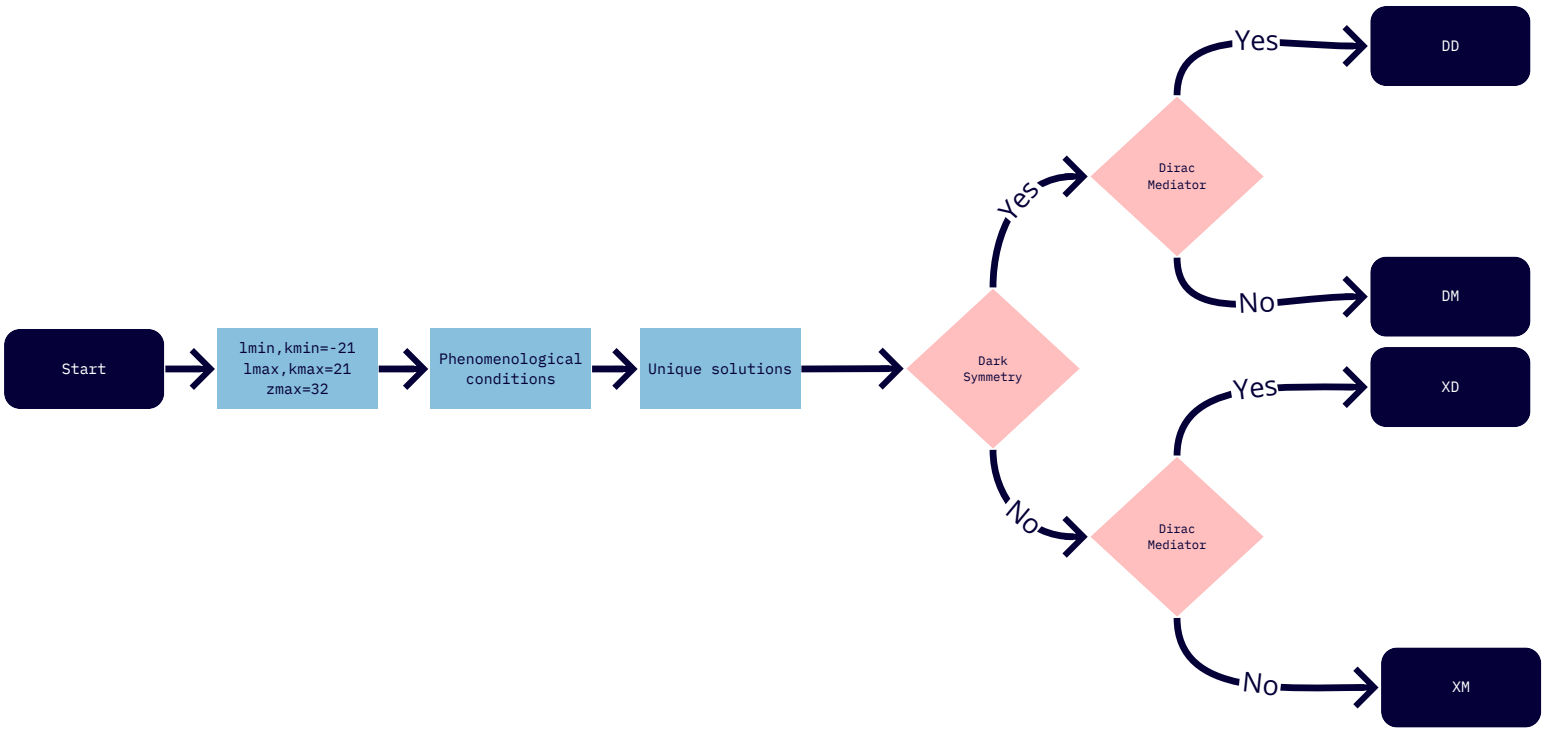
```
>>> from anomalies import anomaly
>>> anomaly.free([-1,1],[4,-2])
array([  3,   3,   3, -12, -12,  15])
>>> anomaly.free.gcd
3
>>> anomaly.free.simplified
array([ 1,  1,  1, -4, -4,  5])
```

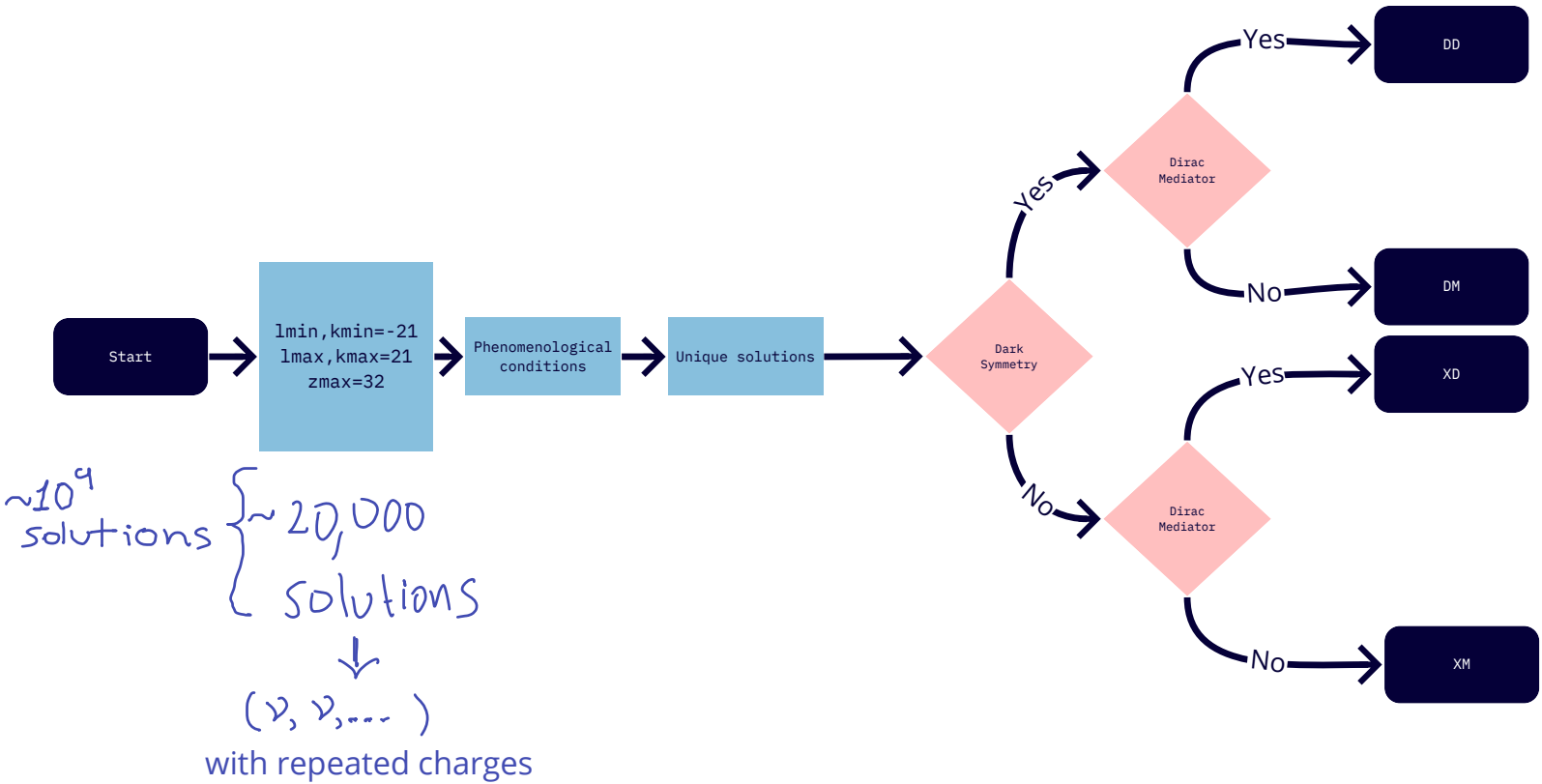
$N=6$

$\alpha=2$

$\vec{l}=(-1,1)$

$\vec{K}=(4,-2)$


Start
lmin,kmin=-21
lmax,kmax=21
zmax=32
Phenomenological conditions
Unique solutions
Dark Symmetry
Yes
No
Dirac Mediator
Yes
No
DD
DM
Dirac Mediator
Yes
No
XD
XM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Unique solutions
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Yes
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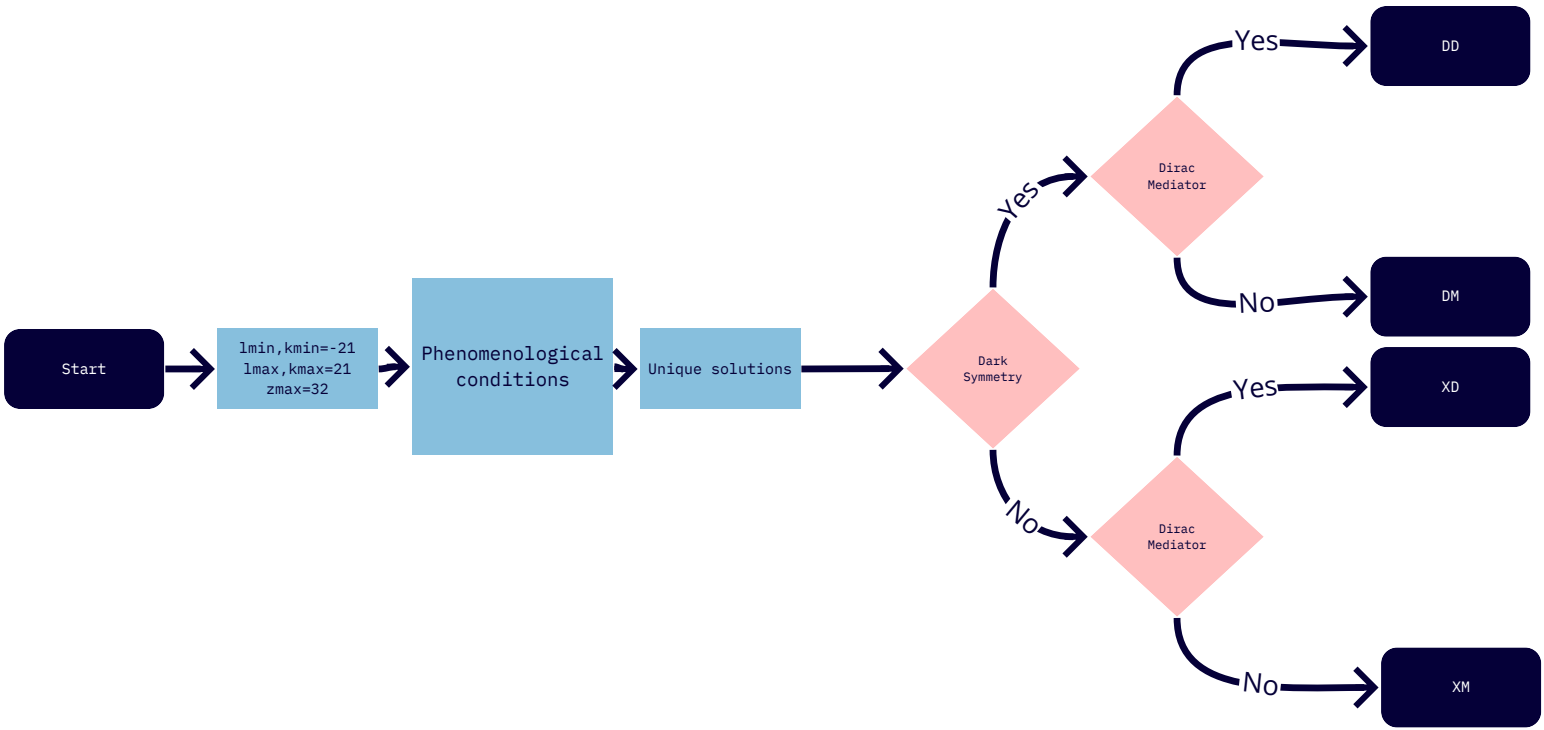
Yes
DD
No
DM
No
Dirac Mediator
Yes
XD
No
XM
~10^9 solutions
~ 20,000 solutions
(ν, ν, ...)
with repeated charges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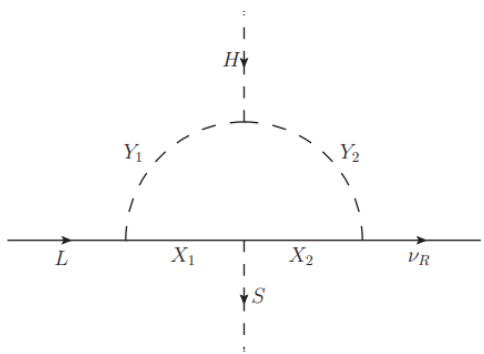
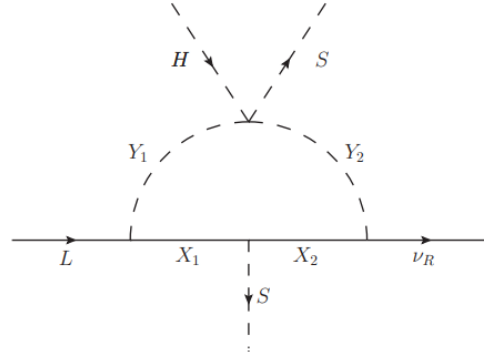
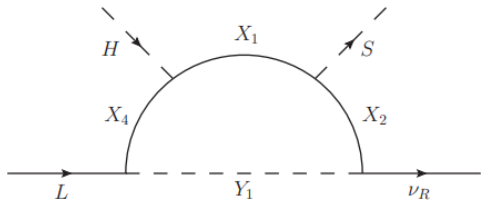


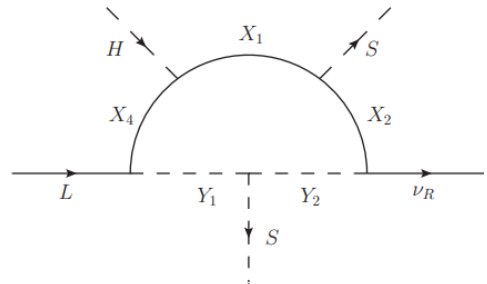
T1-3-E



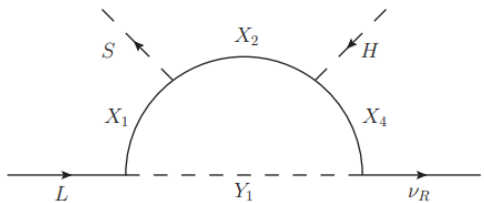
T1-3-E-D-6



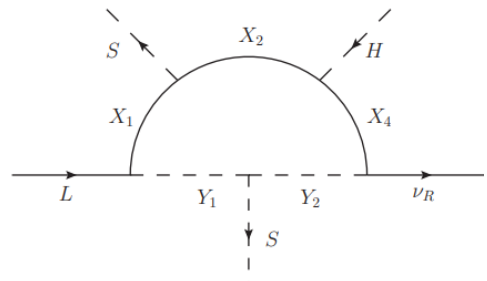
T1-2-A



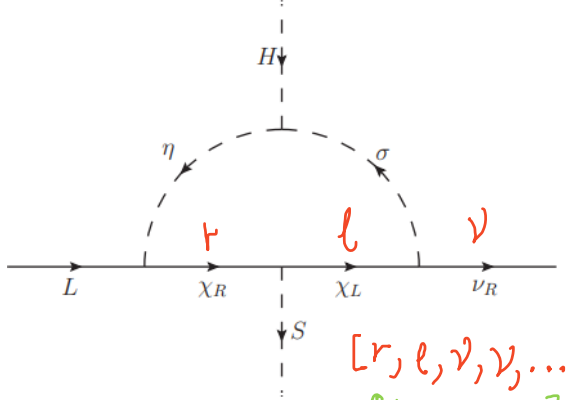
T1-2-A-D-6



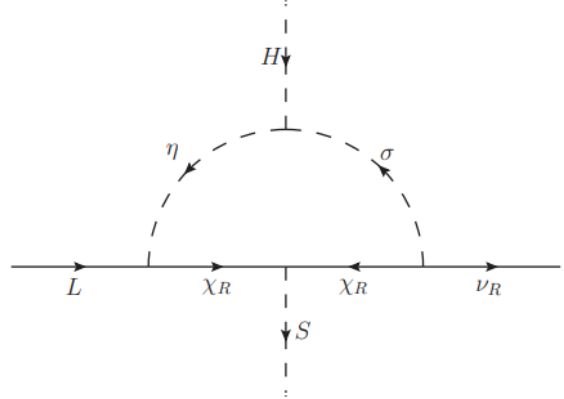
T1-2-B



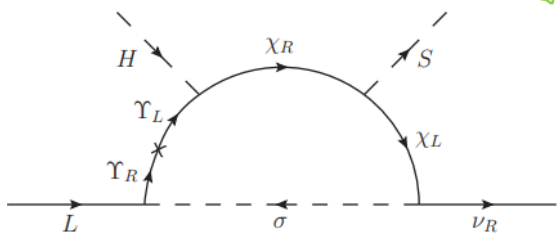
T1-2-B-D-6



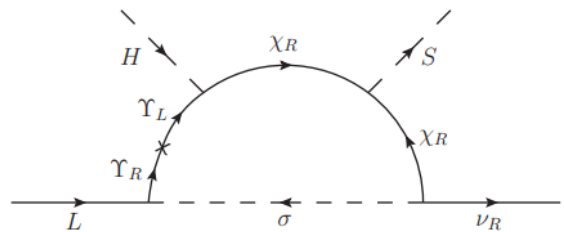
T1-3-E-D



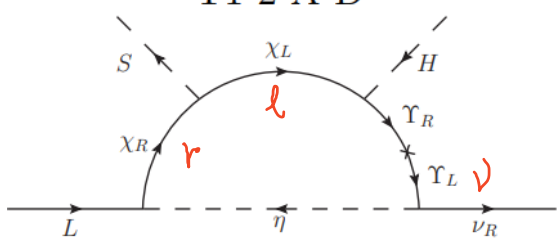
T1-3-E-M



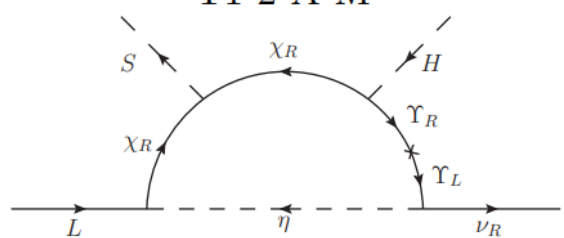
T1-2-A-D



T1-2-A-M



T1-2-B-D



T1-2-B-M

D-5

$$\nu + l + r = \begin{cases} 0\,, & D\text{-Dirac (DD)} \\ -m\,, & X\text{-Dirac (XD)} \end{cases}$$

$$\nu + 2r = \begin{cases} 0\,, & D\text{-Majorana (DM)} \\ -m\,, & X\text{-Majorana (XM)} \end{cases}$$

.

D-6

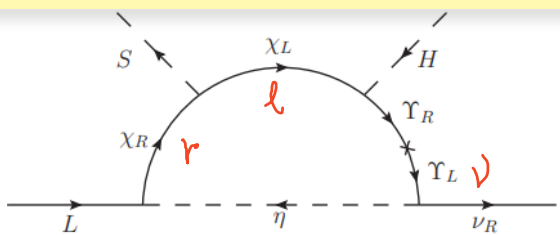
$$\nu + 2l + 2r = \begin{cases} 0\,, & \text{DD} \\ -m\,, & \text{XD} \end{cases}$$

$$\nu + 4r = \begin{cases} 0\,, & \text{DM} \\ -m\,, & \text{XM} \end{cases}$$

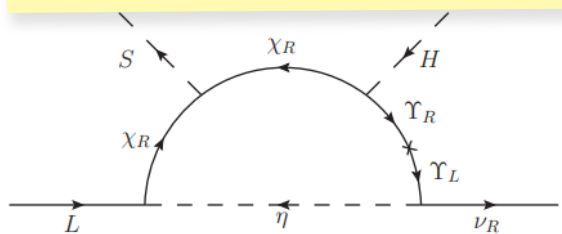
.

Dirac

Majorana



T1-2-B-D



T1-2-B-M

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$$\nu + l + r = \begin{cases} 0\,, & D\text{-Dirac (DD)} \\ -m\,, & X\text{-Dirac (XD)} \end{cases}$$

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•

D-6

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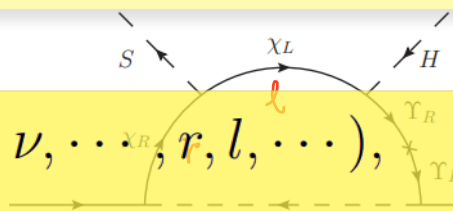
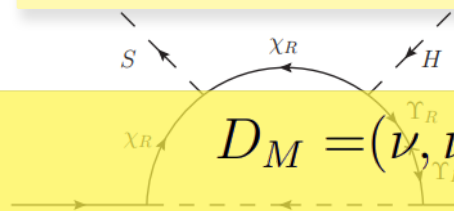


$$D_D = (\nu, \nu, \cdots, r, l, \cdots),$$

$$X_D = (\nu, \nu, \cdots, m, m, m, r, l, \cdots),$$

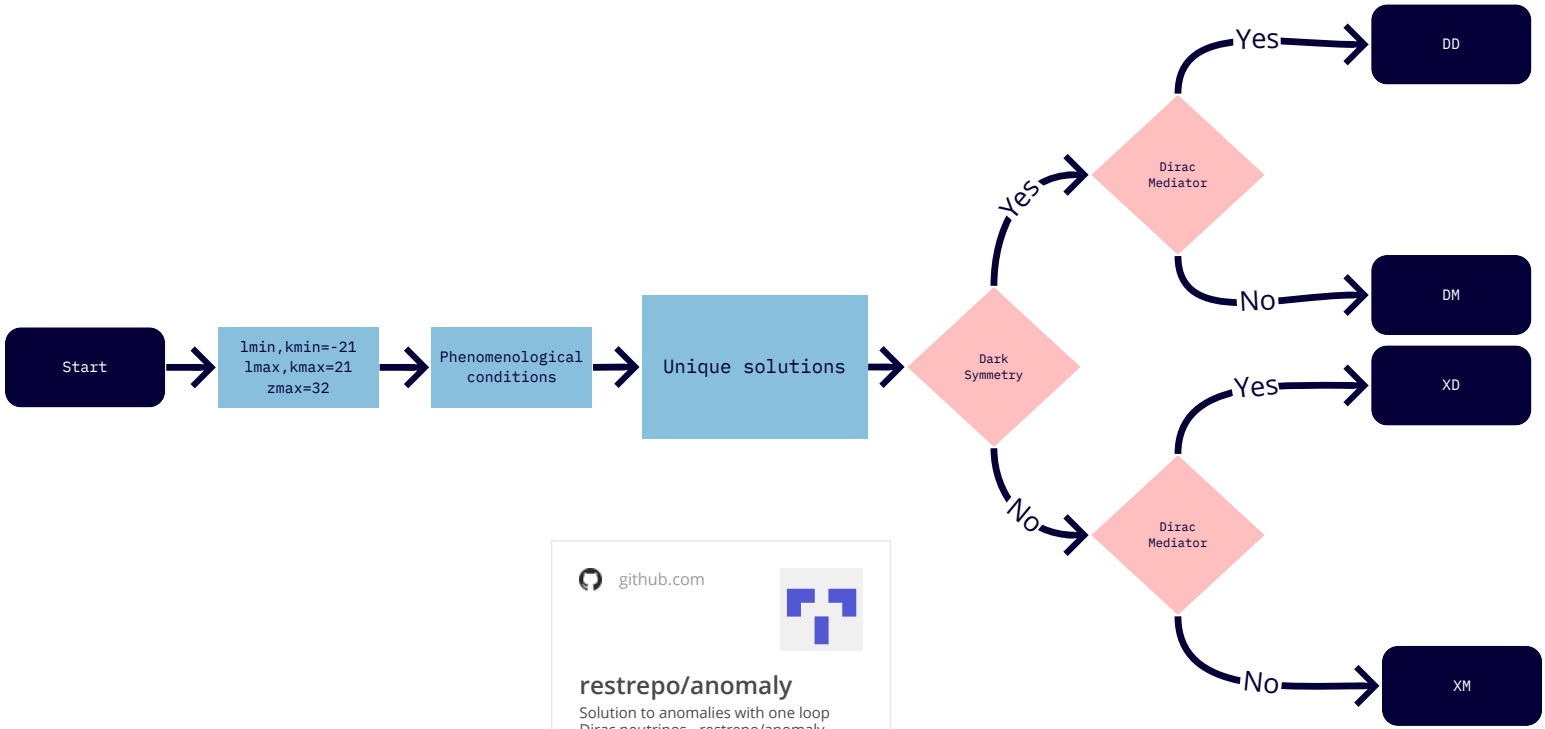
T1-2-B-D



$$D_M = (\nu, \nu, \cdots, r, \cdots),$$

$$X_M = (\nu, \nu, \cdots, m, m, m, r, \cdots),$$

T1-2-B-M



D-5

N	ℓ	k	solution	[Ref]	GCD	DD	ν_R DM	XD	XM
6	$(-21,-14)$	$(-14,-21)$	$(1,-2,-3,5,5,-6)$		36015	(**5**)	0	0	0
7	$(-13,0)$	$(-13,13,-13)$	$(1,2,2,-3,-3,-3,4)$	[6]	28561	(-3)	0	0	0
7	$(-12,-10)$	$(-6,-3,-2)$	$(1,1,-3,-4,6,6,-7)$		264	$(1,6)$	(6)	0	0
7	$(-12,-10)$	$(-12,-9,-5)$	$(1,3,-4,5,-6,-6,7)$		852	(-6)	(-6)	0	0
8	$(-12,-8,-12)$	$(3,-3,-9)$	$(1,1,2,3,-4,-4,-5,6)$		5328	$(-4,1)$	(-4)	0	0
8	$(-12,-9,3)$	$(4,2,-4)$	$(1,2,2,2,-3,-5,-6,7)$		1260	(2)	0	0	0
8	$(-12,-9,3)$	$(-3,-12,-6)$	$(1,2,2,4,-5,-5,-7,8)$		972	(-5)	0	0	0
8	$(-12,-8,-12)$	$(-4,-12,4)$	$(1,3,3,3,-5,-7,-7,9)$		4096	0	0	(**−7**)	0
8	$(-12,-11,-5)$	$(4,12,4)$	$(1,2,3,5,-6,-6,-9,10)$		768	(-6)	(-6)	0	0
8	$(-12,-4,8)$	$(-3,12,9)$	$(2,-5,-5,-5,7,8,8,-10)$	[7]	3456	(8)	0	(8)	0
8	$(-11,0,-11)$	$(-1,1,5)$	$(1,1,1,-5,-7,11,11,-13)$		968	0	0	(11)	0
8	$(-12,-9,3)$	$(-11,-8,-9)$	$(3,-4,-5,8,8,-11,-12,13)$		540	(8)	(8)	0	0
9	$(-6,0,3)$	$(-6,6,0,-6)$	$(1,1,-4,-5,9,9,9,-10,-10)$	[8]	324	(**9**)	0	0	(1)
9	$(-6,-7,-5)$	$(3,-3,0,5)$	$(3,3,-4,5,5,-6,-8,-8,10)$		66	$(-8,3,5)$	0	0	0
9	$(-7,-4,6)$	$(-7,-3,-2,1)$	$(1,1,-5,-7,12,14,14,-15,-15)$		96	$(-15,1,14)$	(14)	0	0
9	$(-6,-7,-5)$	$(-2,2,-6,-2)$	$(1,1,1,2,5,-6,-6,-6,8)$		120	$(-6,1)$	0	0	0
9	$(-6,4,-2)$	$(4,6,4,-2)$	$(1,-3,-3,-3,-5,8,8,8,-11)$		48	$(-3,8)$	0	0	0
9	$(-7,-5,-7)$	$(-6,-4,5,2)$	$(1,-2,-2,-4,7,-9,11,11,-13)$		89	$(-2,11)$	(-2)	0	0
9	$(-6,3,-6)$	$(3,0,3,2)$	$(4,4,4,-5,-9,-10,-10,11,11)$		90	0	0	$(-10,11)$	0
9	$(-6,-4,-7)$	$(-5,-2,1,-3)$	$(1,1,2,2,3,-5,-6,-6,8)$		85	(2)	(-6)	0	0
9	$(-7,-5,-6)$	$(-7,-6,-4,-2)$	$(1,-2,3,4,6,-7,-7,-7,9)$		74	(**−7**)	0	0	0
9	$(-6,-7,-6)$	$(-3,-2,-1,1)$	$(1,2,-3,4,-5,-6,8,8,-9)$		1	(8)	0	0	0
9	$(-7,-4,-6)$	$(1,-2,4,-1)$	$(1,-2,-2,-2,5,-7,8,9,-10)$		106	(**−2**)	(**−2**)	0	0
9	$(-7,-6,-4)$	$(-6,6,0,-4)$	$(2,-3,4,4,4,-6,-7,-7,9)$		204	(4)	0	(-7)	0
9	$(-7,-6,-4)$	$(4,-4,-1,-7)$	$(2,-3,-3,-3,-5,7,7,8,-10)$		140	(-3)	0	0	0
9	$(-6,-1,-5)$	$(2,7,5,-1)$	$(1,-3,-3,-3,-6,8,8,10,-12)$		3	0	0	(8)	0
9	$(-6,-4,-6)$	$(2,-7,-3,-6)$	$(1,2,-6,-6,-6,8,9,9,-11)$		60	(9)	0	(**9**)	0
9	$(-7,-6,2)$	$(4,0,-6,-4)$	$(2,-3,4,6,6,-7,-10,-11,13)$		24	(6)	(6)	0	0
9	$(-7,-6,4)$	$(3,-1,5,-2)$	$(4,4,6,6,-7,-7,-7,-12,13)$		18	0	0	(6)	0
9	$(-4,1,-2)$	$(-1,-2,4,1)$	$(1,-2,-2,-3,-3,-3,14,20,-22)$		2	0	(-2)	0	0
9	$(-7,-4,-6)$	$(1,-7,-4,-1)$	$(1,-2,-2,5,-7,-7,14,18,-20)$		16	0	(-2)	0	0

N	ℓ	k	solution	[Ref]	GCD	DD	DM	XD	XM
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7	$(-13,0)$			[6]	28561	(-3)	0	0	0
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8	$(-12,-8,-12)$		)		4096	0	0	(-7)	0
8	$(-12,-11,-5)$		0)		768	(-6)	(-6)	0	0
8	$(-12,-4,8)$		-10)	[7]	3456	(8)	0	(8)	0
8	$(-11,0,-11)$		$-13)$		968	0	0	(11)	0
8	$(-12,-9,3)$		12, 13)		540	(8)	(8)	0	0
9	$(-6,0,3)$		0, $-10)$	[8]	324	(9)	0	0	(1)
9	$(-6,-7,-5)$		-8, 10)		66	$(-8,3,5)$	0	0	0
9	$(-7,-4,6)$		$-15,-15)$		96	$(-15,1,14)$	(14)	0	0
9	$(-6,-7,-5)$		, 8)		120	$(-6,1)$	0	0	0
9	$(-6,4,-2)$		8, $-11)$		48	$(-3,8)$	0	0	0
9	$(-7,-5,-7)$		11, 11, $-13)$		89	$(-2,11)$	(-2)	0	0
9	$(-6,3,-6)$	$(3,0,3,2)$	$(4,4,4,-5,-9,-10,-10,11,11)$		90	0	0	$(-10,11)$	0
9	$(-6,-4,-7)$	$(-5,-2,1,-3)$	$(1,1,2,2,3,-5,-6,-6,8)$		85	(2)	(-6)	0	0
9	$(-7,-5,-6)$	$(-7,-6,-4,-2)$	$(1,-2,3,4,6,-7,-7,-7,9)$		74	(-7)	0	0	0
9	$(-6,-7,-6)$	$(-3,-2,-1,1)$	$(1,2,-3,4,-5,-6,8,8,-9)$		1	(8)	0	0	0
9	$(-7,-4,-6)$	$(1,-2,4,-1)$	$(1,-2,-2,-2,5,-7,8,9,-10)$		106	(-2)	(-2)	0	0
9	$(-7,-6,-4)$	$(-6,6,0,-4)$	$(2,-3,4,4,4,-6,-7,-7,9)$		204	(4)	0	(-7)	0
9	$(-7,-6,-4)$	$(4,-4,-1,-7)$	$(2,-3,-3,-3,-5,7,7,8,-10)$		140	(-3)	0	0	0
9	$(-6,-1,-5)$	$(2,7,5,-1)$	$(1,-3,-3,-3,-6,8,8,10,-12)$		3	0	0	(8)	0
9	$(-6,-4,-6)$	$(2,-7,-3,-6)$	$(1,2,-6,-6,-6,8,9,9,-11)$		60	(9)	0	(9)	0
9	$(-7,-6,2)$	$(4,0,-6,-4)$	$(2,-3,4,6,6,-7,-10,-11,13)$		24	(6)	(6)	0	0
9	$(-7,-6,4)$	$(3,-1,5,-2)$	$(4,4,6,6,-7,-7,-7,-12,13)$		18	0	0	(6)	0
9	$(-4,1,-2)$	$(-1,-2,4,1)$	$(1,-2,-2,-3,-3,-3,14,20,-22)$		2	0	(-2)	0	0
9	$(-7,-4,-6)$	$(1,-7,-4,-1)$	$(1,-2,-2,5,-7,-7,14,18,-20)$		16	0	(-2)	0	0

D-5

$\nu_{R_i} \to -7$ $\quad S \to 7$

(2,-9) → ψ_{D1}

(3,4) → ψ_{D2} $\quad \langle S \rangle$

(1,6) → ψ_{D3}

3 multicomponent Dirac-fermion DM

D-6

N	ℓ	k	solution	[Ref]	GCD	DD	DM	ν_R XD	XM
6	$(-20,-10)$	$(5,10)$	$(1,1,1,-4,-4,5)$	[1]	15000	0	(-4)	0	0
6	$(-21,21)$	$(-21,-14)$	$(1,-4,-4,9,9,-11)$		43218	0	(-4)	0	0
6	$(-21,14)$	$(-21,3)$	$(1,-4,-8,14,14,-17)$		21168	(14)	0	0	0
7	$(-12,-9)$	$(3,-6,3)$	$(1,1,-4,-4,7,8,-9)$		648	0	(-4)	0	0
7	$(-12,-9)$	$(-12,-11,-2)$	$(2,2,-4,7,-8,-8,9)$		351	(2)	(-8)	0	0
7	$(-12,-9)$	$(-6,-2,6)$	$(3,3,3,-5,-5,-7,8)$		504	0	0	(-5)	0
7	$(-12,-9)$	$(3,-2,7)$	$(4,4,5,-7,-8,-9,11)$		108	(4)	0	0	0
7	$(-12,3)$	$(-6,-7,6)$	$(7,10,-18,25,-28,-28,32)$		135	(-28)	(-28)	0	0
8	$(-12,-4,12)$	$(-12,-6,6)$	$(1,-7,-7,17,17,19,-20,-20)$		1152	(-20)	0	0	0
8	$(-12,-9,3)$	$(-3,-12,-6)$	$(1,2,2,4,-5,-5,-7,8)$		972	(2)	0	0	0
8	$(-12,-9,-12)$	$(4,7,-9)$	$(1,1,-4,-4,-4,12,15,-17)$		2160	0	(-4)	0	0
8	$(-12,6,-9)$	$(-12,-6,-12)$	$(4,4,7,14,-16,-16,-22,25)$		1944	(4)	(-16)	0	0
8	$(-10,-5,10)$	$(-10,-12,12)$	$(5,5,5,-17,-27,-27,28,28)$		400	0	0	$(-27,28)$	0
8	$(-12,-11,-9)$	$(-1,0,6)$	$(1,-3,-3,5,-11,12,12,-13)$		1356	(12)	(12)	0	0
8	$(-12,-9,12)$	$(-2,12,-4)$	$(1,2,2,-8,-8,12,15,-16)$		3024	(2)	(-8)	0	0
8	$(-12,-6,6)$	$(8,4,0)$	$(2,-3,7,-8,-8,11,14,-15)$		1152	(-8)	(-8)	0	0
8	$(-12,-8,-6)$	$(-2,7,4)$	$(1,-2,-4,-4,-4,15,22,-24)$		696	0	(-4)	0	0
8	$(-9,0,-9)$	$(-1,1,-1)$	$(3,3,3,-7,17,-23,-23,27)$		324	0	0	(-23)	0
8	$(-12,0,-12)$	$(-12,-3,-9)$	$(1,-5,-11,15,-16,20,20,-24)$		2592	(20)	(20)	0	0
9	$(-3,1,-3)$	$(-4,5,-4,3)$	$(1,-3,8,8,8,-12,-12,-17,19)$		4	0	0	(-12)	(-12)
9	$(-5,-6,-7)$	$(-4,-3,3,-5)$	$(1,-4,5,5,-9,-9,-9,10,10)$		102	0	0	0	(5)
9	$(-6,-7,-5)$	$(-2,2,-6,-2)$	$(1,1,1,2,5,-6,-6,-6,8)$		120	(-6)	0	0	0
9	$(-5,-4,-7)$	$(-1,4,3,1)$	$(1,1,1,4,-9,-10,-10,11,11)$		4	(-10)	0	$(-10,11)$	0
9	$(-6,0,2)$	$(-2,-1,-3,1)$	$(3,3,3,-4,-4,8,-11,-11,13)$		16	(-4)	0	$(-11,-4)$	0
9	$(-7,-4,6)$	$(-6,-5,0,-3)$	$(2,-3,-3,-8,-9,12,12,14,-17)$		206	(12)	(12)	0	0
9	$(-5,3,2)$	$(-1,-6,2,4)$	$(3,4,-10,-10,-10,12,12,13,-14)$		12	(12)	0	(12)	0
9	$(-6,3,-6)$	$(-5,-4,7,2)$	$(1,1,1,-4,-4,-11,18,26,-28)$		50	0	(-4)	0	0
9	$(16,-1,-6)$	$(1,5,-5,-1)$	$(3,-4,-4,-9,-13,16,16,16,-21)$		32	0	(16)	(-4)	0
9	$(-5,3,-7)$	$(1,-3,-7,4)$	$(5,7,7,-8,-15,-15,-15,17,17)$		98	0	0	$(7,17)$	0
9	$(-7,0,-2)$	$(-4,4,-5,-2)$	$(4,7,-8,9,-16,-16,-16,18,18)$		46	(18)	(-16)	(18)	0
9	$(-6,-7,-2)$	$(6,-2,-1,-5)$	$(4,4,4,5,-6,-6,-6,-10,11)$		158	0	0	$(4,-6)$	0
9	$(-4,-8,5)$	$(2,4,-2,9)$	$(1,-2,-2,-4,-4,-4,17,27,-29)$		18	0	(-4)	0	0
9	$(-5,-4,-2)$	$(2,4,-7,-4)$	$(1,-4,-4,-4,12,-14,15,18,-20)$		50	0	(-4)	0	0
9	$(-3,-2,-5)$	$(1,-5,-4,-2)$	$(1,-4,-4,-5,7,-9,16,23,-25)$		6	(-4)	(-4)	0	0

Conclusions

Dark symmetry for all → simple Diophantine equations

Find the full set of solutions for any phenomenological problem

Extend to multiplets or non-universal *D*-charges