Three-point Gaussian streaming model

and the cosmological information content in relative velocity statistics

(Based on Kuruvilla & Porciani, JCAP, 2020; arXiv:2005.05331
and Kuruvilla & Aghanim, 2020)

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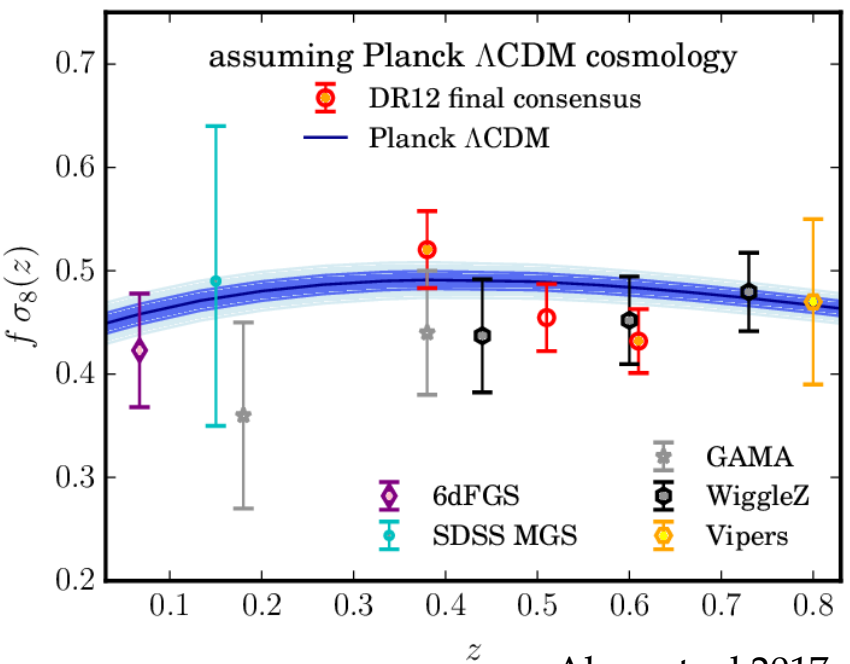
European Research Council
Established by the European Commission

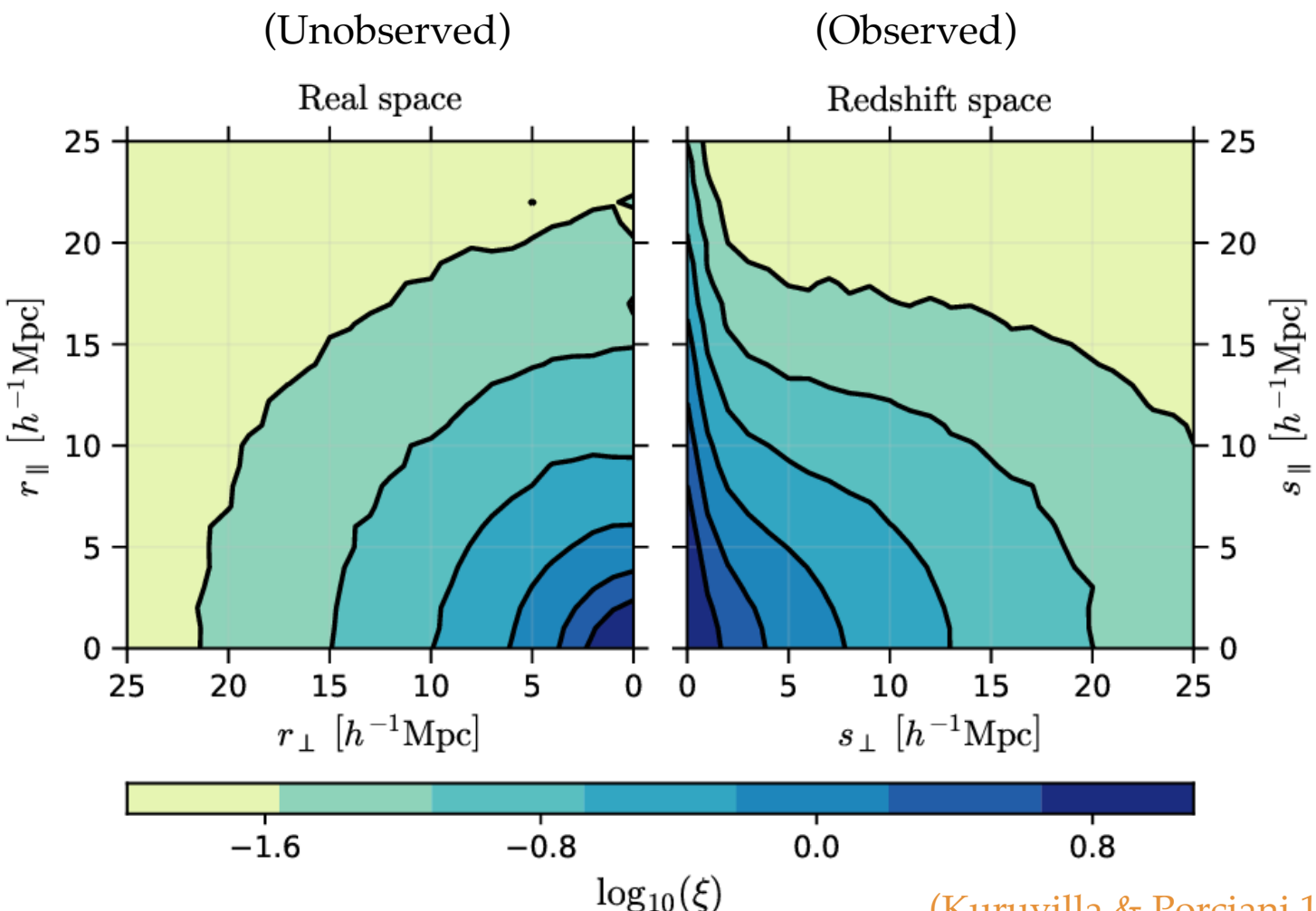
Introduction

- Redshift-space distortions (RSD): a tool to test theories of gravity.
- Growing interest in extending RSD modeling to smaller scales.
- Also to extend to higher order clustering statistics.
  (e.g. Sefusatti et al. 06, Yankelevich & Porciani 19, Gualdi & Verde 20)



Alam et. al 2017

Two-point correlation function



(Kuruvilla & Porciani 18)

Two-point streaming model

$$1 + \xi_{\rm s}(s_\perp, s_\parallel) = \int_{-\infty}^{+\infty} \left[1 + \xi(r)\right] \mathcal{P}^{(2)}_{w_\parallel}(w_\parallel \mid \vec{r}\,)\, {\rm d}r_\parallel$$

(Peebles 80, Fisher 95, Scoccimarro 04)

$\mathcal{P}^{(2)}_{w_\parallel}(w_\parallel \mid \vec{r}\,)$: Relative pairwise line-of-sight velocity distribution

- Current workhorse is the Gaussian streaming model (GSM). (Fisher 95, Reid & White 11)

- GSM used in the BOSS analyses (e.g. Reid et al. 12, Samushia et al. 14; Alam et al. 17, Chuang et al. 17, Satpathy et al. 17) and the recent eBOSS analyses. (e.g. Zarrouk et al. 18, Bautista et al. 20, Tamone et al. 20, Wang et al. 20)

- Various effort in the literature to go beyond the Gaussian approximation. (e.g. Uhlemann et al. 15; Bianchi et al. 15, 16; Kuruvilla & Porciani 18)

Can we generalise it to three-point and higher order statistics?

Can we generalise it to three-point and higher order statistics?

Yes we can!

n-point streaming model

- Generalised the streaming model framework to n-point statistics.

$$\mathcal{G}_n = \int \mathcal{F}_n \, \mathcal{P}^{(n)}_{\boldsymbol{w}_\parallel} \, \mathrm{d}w_{12\parallel} \ldots \mathrm{d}w_{mn\parallel}$$

where,

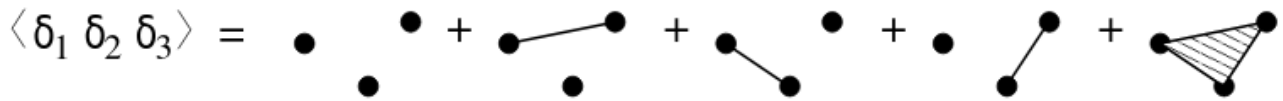
$\mathcal{G}_n$: the (anisotropic) n-point full CF in redshift space

$\mathcal{F}_n$: the (isotropic) n-point full CF in real space

$\mathcal{P}^{(n)}_{\boldsymbol{w}_\parallel}$: the joint probability density of n-1 relative line-of-sight peculiar velocity

The equation is exact under distant observer approximation

Full correlation function in three-point

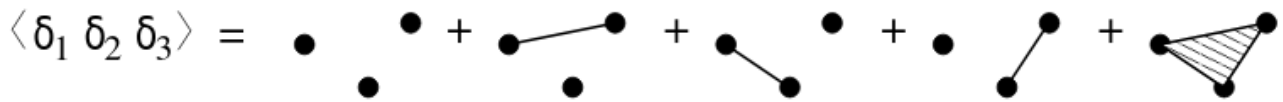


Credits: Bernardeau et al. 01

Thus the three-point streaming model is

$$1+\xi_{\rm s}(s_{12\parallel},s_{12\perp})+\xi_{\rm s}(s_{23\parallel},s_{23\perp})+\xi_{\rm s}(\check{s}_{31\parallel},s_{31\perp})+\zeta_{\rm s}(\boldsymbol{s}_{12},\boldsymbol{s}_{23})$$
$$=\int\left[1+\xi(\check{r}_{12})+\xi(\check{r}_{23})+\xi(\check{r}_{31})+\zeta(\check{r}_{12},\check{r}_{23},\check{r}_{31})\right]$$
$$\mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel}\left(s_{12\parallel}-r_{12\parallel},s_{23\parallel}-r_{23\parallel}\,|\,\check{\boldsymbol{r}}_{12},\check{\boldsymbol{r}}_{23}\right)\,{\rm d}r_{12\parallel}\,{\rm d}r_{23\parallel}$$

Full correlation function in three-point



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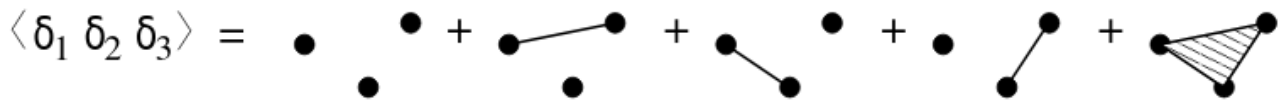
Thus the three-point streaming model is

$$1 + \xi_{\rm s}(s_{12\parallel}, s_{12\perp}) + \xi_{\rm s}(s_{23\parallel}, s_{23\perp}) + \xi_{\rm s}(\check{s}_{31\parallel}, s_{31\perp}) + \zeta_{\rm s}(\boldsymbol{s}_{12}, \boldsymbol{s}_{23})$$
$$= \int \left[1 + \xi(\check{r}_{12}) + \xi(\check{r}_{23}) + \xi(\check{r}_{31}) + \zeta(\check{r}_{12}, \check{r}_{23}, \check{r}_{31})\right]$$
$$\mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel}\left(s_{12\parallel} - r_{12\parallel}, s_{23\parallel} - r_{23\parallel} \,|\, \check{\boldsymbol{r}}_{12}, \check{\boldsymbol{r}}_{23}\right) \mathrm{d}r_{12\parallel}\, \mathrm{d}r_{23\parallel}$$

Full correlation function in three-point



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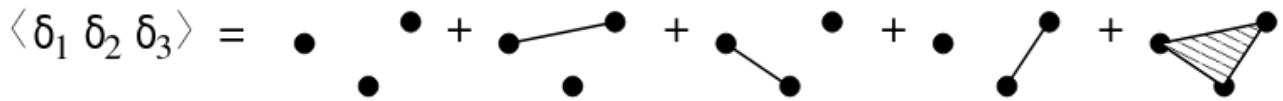
Full correlation function in three-point



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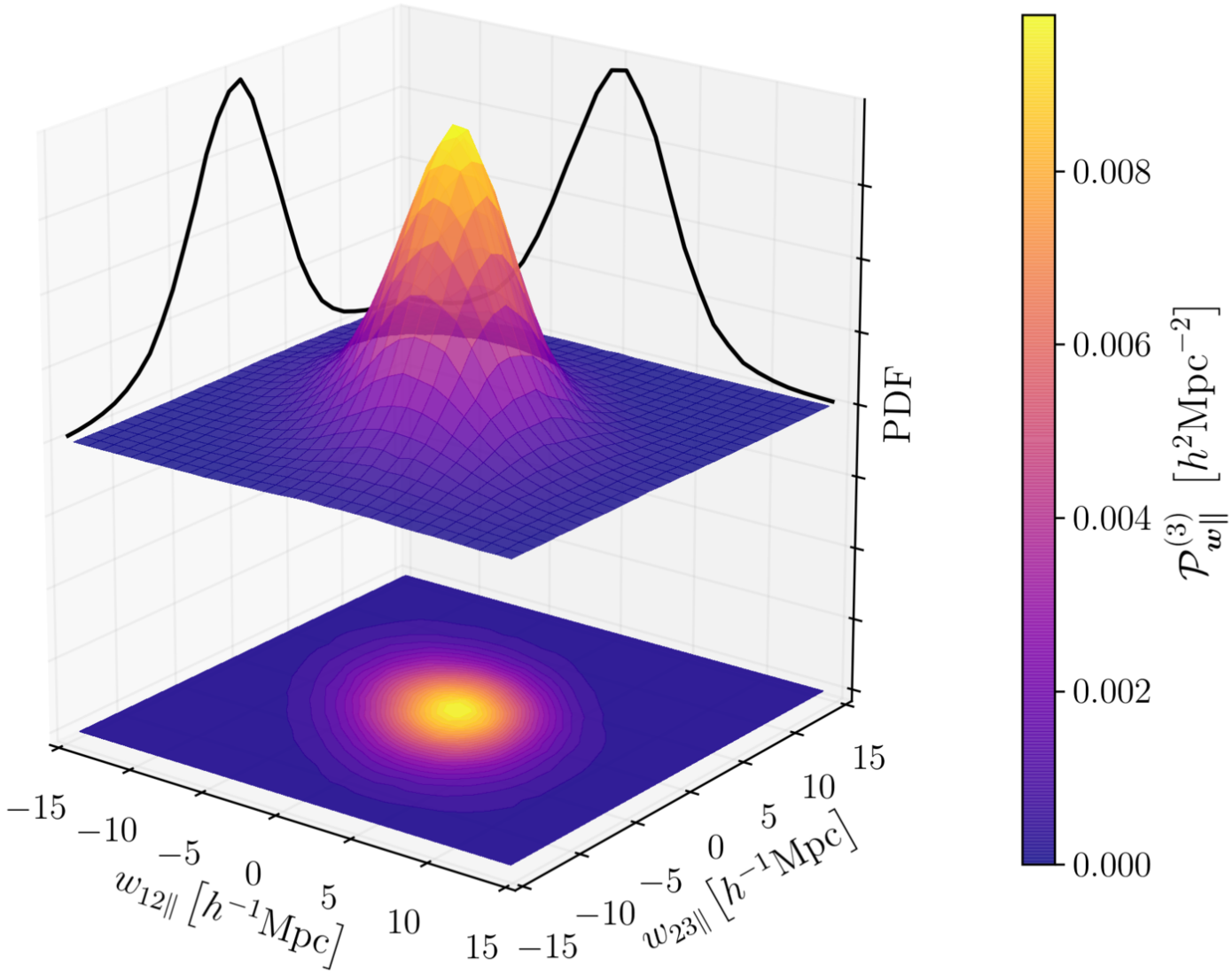
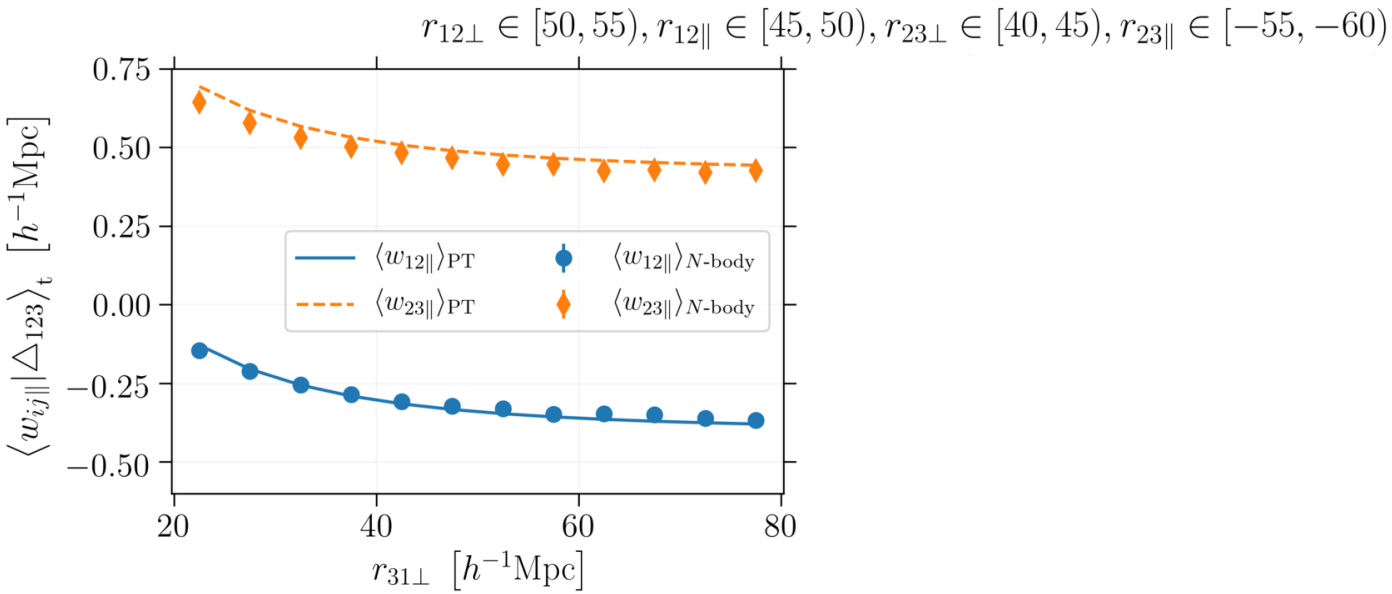
$$\mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel}\left(s_{12\parallel}-r_{12\parallel},s_{23\parallel}-r_{23\parallel}\,|\,\check{\boldsymbol{r}}_{12},\check{\boldsymbol{r}}_{23}\right)\,{\rm d}r_{12\parallel}\,{\rm d}r_{23\parallel}$$

What does $\mathcal{P}^{(3)}_{w_\parallel}$ look like?

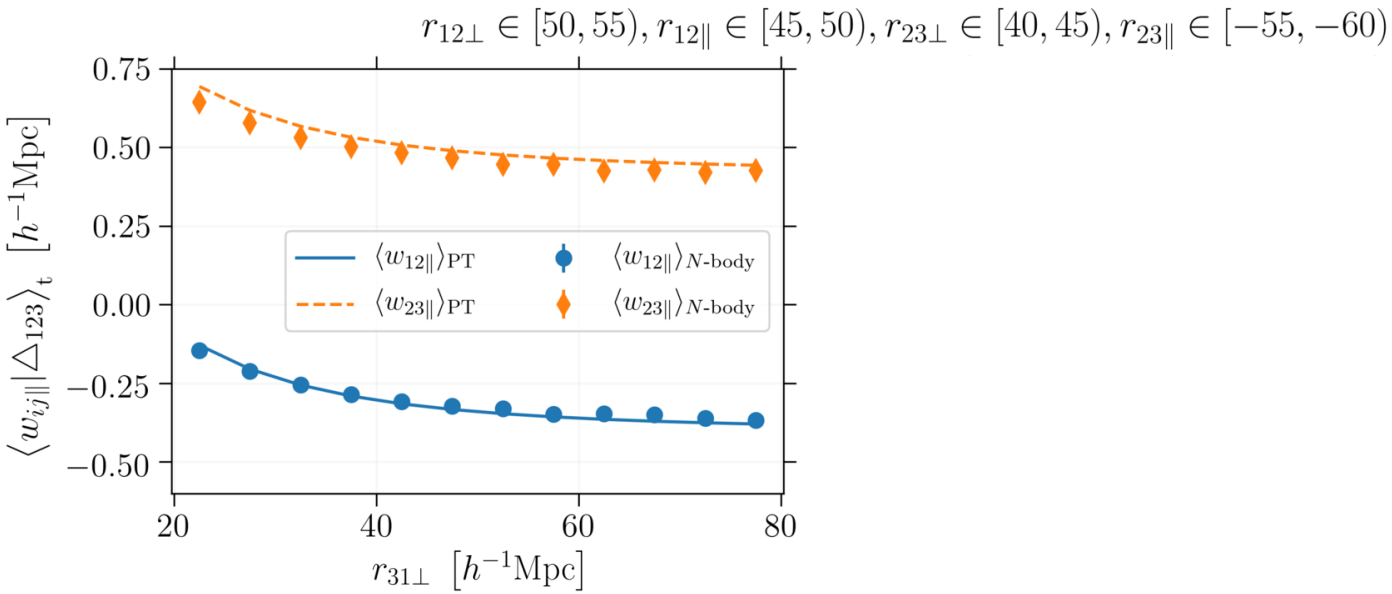

PDF
$w_{12\parallel}\,[h^{-1}\mathrm{Mpc}]$
$w_{23\parallel}\,[h^{-1}\mathrm{Mpc}]$
$\mathcal{P}^{(3)}_{\boldsymbol{w}\parallel}\;[h^2\mathrm{Mpc}^{-2}]$
−15
−10
−5
0
5
10
15
0.000
0.002
0.004
0.006
0.008

Can we predict the velocity moments?

Mean


$r_{12\perp} \in [50, 55), r_{12\parallel} \in [45, 50), r_{23\perp} \in [40, 45), r_{23\parallel} \in [-55, -60)$
$\langle w_{ij\parallel} | \triangle_{123} \rangle_t \ [h^{-1}\mathrm{Mpc}]$
$r_{31\perp} \ [h^{-1}\mathrm{Mpc}]$
$\langle w_{12\parallel} \rangle_{\mathrm{PT}}$
$\langle w_{23\parallel} \rangle_{\mathrm{PT}}$
$\langle w_{12\parallel} \rangle_{N\text{-body}}$
$\langle w_{23\parallel} \rangle_{N\text{-body}}$
0.75
0.50
0.25
0.00
−0.25
−0.50
20
40
60
80

Mean

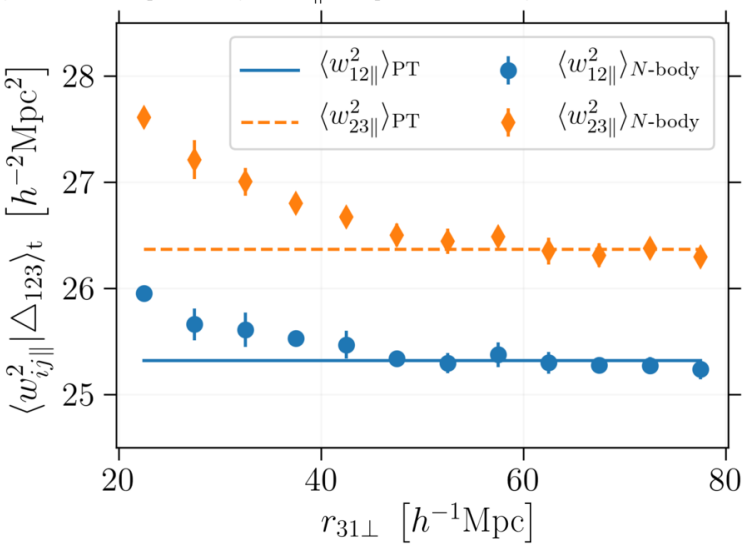


$$\langle w_{12\parallel} | \triangle_{123} \rangle_{\rm t} \simeq \bar{w}(r_{12})\,\mu_{12} - \frac{1}{2}\left[\bar{w}(r_{23})\,\mu_{23} + \bar{w}(r_{31})\,\mu_{31}\right]$$

$$\langle w_{23\parallel} | \triangle_{123} \rangle_{\rm t} \simeq \bar{w}(r_{23})\,\mu_{23} - \frac{1}{2}\left[\bar{w}(r_{12})\,\mu_{12} + \bar{w}(r_{31})\,\mu_{31}\right]$$

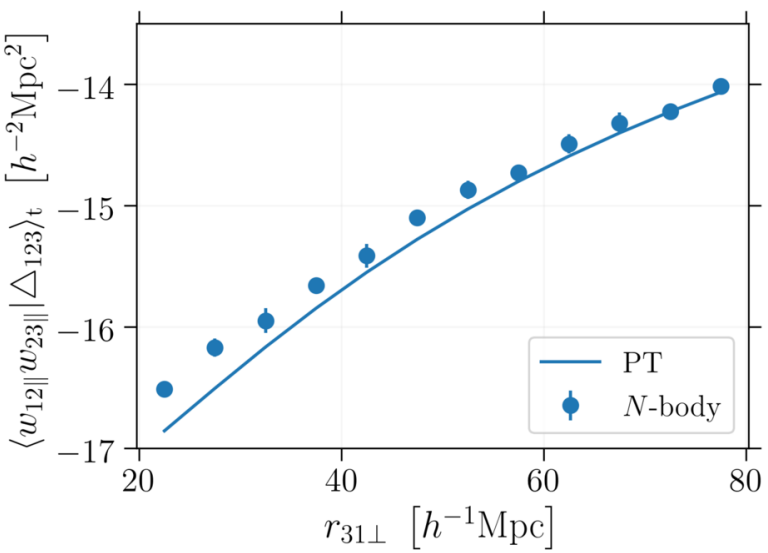
Second moment

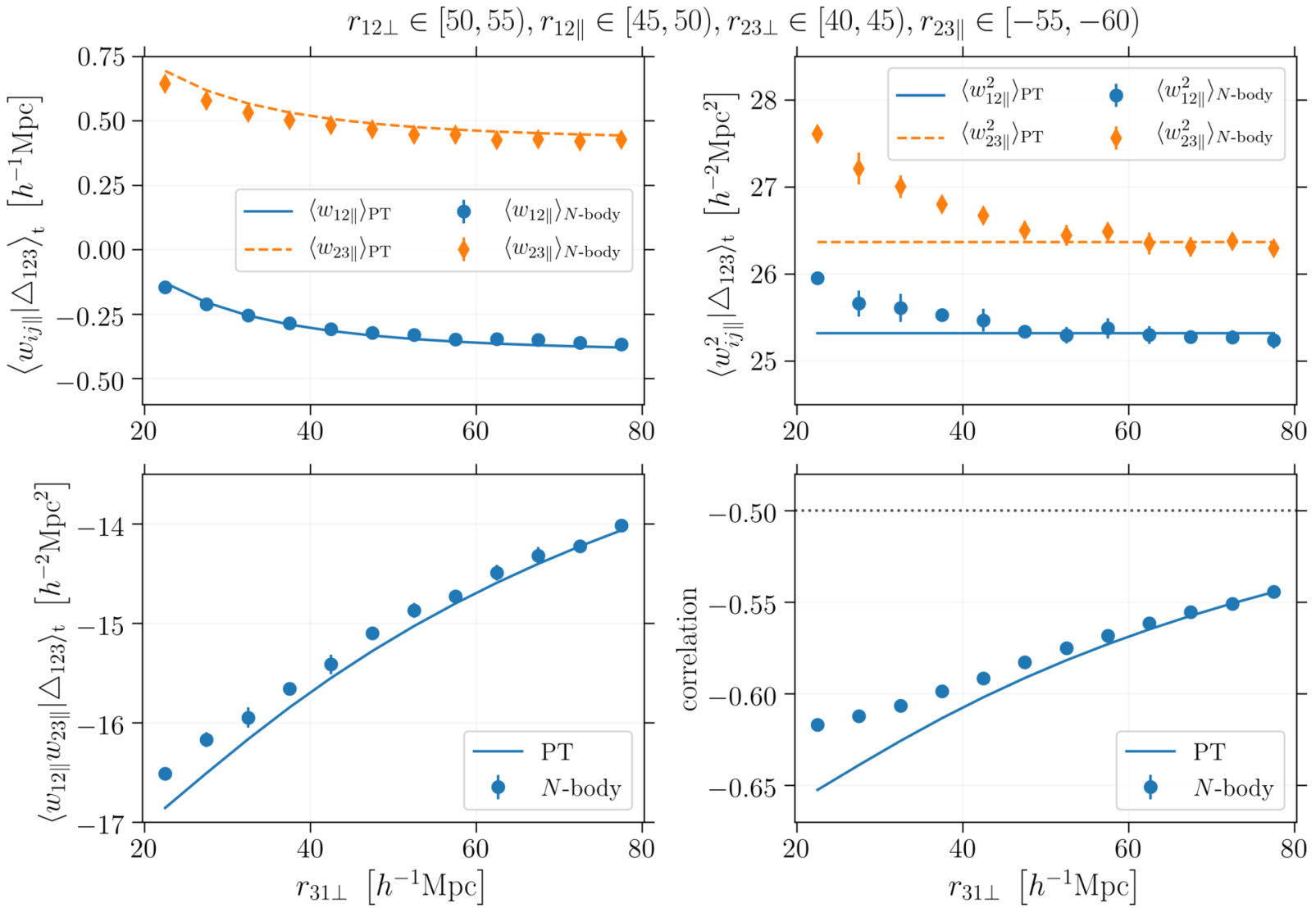
$$r_{12\perp} \in [50, 55), r_{12\parallel} \in [45, 50), r_{23\perp} \in [40, 45), r_{23\parallel} \in [-55, -60)$$



Second cross moment

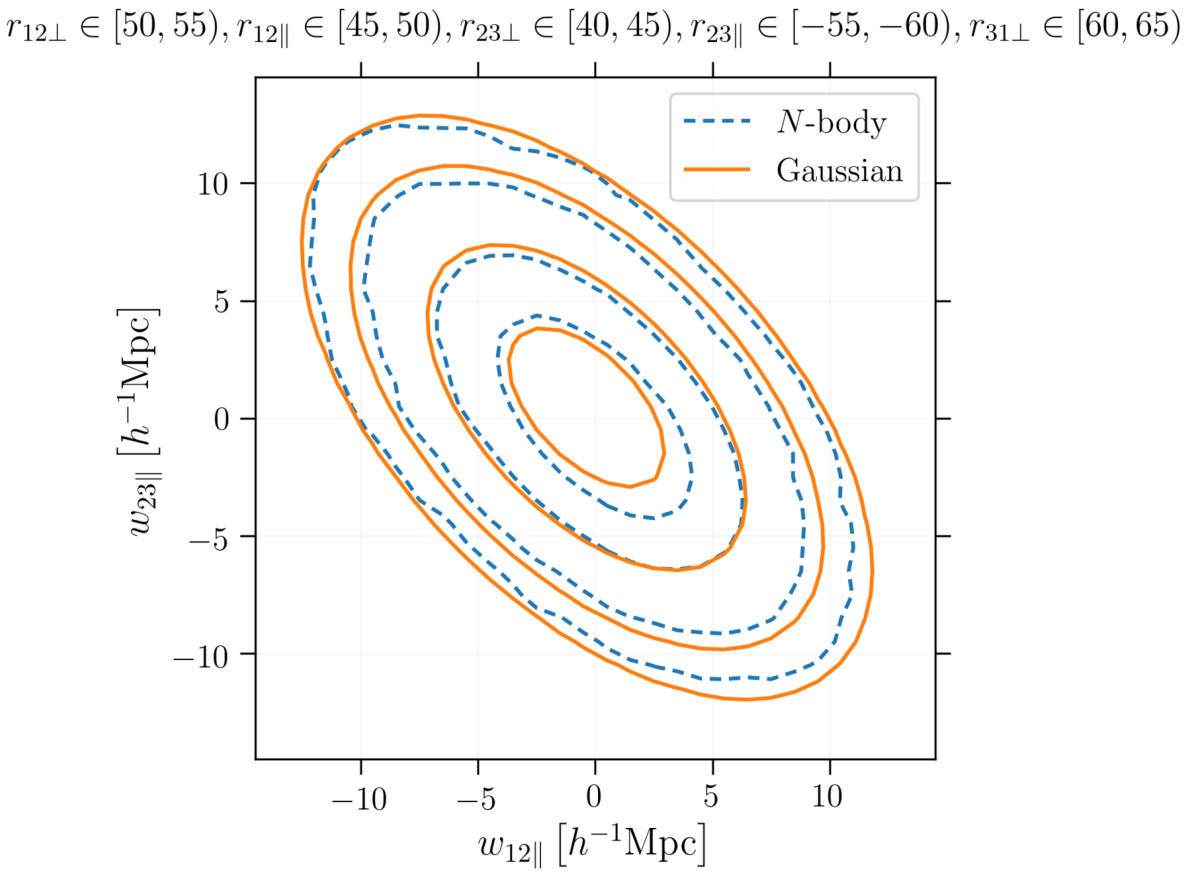
$$r_{12\perp} \in [50, 55), r_{12\parallel} \in [45, 50), r_{23\perp} \in [40, 45), r_{23\parallel} \in [-55, -60)$$




$r_{12\perp} \in [50, 55), r_{12\parallel} \in [45, 50), r_{23\perp} \in [40, 45), r_{23\parallel} \in [-55, -60)$
$\langle w_{ij\parallel} | \triangle_{123} \rangle_t \ [h^{-1}\mathrm{Mpc}]$
$\langle w_{12\parallel} \rangle_{\mathrm{PT}}$
$\langle w_{23\parallel} \rangle_{\mathrm{PT}}$
$\langle w_{12\parallel} \rangle_{N\text{-body}}$
$\langle w_{23\parallel} \rangle_{N\text{-body}}$
$\langle w^2_{ij\parallel} | \triangle_{123} \rangle_t \ [h^{-2}\mathrm{Mpc}^2]$
$\langle w^2_{12\parallel} \rangle_{\mathrm{PT}}$
$\langle w^2_{23\parallel} \rangle_{\mathrm{PT}}$
$\langle w^2_{12\parallel} \rangle_{N\text{-body}}$
$\langle w^2_{23\parallel} \rangle_{N\text{-body}}$
$\langle w_{12\parallel} w_{23\parallel} | \triangle_{123} \rangle_t \ [h^{-2}\mathrm{Mpc}^2]$
PT
N-body
correlation
$r_{31\perp} \ [h^{-1}\mathrm{Mpc}]$

Bivariate Gaussian + velocity moments from PT

"Truth is much too complicated to allow anything but approximations." -- John von Neumann



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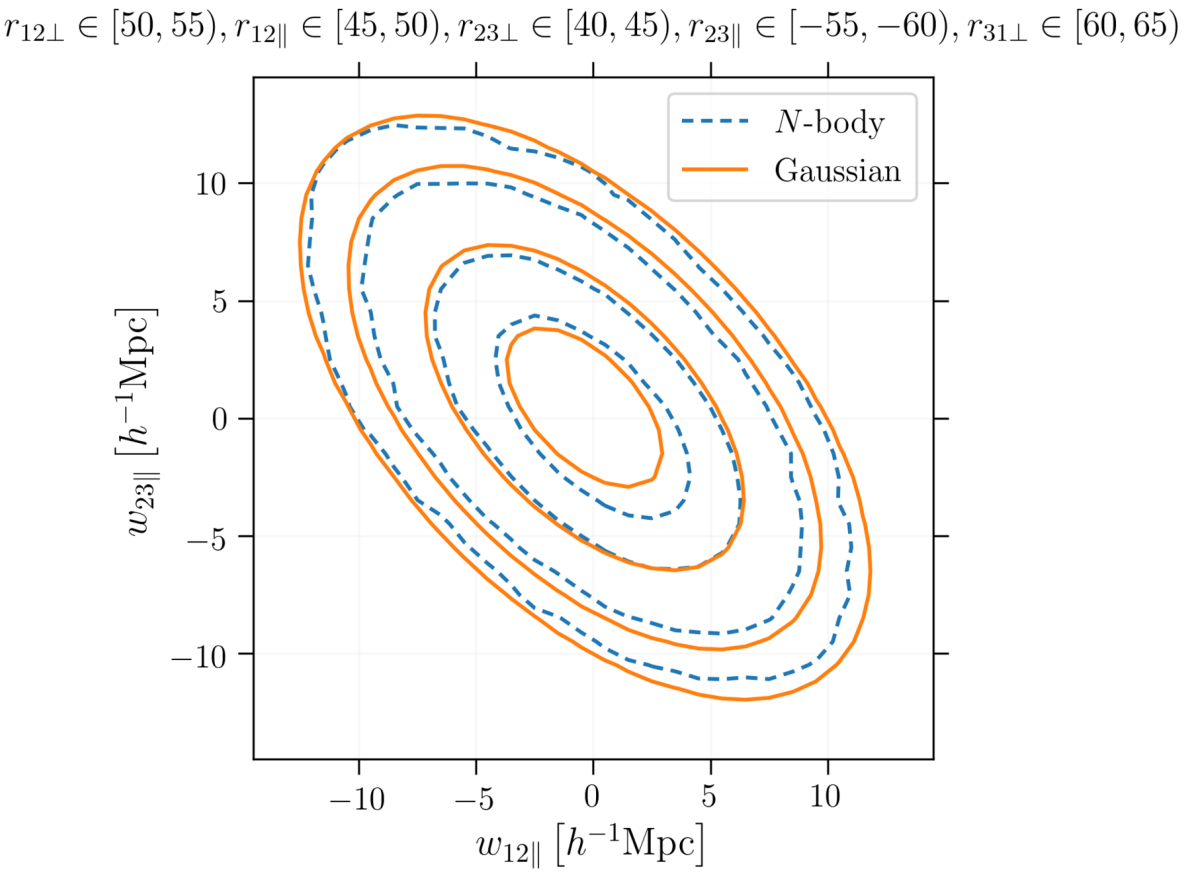


- For these triangular configurations, Gaussian approximation is nearly lossless.
- Jensen-Shannon divergence = 0.005 nats.
- JS divergence is bound between 0 and 0.693.

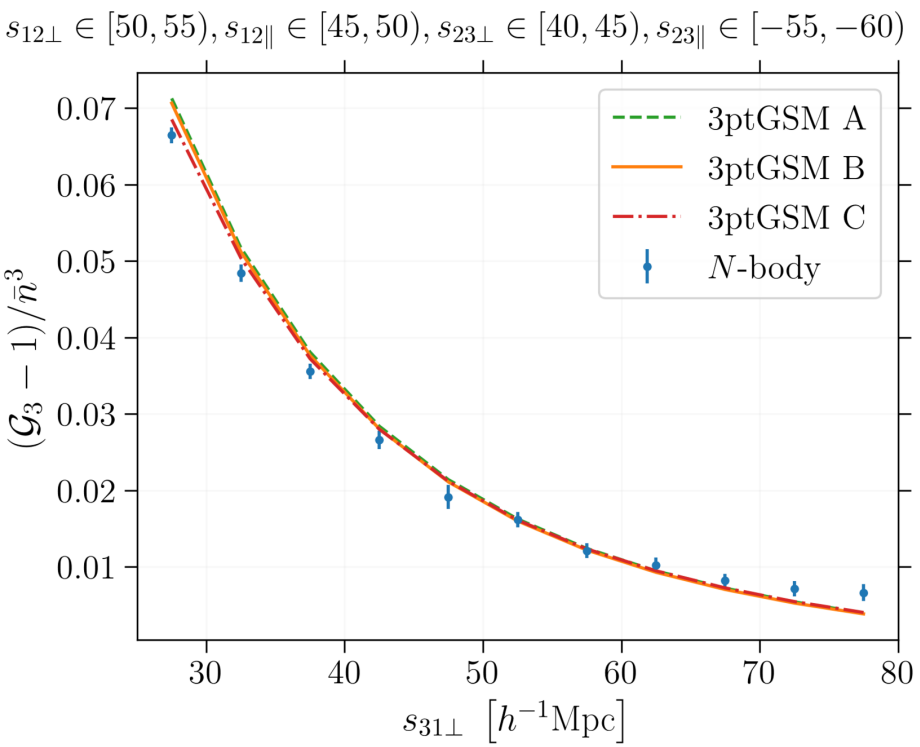
Three-point Gaussian Streaming Model (3ptGSM)

Simplest implementation:

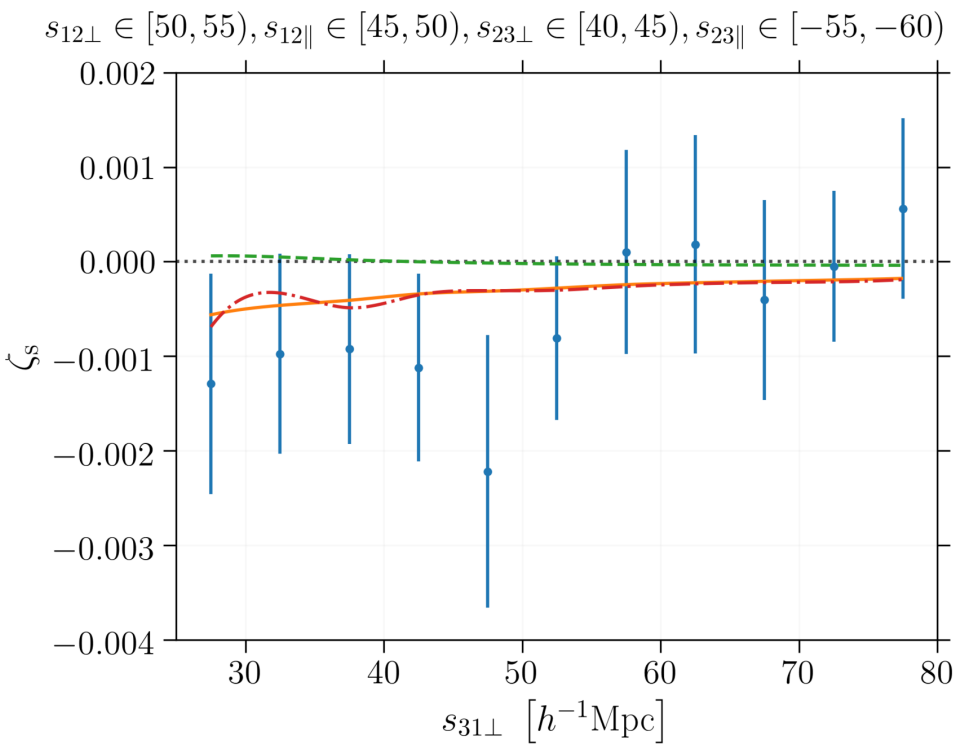
- Velocity moments from the standard perturbation theory at leading order
- Bivariate Gaussian as the approximating PDF for $\mathcal{P}^{(3)}_{\boldsymbol{w}_\parallel}$
- Evaluating real space 2PCF and 3PCF at leading order (LO) in perturbation theory (PT).

Anisotropic 3PCF

(Full 3PCF)

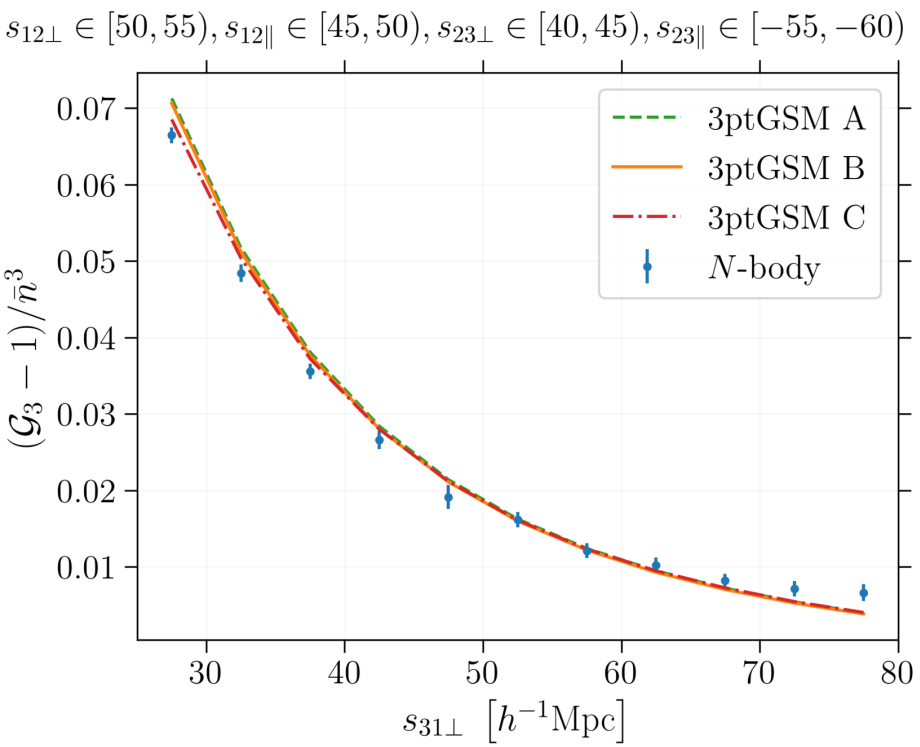


(Connected 3PCF)

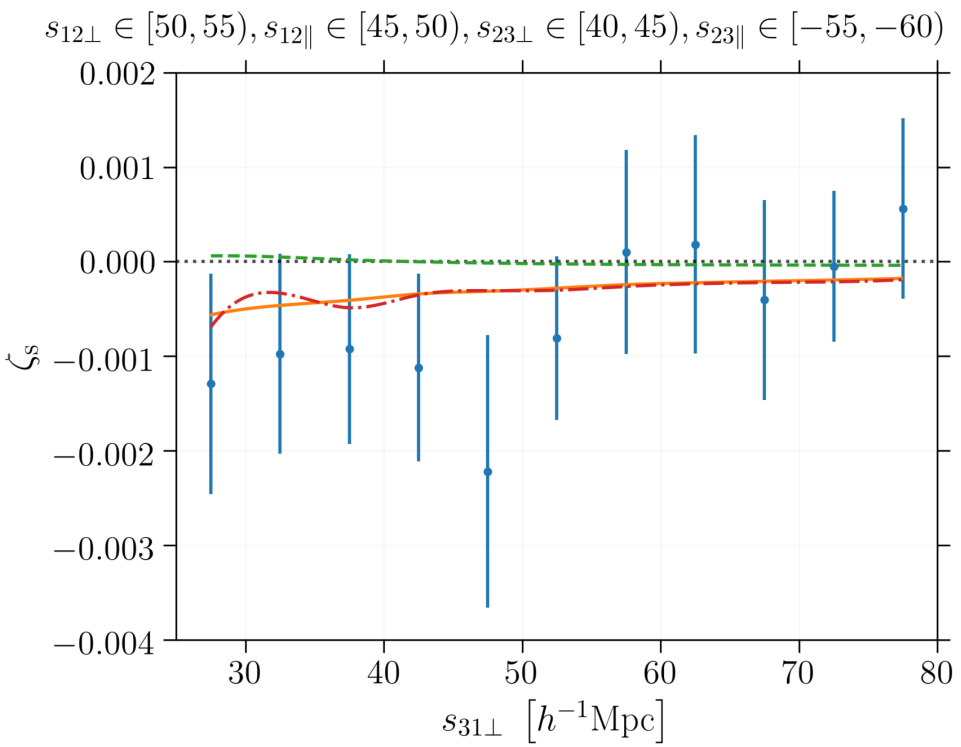


Anisotropic 3PCF

(Full 3PCF)



(Connected 3PCF)



Natural estimator

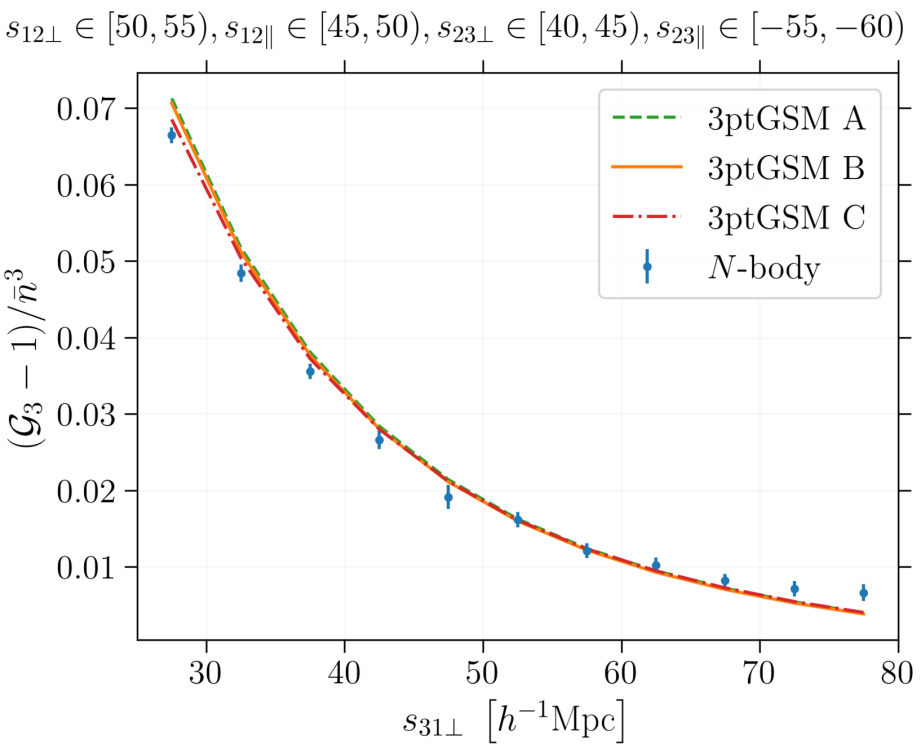
$$\frac{DDD}{RRR}$$

Szapudi-Szalay estimator

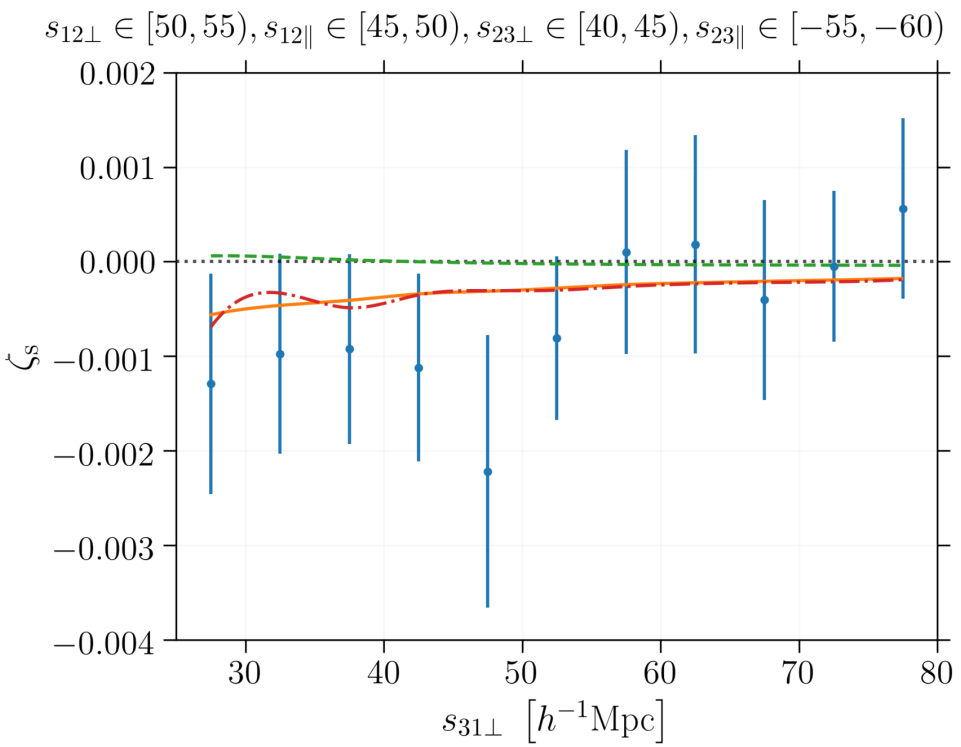
$$\frac{(D-R)(D-R)(D-R)}{RRR}$$

Anisotropic 3PCF

(Full 3PCF)



(Connected 3PCF)



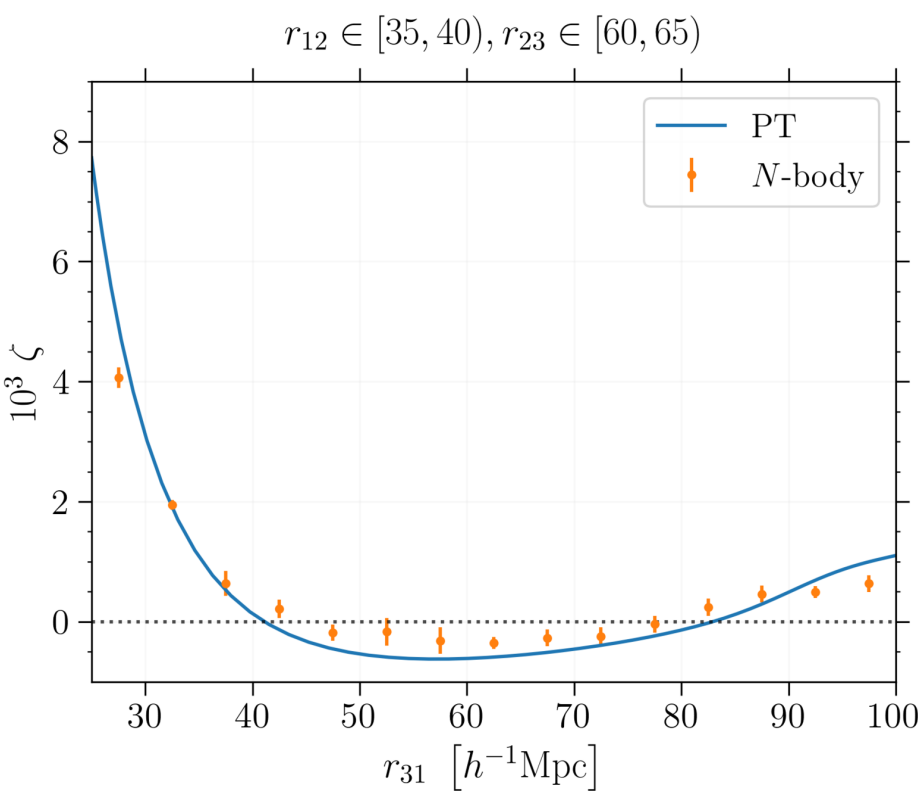
Case A: ξ (LO-PT) and $\zeta = 0$

Case B: ξ (LO-PT) and ζ (LO-PT)

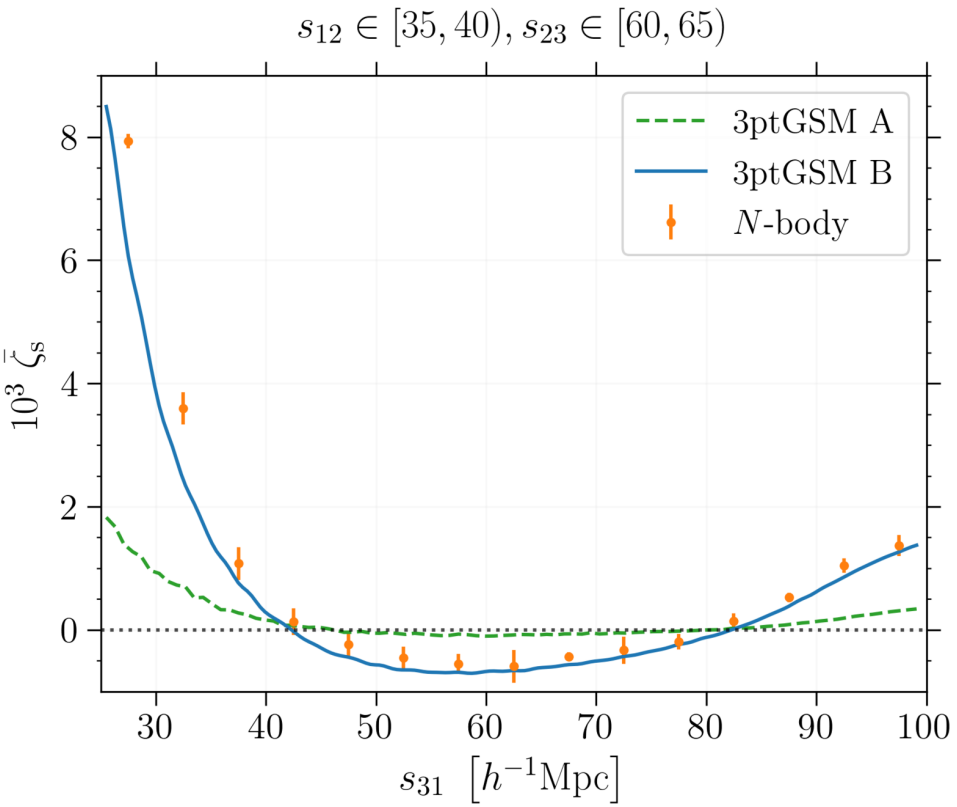
Case C: ξ (halo model) and ζ (LO-PT)

Connected 3PCF: spherically averaged

(real space)



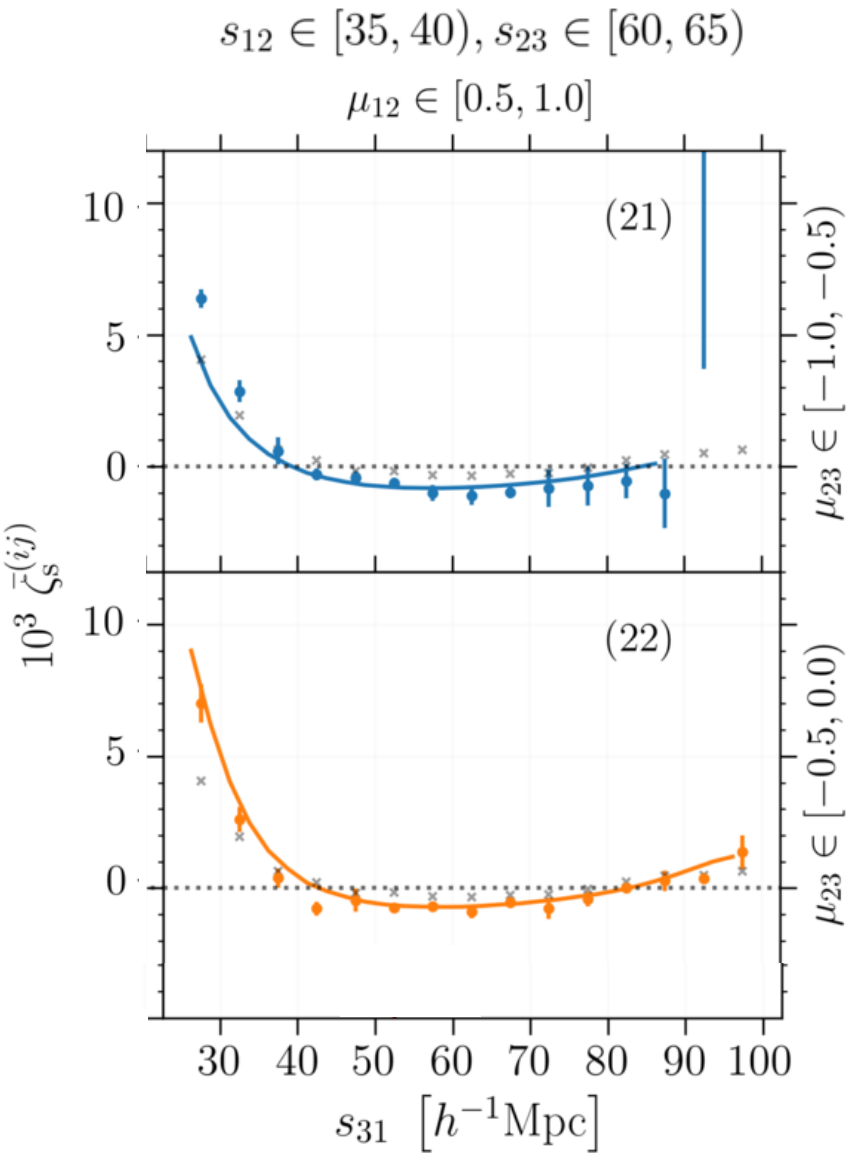
(redshift space)



Case A: ξ (LO-PT) and $\zeta = 0$

Case B: ξ (LO-PT) and ζ (LO-PT)

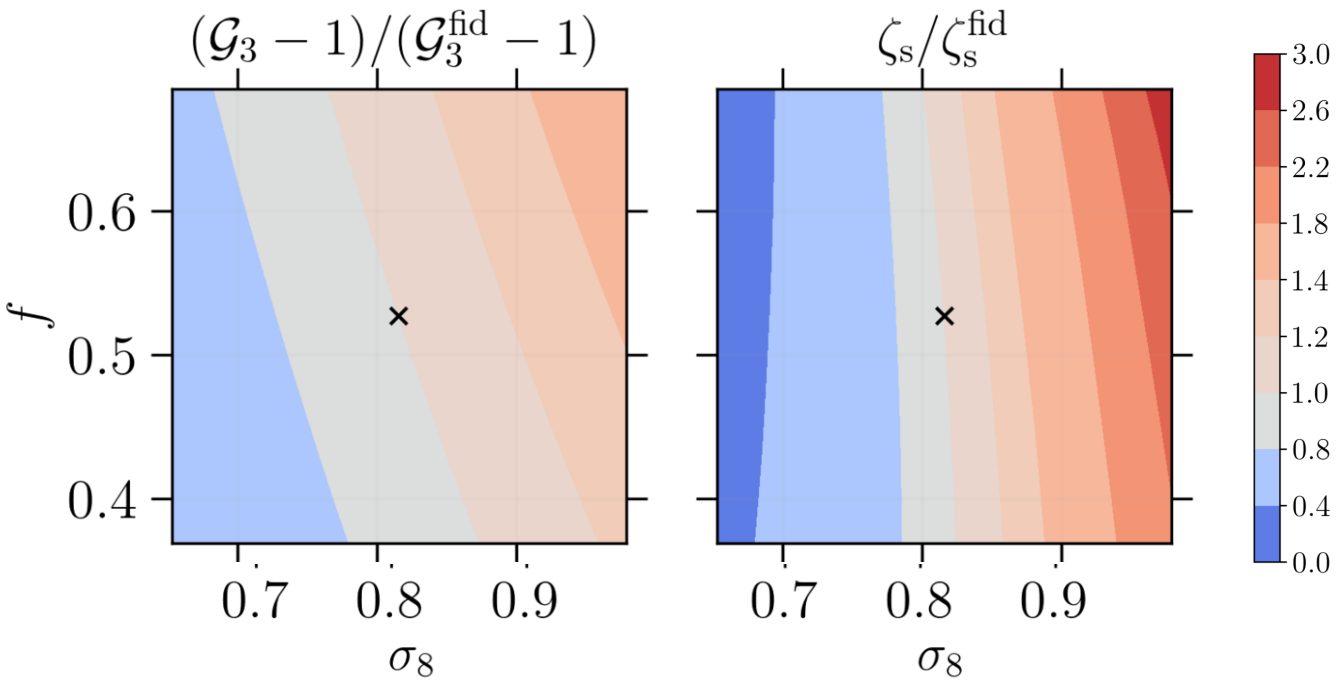
Connected 3PCF: wedges



Dependence on the growth rate of structure

Breaking f-σ_8 degeneracy

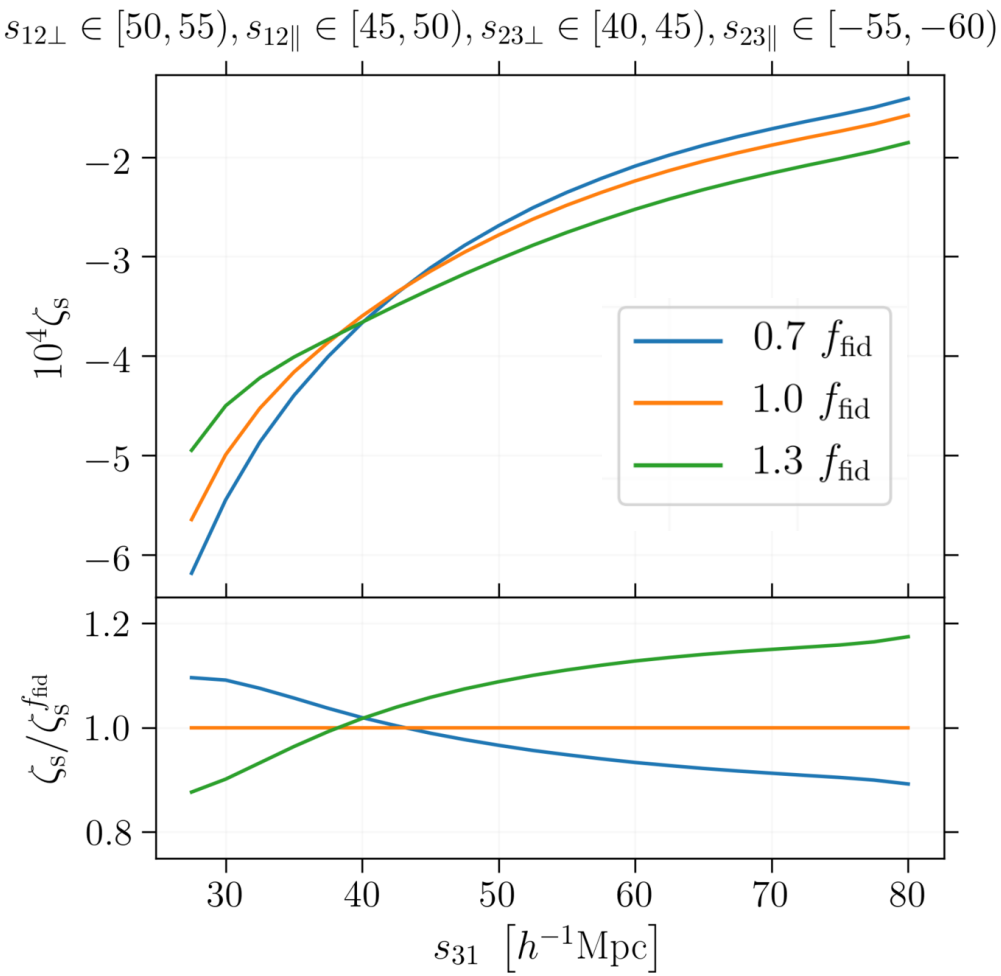
$$s_{12\perp} = 52.5, s_{12\parallel} = 47.5, s_{23\perp} = 42.5, s_{23\parallel} = -57.5, s_{31\perp} = 50.0$$



- Suggests that measuring redshift space 3PCF with sufficient accuracy can help in breaking f-σ_8 degeneracy.
- Similar conclusion was reached based on power spectrum and bispectrum.

(Gil-Marin et al. 14)

Varying growth rate



Galaxy clustering - conclusions

"All models are wrong, but some are useful" -- George Box

- For three-point: velocity moments were predicted using perturbation theory at leading order and compared to simulations.
- Introduced a phenomenological model for 3PCF: the three-point Gaussian streaming model.
- Our results suggests that 3PCF in redshift space can help in breaking f-σ_8 degeneracy

Outlook:

- Apply 3ptGSM to biased tracers.
- Use more sophisticated PT flavours like CLPT or CLEFT

What is the cosmological information content in three-point relative velocity statistics?

Halos - Mean relative velocity between pairs in a triplet

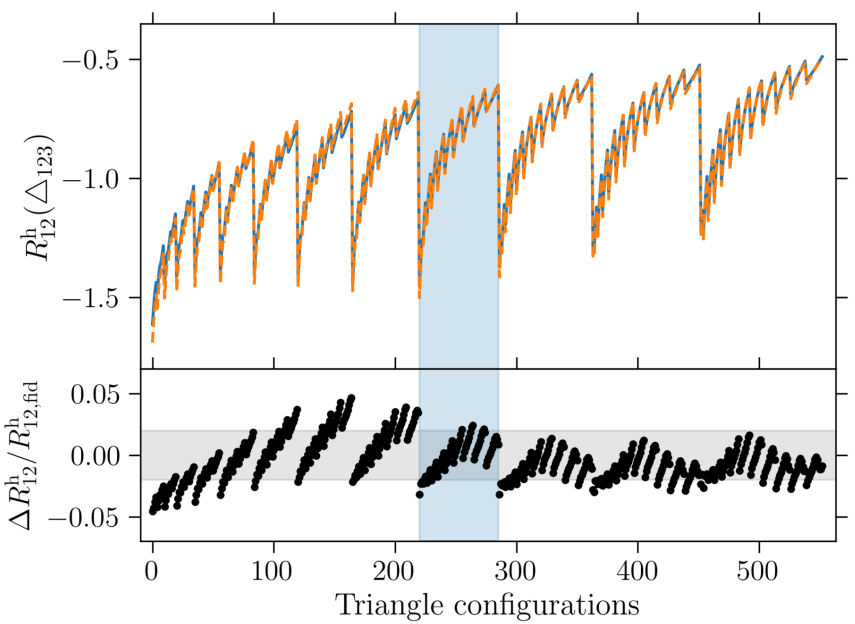
- Fisher matrix formalism
- Compute the covariance matrix for the relative velocity statistics using 15,000 simulations from the Quijote suite.
- Derivatives also computed directly using the Quijote suite.

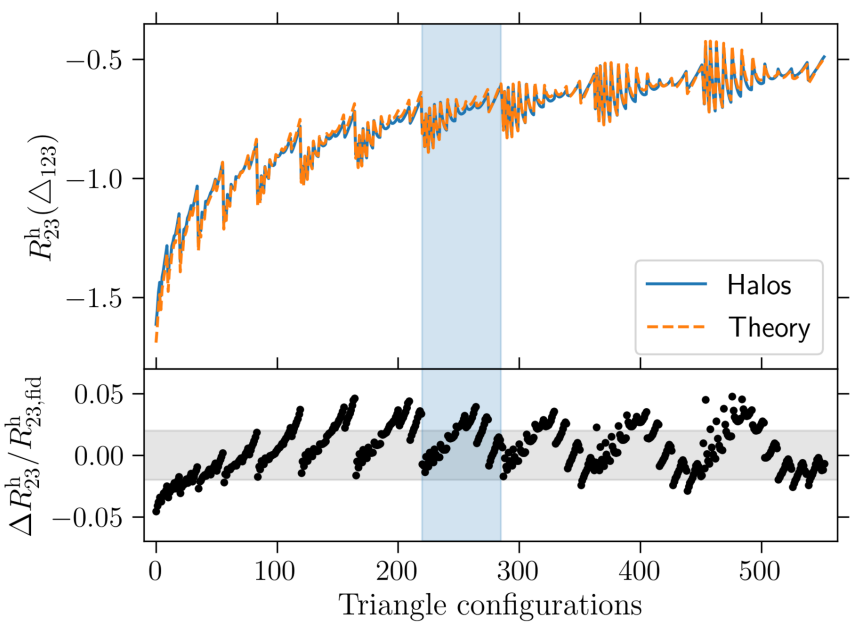
- The Quijote simulations is a suite of 44,100 full N-body simulations. (Villaescusa-Navarro et al. 20)
- This work is similar to the quantification of the information content halo redshift-space bispectrum using the Quijote suite. (Hahn et al. 20)

Halos - Mean relative velocity between pairs in a triplet

Triangular configurations: $50 \leq r_{31} \leq r_{23} \leq r_{12} \leq 120\ h^{-1}\mathrm{Mpc}$

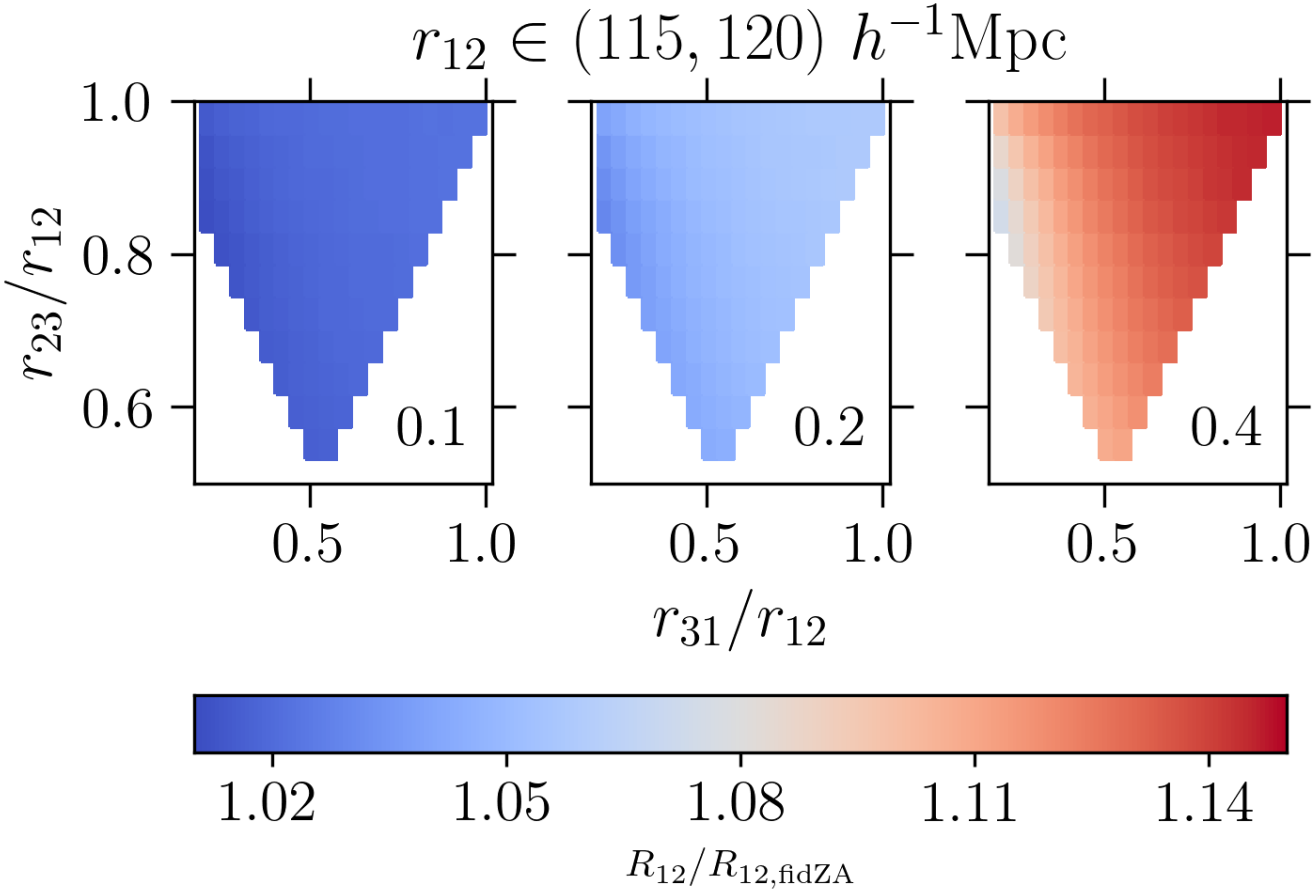
Minimum halo mass: $M_\mathrm{h} > 5 \times 10^{13}\ h^{-1} M_\odot$



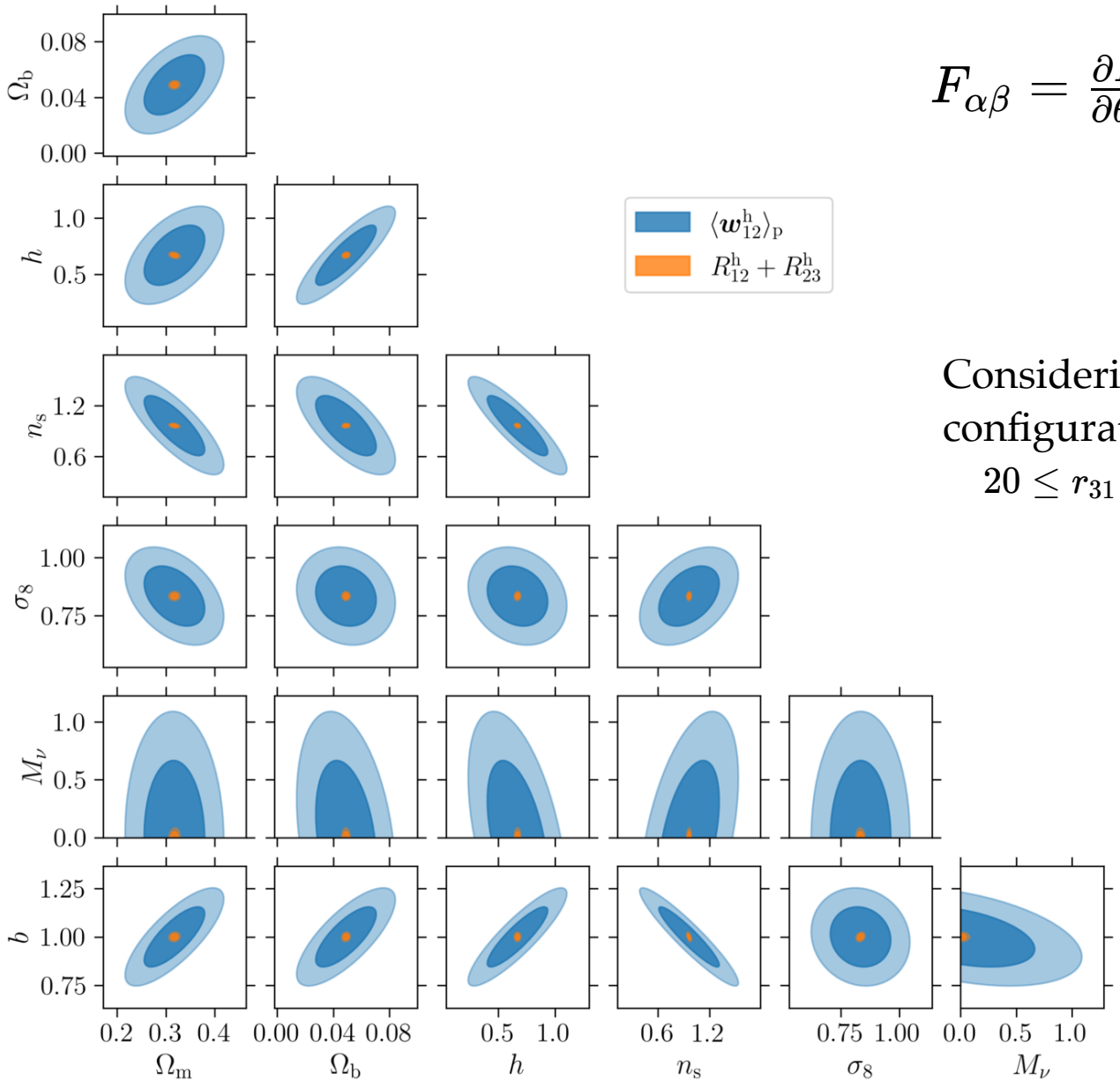


$$R^\mathrm{h}_{ij}(\triangle_{123}, M_\mathrm{h}) = b(M_\mathrm{h})\, R_{ij}(\triangle_{123})$$

Neutrino mass effect



$$F_{\alpha\beta} = \frac{\partial \boldsymbol{D}}{\partial \theta_\alpha} \cdot \hat{\mathbf{C}}^{-1} \cdot \frac{\partial \boldsymbol{D}^{\mathsf{T}}}{\partial \theta_\beta}$$

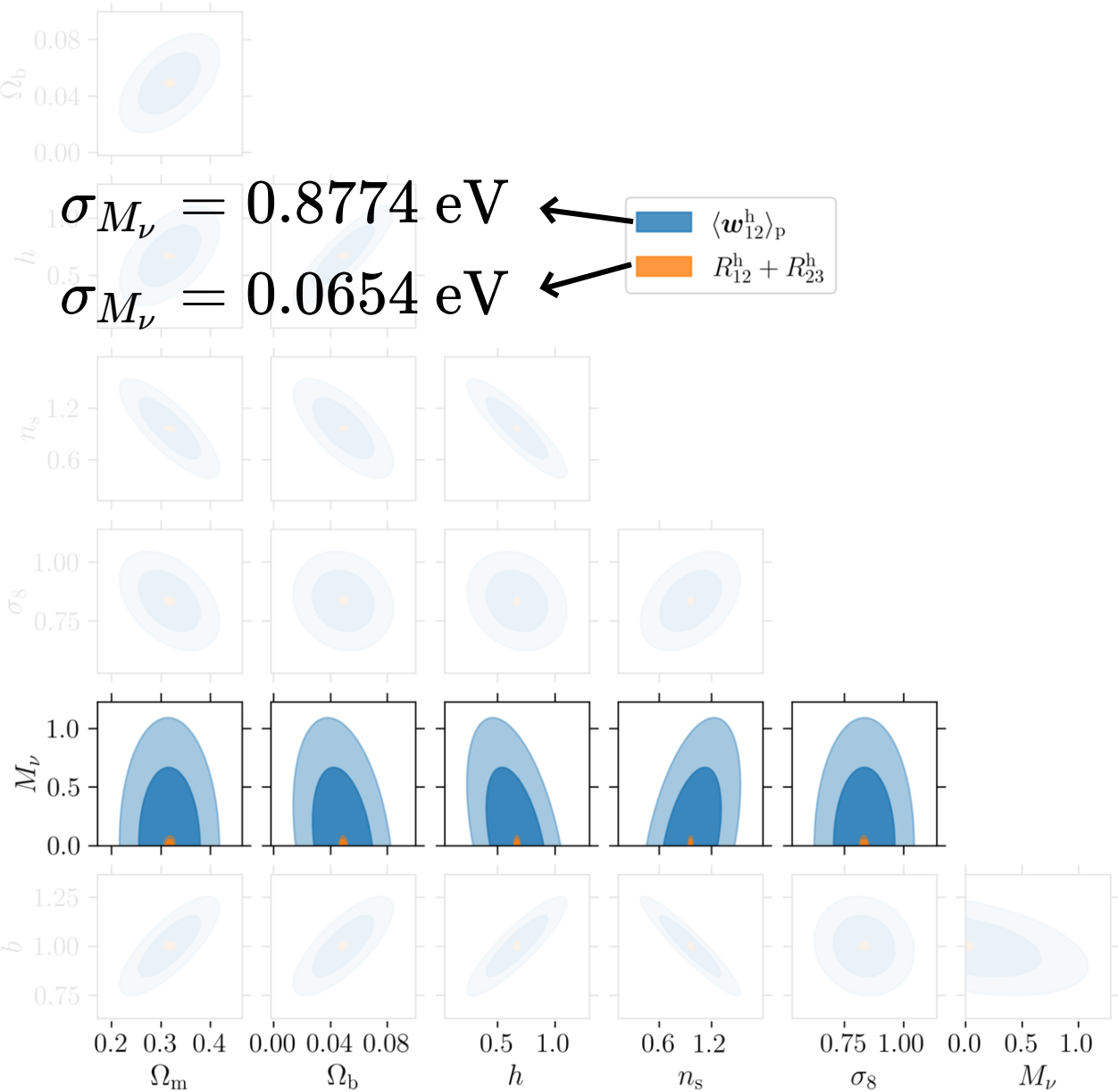


Considering all triangular configurations:

$$20 \leq r_{31} \leq r_{23} \leq r_{12} \leq 120$$

$$\sigma_{M_\nu} = 0.8774 \text{ eV}$$

$$\sigma_{M_\nu} = 0.0654 \text{ eV}$$



Take-away message

- Introduced a phenomenological model for 3PCF: the three-point Gaussian streaming model.

- Three-point mean relative velocity statistics offers substantial cosmological information gain over mean pairwise velocity.

Thank you for listening!